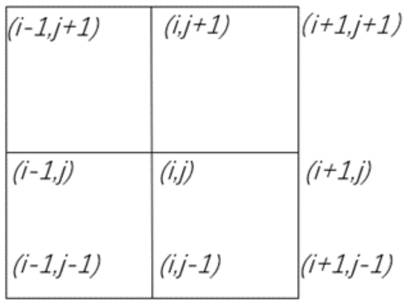
There are the numerical methods for phase field equation:

We applied the method of finite difference (central difference scheme) for space discretization and time discretization.

1. **Space discretization**



We regard the (i,j) as a pint in the space, and the derivative of space are as below.

$$\frac{\partial \varphi}{\partial x_{i,j}} = \frac{\varphi_{i+1,j} - \varphi_{i-1,j}}{2\Delta x} + o(x^2) \tag{1.1}$$

$$\frac{\partial \varphi}{\partial y_{i,j}} = \frac{\varphi_{i,j+1} - \varphi_{i,j-1}}{2\Delta y} + o(y^2) \tag{1.2}$$

$$\frac{\partial^2 \varphi}{\partial {x^2}_{i,j}} = \frac{\varphi_{i+1,j} - 2\varphi_{i,j} + \varphi_{i-1,j}}{\Delta x^2} + o(x^2) \tag{1.3}$$

$$\frac{\partial^2 \varphi}{\partial {y^2}_{i,j}} = \frac{\varphi_{i,j+1} - 2\varphi_{i,j} + \varphi_{i,j-1}}{\Delta y^2} + o(y^2) \tag{1.4}$$

$$\frac{\partial^2 \varphi}{\partial x_{i,j} \partial y_{i,j}} = \frac{\varphi_{i+1,j+1} - \varphi_{i-1,j+1} - \varphi_{i+1,j-1} + \varphi_{i-1,j-1}}{4\Delta x^2} + o(x^2) \tag{1.5}$$

2. **Time discretization**

$$\frac{\partial T}{\partial t} = \frac{T^{n+1} - T^n}{\Delta t} + o(x^2) \tag{2.1}$$