Boundary condition:

For the boundary, we use Zero-Neumann condition, which implies the value of the gradient of the dependent variable normal to the boundary is prescribed on the boundary, and the value is equals to zero.

$$\frac{\partial \varphi}{\partial n} = \frac{\partial T}{\partial n} = 0$$

Initial condition:

Because the crystallization temperature is often less than the melting point of the material and the environment requires a tiny crystal nucleus, this model assumes that the entire area is liquid, with a nucleus in the center. The nucleus is set up as a circle.

$$x^2 + y^2 \leq r^2 : \varphi = 1, T = 0$$
$$x^2 + y^2 \geq r^2 : \varphi = 0, T = -\Delta$$

Stability condition:

In finite element method, it is necessary to control the deviation in calculation, so there is a condition for the time step and space step. m is mobility of phase field, in this model we use 0.035.

$$\Delta t \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right) \leq \frac{2}{5m}$$