ECH 4323 – Spring 2017
Process Dynamics and Control
Mini-Design Project

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Control System for an Anaerobic Digestion System

Introduction:

Control systems are installed on processes so corrective action can be taken when a variable of interest has deviated from its set point. The process discussed in this report is a control system designed for an anaerobic digestion system. This continuous process takes in organic material such as animal waste, food waste, agricultural waste and wastewater sludge and turns it into a biogas that can be used for electricity and fuel for vehicles. Solid coproducts being produced can be used for compost and applied directly to cropland or converted to other products. Liquid coproducts being produced are used in agriculture as fertilizer.

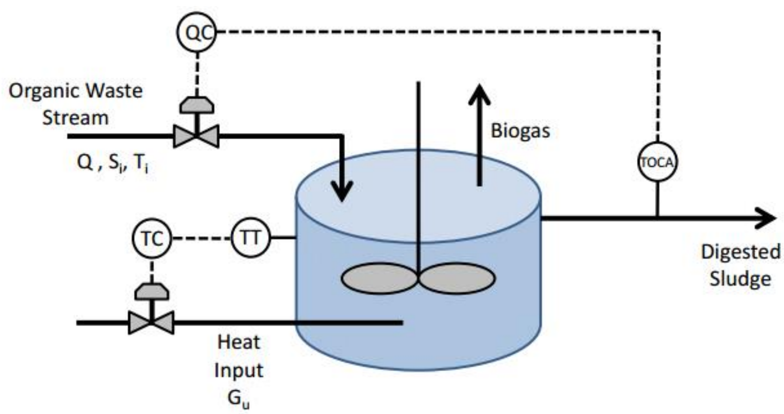

Figure 1: Basic process diagram for an industrial digester (obtained from problem document).

The anaerobic digestion system will be operated for a mesophilic case. Mesophilic digestion occurs at temperatures between 20-45^oC with an optimal digestion temperature between 30 -38^oC. Manipulated inputs for the process are the influent feed rate, Q and the specific heat addition rate, G_u. A temperature sensor (TT) is placed on the process and attached to a temperature controller (TC) that regulates the temperature through the addition of heat provided. A total organic carbon analysis (TOCA) is placed on the effluent stream and is fed to a flow controller (QC) that regulates the amount of influent fed into the process. Disturbance inputs to the process are the varying organic substrate concentration, S_i and a varying input temperature, T_i. A feedback control loop will detect deviations from the steady state process temperature and add heat accordingly. The influent flow rate will be manipulated to account for any deviation from incoming steady state substrate concentration via a feedback control. A feedforward control will be implemented along with the feedback control for a quicker response to any deviation in the incoming substrate concentration.

Abstract:

Enclosed within this document is a very thorough and detailed investigation and development of a control system for an anaerobic digestion system. Process control is a very critical aspect of the design of a process as it allows for operations undertaken to be maintained in a safe manner by implementing safety constraints and automatic start-up or shutdown of process as required. Process control also helps meet the required production rate as well as maintain product quality specifications by minimizing variance within the process. Therefore, it is clear to see the importance of designing an aggressive control system for the digestive biological system to monitor the quality and amount of biogas as well as digested slug being produced.

The mix digester was modelled as a mesophilic digester and the dynamics of the process obtained were utilized in the determination of the control system to minimize the effects of the disturbance coming into the system and maximize the product quality. Two controllers were implemented; one for the organic waste stream and the other for the heat input into the system. Direct synthesis tuning method is utilized and the types of controllers implemented are PI. The tuning constants which yielded a control system that effectively minimized the effects of the disturbance are Kc=18.74 & τ_I=1.601 for the controller on the heat input and Kc=4.185 & τ_I = 1.88 for the controller on the flow rate stream. Based on this control design, the results show that the system will reach steady state after a few days (up to 10 days) when a step change in either the flow rate or the heat input is made.

Results & Discussion:

Problem 1:

Since the equations that represents the system has some non-linear terms in it, we have to linearized these terms because transfers function are only applicable to linear time invariant models. To do the linearization, the Taylor series approximation was utilized. Linearization of the dynamic balance on substrate is shown in detail and the final linear form for the remaining two equations are given. It is left to the reader to verify these for practice.

Linearization of dynamic balances

$$\frac{dS}{dt} = \frac{1}{V}QS_i - \frac{1}{V}QS - \mu_{max}\frac{S}{K+S}\frac{X}{Y} \quad (1)$$

$$\textit{The above equation has 3 non linear terms}: QS_i, QS \textit{ and } \mu_{max}\frac{S}{K+S}X$$
$$\textit{Using Taylor series approximation}:$$

$$QS_i = Q_{ss}S_{i_{ss}} + Q'S_{i_{ss}} + S_i'Q_{ss} \ (2)$$

$$QS = Q_{ss}S_{ss} + Q'S_{ss} + S'Q_{ss} \ (3)$$

$$\mu_{max}\frac{S}{K+S}X = \mu_{\max,ss}\frac{S_{ss}}{K+S_{ss}}X_{ss} + \frac{S_{ss}}{K+S_{ss}}X_{ss} * \mu'_{max} + \mu_{\max,ss}\frac{S_{ss}}{K+S_{ss}}X' + \frac{(K+S_{ss})\mu_{max,ss}X_{ss} - S_{ss}\mu_{max,ss}X_{ss}}{(K+S_{ss})^2}S' \quad (4)$$

$$\textit{plug the above linerarized form of the terms into the dynamic balance and subtract steady state value to get the deviation form as}:$$

$$\frac{dS'}{dt} = \frac{1}{V}\left[Q_{ss}S_i' + Q'S_{i_{ss}}\right] - \frac{1}{V}[Q_{ss}S' + Q'S_{ss}] - \frac{S_{ss}X_{ss}}{(K+S_{ss})Y}\mu'_{max} - \mu_{\max,ss}\frac{S_{ss}}{K+S_{ss}}X' - \frac{K\mu_{max,ss}X_{ss}}{(K+S_{ss})^2Y}S' \ (5)$$

The deviation form for the other dynamic balances are as follows:

$$\frac{dX'}{dt} = -\frac{1}{V}[Q_{ss}X' + Q'X_{ss}] + \frac{S_{ss}X_{ss}}{(K+S_{ss})Y}\mu'_{max} + \mu_{\max,ss}\frac{S_{ss}}{K+S_{ss}}X' + \frac{K\mu_{max,ss}X_{ss}}{(K+S_{ss})^2}S' - K_dX' \quad (6)$$

$$\frac{dT'}{dt} = \frac{1}{V}\left[Q_{ss}T_i' + Q'T_{i_{ss}}\right] - \frac{1}{V}\left[Q_{ss}T' + Q'T_{ss}\right] + Gu' \quad (7)$$

Where:

$$\mu_{max} = 0.013T - 0.129 \quad (8)$$
$$\mu_{max,ss} = 0.013T_{ss} - 0.129 \quad (9)$$

$$\mu'_{max} = 0.013T' \quad (10)$$

Plugging steady state values into equations 5 – 7 and 9-10,

The dynamic balance becomes:

$$\frac{dS'}{dt} = 0.0333S_i' - 0.1148S' + 0.00115Q' - 0.7T' - 0.00488X' \ (11)$$

$$\frac{dX'}{dt} = -0.2321X' - 0.006Q' + 22.91T' + 2.666S' \ (12)$$

$$\frac{dT'}{dt} = 0.0333T_i' - 1.33 * 10^{-5}Q' - 0.0333T' + G_u' \ (13)$$

With the deviation equation obtained, we can now build a state-space model for this system.

$$\dot{x} = \begin{bmatrix} \frac{dS'}{dt} \\ \frac{dX'}{dt} \\ \frac{dT'}{dt} \end{bmatrix}; x = \begin{bmatrix} S' \\ X' \\ T' \end{bmatrix}; u = \begin{bmatrix} Q \\ G_u \end{bmatrix}; d = \begin{bmatrix} S_i \\ T_i \end{bmatrix}$$

$$A = \begin{bmatrix} -0.1148 & -0.00488 & -0.7 \\ 2.666 & -0.2321 & 22.91 \\ 0 & 0 & -0.0333 \end{bmatrix}; B = \begin{bmatrix} 0.00115 & 0 \\ -0.006 & 0 \\ -1.33 * 10^{-5} & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 0.0333 & 0 \\ 0 & 0 \\ 0 & 0.0333 \end{bmatrix}$$

For the output matrix, our system sensor is measuring the outlet concentration in [mgTOC/L] while the dynamic balance is in [mgCOD/L]. It is known that COD measurement is 2.2 times the TOC measurement. The C-matrix is used to convert the COD to TOC by multiplying by a factor of 5/11. The output relation is now shown below:

$$Y = \begin{bmatrix} 5/11 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} S \\ T \end{bmatrix}$$

<u>Problem 2:</u>

Now that the equations that represent the system and state variables have been linearized and put into a state space model. The state space model can be implemented into Simulink and the system can be modeled. This Simulink model will be used to observe how the effluent temperature and substrate concentration from the digester will respond to +/- 10% changes in the input variables, which are Q, Gu, Si, Ti. Where Q and Gu are manipulated input variables and Si and Ti are disturbance input variables. Understanding how the desired state variables (T and S in this case) will respond to changes in the input variables to a system is extremely important when designing a process or building a control system for a process. This model allows us to observe how slight changes or fluctuations in the input variables will change the state variables of the system, the time it will take for the system to reach a new steady state, and in which direction the changes will occur. The figures below represent how the effluent temperature and substrate concentration of the digester will respond to slight changes in the input variables.

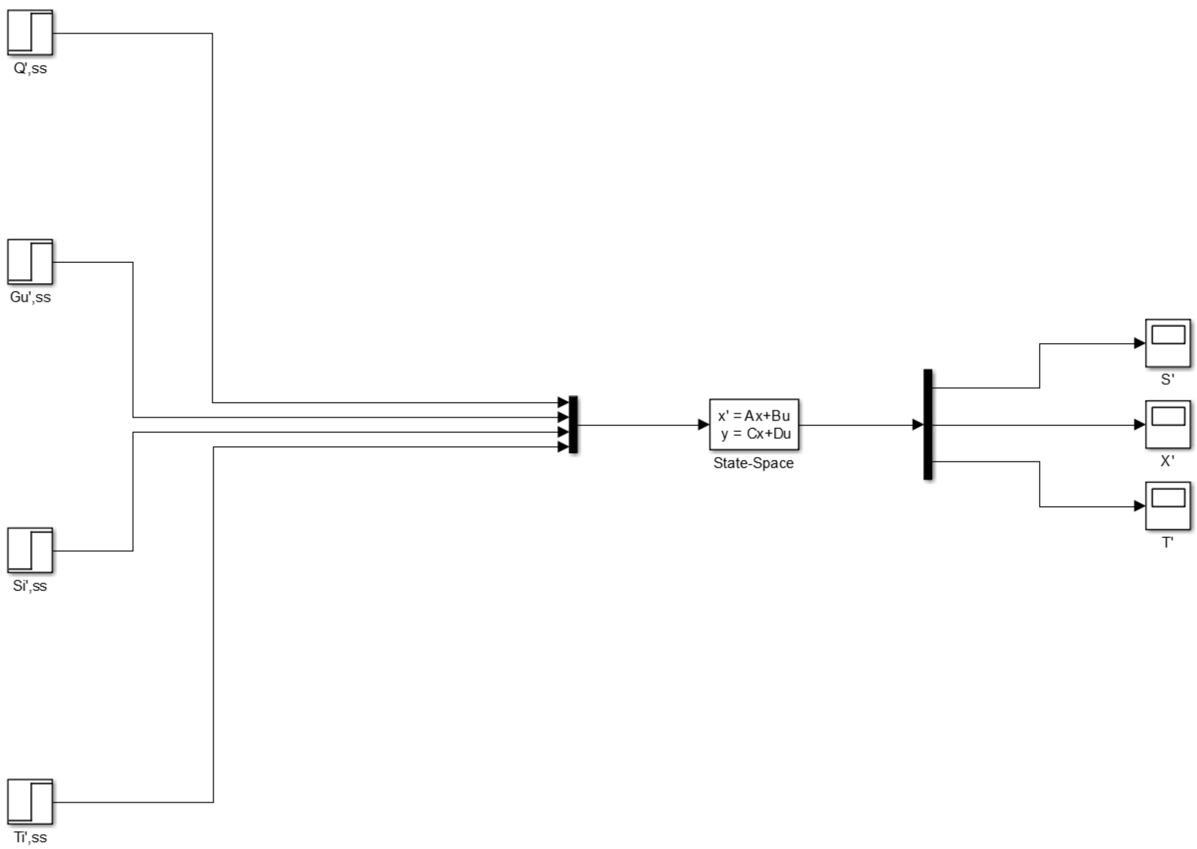

Figure 2. Simulink model used to observe changes in the system's state variables due to changes in the system's input variables.

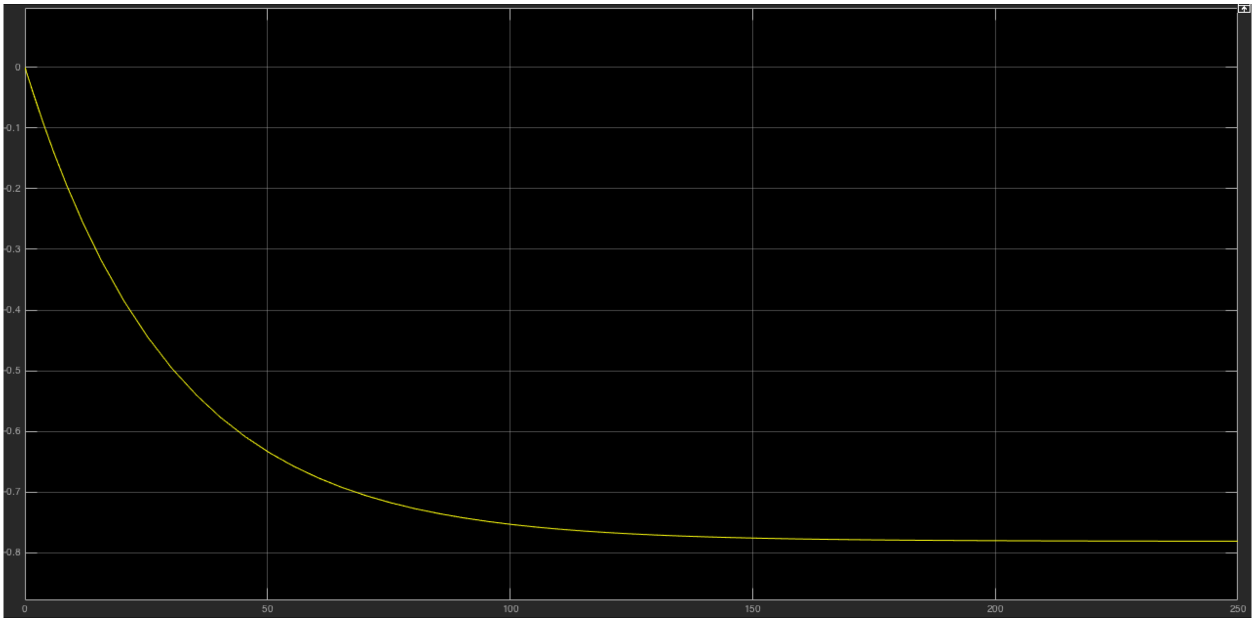

Figure 3. Effluent temperature (T) response to a 10 percent increase in the influent feed rate (Q).

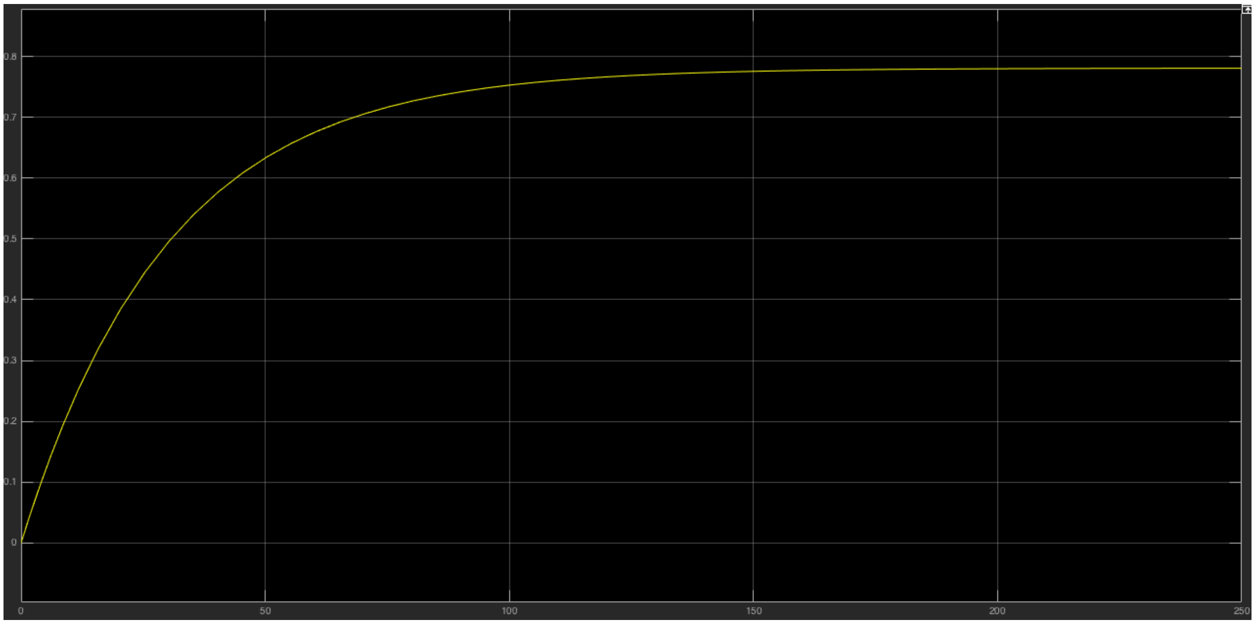

Figure 4. Effluent temperature (T) response to a 10 percent decrease in the influent feed rate (Q).

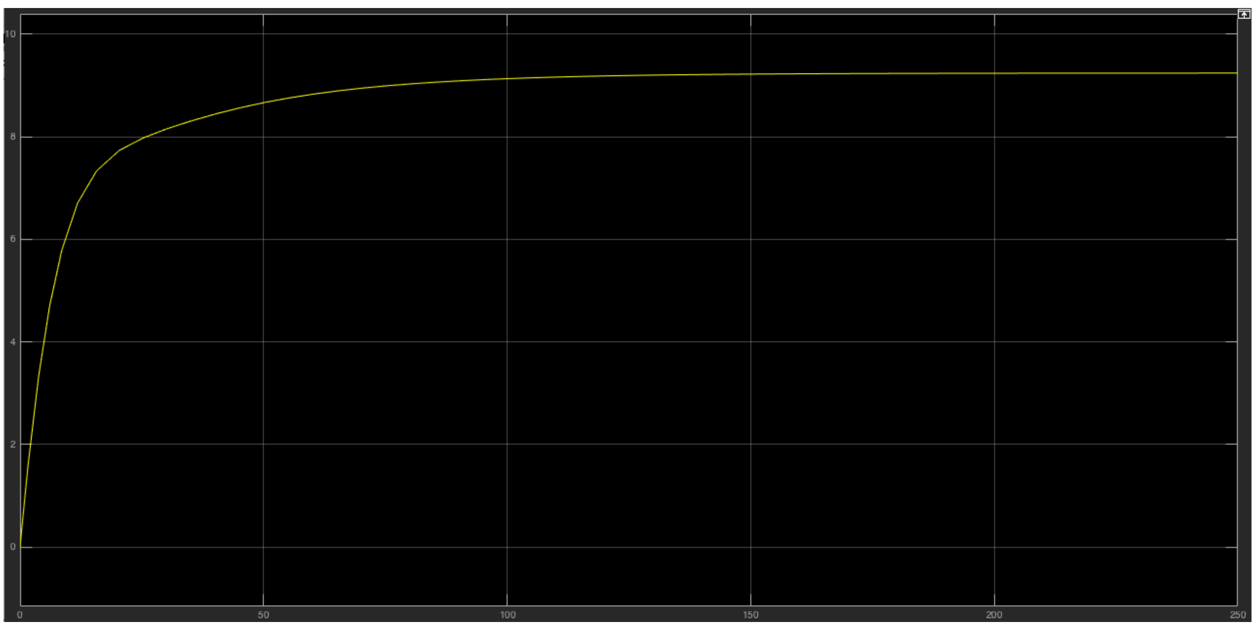

Figure 5. Effluent substrate concentration (S) response to a 10 percent increase in the influent feed rate (Q).

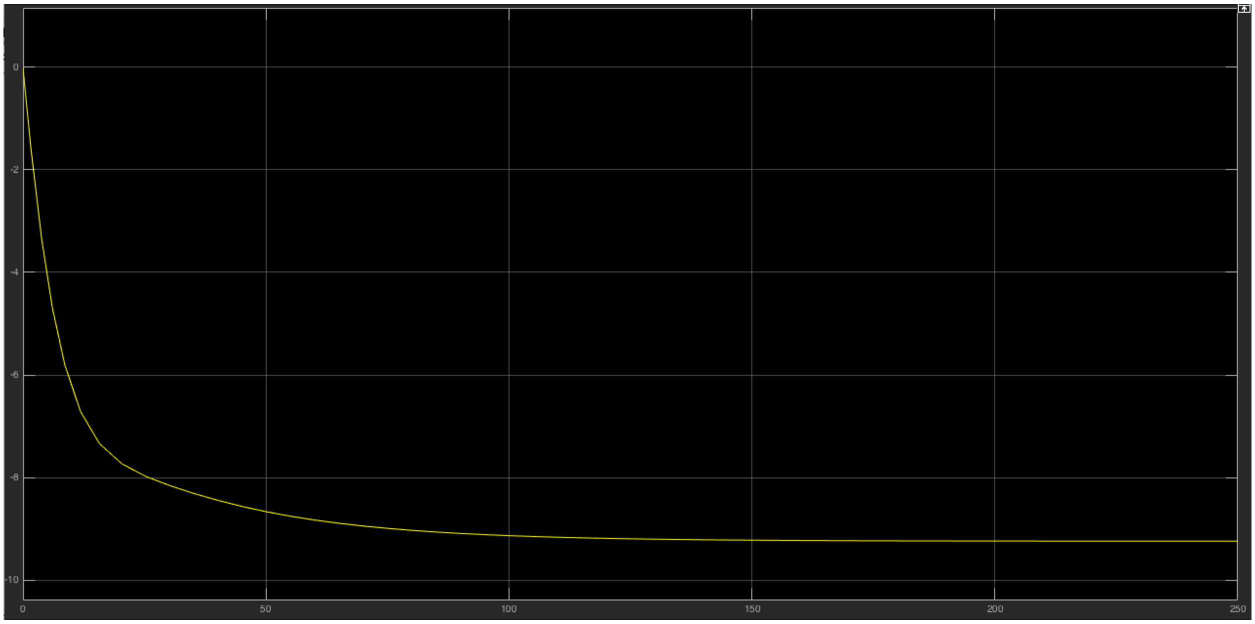

Figure 6. Effluent substrate concentration (S) response to a 10 percent decrease in the influent feed rate (Q).

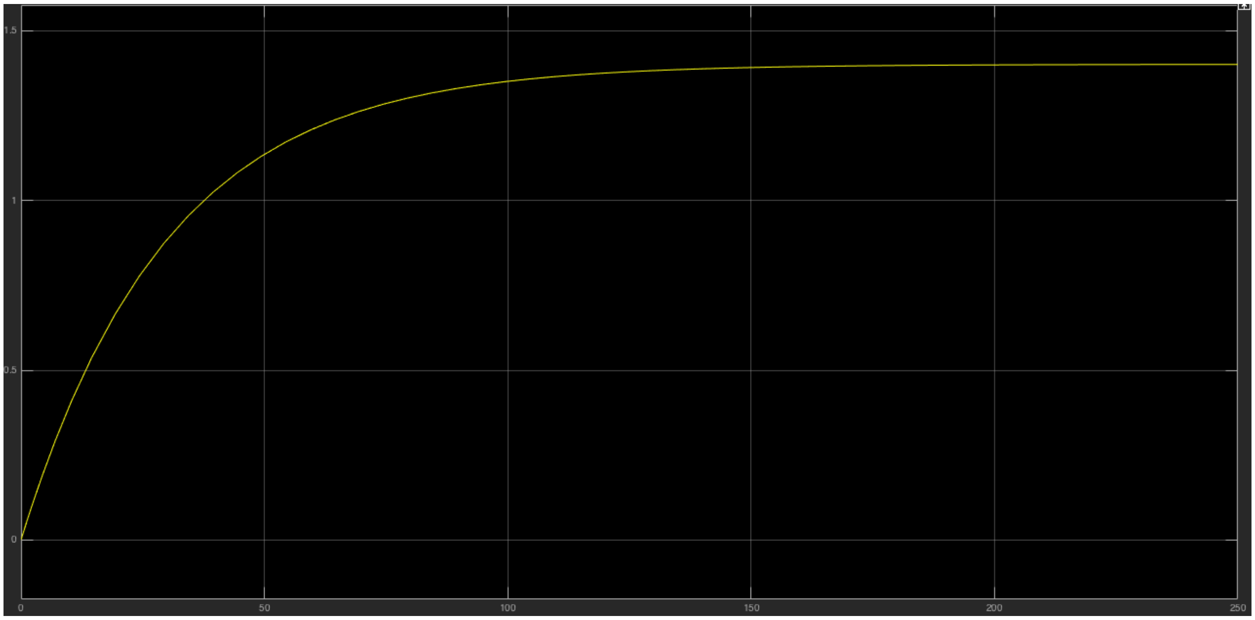

Figure 7. Effluent temperature (T) response to a 10 percent increase in the specific heat addition rate (Gu).

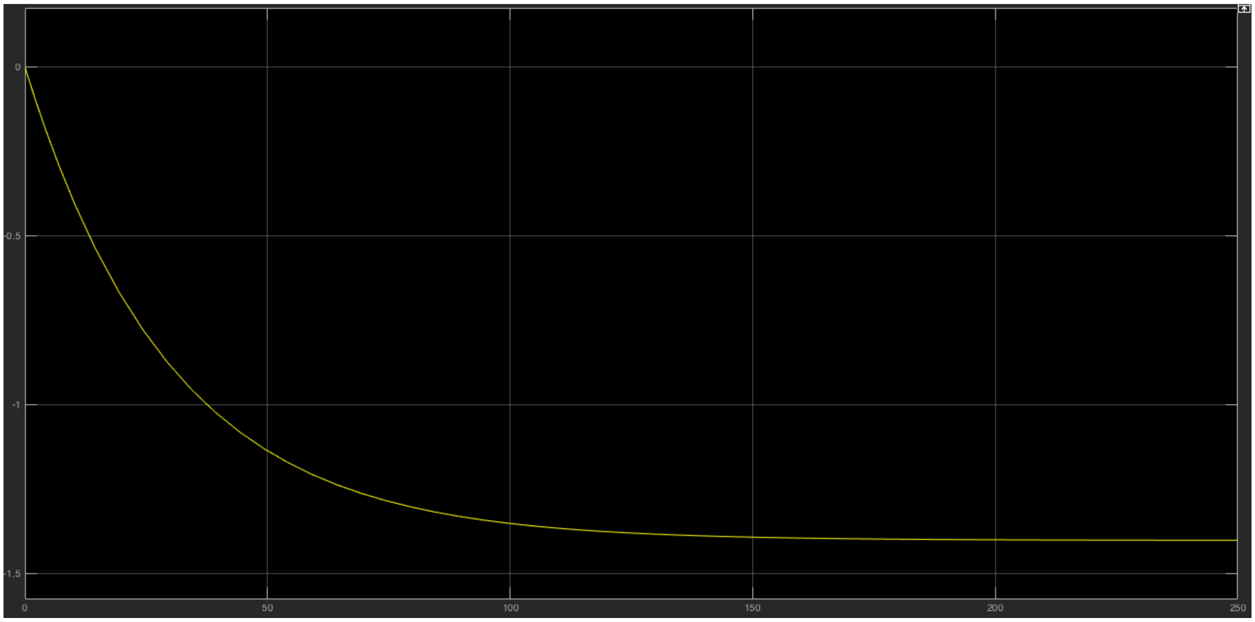

Figure 8. Effluent temperature (T) response to a 10 percent decrease in the specific heat addition rate (Gu).

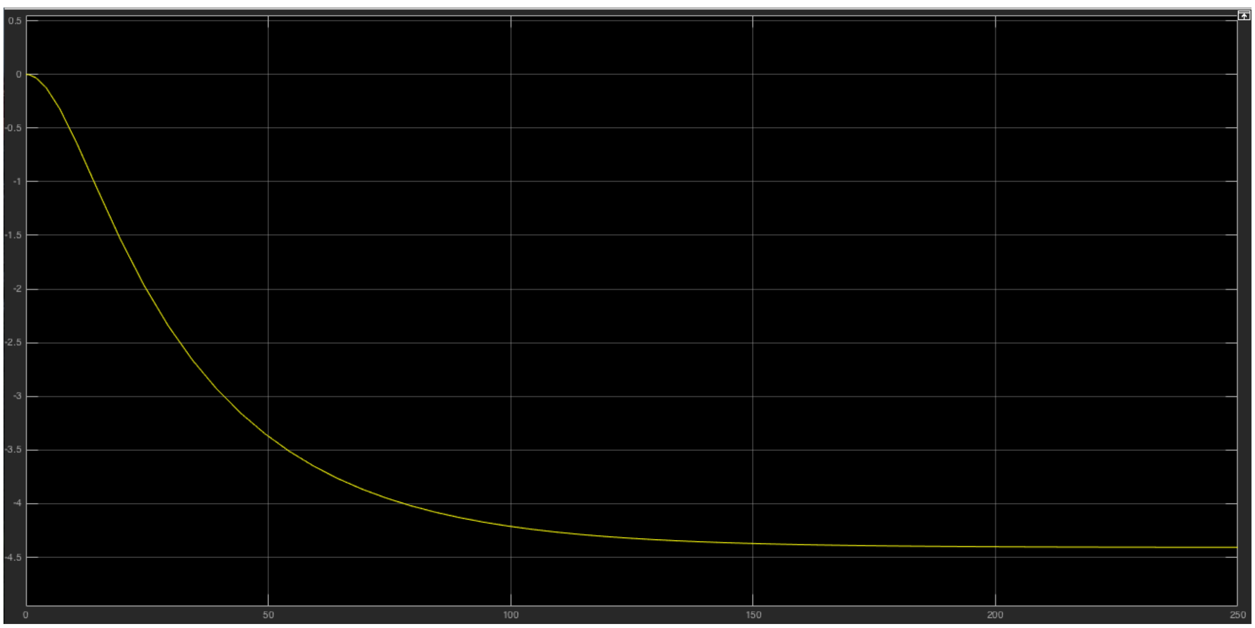

Figure 9. Effluent substrate concentration (S) response to a 10 percent increase in the specific heat addition rate (Gu).

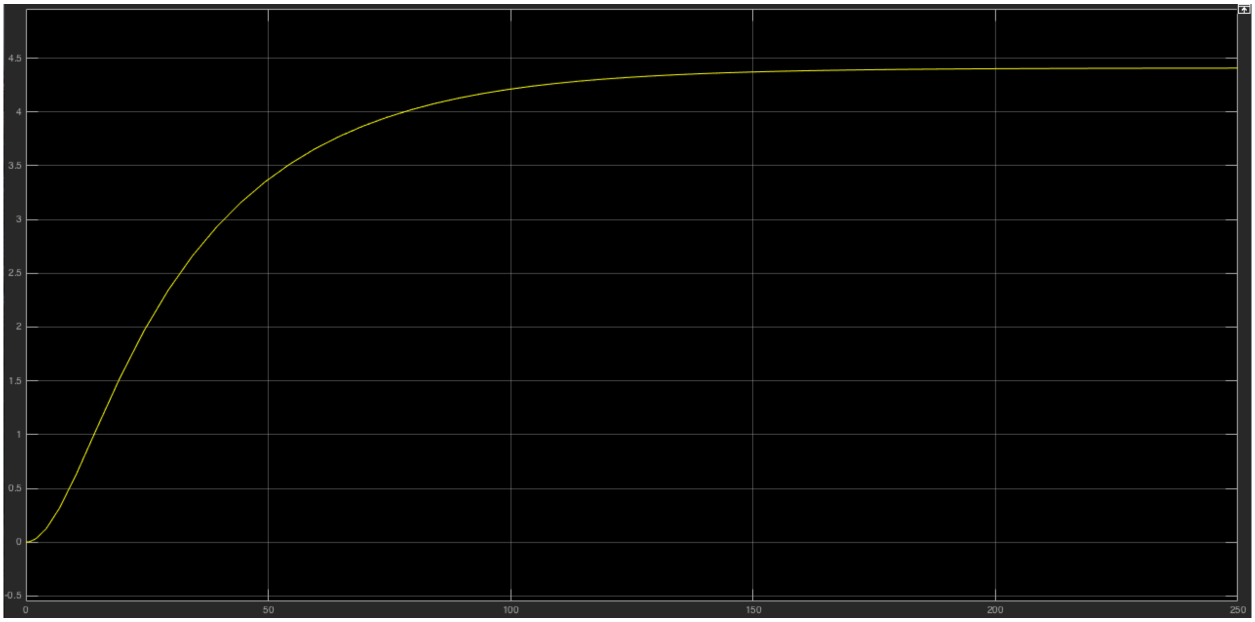

Figure 10. Effluent substrate concentration (S) response to a 10 percent decrease in the specific heat addition rate (Gu).

Figure 11. Effluent temperature (T) response to a 10 percent increase in the influent substrate concentration (Si).

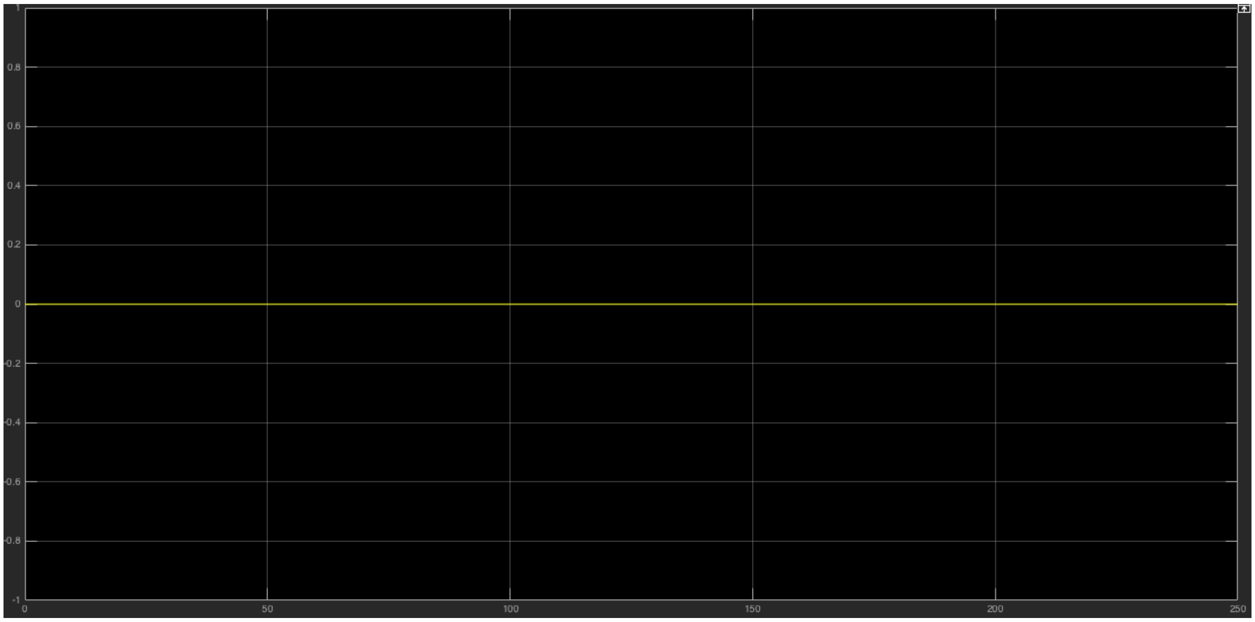

Figure 12. Effluent temperature (T) response to a 10 percent decrease in the influent substrate concentration (Si).

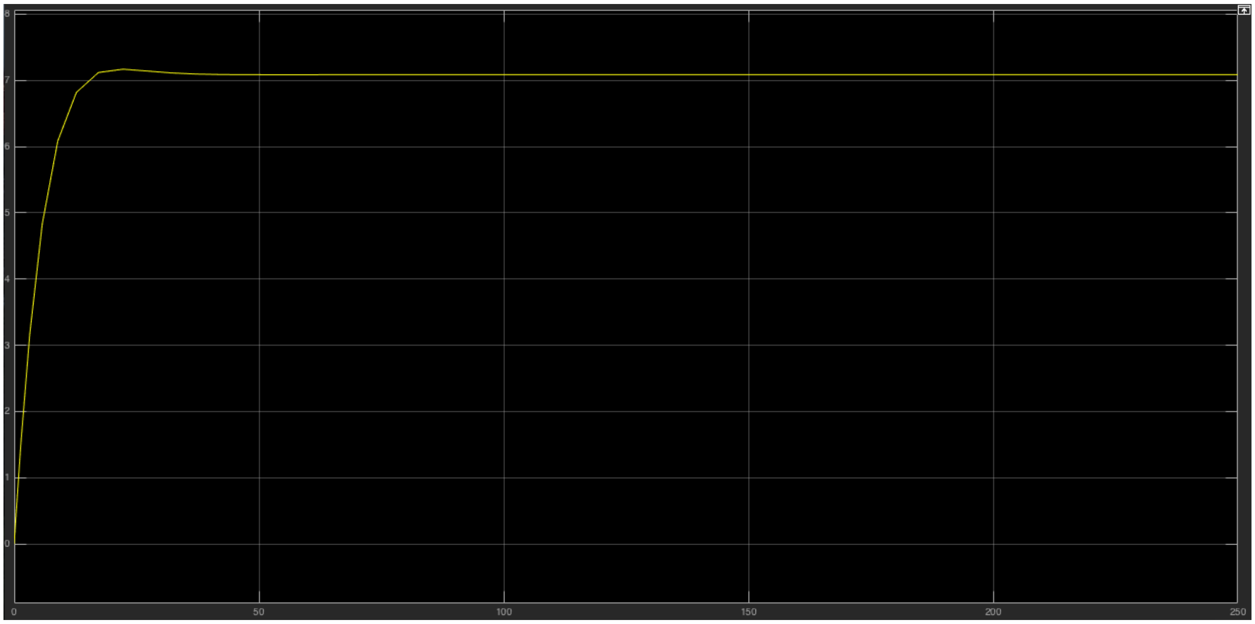

Figure 13. Effluent substrate concentration (S) response to a 10 percent increase in the influent substrate concentration (Si).

Figure 14. Effluent substrate concentration (S) response to a 10 percent decrease in the influent substrate concentration (Si).

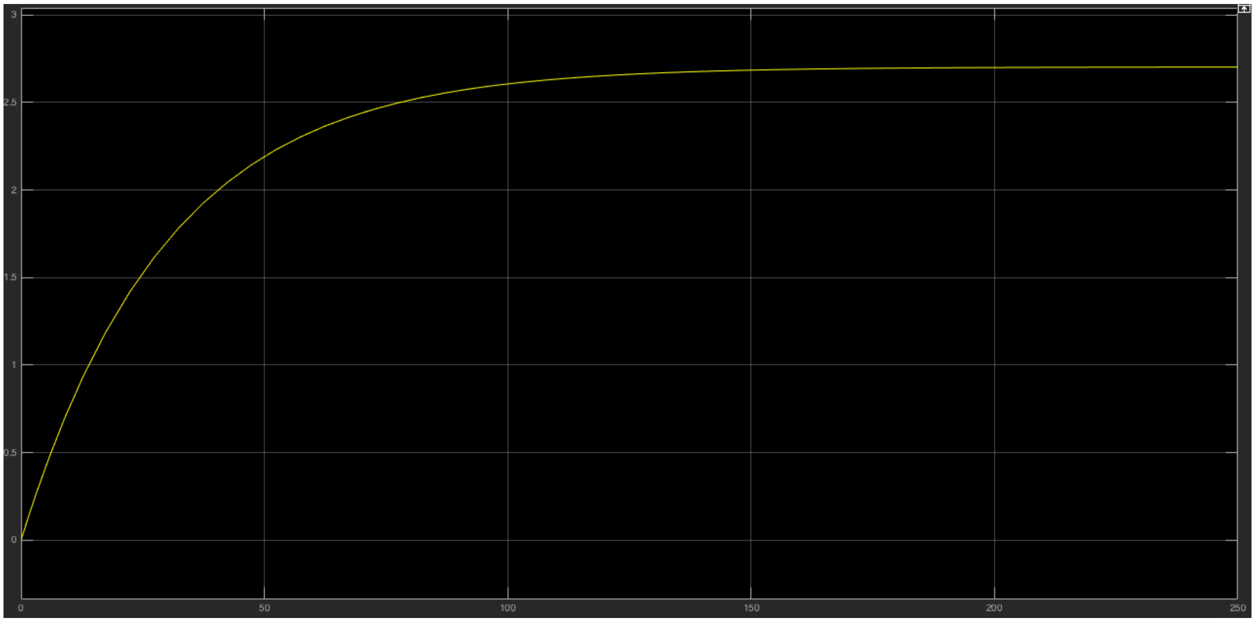

Figure 15. Effluent temperature (T) response to a 10 percent increase in the influent feed temperature (Ti).

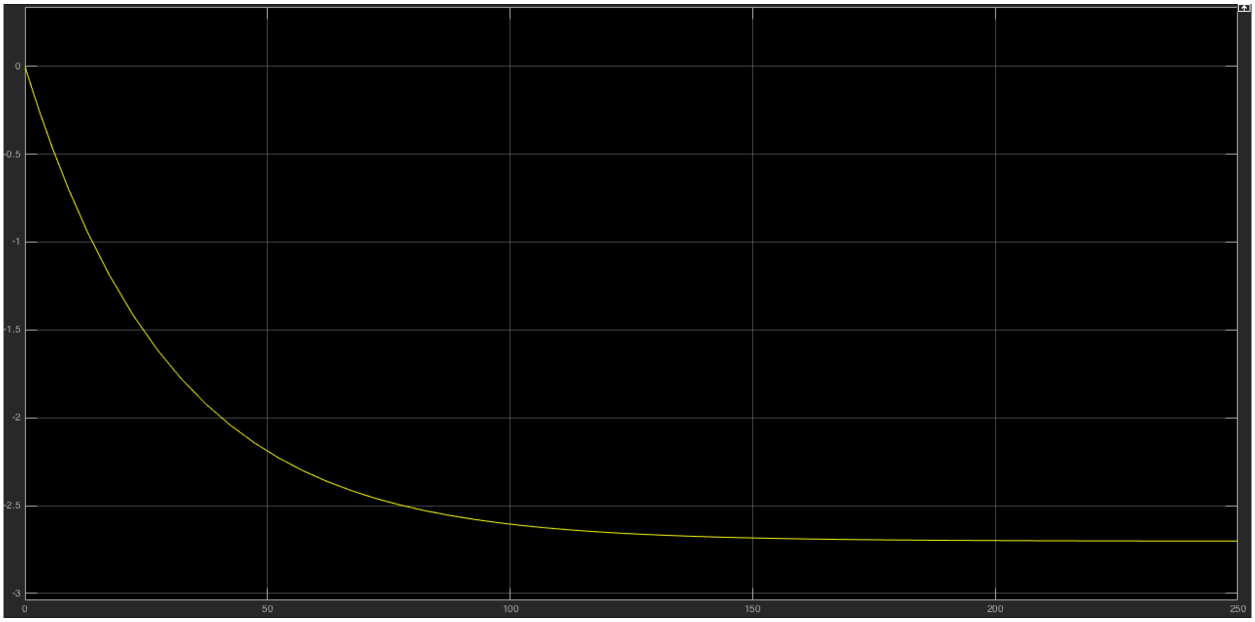

Figure 16. Effluent temperature (T) response to a 10 percent decrease in the influent feed temperature (Ti).

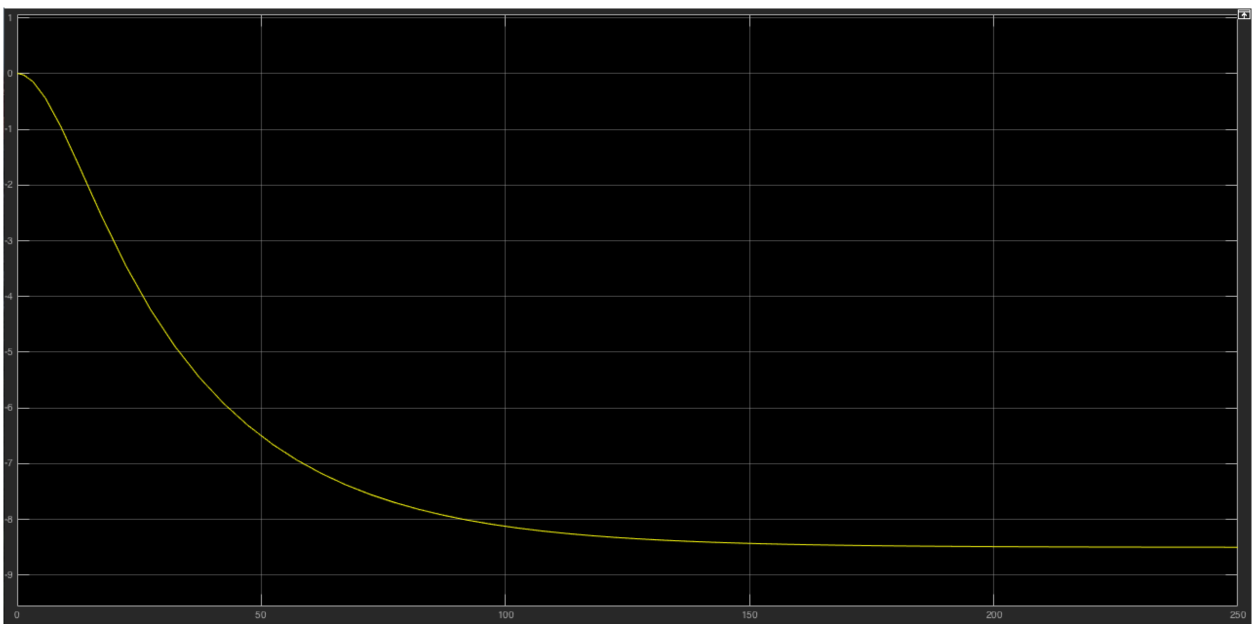

Figure 17. Effluent substrate concentration (S) response to a 10 percent increase in the influent feed temperature (Ti).

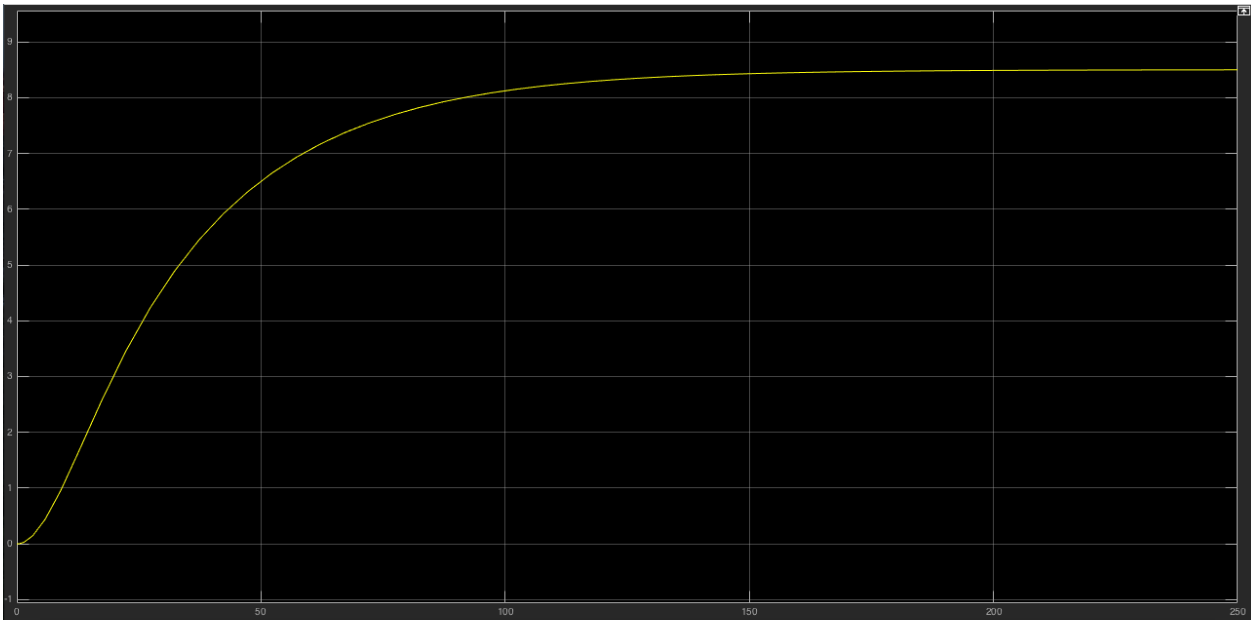

Figure 18. Effluent substrate concentration (S) response to a 10 percent decrease in the influent feed temperature (Ti).

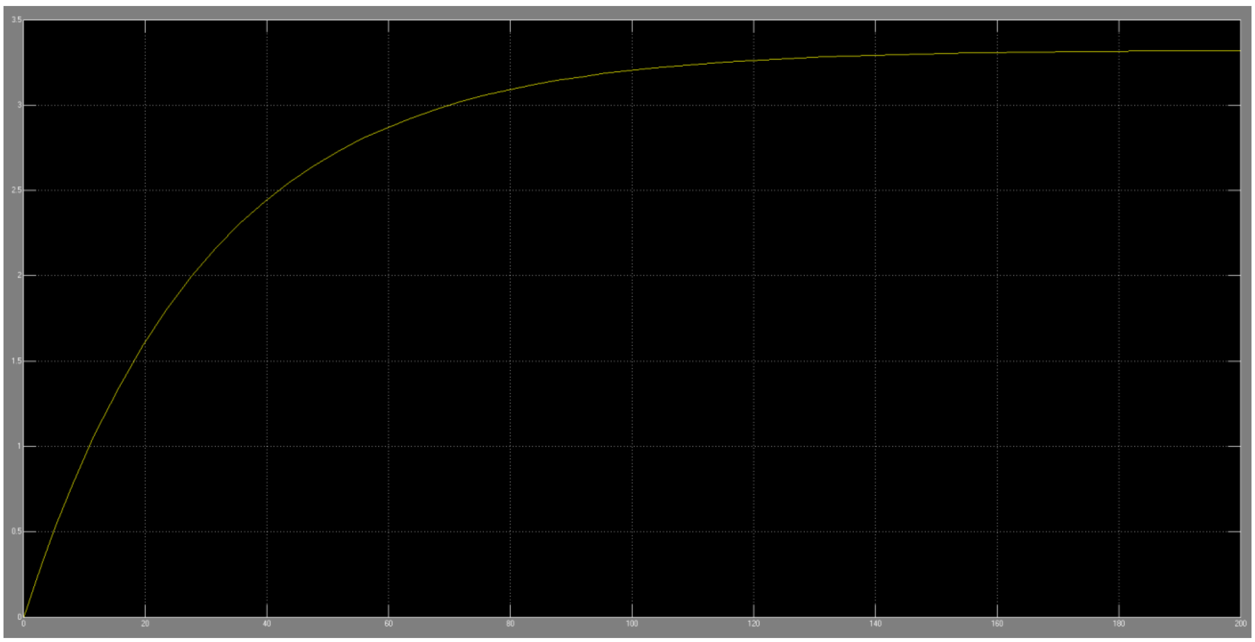

Figure 19. Effluent temperature (T) response to a 10 percent increase in Q’, Gu’, Ti’, and Si’ at the same time.

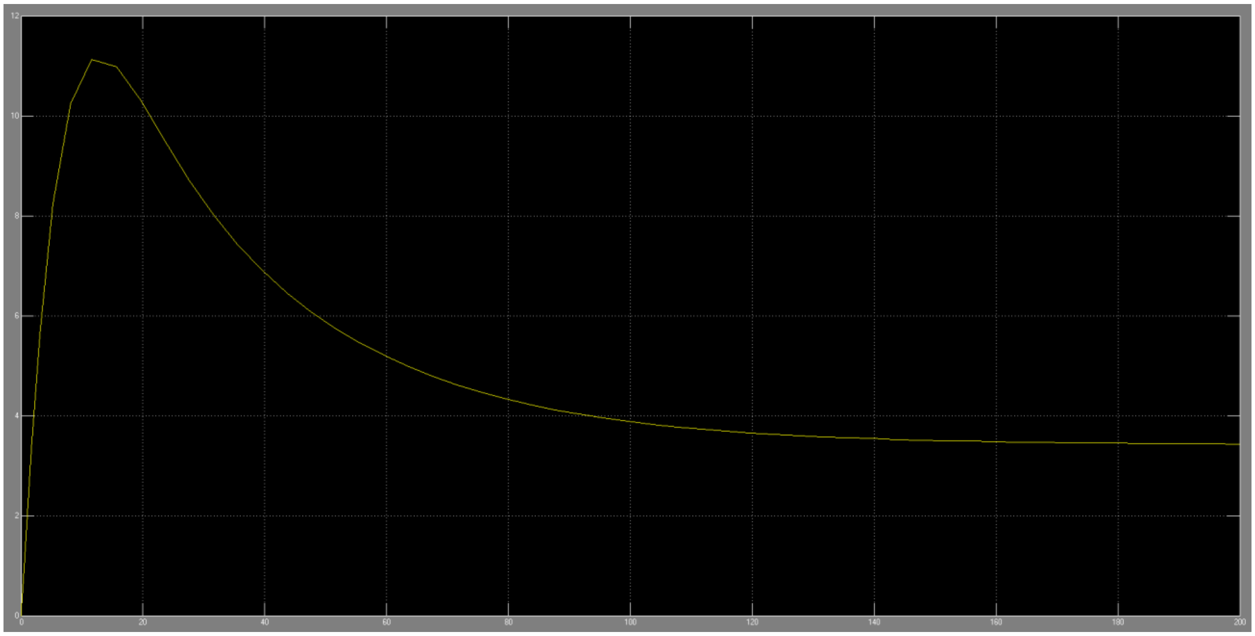

Figure 20. Effluent substrate concentration (S) response to a 10 percent increase in Q', Gu', Ti', and Si' at the same time.

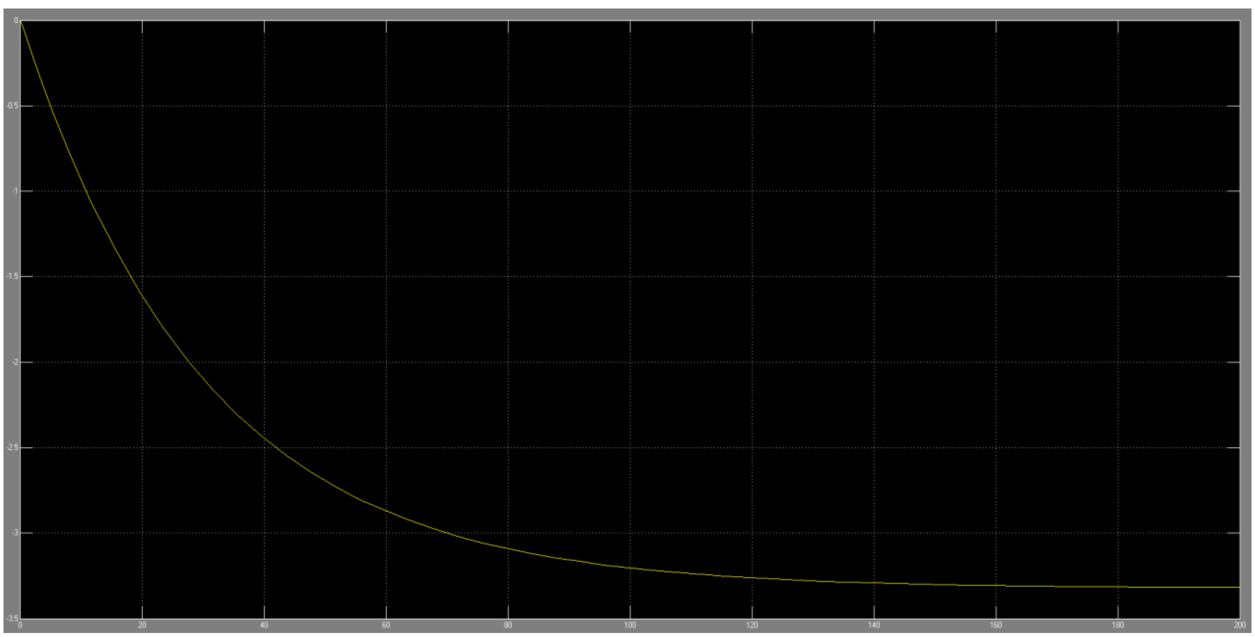

Figure 21. Effluent temperature (T) response to a 10 percent decrease in Q', Gu', Ti', and Si' at the same time.

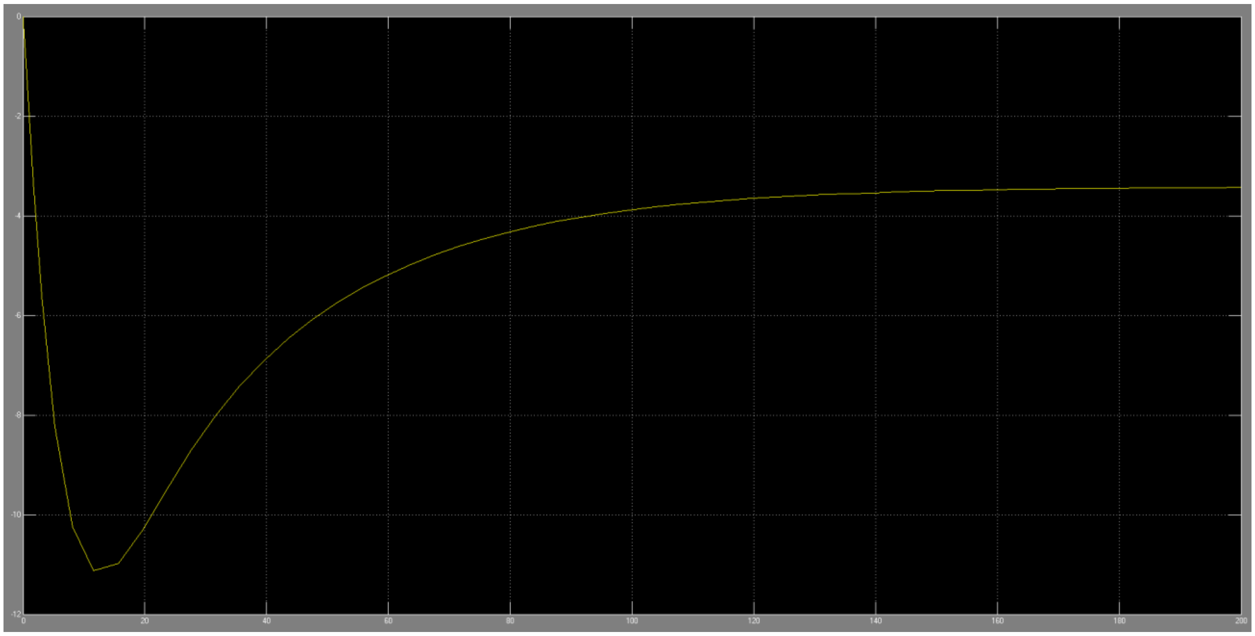

Figure 22. Effluent substrate concentration (S) response to a 10 percent decrease in Q', Gu', Ti', and Si' at the same time.

After reviewing the figures above, it is apparent that slight changes in most of the input variables will have a large and direct effect on the state variables / desired output variables. Also, the time scale for the figures above is in days. Therefore, the time it takes for the system to reach its new steady state can take up to 200 days for only a 10 percent change in some of the input variables. Therefore, it is extremely important to design a control system for this digester which will keep the system in control and allow the system to reach its desired setpoints much faster than 200 days as seen for the uncontrolled system above. It is also important to keep the system operating at its maximum efficiency and within its acceptable limits. The control system that will be designed will aid in keeping the digesters temperature within 30 C to 38 C, and will maintain the effluent substrate concentration below 75 mgTOC/L.

Problem 3:

$$
\begin{array}{cccccccc}
\dot{X} & & A & X & & B & U & & E & D \\
\begin{bmatrix} dS'/dt \\ dX'/dt \\ dT'/dt \end{bmatrix} & = & \begin{bmatrix} -0.1148 & -0.00488 & -0.7 \\ 2.6661 & -0.2321 & 22.9 \\ 0 & 0 & -0.0333 \end{bmatrix} & \begin{bmatrix} S' \\ X' \\ T' \end{bmatrix} & + & \begin{bmatrix} 0.00115 & 0 \\ -0.006 & 0 \\ 1.33E-5 & 1 \end{bmatrix} & \begin{bmatrix} Q' \\ Gu' \\ \ \end{bmatrix} & + & \begin{bmatrix} 0.033 & 0 \\ 0 & 0 \\ 0 & 0.033 \end{bmatrix} & \begin{bmatrix} Si' \\ Ti' \\ \ \end{bmatrix}
\end{array}
$$

$$
\begin{array}{cccc}
Y & & C & X \\
\begin{bmatrix} S' \\ T' \\ \ \end{bmatrix} & = & \begin{bmatrix} 5/11 & 0 & 0 \\ 0 & 0 & 1 \\ & & \end{bmatrix} & \begin{bmatrix} S' \\ X' \\ T' \end{bmatrix}
\end{array}
$$

The following state space models are manipulating into transfer functions as shown below.

$$\dot{X} = AX + BU + ED \tag{1}$$

Take the Laplace of equation (1)

$$s\hat{X}(s) = A\hat{X}(s) + B\hat{U}(s) + E\hat{D}(s) \tag{2}$$

$$\hat{X}(s)(sI - A) = B\hat{U}(s) + E\hat{D}(s) \tag{3}$$

I represent the identity matrix

$$(sI - A)^{-1}[\hat{X}(s)(sI - A) = B\hat{U}(s) + E\hat{D}(s)] \tag{4}$$

$$\hat{X}(s) = (sI - A)^{-1}B\hat{U}(s) + (sI - A)^{-1}E\hat{D}(s) \tag{5}$$

$$Y = CX \tag{6}$$

Take the Laplace of equation (6)

$$\hat{Y}(s) = C\hat{X}(s) \tag{7}$$

Substitute in Equation (5) into Equation (7)

$$\hat{Y}(s) = C[(sI - A)^{-1}B\hat{U}(s) + (sI - A)^{-1}E\hat{D}(s)] \tag{8}$$

$$Gp(s) = C * (s*I - A)^{-1} * B$$

S'/Q'

$$\frac{0.0005227 s^6 + 0.0005189 s^5 + 0.0002189 s^4 + 4.982e\text{-}05 s^3 + 6.34e\text{-}06 s^2 + 4.138e\text{-}07 s + 9.607e\text{-}09}{s^7 + 1.074 s^6 + 0.5148 s^5 + 0.1403 s^4 + 0.02318 s^3 + 0.002271 s^2 + 0.0001169 s + 2.079e\text{-}06}$$

T'/Q'

$$\frac{-1.3e\text{-}05}{s + 0.03333}$$

S'/Gu'

$$\frac{-0.3183 s - 0.1247}{s^3 + 0.3803 s^2 + 0.05122 s + 0.001322}$$

T'/Gu'

$$\frac{1}{s + 0.03333}$$

$$Gd(s) = C * (s*I - A)^{-1} * E$$

S'/Si'

$$\frac{0.01515 s + 0.003517}{s^2 + 0.347 s + 0.03966}$$

T'/Si'

$$0$$

S'/Ti'

$$\frac{-0.01061 s - 0.004155}{s^3 + 0.3803 s^2 + 0.05122 s + 0.001322}$$

T'/Ti'

$$\frac{0.03333}{s + 0.03333}$$

Problem 4:

In order to determine the process interactions and select the best pairing of controlled and manipulated variables, a relative gain array was determined. As we are only concerned with the manipulated and controlled variables, the RGA will be a 2x2 matrix describing the interaction between Gu, Q (manipulated variables) and T, S (controlled/state variables). This interaction along with the steady state gains of for each transfer function is represented below in Figure 23.

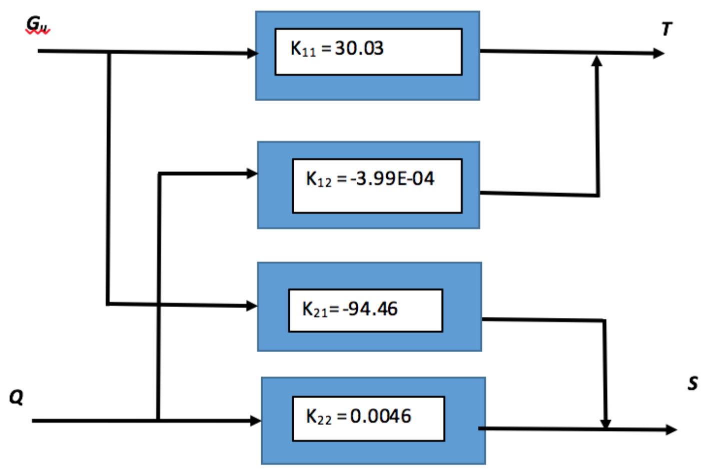

Figure 23: 2x2 multi-loop block diagram

$$\Lambda_{11} = \frac{1}{1 - \frac{K_{12}K_{21}}{K_{11}K_{22}}}$$

$$\Lambda_{12} = \Lambda_{21} = 1 - \Lambda_{11}$$

$$\text{RGA} = \begin{bmatrix} 1.3757 & -0.3757 \\ -0.3757 & 1.3757 \end{bmatrix}$$

From the RGA calculated, we can conclude what the best pairings are. Pairings which correspond to negative pairings should never be selected therefore Λ_{21} and Λ_{21} are not selected and the best pairing are:

Output 1-input 1

Output 2- input 2

Problem 5:

In order to develop an accurate SIMULINK model of the process, the gains and saturation limits of the transducers, valve and sensor had to be calculated. Figure 24 displays the limits for the saturation blocks as well as the gains.

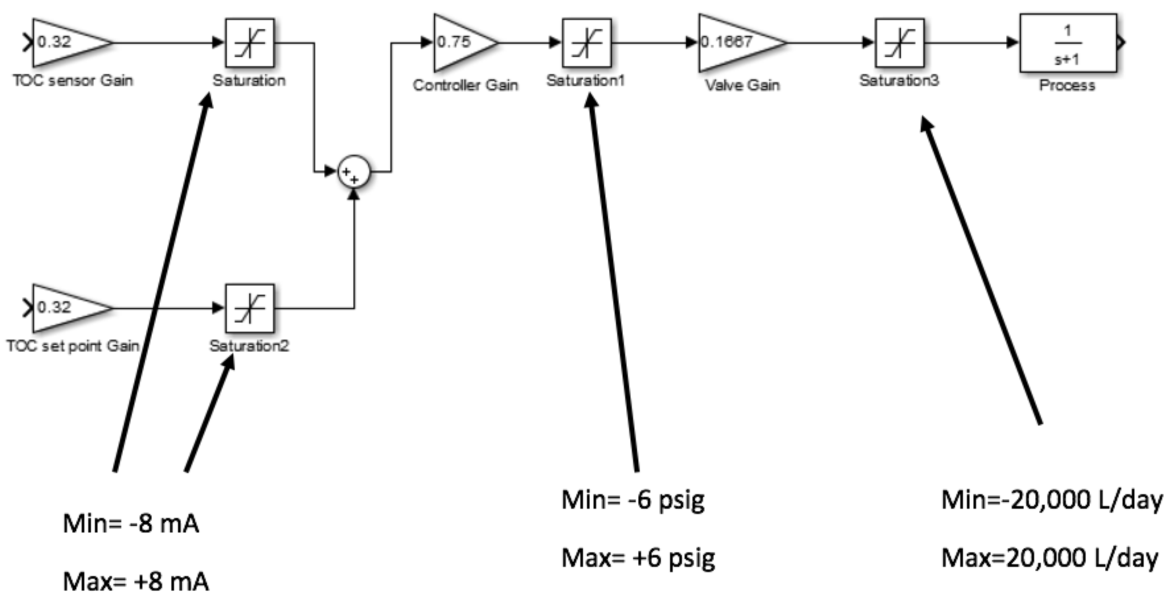

Figure 24: Control system hardware diagram for flow input to process

The gains were calculated by dividing the span of output range of the controller by the input range:

$$\text{Gain} = \frac{\mathit{Span\ of\ output}}{\mathit{span\ of\ input}}$$

To calculate the range for the saturation blocks, we started from the steady state value of the flow rate and worked our way to the temperature sensor/transducer.

Similarly, the steady state gains and saturation limits were calculated for the heat input (Gu') and is displayed below in figure 24.

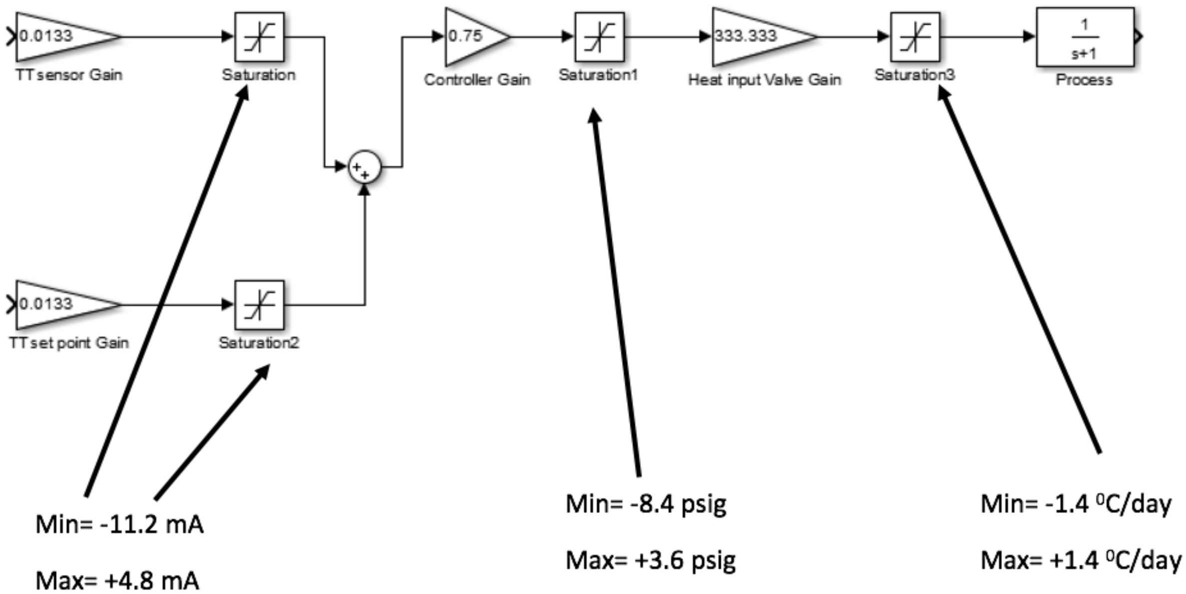

Figure 25. Control system hardware diagram for heat input to process

The transfer functions obtained earlier were implemented into the system along will all the saturation blocks and gains calculated and are displayed below in Figure 26.

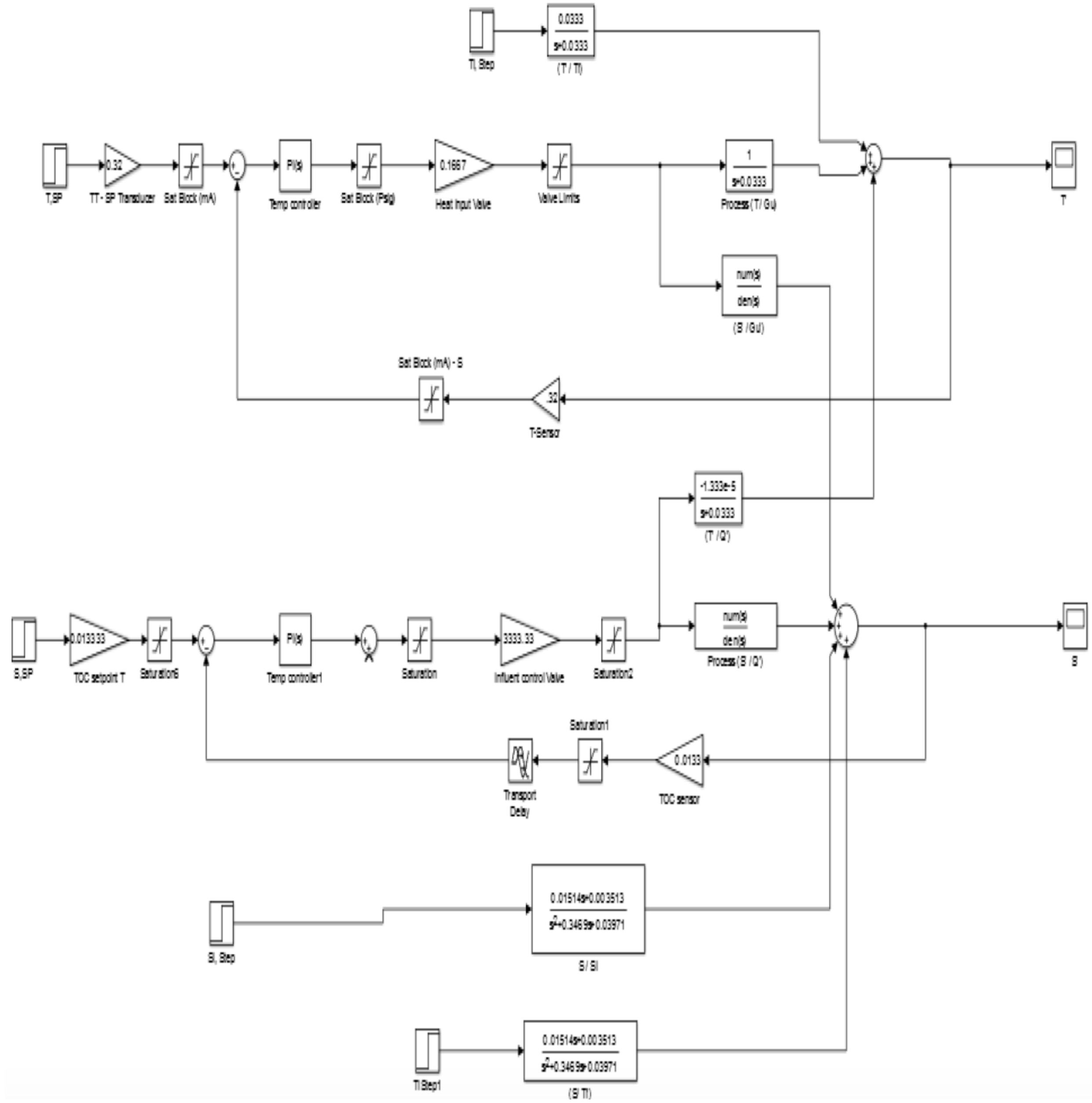

Figure 26: SIMULINK model of biological digestive system

Problem 6:

Using direct synthesis method, two PI controllers were synthesized for the control of the system's temperature and substrate concentration. The synthesis of the temperature and substrate concentration controller is shown below in detail.

$$the\ setpoint\ tracking\ tranfer\ function\ for\ the\ system\ is:$$

$$\frac{T'}{T'_{sp}} = \frac{0.05334G_pG_c}{1 + 0.05334G_cG_p}$$

$$solving\ for\ G_c$$

$$G_c = \frac{\frac{T'}{T'_{sp}}}{0.05334\left(1 - \frac{T'}{T'_{sp}}\right)}\frac{1}{G_p}$$

$$Assuming\ a\ first\ order\ tranfer\ function\ for\ the\ setpoint\ tracking$$

$$\frac{T'}{T'_{sp}} = \frac{1}{\tau_c s + 1}$$

$$plug\ the\ setpoint\ tracking\ into\ the\ G_c$$

$$G_c = \frac{\frac{1}{\tau_c s + 1}}{1 - \frac{1}{\tau_c s + 1}}\left(\frac{1}{\frac{0.05334}{s + 0.0333}}\right)$$

$$simplifying\ the\ equation$$

$$G_c = \frac{1}{0.05334\tau_c} + \frac{1}{0.62425\tau_c s}$$

$$\tau_c = 1\ day$$

The form for the substrate concentration controller is given below:

$$the\ setpoint\ tracking\ tranfer\ function\ for\ the\ system\ is$$

$$\frac{S'}{S'_{sp}} = \frac{44.33G_cGp}{1 + 44.33G_cG_pe^{-\theta s}}$$

$$solving\ for\ G_c$$

$$G_c = \frac{\frac{S'}{S'_{sp}}}{\left(1 - \frac{S'}{S'_{sp}}e^{-\theta s}\right)}\frac{1}{44.33G_p}$$

$$Assuming\ a\ first\ order\ tranfer\ function\ for\ the\ setpoint\ tracking$$

$$\frac{S'}{S'_{sp}} = \frac{1}{\tau_c s + 1}$$

$$plug\ the\ setpoint\ tracking\ into\ the\ G_c$$

$$G_c = \frac{\frac{1}{\tau_c s + 1}}{1 - \frac{1}{\tau_c s + 1} e^{-\theta s}} \left(\frac{1}{44.33 G_p}\right)$$

$$approximate\ the\ time\ delay\ using\ taylor\ series\ approximation$$

$$e^{-\theta s} = 1 - \theta s$$

$$simplifying\ the\ equation\ and\ the\ approximation$$

$$G_c = \frac{1}{\tau_c s + 0.02 s} * \frac{1}{44.33 G_p}$$

$$Substituting\ for\ G_p$$

$$G_c = \frac{1}{\tau_c s + 0.02 s} * \frac{s^7 + 1.1 * s^6 + 0.51 * s^5 + 0.14 * s^4 + 0.023 * s^3 + 2.3e-3 * s^2 + 1.2e-4 * s + 2.1e-6}{0.023 * s^6 + 0.023 * s^5 + 9.7e-3 * s^4 + 2.2e-3 * s^3 + 2.8e-4 * s^2 + 1.8e-5 * s + 4.3e-7}$$

$$\tau_c = 1\ day$$

Problem 7:

With the system modeled in Simulink and the controllers now designed, we can see how well the controllers will perform. The main functions of the controllers are to keep the system as close to the set points as possible, and when there is a disturbance to the system's input variables or a set point change, the controllers will return the system to its desired set point quickly and keep the system stable. The implementation of good controllers is very important as the digester needs to remain at its desired set points to be at the optimum performance and the system’s response to changes without controllers takes a very long time to settle to its new steady state (up to 200 days with only a 10% change in certain inputs). The controllers that have been designed in problem 6 have now been implemented and the controller's performance can be seen from the figures below. The figures below show the effect to the effluent substrate concentration (S’) and effluent temperature (T’) due to a + / - 10% change in the influent feed temperature (T’i) and influent substrate concentration (S’i).

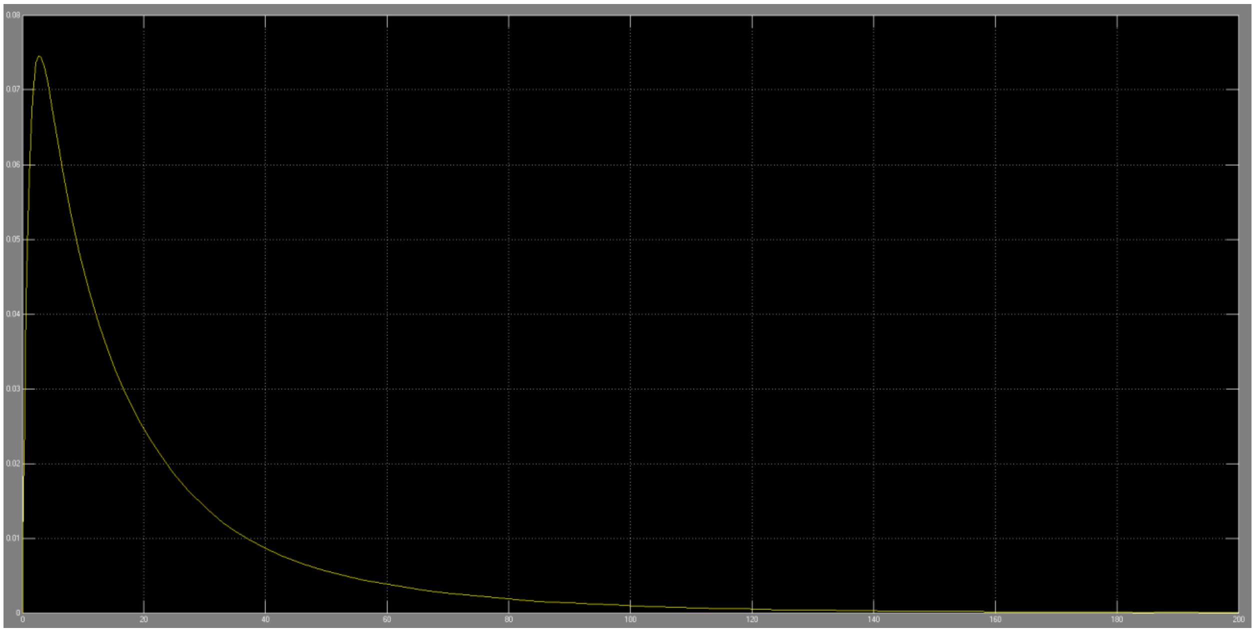

Figure 27. Effluent temperature (T') response to a 10 percent increase in Ti' with PI controllers implemented.

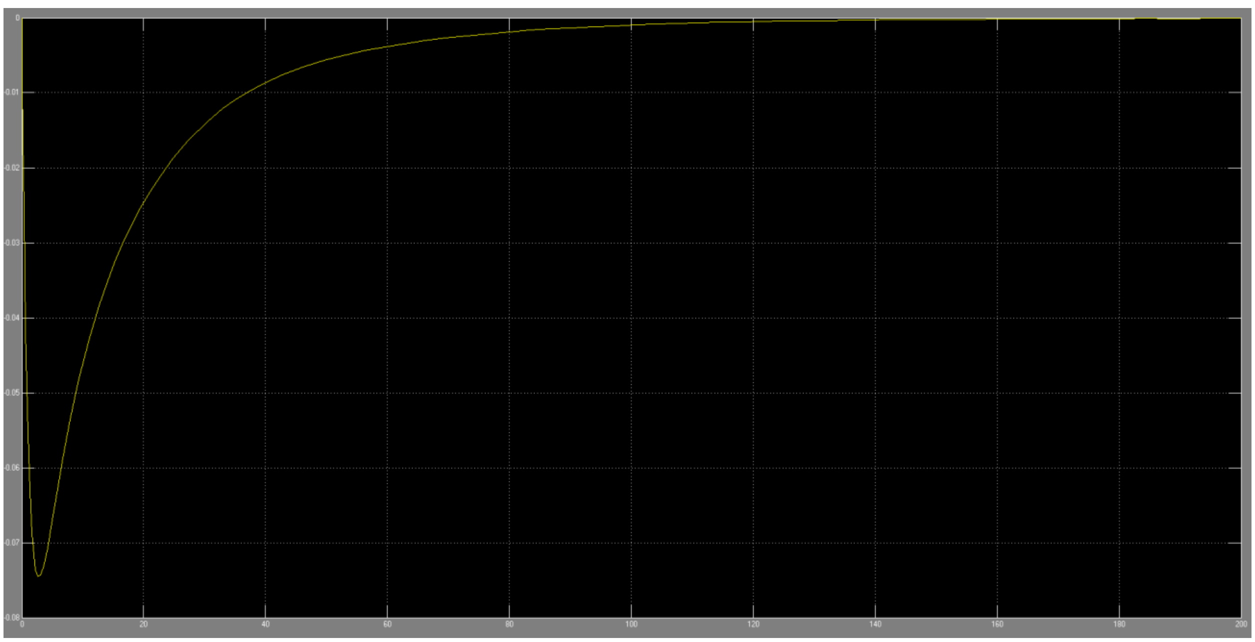

Figure 28. Effluent temperature (T') response to a 10 percent decrease in Ti' with PI controllers implemented.

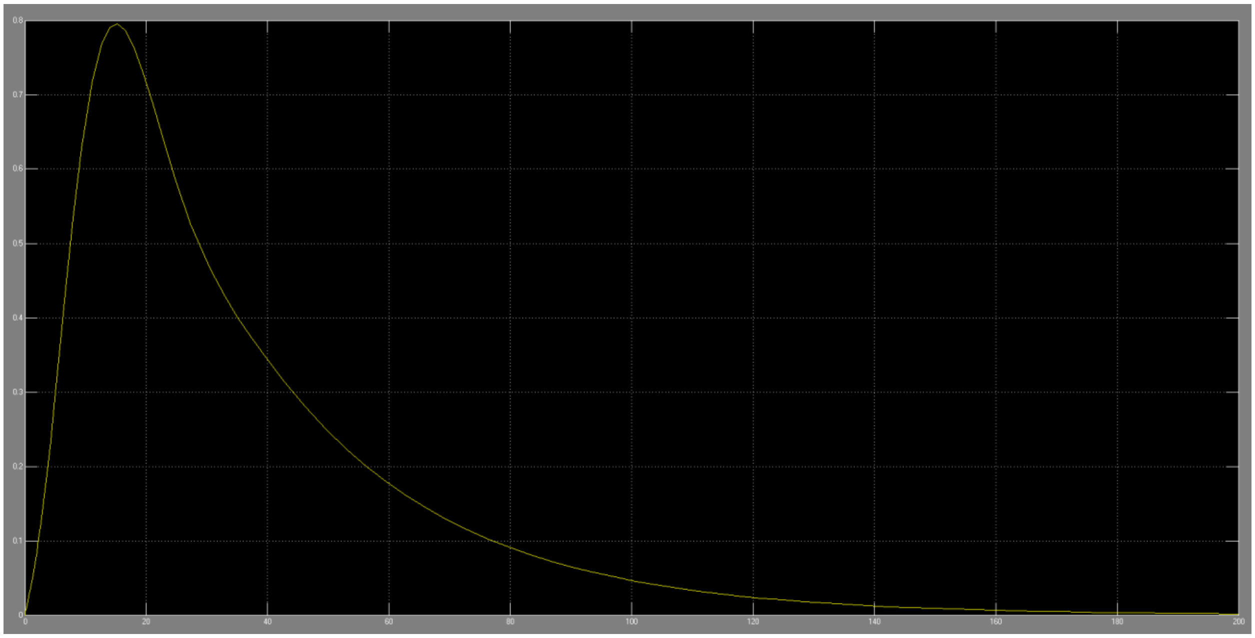

Figure 29. Effluent substrate concentration (S) response to a 10 percent increase in Ti’ with PI controllers implemented.

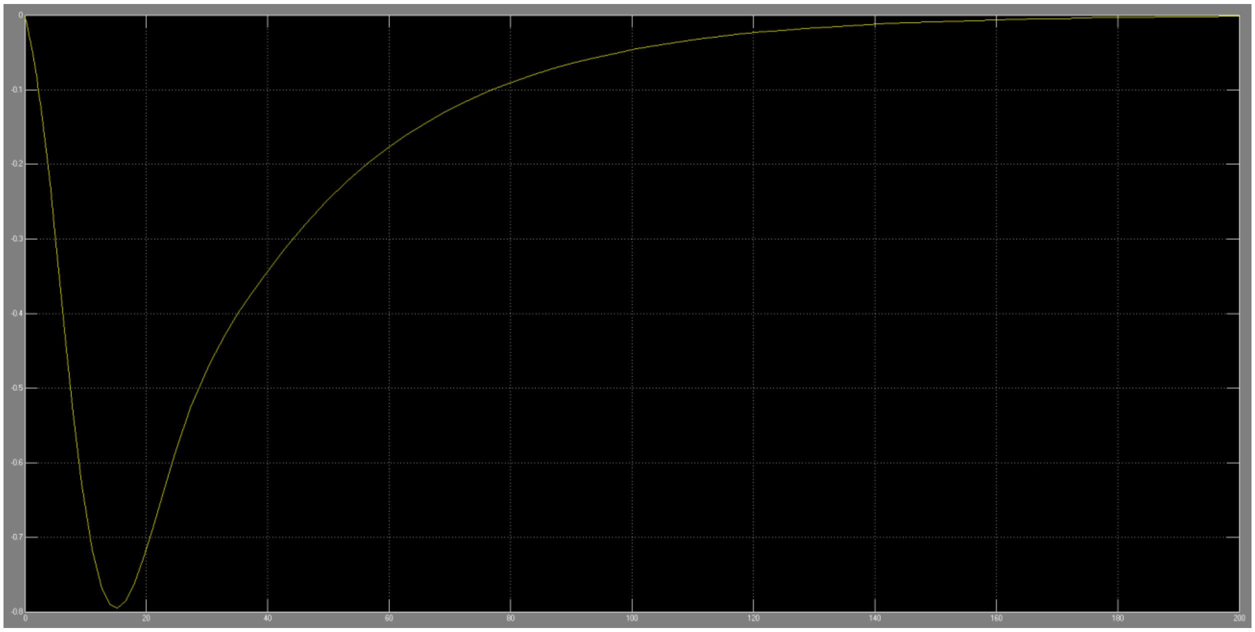

Figure 30. Effluent substrate concentration (S) response to a 10 percent decrease in Ti’ with PI controllers implemented.

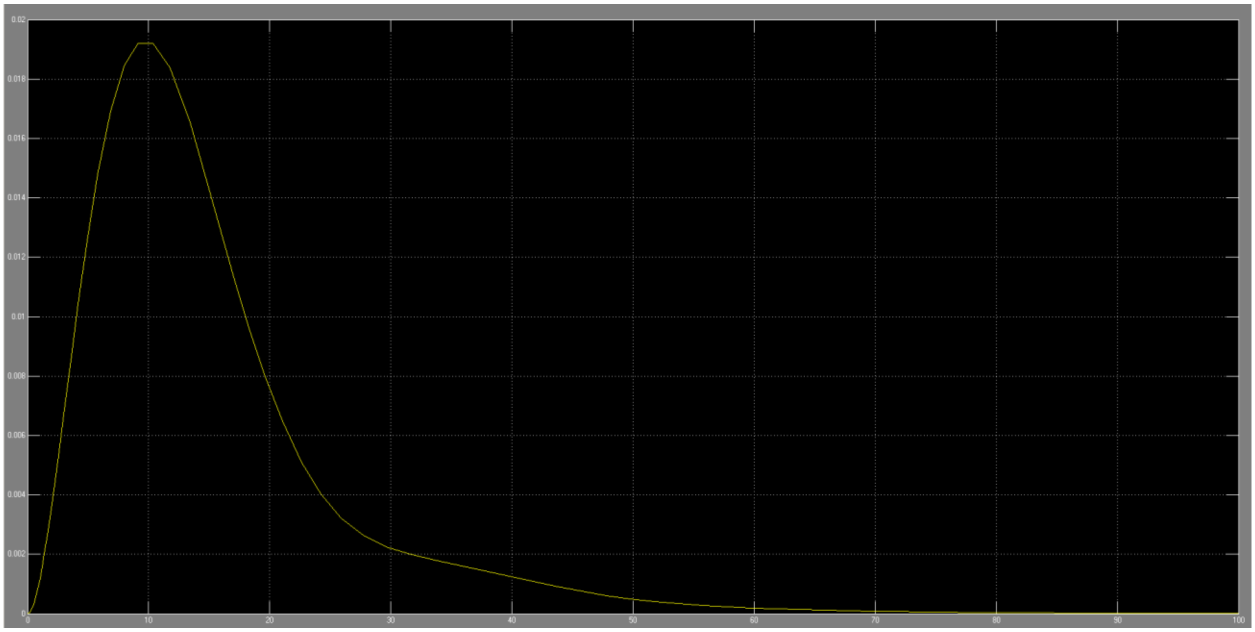

Figure 31. Effluent temperature (T') response to a 10 percent increase in Si' with PI controllers implemented.

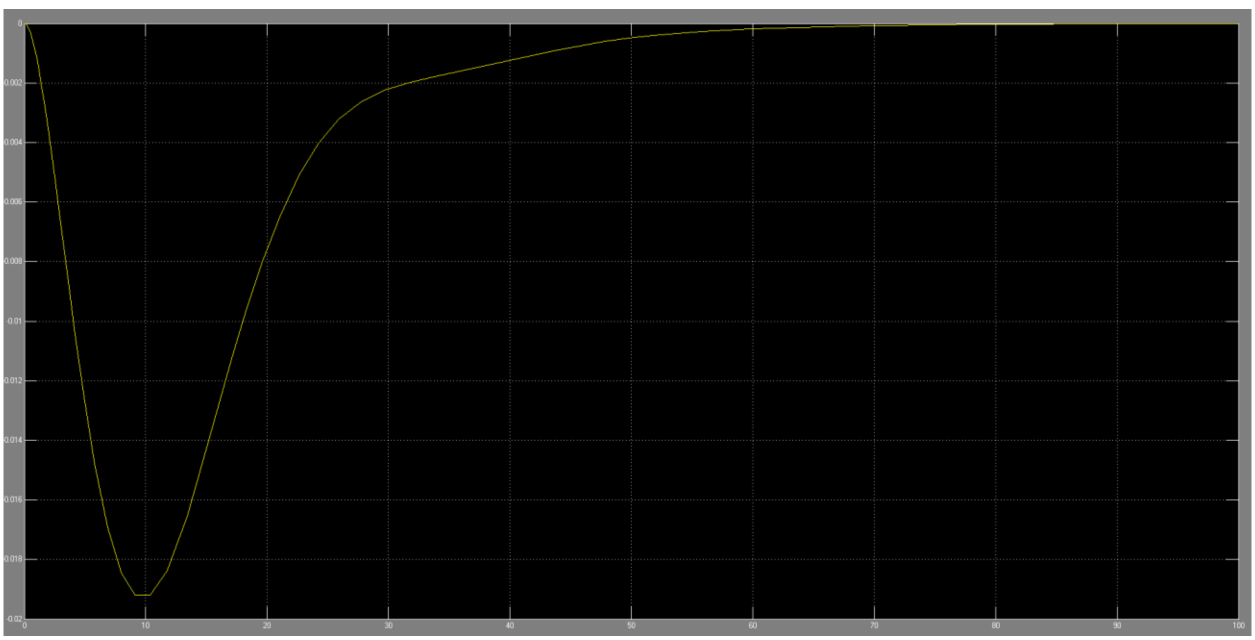

Figure 32. Effluent temperature (T') response to a 10 percent decrease in Si' with PI controllers implemented.

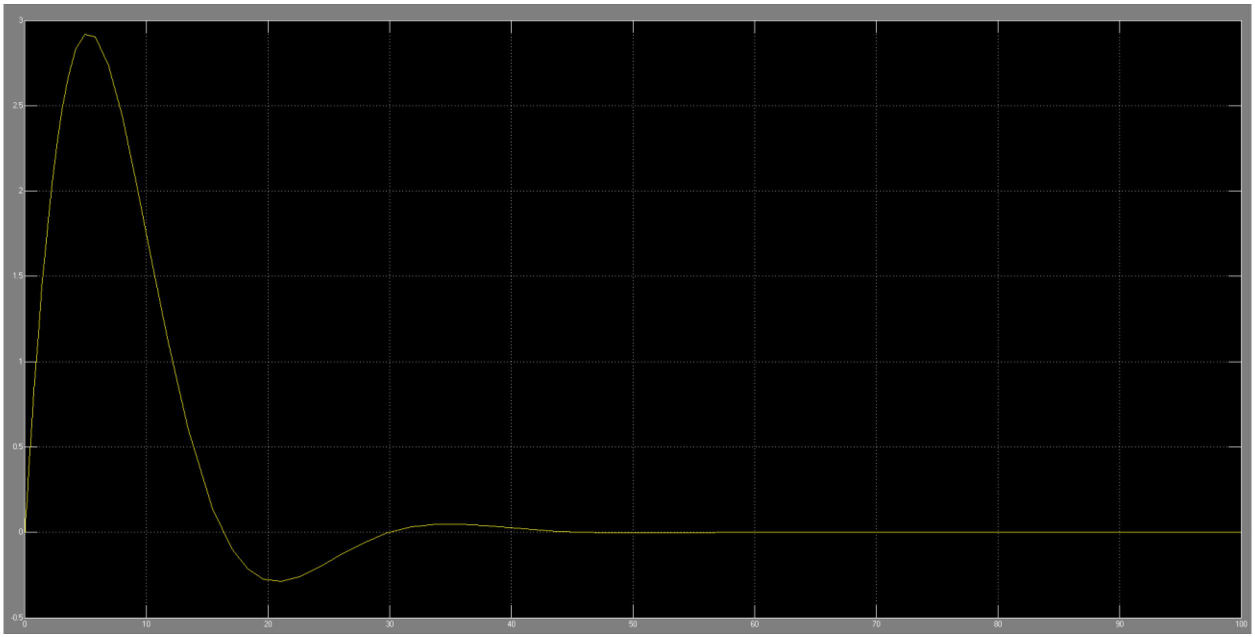

Figure 33. Effluent substrate concentration (S) response to a 10 percent increase in Si’ with PI controllers implemented.

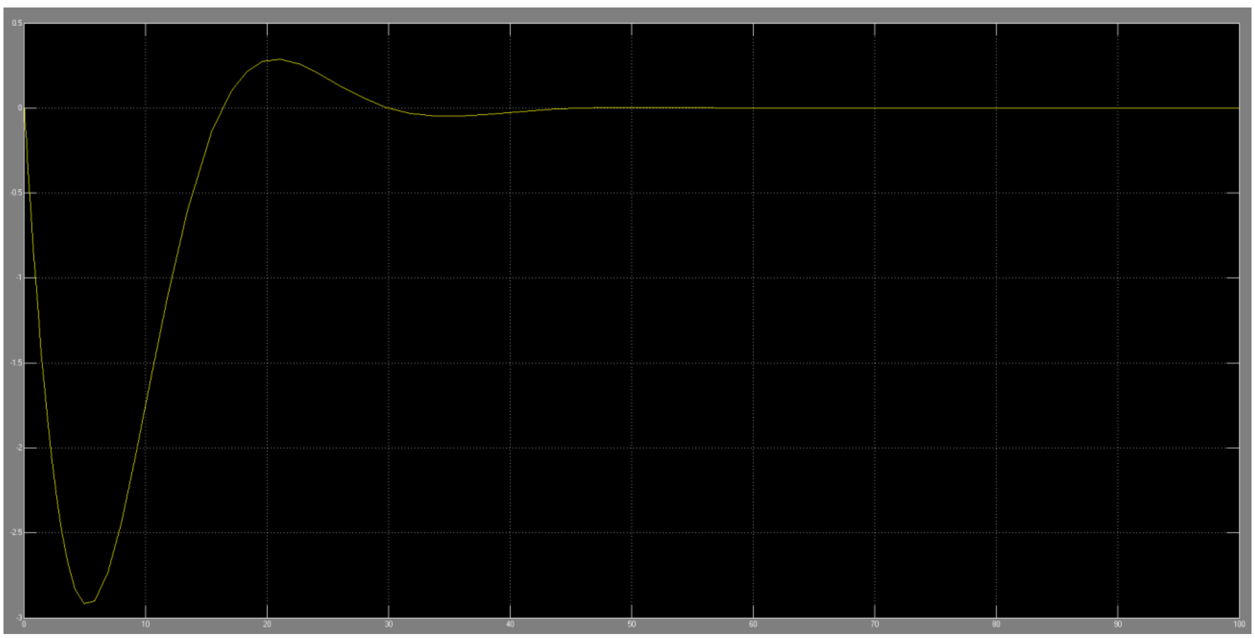

Figure 34. Effluent substrate concentration (S) response to a 10 percent decrease in Si’ with PI controllers implemented.

As seen from the figures above, the PI controllers that have been implemented to control the digester system perform very well. The controllers correct for changes in the disturbance variables by adjusting the influent flow rate and the heat input to the system, these changes to the manipulated variables allow the disturbances to be minimized and the system returns to its desired set point much faster than it did without the controllers. By looking at Figure 14, it can be seen that without a control system, when the influent feed temperature Ti' had a 10 percent increase, the effluent temperature increased by 3 degrees C and it took about 150 days to reach the new steady state value. However, with the control system now implemented, as seen in Figure 28 when the value of Ti' was increased by 10 percent, the effluent temperature T' only changed by 0.075 degrees C and returned to its desired set point after 120 days. Therefore, the controller took action and kept the system at its desired set point, which allowed the digester to remain very close to its optimum temperature.

<u>Bonus Problem Part 1:</u>

Feedforward Control

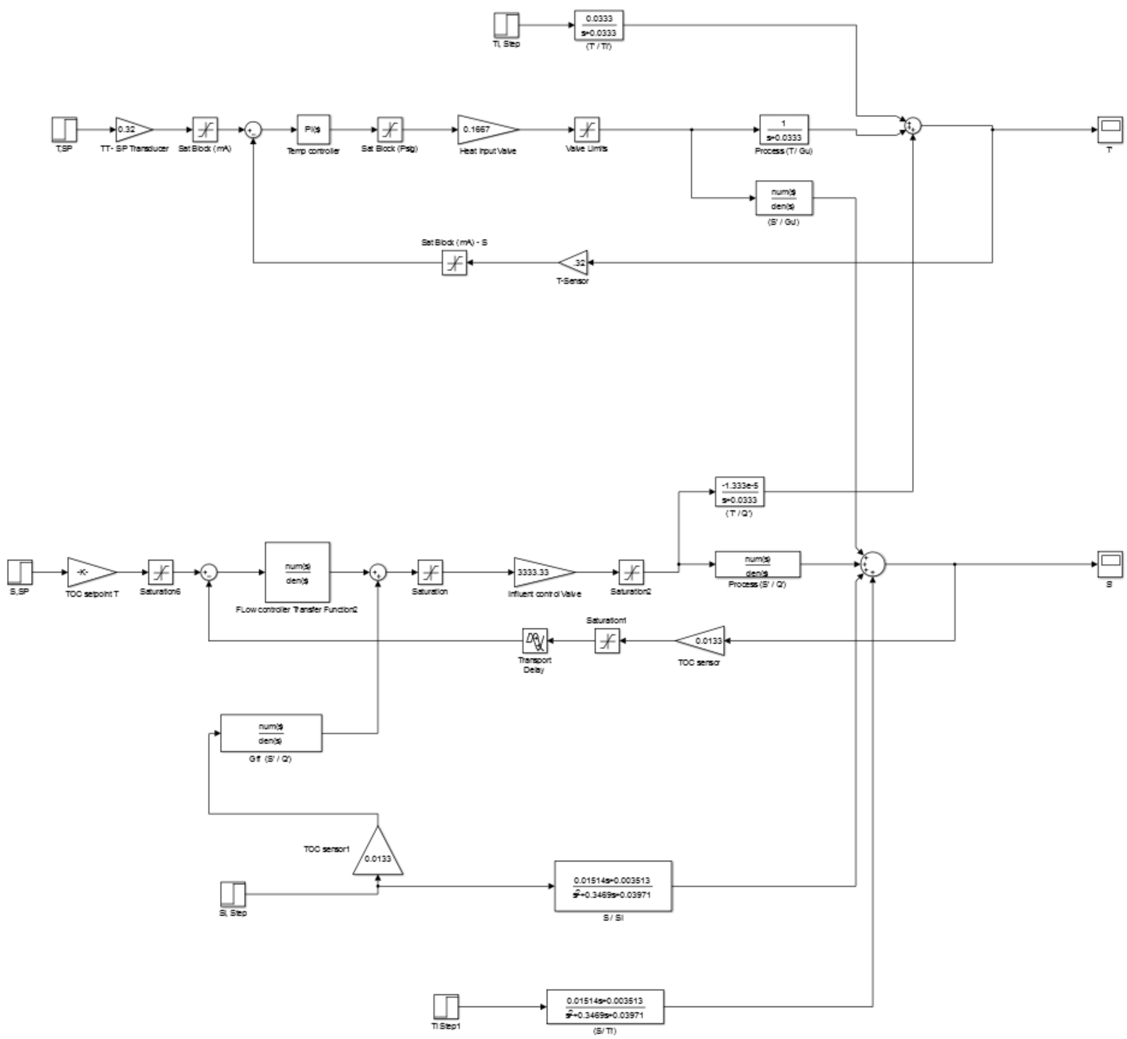

Figure 35. Simulink model with feed forward control implemented.

Calculation and derivation of the feed forward control (G_{ff}) is shown in detail in the Appendix.

$$G_{ff} = \frac{-0.015s^8 - 0.02s^7 - 0.011s^6 - 3.9e\text{-}3s^5 - 8.4e\text{-}4s^4 - 1.2e\text{-}4s^3 - 9.7e\text{-}6s^2 - 4.4e\text{-}7s - 7.3e\text{-}9}{0.023s^8 + 0.031s^7 + 0.019s^6 + 6.5e\text{-}3s^5 + 1.4e\text{-}3s^4 + 2.0e\text{-}4s^3 + 1.8e\text{-}5s^2 + 8.8e\text{-}7s + 1.7e\text{-}8}$$

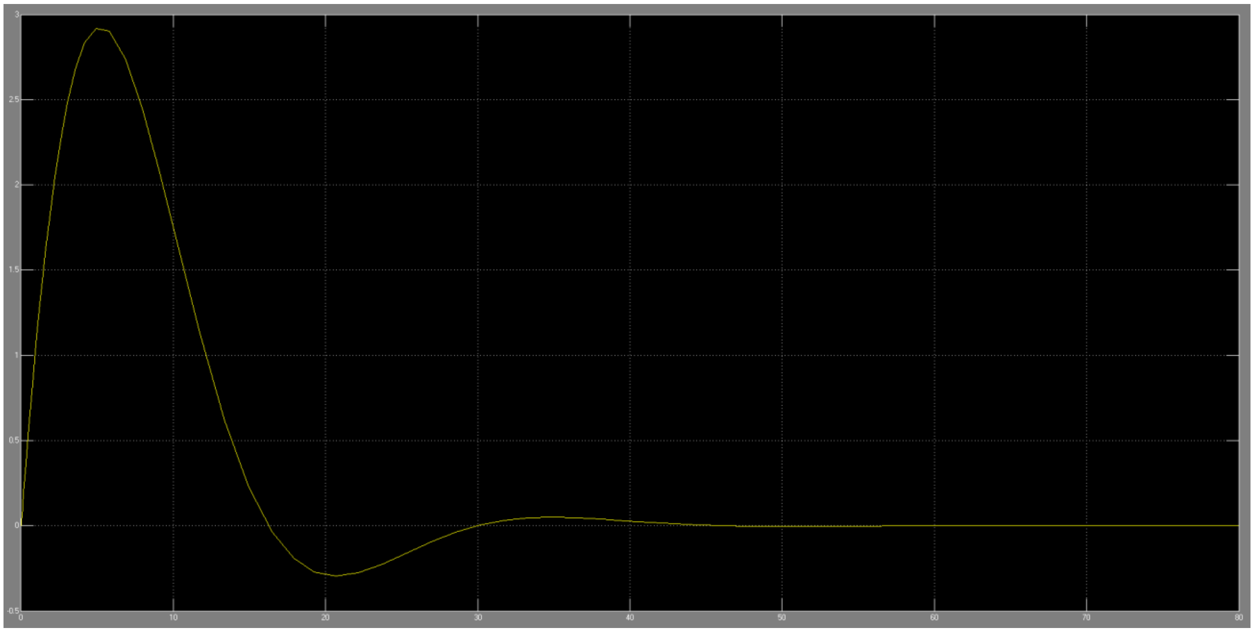

Figure 36. Effluent substrate concentration (S) response to a 10 percent increase in Si' without a feed forward controller implemented.

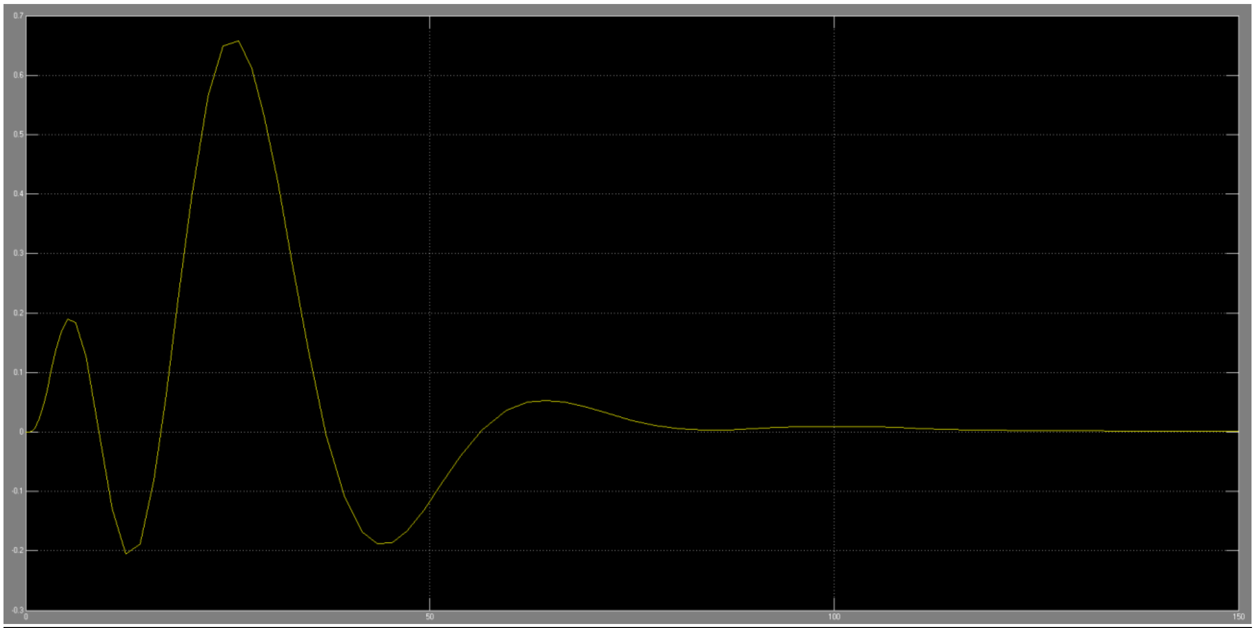

Figure 37. Effluent substrate concentration (S) response to a 10 percent increase in Si' with a feed forward controller implemented.

As seen from Figure 36 and 37 above, the addition of the feedforward controller provides an 80% decrease in the peak of the overshoot when there is a 10 percent increase in the inlet substrate concentration.

Bonus Problem Part 2:

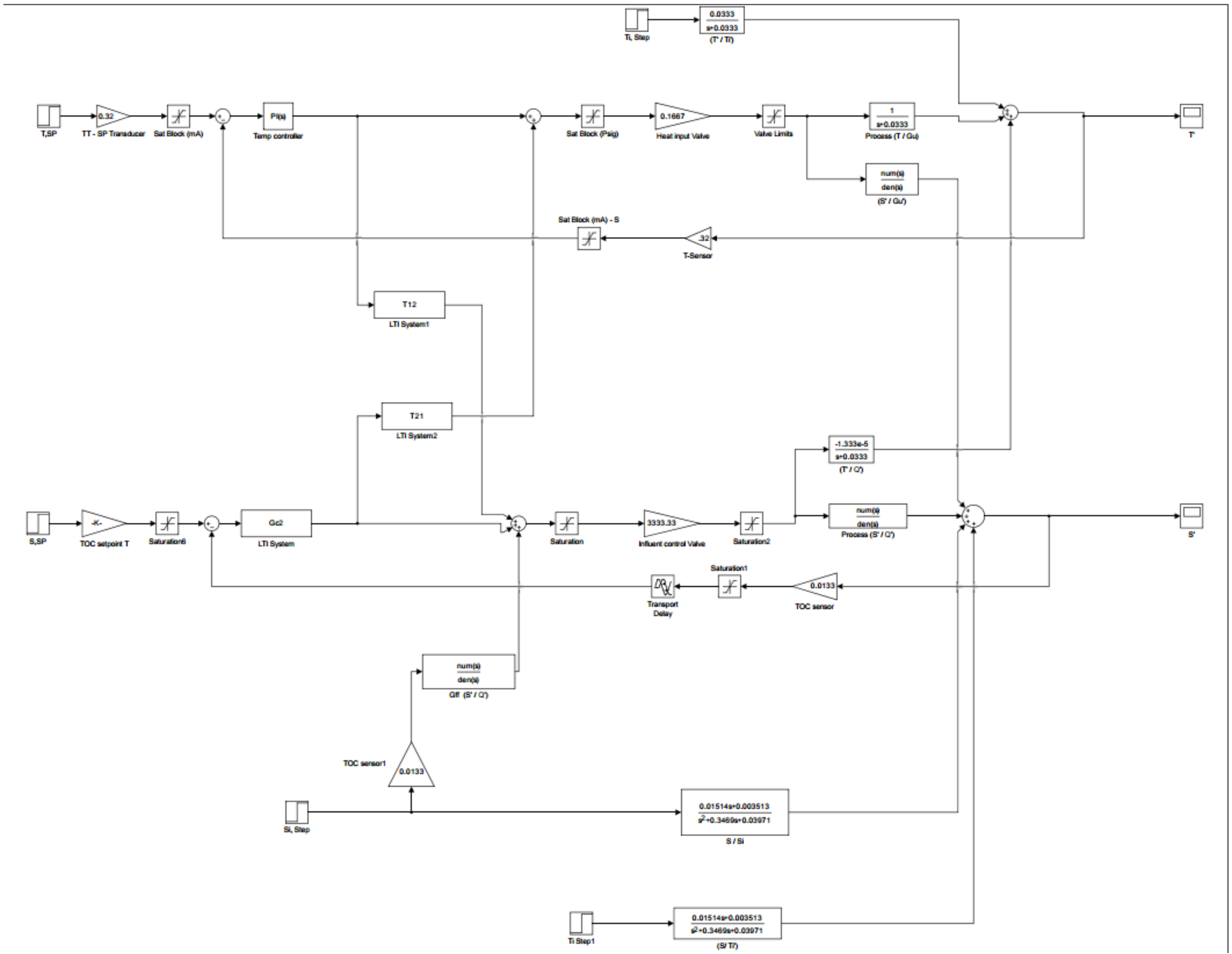

Figure 38: Simulink model of dynamic decouplers.

Dynamic decouplers can be implemented to reduce the interaction between the two states that is being monitored. Using the formula for dynamic decoupling for two systems in the book, shown below

$$G_{ff21} = -\frac{G_{p21}}{G_{p22}}$$

$$G_{ff12} = -\frac{G_{p12}}{G_{p11}}$$

The following decouplers were obtained

$$T21 = \frac{0.4432\, s^7 + 0.476\, s^6 + 0.2282\, s^5 + 0.06223\, s^4 + 0.01029\, s^3 + 0.001009\, s^2 + 5.199e-05\, s + 9.241e-07}{0.0005227\, s^9 + 0.0007177\, s^8 + 0.0004431\, s^7 + 0.0001604\, s^6 + 3.724e-05\, s^5 + 5.68e-06\, s^4 + 5.597e-07\, s^3 + 3.341e-08\, s^2 + 1.046e-09\, s + 1.281e-11}$$

$$T12 = \frac{(1.33e-05\, s + 4.429e-07)}{s + 0.0333}$$

The figure below shows the response of the temperature to a set-point change in the substrate concentration with the decouplers in added to the system.

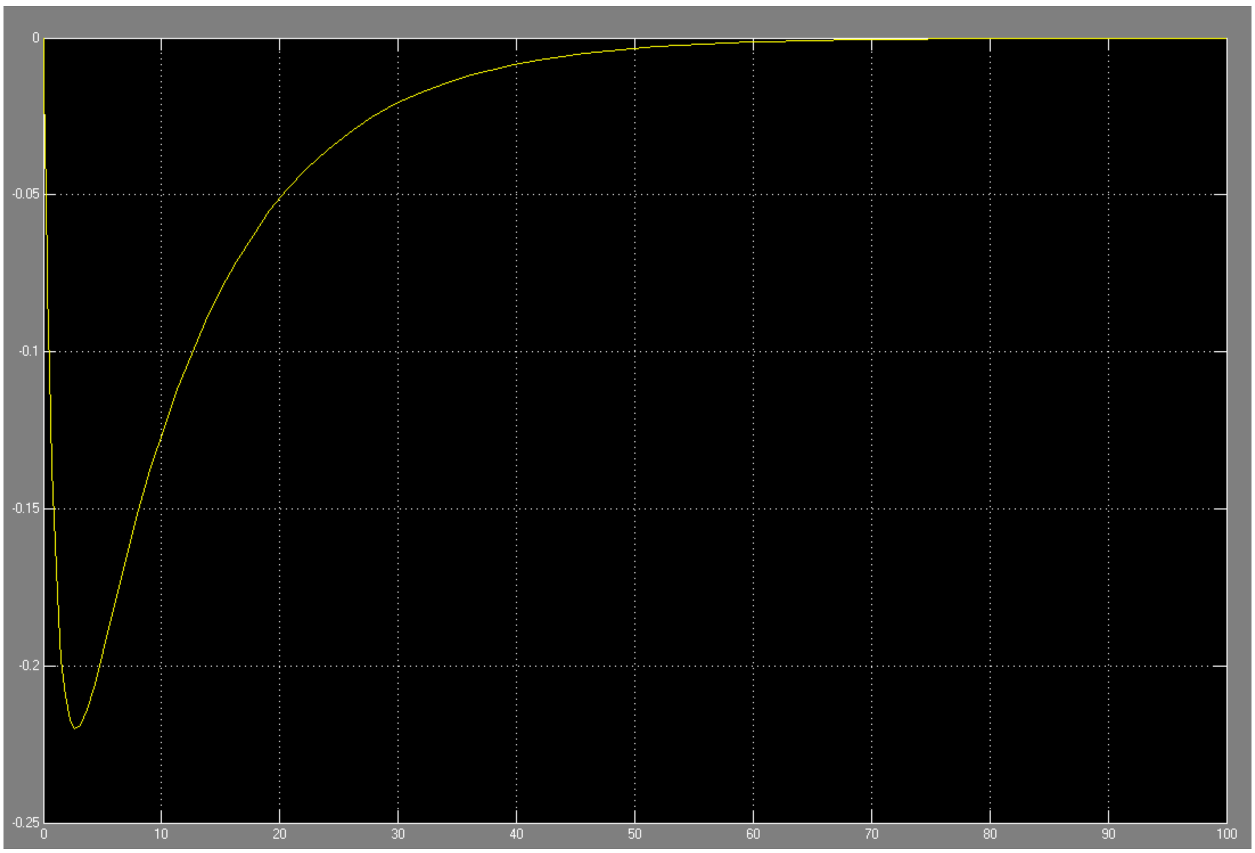

Figure 39. Effluent temperature (T') response to a 10 percent increase in Si' with dynamic decouplers implemented.

```
syms s
clc
Gp_num = 0.0005227*s^6 + 0.000519*s^5 + 0.000219*s^4 + 4.989e-05*s^3 + 6.359e-06*s^2 + 4.158e-07*s+9.691e-09;
Gp_denom = s^7 + 1.074*s^6 + 0.5148*s^5 + 0.1404*s^4 + 0.02321*s^3 + 0.002276*s^2 + 0.0001173*s+2.085e-06;

num_mult = -0.015*s - 0.0035;
denom = s^2 + 0.346*s + 0.0397;
denom_mult = 3333.333*0.0133 * denom;
```

```
Gcff = expand(num_mult*Gp_denom)/expand(denom_mult*Gp_num);
Gcff = vpa(Gcff,2);

% Controller 2
Gp_num = 44.3*(0.0005227*s^6 + 0.000519*s^5 + 0.000219*s^4 + 4.989e-05*s^3 + 6.359e-06*s^2 +
4.158e-07*s + 9.691e-09);
Gp_denom = s^7 + 1.074*s^6 + 0.5148*s^5 + 0.1404*s^4 + 0.02321*s^3 + 0.002276*s^2 + 0.0001173*s +
2.085e-06;

denom = 1.02*s+0.9867;

Gc = Gp_denom/expand(Gp_num*denom);
Gc = vpa(Gc,2);
num = [1 1.1 0.51 0.14 0.023 2.3e-3 1.2e-4 2.1e-6];
denom =[0.024 0.046 0.033 0.012 2.5e-3 3.0e-4 1.9e-5 4.2e-7];
Gc2 = tf(num,denom);
S_Gu = tf([-0.3183 - 0.1249],[1 0.3802 0.05126 0.001322]);
S_Q = tf([0.0005227 0.000519 0.000219 4.989e-05 6.359e-06 4.158e-07 9.691e-09],...
    [1 1.074 0.5148 0.1404 0.02321 0.002276 0.0001173 2.085e-06]);
T_Q = tf([-1.33e-5],[1 0.0333]);
T_Gu = tf([1],[1 0.0333]);

T21 = -S_Gu/S_Q
T12 = -T_Q/T_Gu
```

```
T21 =


  0.4432 s^7 + 0.476 s^6 + 0.2282 s^5 + 0.06223 s^4 + 0.01029 s^3

                                + 0.001009 s^2 + 5.199e-05 s + 9.241e-07

  ------------------------------------------------------------------------

  0.0005227 s^9 + 0.0007177 s^8 + 0.0004431 s^7 + 0.0001604 s^6

          + 3.724e-05 s^5 + 5.68e-06 s^4 + 5.597e-07 s^3 + 3.341e-08 s^2

                                               + 1.046e-09 s + 1.281e-11


Continuous-time transfer function.


T12 =

  1.33e-05 s + 4.429e-07
  ----------------------
        s + 0.0333
```

```
Continuous-time transfer function.
```

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Conclusions:

With the competition of this project, it is apparent that a well designed and implemented control system can greatly benefit an industrial process, such as the digester system that this project is based on. Once the differential equations were linearized that describe the state variables of the system and implemented into state space form. The state space model was then implemented into Simulink and it was immediately apparent that slight changes to the input variables for this system will greatly affect its effluent substrate concentration and effluent temperature. The digester without a controller, will deviate from its desired set points without a control system, and when there are input changes to the manipulated variables, the system can take up to 200 days to reach the new steady state value, which is extremely undesired. Therefore, a control system was designed that would reduce the effect of perturbations on the systems desired set points. As seen in the results above, the control system that was designed for this system greatly reduces the effects of disturbances to the systems set points. The control system also allows the system to reach its new set points much faster when set point changes are made to the system. This control system will allow the digester to remain within its desired specifications when there are + / - 10 percent changes to the systems input variables, the system will also remain in specifications for changes larger than + / - 10 percent in the input variables, however due to the vast amount of variations in disturbances that could occur, only + / - 10 percent variations were shown in the results section.

In order to increase the performance of the control system, a feedforward controller was implemented to reduce the disturbance of influent substrate concentration fluctuations (Si'). It was apparent after comparing Figures 36 and 37 that the feedforward controller will greatly reduce the influent substrate concentration fluctuations.

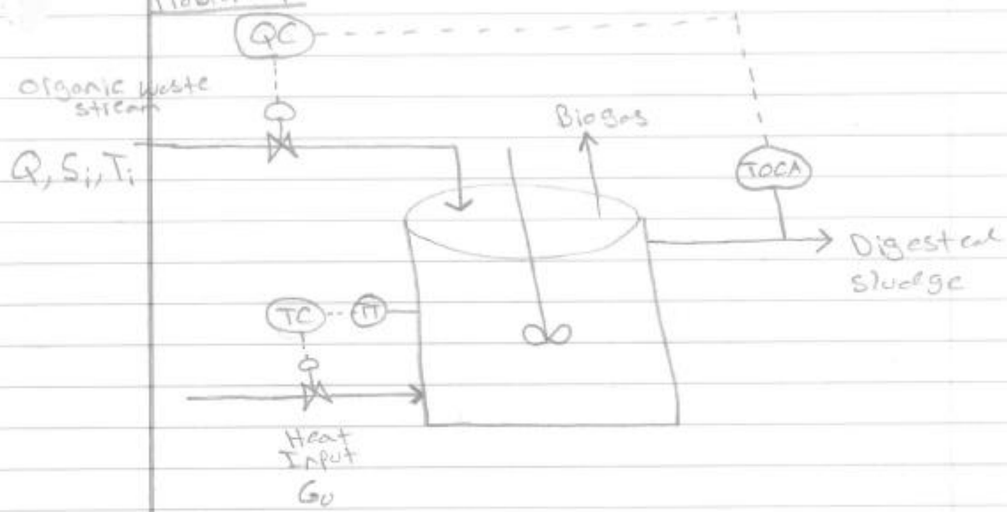
Appendix:

Problem 1 & 2:

Controls Project

1.

Problem 1



- Substrate balance

$$\frac{dS}{dt} = \frac{1}{V} Q S_i - \frac{1}{V} Q S - \mu_{max} \frac{S}{K+S} \frac{X}{Y}$$

- Micro-organisms balance

$$\frac{dX}{dt} = -\frac{1}{V} Q X + \mu_{max} \frac{S}{K+S} X - K_d X$$

- Energy Balance

$$\frac{dT}{dt} = \frac{1}{V} Q T_i - \frac{1}{V} Q T + G_U$$

$T_i, T, Q, S, S_i, G_U, X$ all change with time.

$$\left[\mu_{max} = 0.013T - 0.129\right]$$

2.

Linearize balances

$$\frac{dS}{dt} = \frac{1}{V}\underbrace{QS_i}_{\text{nonlinear}} - \frac{1}{V}\underbrace{QS}_{\text{nonlinear}} - \underbrace{(0.013T-0.129)\frac{S}{K+S}\frac{X}{Y}}_{\text{non-linear}}$$

$$QS_i \approx Q_{ss}S_{i,ss} + \frac{\partial}{\partial Q}\left[QS_i\right]\Big|_{ss}(Q-Q_{ss}) + \frac{\partial}{\partial S_i}\left[QS_i\right]\Big|_{ss}(S_i-S_{i,ss})$$

$$\left[QS_i \approx Q_{ss}S_{i,ss} + S_{i,ss}Q' + Q_{ss}S_i'\right]$$

$$\left[QS \approx Q_{ss}S_{ss} + S_{ss}Q' + Q_{ss}S'\right]$$

$$(0.013T-0.129)\frac{S}{K+S}\frac{X}{Y} \approx (0.013T_{ss}-0.129)\frac{S_{ss}}{K+S_{ss}}\frac{X_{ss}}{Y} +$$

$$\frac{\partial}{\partial T}\left[(0.013T-0.129)\frac{S}{K+S}\frac{X}{Y}\right]\Big|_{ss}(T-T_{ss}) + \frac{\partial}{\partial S}\left[(0.013T-0.129)\frac{S}{K+S}\frac{X}{Y}\right]\Big|_{ss}$$

$$(S-S_{ss}) + \frac{\partial}{\partial X}\left[(0.013T-0.129)\frac{S}{K+S}\frac{X}{Y}\right]\Big|_{ss}(X-X_{ss})$$

$$\Bigg[= (0.013T_{ss}-0.129)\frac{S_{ss}}{K+S_{ss}}\frac{X_{ss}}{Y} + \frac{0.013\,S_{ss}X_{ss}}{Y(K+S_{ss})}(T')$$

$$+ \frac{K(0.013T_{ss}-0.129)X_{ss}}{Y(K+S_{ss})^2}(S') + \frac{S_{ss}(0.013T_{ss}-0.129)}{Y(K+S_{ss})}(X')\Bigg]$$

(3.) Linearized substrate balance

$$\frac{dS}{dt} = \frac{1}{V}\left(Q_{ss}S_{i,ss} + S_{i\,ss}Q' + Q_{ss}S_i'\right)$$

$$-\frac{1}{V}\left(Q_{ss}S_{ss} + S_{ss}Q' + Q_{ss}S'\right) - \left(0.013T_{ss}-0.129\right)\frac{S_{ss}}{K+S_{ss}}\frac{X_{ss}}{Y}$$

$$-\frac{0.013S_{ss}X_{ss}}{Y(K+S_{ss})}T' - \frac{K(0.013T_{ss}-0.129)X_{ss}S'}{Y(K+S_{ss})^2}$$

$$-\frac{S_{ss}(0.013T_{ss}-0.129)X'}{Y(K+S_{ss})}$$

$$-ss\left(\frac{dS}{dt} = \frac{1}{V}\left(Q_{ss}S_{i\,ss} + S_{i\,ss}\underbrace{\cancel{Q'}}_{0} + Q_{ss}\underbrace{\cancel{S_i'}}_{0}\right) - \frac{1}{V}\left(Q_{ss}S_{ss} + S_{ss}\underbrace{\cancel{Q'}}_{0} + Q_{ss}\underbrace{\cancel{S'}}_{0}\right)\right.$$

$$-\left(0.013T_{ss}-0.129\right)\frac{S_{ss}}{K+S_{ss}}\frac{X_{ss}}{Y} - \frac{0.013S_{ss}X_{ss}\overset{0}{\cancel{T'}}}{Y(K+S_{ss})} - \frac{K(0.013T_{ss}-0.129)X_{ss}\underbrace{\cancel{S'}}_{0}}{Y(K+S_{ss})^2}$$

$$-\frac{S_{ss}(0.013T_{ss}-0.129)\underbrace{\cancel{X'}}_{0}}{Y(K+S_{ss})}$$

$$\boxed{\frac{dS'}{dt} = \frac{1}{V}\left(S_{i\,ss}Q' + Q_{ss}S_i'\right) - \frac{1}{V}\left(S_{ss}Q' + Q_{ss}S'\right) - \frac{0.013S_{ss}X_{ss}T'}{Y(K+S_{ss})} - \frac{K(0.013T_{ss}-0.129)X_{ss}S'}{Y(K+S_{ss})^2} - \frac{S_{ss}(0.013T_{ss}-0.129)x'}{Y(K+S_{ss})}}$$

$\mathcal{L}$

$$s\hat{S}'(s) - \overset{0}{\cancel{S'(0)}} = \frac{1}{V}\left(S_{i\,ss}\hat{Q}'(s) + Q_{ss}\hat{S}_i'(s)\right) - \frac{1}{V}\left(S_{ss}\hat{Q}'(s) + Q_{ss}\hat{S}'(s)\right) - \frac{0.013S_{ss}X_{ss}\hat{T}'(s)}{Y(K+S_{ss})}$$

$$-\frac{K(0.013T_{ss}-0.129)X_{ss}\hat{S}'(s)}{Y(K+S_{ss})^2} - \frac{S_{ss}(0.013T_{ss}-0.129)\hat{x}'(s)}{Y(K+S_{ss})}$$

(4.)

$\dfrac{\frac{mgCOD}{L}\ \frac{mg}{L}}{\frac{mg}{mgCOD}\left(\frac{mgCOD}{L}\right)\cdot day}$ $\quad 0.63 = Y^{M}_{X/S},\ M_S = 0.056,\ K_d = 0.035$

Substrate balance in deviation form with given values.

$$\frac{dS'}{dt} = \frac{1}{600{,}000L}\left(800\,\frac{mgCOD}{L}\,Q' + 20{,}000\,\frac{L}{day}\,S_i'\right)$$

$$-\frac{1}{600{,}000L}\left(110\,\frac{mgCOD}{L}\,Q' + 20{,}000\,\frac{L}{day}\,S'\right) - 0.7002\,T'$$

$$-0.081482\,S' - 0.004877\,X'$$

simplify

(all units work out to give $\frac{mgCOD}{L\cdot day}$)

$$\frac{dS'}{dt} = 0.001333Q' + 0.03333S_i' - 0.000183Q' - 0.03333S'$$

$$-0.7002T' - 0.081482S' - 0.004877X'$$

$$\boxed{\frac{dS'}{dt} = 0.00115Q' + 0.03333S_i' - 0.114816S' - 0.7002T' - 0.004877X'}$$

Linearize Micro-organisim Balance & put in deviation form

$$\frac{dX}{dt} = -\frac{1}{V}\underbrace{QX}_{\text{non linear}} + \mu_{max}\underbrace{\frac{S}{K+S}}_{\text{non-linear}}X - K_d X$$

$$\frac{dX}{dt} = -\frac{1}{V}QX + \frac{(0.013T - 0.129)SX}{K+S} - K_d X$$

- $QX \approx Q_{ss}X_{ss} + Q_{ss}X' + X_{ss}Q'$
- $\dfrac{(0.013T - 0.129)SX}{K+S} \approx \dfrac{(0.013T_{ss} - 0.129)S_{ss}X_{ss}}{K + S_{ss}} +$

(5.)

$$\frac{0.013 S_{ss} X_{ss}}{K+S_{ss}} T' + \frac{K(0.013 T_{ss}-0.129) X_{ss}}{(K+S_{ss})^2} S'$$

$$+ \frac{S_{ss}(0.013 T_{ss}-0.129)}{K+S_{ss}} X'$$

Combine all

$$\frac{dX}{dt} = -\frac{1}{V}\left(Q_{ss}X_{ss} + Q_{ss}X' + X_{ss}Q'\right) + \frac{(0.013 T_{ss}-0.129) S_{ss} X_{ss}}{K+S_{ss}}$$

$$+ \frac{0.013 S_{ss} X_{ss}}{K+S_{ss}} T' + \frac{K(0.013 T_{ss}-0.129) X_{ss}}{(K+S_{ss})^2} S'$$

$$+ \frac{S_{ss}(0.013 T_{ss}-0.129)}{K+S_{ss}} X' - K_d X$$

$-ss$

$$\frac{dX}{dt} = -\frac{1}{V}\left(Q_{ss}X_{ss} + Q_{ss}\cancel{X'}_{\to 0} + X_{ss}\cancel{Q'}_{\to 0}\right) + \frac{(0.013 T_{ss}-0.129) S_{ss} X_{ss}}{K+S_{ss}}$$

$$+ \frac{0.013 S_{ss} X_{ss}}{K+S_{ss}} \cancel{T'}_{\to 0} + \frac{K(0.013 T_{ss}-0.129) X_{ss}}{(K+S_{ss})^2} \cancel{S'}_{\to 0} + \frac{S_{ss}(0.013 T_{ss}-0.129)}{K+S_{ss}} \cancel{X'}_{\to 0} - K_d X$$

deviation form & linearized

$$\frac{dX'}{dt} = -\frac{1}{V} Q_{ss} X' - \frac{1}{V} X_{ss} Q' + \frac{0.013 S_{ss} X_{ss}}{K+S_{ss}} T' \qquad \begin{array}{l} -K_d(X-X_{ss}) \\ -K_d X + K_d X_{ss} \end{array}$$

$$+ \frac{K(0.013 T_{ss}-0.129) X_{ss}}{(K+S_{ss})^2} S' + \frac{S_{ss}(0.013 T_{ss}-0.129)}{K+S_{ss}} X' - K_d X'$$

Plug in known values

$$\frac{dX'}{dt} = -0.03333 X' - 0.006 Q' + 22.9105 T' + 2.66611 S' + 0.159591 X' - 0.3584 X'$$

(6.) Micro-organisim balance Linearized & in deviation form

$$\boxed{\frac{dX'}{dt} = -0.232443X' - 0.006Q' + 22.9105T' + 2.66611S'}$$

• Linearize & put in deviation form: Energy balance

$$\frac{dT}{dt} = \frac{1}{V}\underbrace{QT_i}_{\text{non-linear}} - \frac{1}{V}\underbrace{QT}_{\text{non-linear}} + G_u$$

$$\left[QT_i \approx Q_{ss}T_{iss} + Q_{ss}T_i' + T_{iss}Q'\right]$$

$$\left[QT \approx Q_{ss}T_{ss} + Q_{ss}T' + T_{ss}Q'\right]$$

$$\frac{dT}{dt} = \frac{1}{V}\left(Q_{ss}T_{iss} + Q_{ss}T_i' + T_{iss}Q'\right)$$
$$- \frac{1}{V}\left(Q_{ss}T_{ss} + Q_{ss}T' + T_{ss}Q'\right) + G_u$$

−SS

$$\frac{dT}{dt} = \frac{1}{V}\left(\cancel{Q_{ss}T_{iss}} + Q_{ss}\cancel{T_i''}_{0} + T_{iss}\cancel{Q''}_{0}\right)$$
$$- \frac{1}{V}\left(\cancel{Q_{ss}T_{ss}} + Q_{ss}\cancel{T''}_{0} + T_{ss}\cancel{Q''}_{0}\right) + G_u$$

$$\left(\frac{dT'}{dt} = \frac{1}{V}\left(Q_{ss}T_i' + T_{iss}Q'\right) - \frac{1}{V}\left(Q_{ss}T' + T_{ss}Q'\right) + G_u'\right)$$

Plug in Values

$$\frac{dT'}{dt} = 0.03333T_i' + 0.000045Q' - 0.03333T' - 0.000058Q' + G_u'$$

$$\boxed{\frac{dT'}{dt} = 0.03333T_i' - 0.000013Q' - 0.03333T' + G_u'}$$

Energy Balance linearized & in deviation form.

(2)

$$\begin{bmatrix} \cdot\ \dfrac{dS'}{dt} = 0.00115Q' + 0.03333S_i' - 0.114816S' - 0.7002T' - 0.004877X' \\ \cdot\ \dfrac{dX'}{dt} = -0.232143X' - 0.006Q' + 22.9105T' + 2.66611S' \\ \cdot\ \dfrac{dT'}{dt} = 0.03333T_i' - 0.000013Q' - 0.03333T' + G_0' \end{bmatrix}$$

U $\left\{ Q' \text{ \& } G_0' \right.$ - minuliplated Inputs

d $\left\{ S_i' \text{ \& } T_i' \right.$ - disturbance Inputs

X $\left\{ S', X', T' \right.$ ← state Variables

State-Space Model

$\dot{X} = AX + BU + Ed$

$Y = CX + DU$

$$\underbrace{\begin{bmatrix} \frac{dS'}{dt} \\ \frac{dX'}{dt} \\ \frac{dT'}{dt} \end{bmatrix}}_{\dot{X}} = \underbrace{\begin{bmatrix} -0.114816 & -0.004877 & -0.7002 \\ 2.66611 & -0.232143 & 22.9105 \\ 0 & 0 & -0.03333 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} S' \\ X' \\ T' \end{bmatrix}}_{X} +$$

$$\underbrace{\begin{bmatrix} 0.00115 & 0 \\ -0.006 & 0 \\ -0.000013 & 1 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} Q' \\ G_0' \end{bmatrix}}_{U} + \underbrace{\begin{bmatrix} 0.03333 & 0 \\ 0 & 0 \\ 0 & 0.03333 \end{bmatrix}}_{E} \underbrace{\begin{bmatrix} S_i' \\ T_i' \end{bmatrix}}_{d}$$

(8.)

$$S' = \frac{mg\,COD}{L} = 2.2\,\frac{mg\,TOC}{L}$$

$$\overset{\underline{Y}}{\begin{bmatrix} S' \\ X' \\ T' \end{bmatrix}} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S' \\ X' \\ T' \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Q' \\ G_u' \end{bmatrix}$$

$$\overset{\underline{Y}}{\begin{bmatrix} S \\ T \end{bmatrix}} = \begin{bmatrix} \frac{1}{2.2} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} S' \\ X' \\ T' \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Q' \\ G_u' \end{bmatrix}$$

Problem 2

To input the state space model in Simulink, we first need to combine BU and Ed into BU:

$$\overset{\dot{X}}{\begin{bmatrix} \frac{dS'}{dt} \\ \frac{dX'}{dt} \\ \frac{dT'}{dt} \end{bmatrix}} = \overset{\underline{A}}{\begin{bmatrix} -0.114816 & -0.004877 & -0.7002 \\ 2.66611 & -0.232143 & 22.9105 \\ 0 & 0 & -0.03333 \end{bmatrix}} \overset{\underline{X}}{\begin{bmatrix} S' \\ X' \\ T' \end{bmatrix}} +$$

$$\overset{\underline{B}}{\begin{bmatrix} 0.00115 & 0 & 0.03333 & 0 \\ -0.006 & 0 & 0 & 0 \\ -0.000013 & 1 & 0 & 0.03333 \end{bmatrix}} \overset{\underline{U}}{\begin{bmatrix} Q' \\ G_u' \\ S_i' \\ T_i' \end{bmatrix}}$$

9.

$$\overset{Y}{\begin{bmatrix} S' \\ X' \\ T' \end{bmatrix}} = \overset{C}{\begin{bmatrix} 1/2.2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \overset{X}{\begin{bmatrix} S' \\ X' \\ T' \end{bmatrix}} + \overset{D}{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}} \overset{U}{\begin{bmatrix} Q' \\ Gu' \\ S_i' \\ T_i' \end{bmatrix}}$$

Build in Simulink and see how the system will respond to ± 10% changes in the input variables.

Problem 3:

MATLAB code for transfer function and steady state gains

Problem 3 Transfer function and steady state gains

```
clc;
A = [-0.1148 -0.0049 -0.7002;
    2.6661 -0.2321 22.9105;
    0 0 -0.0333;];

U = [0.00115 0 0.03333 0;
    -0.006 0 0 0;
    -0.000013 1 0 0.03333];

B = [0.00115 0;
    -0.0060 0;
    -0.0000133 1];

C = eye(3);
C(1) = 1/2.2;
CC = C;
CC(2,2) = 0;

E = [0.0333 0;
    0 0;
    0 0.0333];
%{
    GP:
        input 1 - output 1: S(s)/Q(s)
        input 1 - output 2: X(s)/Q(s)
        input 1 - output 3: T(s)/Q(s)


    Gp:
        input 2 - output 1: S(s)/Gu(s)
        input 2 - output 2: X(s)/Gu(s)
```

```
        input 2 - output 3: T(s)/Gu(s)
%}
s = tf('s');
Gp = C*(s*eye(3)-A)^-1 * B

% Steady State gains
disp('Steady state gains')
S_Q = dcgain(Gp(1,1))
X_Q = dcgain(Gp(2,1))
T_Q = dcgain(Gp(3,1))


S_Gu = dcgain(Gp(1,2))
X_Gu = dcgain(Gp(2,2))
T_Gu = dcgain(Gp(3,2))


%{
    Gd:
        input 1 - output 1: S(s)/Si(s)
        input 1 - output 2: X(s)/Si(s)
        input 1 - output 3: T(s)/Si(s)


    Gd:
        input 2 - output 1: S(s)/Ti(s)
        input 2 - output 2: X(s)/Ti(s)
        input 2 - output 3: T(s)/Ti(s)
%}
Gd = C*(s*eye(3)-A)^-1 * E

%Steady state gains
disp('Steady state gains')
S_Si = dcgain(Gd(1,1))
X_Si = dcgain(Gd(2,1))
T_Si = dcgain(Gd(3,1))



S_Ti = dcgain(Gd(1,2))
X_Ti = dcgain(Gd(2,2))
T_Ti = dcgain(Gd(3,2))
```

Gp =

From input 1 to output...

0.0005227 s^6 + 0.000519 s^5 + 0.000219 s^4 + 4.989e-05 s^3 + 6.359e-06 s^2 + 4.158e-07 s + 9.691e-09
1: --
s^7 + 1.074 s^6 + 0.5148 s^5 + 0.1404 s^4 + 0.02321 s^3 + 0.002276 s^2 + 0.0001173 s + 2.085e-06

```
       -0.006 s^6 - 0.00229 s^5 + 0.0001697 s^4 + 0.0002567 s^3 + 5.592e-05 s^2 + 4.854e-06 s +
1.088e-07
   2:  ----------------------------------------------------------------------------------------------------
        s^7 + 1.074 s^6 + 0.5148 s^5 + 0.1404 s^4 + 0.02321 s^3 + 0.002276 s^2 + 0.0001173 s +
2.085e-06

       -1.33e-05
   3:  ----------
       s + 0.0333

  From input 2 to output...
            -0.3183 s - 0.1249
   1:  --------------------------------------
       s^3 + 0.3802 s^2 + 0.05126 s + 0.001322

             22.91 s + 0.7633
   2:  --------------------------------------
       s^3 + 0.3802 s^2 + 0.05126 s + 0.001322

           1
   3:  ----------
       s + 0.0333
```

Continuous-time transfer function.

Steady state gains

S_Q =

    0.0046

X_Q =

    0.0522

T_Q =

  -3.9940e-04

S_Gu =

  -94.4553

X_Gu =

  577.2647

```
T_Gu =

  30.0300


Gd =

  From input 1 to output...
       0.01514 s + 0.003513
   1:  ------------------------
       s^2 + 0.3469 s + 0.03971

             0.08878
   2:  ------------------------
       s^2 + 0.3469 s + 0.03971

   3:  0

  From input 2 to output...
               -0.0106 s - 0.004159
   1:  ---------------------------------------
       s^3 + 0.3802 s^2 + 0.05126 s + 0.001322

                0.7629 s + 0.02542
   2:  ---------------------------------------
       s^3 + 0.3802 s^2 + 0.05126 s + 0.001322

        0.0333
   3:  ----------
       s + 0.0333

Continuous-time transfer function.

Steady state gains

S_Si =

    0.0885


X_Si =

    2.2358


T_Si =

     0
```

S_Ti =

-3.1454

X_Ti =

19.2229

T_Ti =

1.0000

Problem 4:

```
clc;clear all
% RGA

K= [30.03 -3.994E-4; -94.455 0.0046]; %Steady state gain
A=inv(K)
H=A'
L=K.*H %RGA matrix
```

A =

```
   0.0458    0.0040
 940.6681  299.0658
```

H =

```
   0.0458  940.6681
   0.0040  299.0658
```

L =

```
   1.3757   -0.3757
  -0.3757    1.3757
```

Problem 5:

Problem 5 Controller Transfer function

```
sys=tf([1 1.074 0.158 0.1404 0.02321 0.002276 0.0001173 2.08E-6],[0.0005227
0.000519 0.000219 4.99e-5 6.359e-6 4.158e-7 9.69e-9])
% sysr=sminreal(sys);
A=tf(1,[1 0])
```

```
B=tf(1, [0.02 0])
C=4.433
D=(A+B)*1/C
E=D*sys
```

```
sys =

    s^7 + 1.074 s^6 + 0.158 s^5 + 0.1404 s^4 + 0.02321 s^3 + 0.002276 s^2 + 0.0001173 s + 2.08e-06
  -----------------------------------------------------------------------------------------------------
  0.0005227 s^6 + 0.000519 s^5 + 0.000219 s^4 + 4.99e-05 s^3 + 6.359e-06 s^2 + 4.158e-07 s + 9.69e-09

Continuous-time transfer function.


A =

  1
  -
  s

Continuous-time transfer function.


B =

    1
  ------
  0.02 s

Continuous-time transfer function.


C =

    4.4330


D =

    1.02 s
  -----------
  0.08866 s^2

Continuous-time transfer function.


E =
```

1.02 s^8 + 1.095 s^7 + 0.1612 s^6 + 0.1432 s^5 + 0.02367 s^4 + 0.002322 s^3 + 0.0001196 s^2 + 2.122e-06 s
----
4.634e-05 s^8 + 4.601e-05 s^7 + 1.942e-05 s^6 + 4.424e-06 s^5 + 5.638e-07 s^4 + 3.686e-08 s^3 + 8.591e-10 s^2

Continuous-time transfer function.

Bonus Problem 1:

Feedforward control

Feedforward

$$\frac{S}{S_i} = \frac{G_D + G_S G_{FF} G_V G_P}{1 + loop} = 0$$

$$G_D + G_S G_{FF} G_V G_P = 0$$

$$G_{FF} = \frac{-G_D}{G_S G_V G_P}$$

$$G_D = \frac{0.0152 s + 0.00352}{s^2 + 0.347 s + 0.0397} \qquad G_S = K_S = 0.0133$$

$$G_V = K_V = 3333.33$$

$$G_P = \frac{5.0e{-4} s^6 + 5.0e{-4} s^5 + 2.0e{-4} s^4 + 4.989e{-5} s^3 + 6.359e{-6} s^2 + 4.158e{-7} s + 9.691e{-9}}{s^7 + 1.074 s^6 + 0.5148 s^5 + 0.1404 s^4 + 0.02321 s^3 + 2.28e{-3} s^2 + 1.17e{-4} s + 2.085e{-6}}$$

Plugging in the transfer functions into MATLAB calculated G_{FF}:

$$G_{FF} = \frac{-1.0(0.015 s^8 + 0.02 s^7 + 0.011 s^6 + 3.9e{-3} s^5 + 8.4e{-4} s^4 + 1.2e{-4} s^3 + 9.7e{-6} s^2 + 4.4e{-7} s + 7.3e{-9}}{0.023 s^8 + 0.031 s^7 + 0.019 s^6 + 6.5e{-3} s^5 + 1.4e{-3} s^4 + 2.0e{-4} s^3 + 1.8e{-5} s^2 + 8.8e{-7} s + 1.7e{-8}}$$

Feedfoward Transfer Function and Controller 2

```
syms s
clc
```

```
Gp_num = 0.0005227*s^6 + 0.000519*s^5 + 0.000219*s^4 + 4.989e-05*s^3 +
6.359e-06*s^2 + 4.158e-07*s+9.691e-09;
Gp_denom = s^7 + 1.074*s^6 + 0.5148*s^5 + 0.1404*s^4 + 0.02321*s^3 +
0.002276*s^2 + 0.0001173*s+ 2.085e-06;

num_mult = -0.015*s - 0.0035;
denom = s^2 + 0.346*s + 0.0397;
denom_mult = 3333.333*0.0133 * denom;

Gcff = expand(num_mult*Gp_denom)/expand(denom_mult*Gp_num);
Gcff = vpa(Gcff,2)

% Controller 2
Gp_num = 4.43*(0.0005227*s^6 + 0.000519*s^5 + 0.000219*s^4 + 4.989e-05*s^3 +
6.359e-06*s^2 + 4.158e-07*s + 9.691e-09);
Gp_denom = s^7 + 1.074*s^6 + 0.5148*s^5 + 0.1404*s^4 + 0.02321*s^3 +
0.002276*s^2 + 0.0001173*s + 2.085e-06;

denom = 1.02*s+0.9867;

Gc = Gp_denom/expand(Gp_num*denom);
Gc = vpa(Gc,2)
```

Gcff =

-(1.0*(0.015*s^8 + 0.02*s^7 + 0.011*s^6 + 3.9e-3*s^5 + 8.4e-4*s^4 + 1.2e-4*s^3 + 9.7e-6*s^2 + 4.4e-7*s + 7.3e-9)) / (0.023*s^8 + 0.031*s^7 + 0.019*s^6 + 6.5e-3*s^5 + 1.4e-3*s^4 + 2.0e-4*s^3 + 1.8e-5*s^2 + 8.8e-7*s + 1.7e-8)

Gc =

(s^7 + 1.1*s^6 + 0.51*s^5 + 0.14*s^4 + 0.023*s^3 + 2.3e-3*s^2 + 1.2e-4*s + 2.1e-6) / (2.4e-3*s^7 + 4.6e-3*s^6 + 3.3e-3*s^5 + 1.2e-3*s^4 + 2.5e-4*s^3 + 3.0e-5*s^2 + 1.9e-6*s + 4.2e-8)
