Land bucket model

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This model is due to Manabe 1969, with changes as noted.

1 Code proposal

- Create the below model in ClimaLSM.jl. Internal implementation details are up to the land team, but the outward facing interface should be consistent with what we plan to use for all land models when coupled to other models.
- This model is a set of ODEs at each point on the surface of the Earth. We will support a 2D domain which is the surface of a sphere (with the same resolution as the atmosphere; this is a cubed sphere with a single element in the vertical, I think), as well as being able to run at a single point, for column simulations.
- This bucket model will be both an stand-in for the land model for the coupler to run tests with, as well as a backup if the implicit timestepping is not completed in time for AMIP.
- **This model exhibits some complexity for the coupler already, in that different surface cover types lead to different flux computations; see text below. That is, we need to pass T, q, and a coefficient for soil covered regions, to the coupler, as well as an indication of the surface cover type, in order for it to compute fluxes correctly.**

2 Soil moisture

The soil moisture is treated via a bucket model. The prognostic variable is the integrated water height in the first meter of soil, which, in terms of the volumetric soil water content θ_l, is given by

$$W = \theta_l \times \frac{V_{soil}}{A_g} = \theta_l \times \frac{d_{soil} A_g}{A_g} = \theta_l|_{d_{soil}=1}. \quad (1)$$

In words: precipitation in the form of liquid water, as well as snowmelt, replenish the soil moisture. Evaporation dries the soil *when it is not snow covered*. The soil at each grid has a maximum capacity (W_f), above which, all excess runs off. The equation for soil moisture is a depth integrated form of Richards equation, with depth set to 1m, with water volume fluxes of

- $\tilde{P}$ is the water volume flux due to liquid precipitation: positive by definition,
- $\tilde{M}$ is the water volume flux due to snowmelt: positive by definition,
- $\tilde{E}_{soil}$ is the water volume flux from evaporation; positive meaning water transfer from the soil to the atmosphere.

When snow is present, this variable satisfies

$$\frac{dW}{dt} = [\tilde{P} + \tilde{M}]\mathcal{H}(W_f - W), \tag{2}$$

and when snow is not present (snow water equivalent $S \sim 0$), we have

$$\frac{dW}{dt} = \begin{cases} 0, & \text{if } W = W_f \text{ and } \tilde{P} > \tilde{E}_{soil} \\ \tilde{P} - \tilde{E}_{soil}, & \text{otherwise} \end{cases} \tag{3}$$

That is, when soil is at capacity AND $\tilde{P} - \tilde{E}_{soil} > 0$, $dW/dt = 0$, while if $\tilde{P} - \tilde{E}_{soil} < 0$ (net loss of water), $dW/dt = \tilde{P} - \tilde{E}_{soil}$. When the soil is not at capacity, we have $dW/dt = \tilde{P} - \tilde{E}_{soil}$.

We can combine these by ensuring $\tilde{M} = 0$ when $S = 0$ and that $E_{soil} = 0$ when $S > 0$, in which case we would have:

$$\boxed{\frac{dW}{dt} = \begin{cases} 0, & \text{if } W = W_f \text{ and } \tilde{P} > \tilde{E}_{soil} \\ \tilde{P} + \tilde{M} - \tilde{E}_{soil}, & \text{otherwise} \end{cases}} \tag{4}$$

This reduces to Equation (2) if $S > 0$ ($\tilde{E}_{soil} = 0$, noting that precip is positive by definition), and it reduces to Equation (3) when $\tilde{M} = 0$.

To keep track of runoff, we would have a storage of water on the land surface W_s satisfying

$$\boxed{\frac{dW_s}{dt} = \begin{cases} \tilde{P} + \tilde{M} - \tilde{E}_{soil}, & \text{if } W = W_f \text{ and } \tilde{P} > \tilde{E}_{soil} \\ 0, & \text{otherwise} \end{cases}} \tag{5}$$

In this equation, $\tilde{P}$ is provided by the atmosphere model and $\tilde{E}_{soil}$ is given by

$$\boxed{\tilde{E}_{soil} = \frac{W}{0.75W_f}\tilde{E}_0(T_{sfc};\text{atmos...})\mathcal{H}(\epsilon - S),} \tag{6}$$

where $\tilde{E}_0 = E_0/\rho_l$ is referred to as the potential evaporation (water mass flux)at the ground surface:

$$E_0(T;\text{atmos...}) = (\rho_{air}C_d|u|)|_h(q_{sat}(T_{sfc}, \rho_{air,sfc}) - q(h)). \tag{7}$$

This is the mass flux between height h in the atmosphere and the surface. It is defined such that evaporation into the atmosphere is positive. For soil, we would take T_{sfc} to be the surface temperature of the soil, and we presumably can infer $\rho_{air,sfc}$) from $\rho_{air}(h)$ and h...Note the additional prefactor in terms of E_0, which (I believe) must be included in the Monin-Obukhov solve. Note that as $W \to 0$, evaporation also goes to zero. If snow is present, evaporation goes to zero by using $\mathcal{H}(\epsilon - S)$.

3 Soil energy

Beginning from the heat equation in one spatial dimension, and integrating each side with respect to z yields an equation for the bulk energy of the soil

$$\boxed{\tilde{c}\frac{dT}{dt} = \frac{1}{d_{soil}}(F_{sfc} - F_{bot})} \tag{8}$$

where $\vec{F}_{sfc} \cdot \hat{n} = F_{sfc}$, and

$$\boxed{F_{sfc} = \begin{cases} -\Big[(1-\alpha_{soil})SW^{\downarrow} + LW^{\downarrow} - \sigma T_{sfc}^4 - L_{v,0}E_{soil} - SHF_{soil}\Big], & \text{if } S < \epsilon \\ 0, & \text{if } S \geq \epsilon \end{cases}} \tag{9}$$

is the net heat flux at the surface of the soil, where $\hat{n} = \hat{z}$ for a flat surface. If the soil is under snow, we take the surface temperature to be $T_{sfc} = T_f$, which is the temperature of the snow, such that $F_{sfc} = 0$.

The bottom flux is

$$\boxed{F_{bot} = -\kappa_{soil} \frac{T_{sfc} - T_0}{d_{soil}}.} \tag{10}$$

If the soil is under snow, the soil temperature varies linearly between T_f and T_0 within the soil layer.

Here, we have

- T_{sfc} is the soil surface temperature,
- $SW^{\downarrow}$ is the absorbed downward shortwave radiation, and α_{soil} is the soil albedo, which can be a function of space
- $LW^{\downarrow}$ is the absorbed downward longwave radiation, assuming an absorptivity of one,
- σT_{sfc}^4 is the blackbody radiation from the soil, assuming an emissivity of one,
- $L_{v,0}$ is the latent heat of vaporization at the reference temperature, such that $L_{v,0}E_{soil}$ is the latent heat flux,
- SHF_{soil} is the sensible heat flux,
- κ_{soil} and $\tilde{c}$ are the thermal conductivity and volumetric heat capacity of the soil, which are taken to be independent of the water content in the soil,
- T_0 is the soil temperature at depth d_{soil}. Both of these need to be specified, but are taken as constant across the globe.

Some comments:

- The soil temperature T varies linearly from T_{sfc} at the surface to T_0 at depth d_{soil}.
- We assume the soil does not freeze.
- Setting the specific heat capacity of the soil to zero yields the form of the equation used in Manabe (1969).
- The sensible heat flux is given by Monin-Obukhov theory, similar to the evaporation expression, using a surface temperature of T_{sfc}.
- If the soil becomes dry, the latent heat flux goes to zero, but we still track the temperature of the surface and sensible heat fluxes.

4 Snow

We additionally track the snow water equivalent S: we will evaluate a differential equation for snow everywhere, but it will return $S = 0$ in regions without snow. We have

$$\boxed{\frac{dS}{dt} = \tilde{P}_{snow} - \tilde{E}_{snow} - \tilde{M}} \tag{11}$$

where

- $\tilde{P}_{snow} = P_{snow}/\rho_l$ is the liquid water volume flux contained in precipitating snow: positive by definition,
- $\tilde{E}_{snow}$ is the water volume flux from sublimation, in terms of liquid water, which should be zero if there is no snow ($S = 0$),
- $\tilde{M}$ is the liquid water volume flux in snowmelt, which is assumed to immediately run off to the soil store or to the runoff store, and which also should be zero if there is no snow ($S = 0$). This is also positive by definition.

To ensure that $dS/dT \to 0$ as $S \to 0$ and $\tilde{P}_{snow} = 0$, we will make use of heaviside functions in the melt and sublimation term, below. Manabe (1969) does not do this, but I don't see how his expression tend to zero in the limit of no snow, $S = 0$.

The snow does not contain liquid water in this approximation. Additionally, the snow temperature is fixed at T_f, such that we can define

$$\boxed{\tilde{M} = -\frac{F_{sfc,snow}}{L_{f,0}}\mathcal{H}(-F_{sfc,snow})\mathcal{H}(S-\epsilon),} \tag{12}$$

where $F_{sfc,snow}$ is the net surface flux at the surface of the snow, note that a positive surface flux means that the surface is losing energy. The heaviside functions ensure (1) that melting is zero if the snow is not gaining energy and (2) that if the snow water equivalent $S \approx 0$, $\tilde{M} = 0$.

Note that if the snow is losing energy, we do not track the temperature change..and always keep it T_f.

Here, we have

$$\boxed{F_{sfc,snow} = -\left[(1-\alpha_{snow})SW^{\downarrow} + LW^{\downarrow} - \sigma T_f^4 - L_{s,0}E_{snow}(T_f) - SHF_{snow}(T_f)\right].} \tag{13}$$

The surface fluxes over snow are computed similarily to soil, except the snow sublimates at the potential rate (Equation (7)), the latent heat of sublimation enters in the latent heat flux, and we use a temperature of T_f. To ensure that sublimation goes to zero as $S \to 0$, we write

$$\boxed{E_{snow} = E_0(T_f; \text{atmos...})\mathcal{H}(S-\epsilon)} \tag{14}$$

5 Parameters and supplied functions

- $\kappa_{soil} = 2$ W/m/K
- $\tilde{c} = 2\text{e}6$ J/m^3/K
- $W_f = 15$cm
- $\alpha_{snow} = 0.7$,
- $\tilde{P}$, $\tilde{P}_{snow}$ are supplied (prescribed or from the coupler)
- We assume we can call surface fluxes with (1) T_{soil}, (2) q_{soil} as a function of T_{soil}, (3) the prefactor $W/(0.75W_f)$, and (4) the supplied values of $u(h), T(h), q(h), h$ in order to compute the correct fluxes when the surface is covered by soil, or that the coupler will return the evaporation flux and the net surface energy flux...
- We assume we can call surface fluxes with (1) T_f, (2) $q_{sat}(T_f)$, and (3) the supplied values of $u(h), T(h), q(h), h$ in order to compute the correct fluxes when the surface is covered by snow, or that the coupler will return the sublimation flux and the net surface energy flux...
- T_0 and d_{soil} need to be specified.
- $\alpha_{soil}(\vec{x})$ needs to be specified, but a representative value is ~ 0.15. This could account for vegetation, though the evaporative flux would be due to bare soil evaporation and not include transpiration.

We also internally to the land model need to be able to digest a mask indicating where (in lat/lon space) the ocean is, such that we do not solve our equations where they are not needed.