$P\left(\frac{B}{E_1}\right)=\frac{3}{7}$ $\quad P\left(\frac{B}{E_2}\right)=\frac{4}{7}$

$\therefore$ **Probability of selecting balck ball**

$$P(B)=P(E_1).P\left(\frac{B}{E_1}\right)+P(E_2).P\left(\frac{B}{E_2}\right)=\frac{1}{3}\cdot\frac{3}{7}+\frac{2}{3}\cdot\frac{4}{7}=\frac{3}{21}+\frac{8}{21}=\frac{11}{21}$$

*****10. A, B, C are 3 newspapers from a city. 20% of the population read A, 16% read B, 14% read C, 8% read both A and B, 5% read both A and C, 4% read both B and C and 2% read all the three. Find the percentage of the population who read atleast one newspaper (M**

Sol: Let A, B, C are events of reading three news papers A, B, C respectively.

$$P(A)=\frac{20}{100},\quad P(B)=\frac{16}{100},\quad P(C)=\frac{14}{100},$$

$$P(A\cap B)=\frac{8}{100},\quad P(A\cap C)=\frac{5}{100},\quad P(B\cap C)=\frac{4}{100},\quad P(A\cap B\cap C)=\frac{2}{100}$$

$$P(A\cup B\cup C)=P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(A\cap C)+P(A\cap B\cap C)$$

$$=\frac{20}{100}+\frac{16}{100}+\frac{14}{100}-\frac{8}{100}-\frac{5}{100}-\frac{4}{100}+\frac{2}{100}=\frac{52}{100}-\frac{17}{100}=\frac{35}{100} \quad (4M)$$

$\therefore$ The percentage of the population read at least one newspaper 35%

*****11. In a box contaning 15 bulbs, 5 are defective. If 5 bulbs are selected at random from box. Find the probability of the event that**

i) none of them is defective ii) only one of them is defective

iii) atleast one of them is defective.

Sol: No. of bulbs $=15$; Non-Defective bulbs $=10$; Defective bulbs $=5$

Sample space of experiment is selecting 5 bulbs from 15 bulbs $\Rightarrow n(S)={}^{15}C_5$ (1M

i) Let 'A' be the event so that all the selected bulbs and non - defective

$\therefore n(A)={}^{10}C_5$ $\qquad P(A)=\frac{n(A)}{n(S)}=\frac{{}^{10}C_5}{{}^{15}C_5}=\frac{12}{143}$ (2M)

ii) Let 'B' be the event so that there is one defective bulb and four non - defective

$\therefore n(B)={}^{5}C_1\times{}^{10}C_4$ $\qquad P(B)=\frac{n(B)}{n(S)}=\frac{{}^{5}C_1\times{}^{10}C_4}{{}^{15}C_5}$ (2M)

iii) Let 'C' be the event so that there is atleast one defect bulb

$$P(C)=1-P[\textit{none of them defective}] \quad =1-\frac{12}{143}=\frac{131}{143}$$

****12. Two persons A and B are rolling a die on the condition that the person who gets first will win the game. If A starts the game, then find the probabilities of A an respectively to win the game. (May**

Sol: Let 'p' be the probability of success and 'q' be the probability of failure when rolling a di getting '3' and not getting '3' respectively.

$p=\frac{1}{6}$, and $q=1-p\Rightarrow q=\frac{5}{6}$

A	B
p	qp
qqp	qqqp

$\therefore$ P[A to win game] = p + qqp + qqqqp + ...

26. If $|x|$ is so small that x^3 and higher powers of x can be neglected, find the approximate value of $\frac{(4-7x)^{1/2}}{(3+5x)^3}$

Sol: $\frac{(4-7x)^{1/2}}{(3+5x)^3} = (4-7x)^{1/2}(3+5x)^{-3}$

$$= 4^{1/2}\left(1-\frac{7x}{4}\right)^{1/2} \cdot 3^{-3}\left(1+\frac{5x}{3}\right)^{-3}$$

$$= 2\left(1-\frac{7x}{4}\right)^{1/2} \cdot \frac{1}{27}\left(1+\frac{5x}{3}\right)^{-3}$$

$\boxed{(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2}$

$$= \frac{2}{27}\left(1+\frac{1}{2}\left(\frac{-7x}{4}\right)+\frac{\frac{1}{2}\cdot\left(\frac{1}{2}-1\right)}{2!}\cdot\left(\frac{-7x}{4}\right)^2\right)\cdot\left(1+(-3)\left(\frac{5x}{3}\right)+\frac{(-3).(-3-1)}{2!}\cdot\left(\frac{5x}{3}\right)^2\right)$$

$$= \frac{2}{27}\left(1-\frac{7x}{8}-\frac{49}{128}x^2\right)\cdot\left(1-5x+\frac{50x^2}{3}\right)$$ (since x^3 and higher power of x can be neglected)

$$= \frac{2}{27}\left(1-5x+\frac{50x^2}{3}-\frac{7x}{8}+\frac{35x^2}{8}-\frac{49x^2}{128}\right)$$

$$= \frac{2}{27}\left(1-\frac{47x}{8}+x^2\left(\frac{50}{3}+\frac{35}{8}-\frac{49}{128}\right)\right) = \frac{2}{27}\left(1-\frac{47x}{8}+\frac{7933}{384}x^2\right)$$

VERY SHORT ANSWER QUESTIONS (2 M)

1. Find the number of terms in the expansion of

(i) $(2x+3y+z)^7$ $\quad$ (ii) $\left(\frac{3a}{4}+\frac{b}{2}\right)^9$ $\quad$ **(Mar-12,14), (May-1**

Sol:i) The no.of terms in the expansion of

$$(2x+3y+z)^7 = \frac{(n+1)(n+2)}{2} = \frac{(7+1)(7+2)}{2} = 4\times 9 = 36$$

$\because$ **The no.of terms in $(x+y+z)^n$ is $\frac{(n+1)(n+2)}{2}$**

ii) The no.of terms in the expansion of $\left(\frac{3a}{4}+\frac{b}{2}\right)^9 = n+1 = 9+1 = 10$

2. Find the number of terms with non-zero coeffecents in $(4x-7y)^{49}+(4x+7y)^{49}$

Sol: The number terms in $(x-y)^n+(x+y)^n$ is $\frac{n+1}{2}$ if n is odd

$\therefore n=49$, total number of terms $=\frac{49+1}{2}=25$

Q. Find the set of values of 'x' for which

(i) $(2+3x)^{\frac{-2}{3}}$ is valid

(ii) $(7+3x)^{-5}$ is valid

(iii) $(3-4x)^{3/4}$ is valid

(Mar-11, 16 …) (May-…) (Mar-…)

Sol: (i) Given $(2+3x)^{\frac{-2}{3}}=2^{\frac{-2}{3}}\left(1+\frac{3x}{2}\right)^{\frac{-2}{3}}$ here $X=\frac{3x}{2}$

$\boxed{\text{Binomial expansion is valid for } |X|<1}$ $\therefore \left|\frac{3x}{2}\right|<1 \Rightarrow |x|<\frac{2}{3} \Rightarrow -\frac{2}{3}<x<\frac{2}{3}$

$\therefore$ for $x\in\left(-\frac{2}{3},\frac{2}{3}\right)$ the given expansion is valid

(ii) Given $(7+3x)^{-5}=7^{-5}\left(1+\frac{3x}{7}\right)^{-5}$ here $X=\frac{3x}{7}$

$\boxed{\text{Binomial expansion is valid for } |X|<1}$

$\Rightarrow\left|\frac{3x}{7}\right|<1 \Rightarrow |x|<\frac{7}{3} \Rightarrow -\frac{7}{3}<x<\frac{7}{3} \Rightarrow$ for $x\in\left(-\frac{7}{3},\frac{7}{3}\right)$ the given expansion is valid

(iii) Given $(3-4x)^{3/4}=3^{3/4}\left(1-\frac{4x}{3}\right)^{3/4}$ Here $X=\frac{4x}{3}$

$\boxed{\text{Binomial expansion is valid for } |X|<1}$

$\Rightarrow\left|\frac{4x}{3}\right|<1 \Rightarrow |x|<\frac{3}{4} \Rightarrow -\frac{3}{4}<x<\frac{3}{4}$

$\therefore$ for $x\in\left(-\frac{3}{4},\frac{3}{4}\right)$ the given expansion is valid

4. **If ${}^{22}C_r$ is the largest binomial coefficient in the expansion of $(1+x)^{22}$ find the value ${}^{13}C_r$**

Imp

(May-11, 15 T.S, A.P), (Mar-16 …)

Sol: Given expanison is $(1+x)^{22}$ here $n=22$ is an even integer

$\Rightarrow$ $\boxed{\text{If 'n' is even then largest binomial co-efficient is } {}^nC_{n/2}={}^{22}C_{11}}$

Given that ${}^{22}C_r$ is the largest binomial co-efficient in $(1+x)^{22}$

$\Rightarrow 22_{C_{11}} = {}^{22}C_r \Rightarrow r = 11$ ${}^{13}C_r = {}^{13}C_{11} = {}^{13}C_2 = \frac{13\times 12}{1\times 2} = 78$

5. **Find the largest binomial coefficient (s) in the expansion of $(1+x)^{19}$**

Sol: Here n=19(odd)

if n is odd, the largest binomial coefficient in $(1+x)^n$ is

$n_{C_{\frac{n-1}{2}}} (or) n_{C_{\frac{n+1}{2}}} = 19_{C_{\frac{19-1}{2}}} = 19_{C_9}$

6. **Find the Coefficient of x^{-7} in $\left(\frac{2x^2}{3} - \frac{5}{4x^5}\right)^7$** (Mar-10)

Sol: Given $\left(\frac{2x^2}{3} - \frac{5}{4x^5}\right)^7$

$$\boxed{\text{The Coefficient of } x^m \text{ in } \left(ax^p + \frac{b}{x^q}\right)^n \text{ is } T_{r+1} \text{ ; where } r = \frac{np-m}{p+q}}$$

here $n = 7; p = 2; q = 5, m = -7$

$$\therefore r = \frac{7(2)-(-7)}{2+5} = \frac{14+7}{7} = \frac{21}{7} = 3$$

$$\therefore T_{r+1} = T_{3+1} = n_{C_r} x^{n-r} a^r = {}^7C_3\left(\frac{2x^2}{3}\right)^{7-3}\left(\frac{-5}{4x^5}\right)^3 = {}^7C_3\left(\frac{2x^2}{3}\right)^4\left(\frac{-5}{4x^5}\right)^3$$

$$= {}^7C_3\left(\frac{2}{3}\right)^4 x^8\left(\frac{-5}{4}\right)^3 \cdot \frac{1}{x^{15}} = {}^7C_3\left(\frac{2}{3}\right)^4\left(\frac{-5}{4}\right)^3 x^{-7}$$

$$\therefore \text{Coefficient of } x^{-7} = {}^7C_3\left(\frac{2}{3}\right)^4\left(\frac{-5}{4}\right)^3$$

7. **Find the middle terms in the expansion of**

(i) $\left(\frac{3x}{7} - 2y\right)^{10}$ **(Jun-10, Mar-12)** (ii) $\left(4a + \frac{3}{2}b\right)^{11}$ **(Jun-05) (May--15**

Sol: (i) Given $\left(\frac{3x}{7} - 2y\right)^{10}$, $n = 10$

If n is even, middle term is $T_{\frac{n+2}{2}} = T_{\frac{10+2}{2}} = T_{\frac{12}{2}} = T_6$

$$T_{r+1} = {}^nC_r x^{n-r} a^r \Rightarrow T_6 = T_{5+1} = {}^{10}C_5\left(\frac{3x}{7}\right)^{10-5}(-2y)^5$$

remaining 4 ... in the upper deck. Now we can arrange ... people in ${}^{11}C_{...}$ ways. The others) in the upper deck and 8 persons (4 old people and 4 others) in the lower deck in 10! and 8! ways respectively.

Hence, the required number of arrangements $= {}^{11}C_7 \times 10! \times 8!$

*25. **Find the sum of all 4 digited numbers that can be formed using digits 0,2,4,7,8 without repetition.** (Mar

Sol. Given digits are 0, 2, 4, 7, 8. Here r = 4, n = 5

Sum of all r digited numbers $= {}^{n-1}P_{r-1}$(sum of given digits)(111...r times) $- {}^{n-2}P_{r-2}$(sum of given digits)(11...

Sum of all 4 digited numbers

$= {}^{(5-1)}P_{4-1}(0+2+4+7+8)(1111) - {}^{(5-2)}P_{4-2}(0+2+4+7+8)(111)$

$= {}^{4}P_3 \times 21 \times 1111 - {}^{3}P_2 \times 21 \times 111 = 21 \times ({}^{4}P_3 \times 1111 - {}^{3}P_2 \times 111)$

$= 21 \times (24 \times 1111 - 6 \times 111) = 21 \times (26664 - 666) = 21 \times 25998 = 5,45,958$

*26. **If a set A has 8 elements. Find the number of subsets of A containing atleast 6 ele...**

Sol. We have to fix the number of subsets of A, containing 6 or 7 or 8 elements.

Number of subsets of A, containing exactly 6 elements $= {}^{8}C_6$

Number of subsets of A, containing exactly 7 elements $= {}^{8}C_7$

Number of subsets of A, containing exactly 8 elements $= {}^{8}C_8$

Required number of subsets of A $= {}^{8}C_6 + {}^{8}C_7 + {}^{8}C_8 = 28 + 8 + 1 = 37$

*27. **14 persons are seated at a round table. Find the number of ways of selecti... persons out of them who are not seated adjacent to each other.** (May-1

Sol. No.of ways of selecting 2 persons out of 14 persons $= 14c_2 = 91$ ways

In the above arrangaments, 2 persons sitting adjacent to each other can be selected ... ways.

$\therefore$ The Req. no.of ways = 91-14 = 77 ways.

VERY SHORT ANSWER QUESTIONS (2M)

01) **(i) If ${}^{n}p_4 = 1680$ then find 'n'**

(ii) If ${}^{n}p_3 = 1320$ find 'n'

Sol. (i) Given that ${}^{n}P_4 = 1680$

$n(n-1)(n-2)(n-3) = 8 \times 7 \times 6 \times 5 \Rightarrow {}^{n}P_4 = {}^{8}P_4 \Rightarrow n = 8$

(ii) Given that ${}^{n}p_3 = 1320$

${}^{n}p_3 = 12 \times 11 \times 10 \Rightarrow {}^{n}p_3 = {}^{12}P_3 \Rightarrow n = 12$

02) **(i) If ${}^{(n+1)}P_5 : {}^{n}P_6 = 2:7$, find n.**

(ii) If ${}^{(n+1)}P_5 : {}^{n}P_5 = 3:2$ then find n.

Sol. (i) Given that ${}^{(n+1)}P_5 : {}^{n}P_6 = 2:7$

$\Rightarrow \dfrac{(n+1)!}{(n-4)!} \times \dfrac{(n-6)!}{n!} = \dfrac{2}{7} \Rightarrow \dfrac{n+1}{(n-4)(n-5)} = \dfrac{2}{7}$ $\boxed{\because {}^{n}P_r = \dfrac{n!}{(n-r)!}}$

$\Rightarrow 7n+7 = 2(n^2-9n+20) \Rightarrow 2n^2 - 25n + 33 = 0$

$(n-11)(2n-3)=0 \quad n = 11 \quad \left(\because n \neq \frac{3}{2}\right)$

(ii) Given that ${}^{(n+1)}P_5 : {}^{n}P_5 = 3:2$

$\frac{{}^{(n+1)}P_5}{{}^{n}P_5} = \frac{3}{2}$ $\quad \boxed{\because {}^{n}P_r = \frac{n!}{(n-r)!}}$

$\Rightarrow \frac{\frac{(n+1)!}{(n-4)!}}{\frac{n!}{(n-5)!}} = \frac{3}{2} \Rightarrow \frac{\frac{(n+1)n!}{(n-4)(n-5)!}}{\frac{n!}{(n-5)!}} = \frac{3}{2} \Rightarrow \frac{n+1}{n-4} = \frac{3}{2} \Rightarrow 2n+2 = 3n-12 \Rightarrow n=14$

03) If ${}^{n}P_7 = 42.{}^{n}P_5$ find n. (May-09,11), (Mar-15 T

Sol. Given that ${}^{n}P_7 = 42.{}^{n}P_5$ $\quad \boxed{\because n! = n(n-1)(n-2)(n-3)\ldots\ldots 3.2.1}$

$n(n-1)(n-2)(n-3)(n-4)(n-5)(n-6) = 42n(n-1)(n-2)(n-3)(n-4)$

$\Rightarrow (n-5)(n-6) = 42 \Rightarrow (n-5)(n-6) = 7\times 6$

$\Rightarrow n-5 = 7 \text{ or } n-6 = 6 \quad \therefore n = 12$

04) If ${}^{12}P_5 + 5.{}^{12}P_4 = {}^{13}P_r$ find 'r' (May-15

Sol. Given that We have ${}^{12}P_5 + 5.{}^{12}P_4 = {}^{13}P_r$ $\quad \boxed{\because {}^{n}P_r + r.{}^{n}P_{r-1} = {}^{(n+1)}P_r}$

${}^{13}P_5 = {}^{13}P_r \Rightarrow r = 5$

05) If $10.{}^{n}C_2 = 3.{}^{(n+1)}C_3$ then find n (May-12,Mar-15

Sol. Given that $10.{}^{n}C_2 = 3.{}^{(n+1)}C_3$

$\Rightarrow 10.\frac{n.(n-1)}{1\times 2} = 3\times\frac{(n+1)n.(n-1)}{1\times 2\times 3} \Rightarrow 10 = n+1$

$\therefore n = 9$

06) If ${}^{n}C_{21} = {}^{n}C_{27}$, then find ${}^{50}C_n$ (M

Sol. Given that ${}^{n}C_{21} = {}^{n}C_{27}$ $\quad \boxed{\because {}^{n}C_r = {}^{n}C_s \Rightarrow r = s \text{ or } n = r+s}$

${}^{n}C_{21} = {}^{n}C_{27} \Rightarrow n = 21+27 \Rightarrow n = 48$

Now ${}^{50}C_n = {}^{50}C_{48} = {}^{50}C_2 = \frac{50\times 49}{2\times 1} = 1225$ $\quad \boxed{\because {}^{n}C_r = {}^{n}C_{n-r}}$

07) If ${}^{n}C_5 = {}^{n}C_6$ then find the value of ${}^{13}C_n$. (Mar-13) (June-10)

Sol. Given that ${}^{n}C_5 = {}^{n}C_6$ $\quad \boxed{\because {}^{n}C_r = {}^{n}C_s \Rightarrow r = s \text{ or } n = r+s}$

$\Rightarrow n = 6+5 \Rightarrow n = 11.$

Now ${}^{13}C_n = {}^{13}C_{11} = {}^{13}C_2 = \frac{13\times 12}{1\times 2} = 78$ We have $\boxed{\because {}^{n}C_r = {}^{n}C_{n-r}}$

08) If ${}^{12}p_r = 1320$, find 'r'

Sol. ${}^{12}p_r = 1320 = 132\times 10 = 12\times 11\times 10$

$1-3x+7x^2+5x^3-2x^4=0 \Rightarrow 2x^4-5x^3-7x^2+3x-1=0$

11) (i) If α,β,γ are the roots of the equation $x^3+2x^2-4x-3=0$, find the equ... (Ma
whose roots are 3 times the roots of given equation.
(ii) Find the algebric equation whose roots are two times the roots of
$x^5-2x^4+3x^3-2x^2+4x+3=0$.

Sol. (i) Given equation is $f(x)= x^3+2x^2-4x-3=0$

Required equation is $f\left(\frac{x}{3}\right)=0$

= 9-16

$$\boxed{\because 3\alpha_1, 3\alpha_2, \ldots\ldots, 3\alpha_n \text{ are the roots of } f\left(\frac{x}{3}\right)=0}$$

$$\Rightarrow \left(\frac{x}{3}\right)^3+2\left(\frac{x}{3}\right)^2-4\left(\frac{x}{3}\right)-3=0$$

$$\Rightarrow \frac{x^3}{27}+2\frac{x^2}{9}-\frac{4x}{3}-3=0 \Rightarrow x^3+6x^2-36x-81=0$$

(ii) Given equation is $f(x)=x^5-2x^4+3x^3-2x^2+4x+3=0$

required equation is $f\left(\frac{x}{2}\right)=0$

$$\because 2\alpha_1, 2\alpha_2, \ldots\ldots, 2\alpha_n \text{ are the roots of } f$$

$$\Rightarrow \left(\frac{x}{2}\right)^5-2\left(\frac{x}{2}\right)^4+3\left(\frac{x}{2}\right)^3-2\left(\frac{x}{2}\right)^2+4\left(\frac{x}{2}\right)+3=0$$

$$\Rightarrow \frac{x^5}{32}-\frac{2x^4}{16}+\frac{3x^3}{8}-\frac{2x^2}{4}+\frac{4x}{2}+3=0 \quad \Rightarrow x^5-4x^4+12x^3-16x^2+64x+$$

... equation whose roots are the squares of the roots