Carefully read each question and circle the correct answer. If your answer is not [illegible] alternatives provided, write your answer beside the question. Each question under this section carries one (1) mark

1. Numerical Analysis is a branch of Mathematics that deals with devising efficient methods for obtaining _____ solutions to difficult mathematical problems.
   a. exact
   b. analytical
   c. numerical
   d. accurate

2. There are at least three parts in numerical analysis. Which of the following is **not** part?
   a. Cross validation of method
   b. Analysis of method
   c. Development of an algorithm
   d. Development of method

3. There are at least three parts in numerical analysis. What is the correct order of these parts?
   i. Cross validation of method
   ii. Analysis of method
   iii. Development of an algorithm
   iv. Development of method
   a. i, ii, iii
   b. ii, iii, iv
   c. iv, ii, i
   d. iv, ii, iii

4. The following are some of the benefits of Numerical Analysis to an engineer, except _____?
   a. more methods for a given problem
   b. help in obtaining exact solutions
   c. proper utilisation of methods
   d. easy to design new methods

5. _____ refers to how closely individual computed or measured values agree with each other.
   a. Biasness
   b. Imprecision
   c. Accuracy
   d. Precision

6. _____ refers to how closely a computed or measured value agrees with the true value.
   a. Precision
   b. Imprecision
   c. Accuracy
   d. Biasness

7. _____ is defined as systematic deviation from the truth.
   a. Precision
   b. Biasness
   c. Imprecision
   d. Accuracy

8. _____ errors result from using an approximation in place of an exact mathematical procedure.
   a. Truncation
   b. Roundoff
   c. Measurement
   d. Accumulation

9. _____ appears when accumulating (with additions, essentially) small errors of the same sign a lot of times.
   a. Roundoff
   b. Measurement
   c. Accumulation
   d. Truncation

10. _____ errors are errors arising from the process of rounding off during computation.
   a. Machine
   b. Measurement
   c. Truncation
   d. Accumulation

SECTION B

Carefully read each question and circle the correct answer. Show all workings in the Answer Booklet provided. If your answer is not found in the alternatives provided, write your answer beside the question. Each question under this section carries one (1) mark

Use the following information to answer Questions 11 to 15.

PROBLEM I: As a crime scene investigator and you must predict the temperature of a homicide victim over a 2.5-hr period. From the preliminary investigation, it was established that the room where the victim was found was at 10 °C when the body was discovered. Use Newton's law of cooling $\frac{dT}{dt} = -k(T - T_a)$ and Euler's method to compute the victim's body temperature for the 2.5-hr period using values of k = 0.12/hr and Δt = 0.5 hr. Assume that the victim's body temperature at the time of death was 37 °C, and that the room temperature T_a, was at a constant value of 20 °C over the 2.5-hr period. [*Perform all computations in* ***radians*** *and in 4 decimal places. Error margin of ±0.0001 may apply*].

11. What will be the victim's body temperature T, at time t = 0.5? 37.6000
   a. 35.3628
   b. 35.5894
   c. 35.0212
   d. 35.9800

12. What will be the victim's body temperature T, at time $t = 1.0$?

a. 35.3628
c. 35.5894
b. 35.0212
d. 35.9800

13. What will be the victim's body temperature T, at time $t = 1.5$?

a. 34.6894
c. 34.9212
b. 34.8800
d. 34.1199

14. What will be the victim's body temperature T, at time $t = 2.0$?

a. 33.1599
c. 33.2727
b. 33.5800
d. 33.7212

15. What will be the victim's body temperature T, at time $t = 2.5$?

a. 32.5454
c. 32.5642
b. 32.4763
d. 32.9212

Use the following information to answer Questions 16 to 20.

PROBLEM II: A storage tank contains a liquid at depth y where $y = 0$ when the tank is half full. Liquid is withdrawn at a constant flow rate Q to meet demands. The contents are resupplied at a sinusoidal rate $3Q\sin^2(t)$. The mathematical expression for this system can be written as $\underbrace{\frac{d(Ay)}{dt}}_{\text{change in volume}} = \underbrace{3Q\sin^2(t)}_{\text{inflow}} - \underbrace{Q}_{\text{outflow}}$. However, since the surface area A is constant, the mathematical expression for the system is modified as $\frac{dy}{dt} = 3\frac{Q}{A}\sin^2(t) - \frac{Q}{A}$. Obtain the numerical solution for the depth y from $t = 0$ to 2.5 d with a step size of 0.5 d. The parameter values are $A = 1200$ m^2 and $Q = 400$ m^3/d. Assume that the initial condition is $y = 0$. [*Perform all computations in **radians** and in 4 decimal places. Error margin of ±0.0001 may apply*].

16. What will be the depth y, of the liquid in the storage tank at time $t = 0.5$?

a. -0.1667
c. -0.1166
b. -0.1176
d. -0.1177

17. What will be the depth y, of the liquid in the storage tank at time $t = 1.0$?

a. -0.2481
c. -0.2184
b. -0.2148
d. -0.2814

18. What will be the depth y, of the liquid in the storage tank at time $t = 1.5$?

a. -0.0310
c. 0.3010
b. 0.0311
d. -0.0311

19. What will be the depth y, of the liquid in the storage tank at time $t = 2.0$?

a. 0.2999
c. 0.2989
b. 0.2898
d. 0.2998

20. What will be the depth y, of the liquid in the storage tank at time $t = 2.5$?

a. 0.5546
c. 0.5456
b. 0.5465
d. 0.5645

Use the following information to answer Questions 21 to 23.

PROBLEM III: To estimate the speed of wind required to turn the wind turbine located behind the Mathematical Sciences Department, an engineer will have to approximate the first derivative of the function $f(x) = 75x^2 - 12x + 7$ using a step size of 0.25. [*Perform all computations in 4 decimal places. Error margin of ±0.0001 may apply*].

21. Using the centered difference approximation, what will be the approximated value at $x = 2.0$ and its corresponding absolute percentage relative error respectively?

a. 306.75 with 6.5104% error
c. 269.25 with 6.5104% error
b. 576 with 100% error
d. 288 with 0% error

22. Using the backward difference approximation, what will be the approximated value at $x = 2.0$ and its corresponding absolute percentage relative error respectively?

a. 306.7500 with 6.5104% error
c. 269.25 with 6.5104% error
b. 288 with 0% error
d. 576 with 100% error

23. Using the forward difference approximation, what will be the approximated value at $x = 2.0$ and its corresponding absolute percentage relative error respectively?

a. 288 with 0% error
c. 576 with 100% error
b. 306.75 with 6.5104% error
d. 269.25 with 6.5104% error

Use the following information to answer Questions 24 and 25.

PROBLEM IV: Many fields of engineering require accurate estimates using natural logarithms. For example, road construction engineers might find it necessary to determine the required level for roads above or below sea level. Suppose a graduate from UMaT wants to determine the level (x) at which the road beside the University's sport complex could be constructed with a

corresponding road thickness of 0.7 expressed mathematically as $f(x) = \ln(x^2) - 0.7$. [*Perform all computations in 4 decimal places. Error margin of ±0.0001 may apply*]

24. Using the Bisection method with initial guesses of x_l = 0.5 and x_u = 2, what will be the required level (x) and its corresponding error after the **second** iteration?

a. 1.6250 with 23.0769% error
b. 1.6287 with 13.0435 error
c. 1.4970 with 8.7976% error
d. 1.2500 with 3.3554% error.

25. Using the method of Regula Falsi (False Position) with same initial guesses as Question 24, what will be the required level (x) and its corresponding error after the **second** iteration?

a. 1.6287 with 13.0435 error
b. 1.6250 with 23.0769% error
c. 1.4970 with 8.7976% error
d. 1.2500 with 3.3554% error

SECTION C

Carefully read each question and circle the correct answer. Show all workings in the Answer Booklet provided. If your answer is not found in the alternatives provided, write your answer beside the question. Each question under this section carries one (1) mark

Use the following information to answer Questions 26 and 38.

PROBLEM V: The average time t required by a mechanic to fix a car tyre is expressed mathematically as $f(t) = t^3 - 6t^2 + 11t - 6.1$. Suppose that the average time (t_0) required by a fresh mechanical engineering graduate from UMaT is 3.5 minutes.

Based on the information from **PROBLEM V** above, use the **Newton-Raphson** method to answer Questions 26 and 29. [*Perform all computations in 6 decimal places. Error margin of ±0.000001 may apply*].

26. What will be the average time t required by the engineer after gaining one-week experience (1^{st} iteration) on the job?

a. 3.191304
b. 3.193104
c. 3.191430
d. 3.191340

27. What will be the average time t required by the engineer after gaining two-weeks experience (2^{nd} iteration) on the job?

a. 3.068969
b. 3.068996
c. 3.068968
d. 3.068699

28. What will be the average time t required by the engineer after gaining three-weeks experience (3^{rd} iteration) on the job?

a. 3.047137
b. 3.047317

c. 3.047731

d. 3.047173

29. What will be the corresponding error $(|\varepsilon_a|)$ of the average time t after gaining three-week experience (3rd iteration) on the job? 0.269

a. 0.702000%

b. 0.601672%

c. 0.602000

d. 0.701666%

Based on the information from **PROBLEM V** above, use the **Secant** method with $t_{-1} = 2.5$ to answer Questions 30 and 33. [*Perform all computations in 6 decimal places. Error margin of ±0.000001 may apply*].

30. What will be the average time t required by the engineer after gaining one-week experience (1st iteration) on the job?

a. 2.711211

b. 2.711011

c. 2.711111

d. 2.711120

31. What will be the average time t required by the engineer after gaining two-weeks experience (2nd iteration) on the job?

a. 2.870115

b. 2.871091

c. 2.870091

d. 2.871109

32. What will be the average time t required by the engineer after gaining three-weeks experience (3rd iteration) on the job?

a. 3.221923

b. 3.222931

c. 3.229213

d. 3.223291

33. What will be the corresponding error $(|\varepsilon_a|)$ of the average time t after gaining three-weeks experience (3rd iteration) on the job?

a. 10.889100%

b. 10.889000%

c. 10.888901%

d. 10.989012%

Based on the information from **PROBLEM V** above, use the **Modified Secant** method with a fractional perturbation $\delta = 0.01$ to answer Questions 34 and 38. [*Perform all computations in 6 decimal places. Error margin of ±0.000001 may apply*].

34. What will be the average time t required by the engineer after gaining one-week experience (1st iteration) on the job?

a. 3.207573

b. 3.215737

c. 3.270235

d. 3.205737

35. What will be the average time t required by the engineer after gaining two-weeks experience (2nd iteration) on the job?

a. 3.028403

b. 3.080423

c. 3.002423

d. 3.082034

36. What will be the corresponding error ($|\varepsilon_a|$) of the average time t after gaining two-weeks experience (2nd iteration) on the job?

a. 4.074105%

b. 4.073252%

c. 4.073000%

d. 4.070054%

37. What will be the average time t required by the engineer after gaining three-weeks experience (3rd iteration) on the job?

a. 3.051208

b. 3.051280

c. 3.012580

d. 3.050812

38. What will be the corresponding error ($|\varepsilon_a|$) of the average time t after gaining three-weeks experience (3rd iteration) on the job?

a. 1.234105

b. 1.123000%

c. 1.023400%

d. 1.210054%

SECTION D

Carefully read each question and circle the correct answer. Show all workings in the Answer Booklet provided. If your answer is not found in the alternatives provided, write your answer beside the question. Each question under this section carries 1.5 marks

Use the following information to answer Questions 39 and 42.

PROBLEM VI: The following system of equations is designed to determine concentrations (the $c's$ in g/m^3) in a series of coupled reactors as a function of the amount of mass input to each reactor (the right-hand sides in g/day):

$$15c_1 - 3c_2 - c_3 = 3800$$
$$-3c_1 + 18c_2 - 6c_3 = 1200$$
$$-4c_1 - c_2 + 12c_3 = 2350$$

Determine the concentrations (the $c's$ in g/m^3) using the **Gauss-Seidel** method. [*Perform all computations in 4 decimal places. Error margin of ±0.0001 may apply*].

39. What is the concentration c_1, c_2 and c_3 respectively, in the series of coupled reactors after the first iteration? 253.2333, 108.8889, 289.6664

a. 253.3232, 108.8989, 289.1953

b. 253.3333, 108.8889, 289.3519

c. 253.2323, 108.9898, 289.5319

d. 253.3223, 108.9998, 289.1539

40. What is the concentration c_1 in the series of coupled reactors with its corresponding error ($|\varepsilon_a|$) after the second iteration? 166.8887 with 51.7776%

a. 294.4012 with 13.9496%
b. 294.1204 with 13.6445%
c. 294.0421 with 13.9646%
d. 294.0241 with 13.6943%

41. What is the concentration c_2 in the series of coupled reactors with its corresponding error ($|\varepsilon_a|$) after the second iteration?

a. 212.8421 with 48.6189%
b. 212.8241 with 48.1986%
c. 212.1842 with 48.6819%
d. 212.4812 with 48.1896%

42. What is the concentration c_3 in the series of coupled reactors with its corresponding error ($|\varepsilon_a|$) after the second iteration?

a. 311.6491 with 7.1546%
b. 311.4619 with 7.5641%
c. 311.1946 with 7.4561%
d. 311.4961 with 7.6415%

SECTION E

Carefully read each question and circle the correct answer. Show all workings in the Answer Booklet provided. If your answer is not found in the alternatives provided, write your answer beside the question. Each question under this section carries 1.5 marks

43. Given the two points $[a, f(a)]$, $[b, f(b)]$, the linear Lagrange polynomial $f_1(x)$ that passes through these two points is given by

a. $f_1(x) = \frac{x}{b-a} f(a) + \frac{x}{b-a} f(b)$
b. $f_1(x) = f(a) + \frac{f(b) - f(a)}{b-a}(b-a)$
c. $f_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{b-a} f(b)$
d. $f_1(x) = \frac{x-b}{a-b} f(a) + \frac{x-a}{a-b} f(b)$

44. Use the Lagrange interpolating polynomial of the first order to evaluate the density of unused motor oil at a temperature of 15 degree Celsius based on the following data:

$x_1 = 0; \quad f(x_1) = 3.85$
$x_2 = 20; \quad f(x_2) = 0.8$
$x_3 = 40; \quad f(x_3) = 2.12$

a. 1.3317
b. 1.5625
c. 2.5694
d. 2.5625

45. Use the Lagrange interpolating polynomial of the second order to evaluate the density of unused motor oil at a temperature of 15 degree Celsius based on the following data:

$x_1 = 0; \quad f(x_1) = 3.85$

$x_2 = 20; \quad f(x_2) = 0.8$

$x_3 = 40; \quad f(x_3) = 2.12$

a. 1.5625

b. 2.5694

c. 2.5625

d. 1.3317

46. Use the second-order Newton polynomial to evaluate $\ln 2$ based on the following data:

$x_1 = 1; \quad f(x_1) = 0$

$x_2 = 4; \quad f(x_2) = 1.386294$

$x_3 = 6; \quad f(x_3) = 1.791759$

a. 0.5625453

b. 0.3317153

c. 0.5658444

d. 0.5694426

SECTION F

Carefully read each question and circle the correct answer. Show all workings in the Answer Booklet. If your answer is not found in the alternatives provided, write your answer beside the question. Each question under this section carries two (2) marks

Use the following information to answer Questions 47 and 50.

PROBLEM VII: The growth of populations of organisms has many engineering and scientific applications. One of the simplest instances is the growth of pollutant bacteria y in a drainage system increases according to $y' = 4e^{0.8t} - 0.5y$. Use the 4th Order Runge-Kutta (RK4) method to determine the growth of the pollutant bacteria from time $t = 0$ to 2 using a step size of 1.0 with $y(0) = 2.0$ (in million). [*Perform all computations in 6 decimal places. Error margin of ±0.000001 may apply*].

47. For iteration 1, what are the values for k_1, k_2, k_3 and k_4 respectively?

a. 3, 4.217929, 3.912974, 5.945677

b. 3, 4.217299, 3.912974, 5.945677

c. 3, 4.217299, 3.912974, 5.946577

d. 3, 4.217299, 3.912947, 5.945677

48. What will be the growth of pollutant bacteria y at time $t = 1.0$?

a. 6.201037
c. 6.201247
b. 6.201028
d. 6.201187

49. For iteration 2, what are the values for k_1, k_2, k_3 and k_4 respectively?

a. 5.801645, 8.739538, 7.999565, 12.812829
c. 5.801645, 8.729538, 7.997565, 12.712829
b. 5.701645, 8.729538, 7.897565, 12.712829
d. 5.701645, 8.729538, 7.797565, 12.812829

50. What will be the growth of pollutant bacteria y at time $t = 2.0$?

a. 14.862484
c. 14.862438
b. 14.864283
d. 14.864238

Good Luck

A. Buabeng/P. K. Nyarko/P. Boye/J. Acquah

UNIVERSITY OF MINES AND TECHNOLOGY, TARKWA

FIRST SEMESTER EXAMINATION, APRIL 2023

COURSE NO: GL/GM/ES/MN/MR/MC/EL/CE/RN/ PE/NG/PG/RP 361
COURSE NAME: PROBABILITY AND STATISTICS
CLASS: GL/GM/ES/MN/MR/MC/EL/CE/RN/ PE/NG/PG/RP III
TIME: 3 HOURS

Answer all Questions in Section A and any two (2) Questions from Section B

SECTION A

1. Given the cumulative distribution function (CDF) $F(x)=\frac{x+k}{6}, for \quad x=1,2,3,4$

   Find k.

   A. 1
   B. 2
   C. 3
   D. 4

2. Consider the probability distribution function. What is the value of $F(3.3)$?

x	2	3	4	5	6
$P(X=x)$	k	$2k$	$4k$	$3k$	$2k$

   A. 0
   B. 2/3
   C. 1/4
   D. 2/5

3. Consider the probability distribution function

x	-1	0	1	2
$P(x)$	0.2	0.3	a	b

Given that $E(X)=0.6$; find the values of a and b.

   A. $a=0.3, b=0.4$
   B. $a=0.3, b=0.1$
   C. $a=0.1, b=0.2$
   D. $a=0.2, b=0.3$

4. The expectation of an estimated parameter minus the parameter is equal to zero. This concept is statistics is called?
   A. biasedness
   B. efficiency
   C. unbiasedness
   D. sufficiency

5. Consider the following marks: 45, 32, 37, 46, 39, 36, 41, 48, 36. Calculate the arithmetic mean.
   A. 40 marks
   B. 42 marks
   C. 50 marks
   D. 52 marks

6. Calculate the geometric mean of the following marks: 45, 32, 37, 46, 39, 36, 41, 48, 36
   A. 37.68
   B. 38.68
   C. 39.68
   D. 40.68

7. A property of a point estimator that occurs whenever larger sample sizes tend to provide point estimates closer to the population parameter is known as
   A. efficiency
   B. unbiased sampling
   C. consistency
   D. relative estimation

8. A numerical description of the outcome of an experiment is called a
   A. descriptive statistic
   B. probability function
   C. variance
   D. random variable

9. In point estimation
   A. data from the population is used to estimate the population parameter
   B. data from the sample is used to estimate the population parameter
   C. data from the sample is used to estimate the sample statistic
   D. the mean of the population equals the mean of the sample

10. The *t distribution* is used when the sample size is
   A. less than 30 and the population variance is unknown.
   B. greater than 30 and the population variance is unknown
   C. less than 30 and the population variance is known
   D. at least 30 and the population variance

11. Consider the following paddy yields (kg/plot). What is the 25th percentile?
   A. 36.15 kg
   B. 37.25 kg
   C. 38.35 kg
   D. 39.45 kg

12. Calculate the interquartile range for the following data (grain/panicles) 25, 18, 30, 8, 15, 5, 10, 35, 40, 45.
   A. 26.75
   B. 27.85
   C. 28.85
   D. 29.95

13. An unbiased point estimate that has a smaller variance than any other unbiased estimate is called?
   A. minimum variance unbiased estimate (MVUE)
   B. variance
   C. kurtosis
   D. skewness

14. Measurable characteristic obtained from the population is called
   A. Statistic
   B. Parameter
   C. Estimation
   D. Efficiency

15. Decile divides the group into ten equal parts.
   A. True
   B. False

$\bar{X} - Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \leq \mu \leq \bar{X} - Z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)$

16. The way we classify variables affects how we use them in our analysis.
A. True B. False

17. The relationship between red, black, and white pigs is 8:4:4. What is the probability that there among 8 pigs are 5 red, 2 black and 1 white is found by using the multinomial distribution.
A. 0.052 C. 0.072
B. 0.062 D. 0.082

18. A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%. The inspector of the retailer randomly picks 20 items from a shipment. What is the probability that there will be at least one defective item among these 20?
A. 0.4552 C. 0.4572
B. 0.4562 D. 0.4572

19. The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease. Find the variance of this distribution.
A. 3.6 C. 6.6
B. 4.6 D. 7.6

20. Mass-produced needles are packed in boxes of 1000. It is believed that 1 needle in 2000 on average is substandard. What is the probability that a box contains 2 or more defectives?
A. 0.0902 C. 0.0702
B. 0.0802 D. 0.0602

21. Statistics is the science of data.
A. True B. False

22. In statistics, a *variable* has two defining characteristics thus, a variable is an attribute that describes a person, place, thing, or idea, and the value of a variable can "vary" from one entity to another.
A. True B. False

23. A basic tool in the study of inferential statistics is *probability*.
A. True B. False

24. Variables can be classified as *qualitative* (aka, categorical) or *quantitative* (aka, numeric).
A. True B. False

25. The main difference between a population and a sample has to do with how observations are assigned to the data set.
A. True B. False

26. The two most common unusual features are gaps and outliers.
A. True B. False

27. Sampling technique is one of the most important procedures to consider in choosing the observations that will constitute the random samples of the population as a Field Engineer.
A. True B. False

28. A table with all possible values of a random variable and its corresponding probabilities is called

$SS_{xy} = \Sigma(xy) - \frac{\Sigma x(\Sigma y)}{n}$

A. Probability Mass Function
B. Probability Density Function
C. Cumulative distribution function
D. Probability Distribution

29. If a variable can have certain integer values between two given points is called
A. Continuous random variable
B. Discrete random variable
C. Irregular random variable
D. Uncertain random variable

30. Out of the following values, which one is not possible in probability?
A. P(x) = 1
B. $\sum$ x P(x) = 3
C. P(x) = 0.5
D. P(x) = – 0.5

31. If E(x) = 2 and E(z) = 4, then E(z – x) =?
A. 2
B. 6
C. 0
D. Insufficient data

32. The exponential distribution is *bi-variate*.
A. True
B. False

33. Consider a random variable with exponential distribution with λ=1. Compute the probability for P (X>3).
A. e-3
B. e-1
C. e-2
D. e-4

34. Normal Distribution is applied for
A. Continuous Random Distribution
B. Discrete Random Variable
C. Irregular Random Variable
D. Uncertain Random Variable

35. Normal Distribution is also known as
A. Cauchy's distribution
B. Laplacian distribution
C. Gaussian distribution
D. Lagrangian distribution

36. In a Binomial Distribution, if p, q, and *n* are the probability of success, failure, and number of trials respectively then the variance is given by
A. np
B. npq
C. 2npq
D. npq^2

37. In a Binomial Distribution, the mean and variance are equal.
A. True
B. False

38. For larger values of 'n', Binomial Distribution
A. loses its discreteness
B. tends to Poisson Distribution
C. stays as it is
D. gives oscillatory values

39. The shape of the Normal Curve is
A. Bell Shaped
B. Flat
C. Circular
D. Spiked

40. Two events, A and B, are mutually exclusive and each has a nonzero probability. If event A is known to occur, the probability of the occurrence of event B is
A. one
B. any positive value
C. zero
D. any value between 0 to 1