NITK SURATHKAL

Digital Signal Processing

Mini Project

Adaptive Beamforming

Authors:
Aniket Rege
Prabhanjan Mannari
Ritesh Waykole

Supervisor:
Dr. Pathipati Srihari

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Abstract

Adaptive array antennas use smart signal processing algorithms to allow the antenna to steer the beam to any direction of interest while simultaneously nulling interfering signals. Beamforming is the technique used to create the radiation pattern of the array by constructively adding the phases of the signals in the direction of targets and nulling the pattern of undesired/interfering targets, thus providing directional sensitivity without physically moving an array of receivers and transmitters. This can be implemented with a simple FIR tapped delay line filter, whose weights may be changed adaptively to provide optimal beamforming, which reduced the Minimum Mean Square Error (MMSE) between the actual and desired beam pattern. The most common Adaptive beamforming algorithms explored are LMS algorithm and RLS algorithm, of which the LMS algorithm is explored in this project. The LMS algorithm is described by a recursive equation which updates filter weights in such a manner that they converge to the optimum filter weight in an inverse accordance with the gradient of the mean square error vs. filter weight curve, i.e. changing the weights in a direction opposite to that of gradient slope. We hope to explore these techniques and create our own modification to existing algorithms for a more efficient beamforming process.

1 Preliminaries

The following results are given as preliminaries for the purpose of foresight into the theoretical principles behind adaptive beamforming and its use in antenna arrays.

1.1 Random Process

A random process X(t,s) is an ensemble of time functions from the sample space S.
The samples $X_{ti} = x(ti), i = 1, 2, 3, \cdots n$ are n random variables characterized by their joint probability density function $= p(x_{t1}, x_{t2}, \cdots x_{tn})$

1.2 Stationary Random Process

Consider random process $X(ti) = x(ti), i = 1, 2, 3, \cdots n$ and
$X(ti + \tau) = X(ti + \tau, i = 1, 2, 3, \cdots n$ for some parameter τ with probability density functions

$p(x_{t1}, x_{t2}, x_{t3}, \cdots x_t n) = p_1$

$p(x_{t1+\tau}, x_{t2+\tau}, x_{t3+\tau}, \cdots x_{tn+\tau}) = p_2$

We say the process x(ti) is stationary if $p_1 = p_2 \forall n$

1.3 Statistical [Ensemble] Averages

Consider a random process $X(t_i) = x(t_i)$ sampled at $t = t_i$
$X(t_i)$ is a random variable. The l^{th} moment of the random variable is defined as the expected value of $x^l(t_i)$

$$E[X^l(t_i)] = \int_{-\infty}^{\infty} X_{ti}^l p(x_{ti}) dx_{ti}$$

If the process is stationary, $p(x_{ti}) = p(x_{ti+\tau})$ and hence the l^{th} moment is a constant. The statistical mean or expected value of a random process is defined as

$$\mu_x(t_i) = \int_{-\infty}^{\infty} x_{ti} p(x_{ti}) dx_{ti}$$

1.4 Correlation

The statistical correlation between two random variables of the process is defined as

$$E(X_{t1}X_{t2}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_{t1}x_{t2}p(x_{t1}x_{t2})dx_{t1}dx_{t2}$$

if $t_2 = t_1 + \tau$, the above relation is known as the autocorrelation of the random process $x(t_i)$ denoted as $r_{xx}(t_1, t_{1+\tau})$

$$r_{xx}(t_1, t_{1+\tau}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_{t1}x_{t1+\tau}p(x_{t1}x_{t1+\tau})dx_{t1}dx_{t1+\tau}$$

Suppose the mean $\mu_x(t_i)$ is constant, i.e. independent of t_i and the autocorrelation depends only on the lag τ and is independent of t_i, then the random process $X(t_i)$ is said to be stationary in the wide sense, or **Wide Sense Stationary**.

1.5 Auto Covariance of a Random Process

$$C_{xx}(t_1, t_2) = E([X_{t1} - \mu_{t1}][X_{t2} - \mu_{t2}]) = r_{xx}(t_1, t_2) - \mu_{t1}\mu_{t2}$$

For a stationary random process, $e_{xx}(\tau) = r_{xx}(\tau) - \mu^2$

1.6 Statistical Averages for Joint Random Processes

Let X(t) and Y(t) be two random processes characterized by joint PDF $P(x_{t1}, x_{t2}, \cdots y_{t1}, y_{t2}, \cdots)$ We define

Cross Correlation:

$$r_{xy}(t_1, t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_{t1} y_{t2} p(x_{t1} y_{t2}) dx_{t1} dy_{t2}$$

Cross Covariance:

$$r_{xy}(t_1, t_2) = \mu_y(t_2)\mu_x(t_1)$$

1.7 Power Density Spectrum

A stationary random process is an infinite energy signal, and hence does not have a Fourier transform. Its spectral characteristic is obtained from the Fourier Transform of it auto-correlation according to Wiener-Khinchin theorem.

State Theorem:

$$\Gamma_{xx}(F) = \int_{-\infty}^{\infty} r_{xx}(\tau) e^{-j2\pi F\tau} d\tau$$

$$\begin{aligned} r_{xx}(\tau) &= \int_{-\infty}^{\infty} \Gamma_{xx}(F) e^{-j2\pi F\tau} dF \\ &= \int_{-\infty}^{\infty} \Gamma_{xx}(F) dF \\ &= E(x_t^2) \geq 0 \end{aligned}$$

$r_{xx}(0)$ is equal to average power of the random process

1.8 Discrete Time Random Signals

We model a discrete time random signal using a discrete time random process which is atleast wide sense stationary (possible with zero mean). Consider a random signal x[n] of length N, we define its auto-correlation as

$$r_{xx}(l) = \sum_{n=0}^{N} x[n]x[n-l]$$

$$r_{xx}(m) = E(x[n]x * [n-m])$$