***Essence of the project:*** Take the initial wave function $\Psi(x,0) = Ax\sin(\pi x/a)$ in the **standard infinite square well**, and study (through plotting) how the **probability density function** varies with time.

Define the **Characteristic angular frequency** (angular frequency for ground state): $\omega = \pi^2\hbar/2ma^2$.

Recall that the revival time for any wave function in the square well is given by $T = 2\pi/\omega$.

For your convenience, the normalization constant and the expansion coefficients for the given wave function are presented below:

$$A = \frac{1}{a^{3/2}}\sqrt{\frac{12\pi^2}{2\pi^2-3}} \;;$$

$$c_1 = \frac{1}{2}\sqrt{\frac{6\pi^2}{2\pi^2-3}}$$

$$c_n = -\frac{1}{\pi^2}\sqrt{\frac{24\pi^2}{2\pi^2-3}}\left[\frac{1}{(n-1)^2}-\frac{1}{(n+1)^2}\right] \quad \text{for even } n;$$

$$c_n = 0 \qquad \text{for odd } n \text{ except } n=1.$$

**Hence, the wave function at a later time $t$ will be given by:**

$$\Psi(x,t) = \frac{1}{2}\sqrt{\frac{6\pi^2}{2\pi^2-3}}\sqrt{\frac{2}{a}}\sin(\pi x/a)e^{-i\omega t} - \frac{1}{\pi^2}\sqrt{\frac{24\pi^2}{2\pi^2-3}}\sqrt{\frac{2}{a}}\sum_{\text{even } n}\left[\frac{1}{(n-1)^2}-\frac{1}{(n+1)^2}\right]\sin(n\pi x/a)e^{-in^2\omega t}$$

## *Task*

(a) Using the expansion coefficients from $n$=1 up to $n$=20 (instead of worrying about the infinite sum), plot the regenerated initial wave function. Take $\hbar = 1; \;\; m = 1; \;\; a = 1.$ Check if this indeed matches the given initial wave function. **(3 points)**

(b) Plot the initial probability density function. **(1 points)**

(c) Plot the probability density function at times

$t = T/8;$

$t = T/4;$

$t = 3T/8;$

$t = T/2;$

$t = 5T/8;$

$t = 3T/4;$

$t = 7T/8;$

$t = T.$

Plot all these on the same graph along with the plot at $t$=0! **(5 points)**

Check if indeed the density function at $t$=$T$ matches the density function at $t$=0.

(d) For which value/s of $t$ is the density function an even function about the midpoint ($x$=$a$/2)?

**Mathematically explain why!** **(3 points)**

(e) For which value/s of $t$ is the density function a mirror reflection (about the line $x$=$a$/2)

of the initial density function?

**Mathematically explain why!** **(3 points)**

If you can make an actual movie of the evolution (which I cannot, alas!) with plots of the density function at, say, 100 equally spaced values of $t$ (instead of the eight plotted) between $t$=0 and $t$=$T$, you get **3 extra bonus points!**