A comparison of time series similarity measures

https://www.sciencedirect.com/science/article/pii/S0034425711002446

- What did the authors try to accomplish?

  Present an overview and quantitative comparision of different similarity measures in function of varying time series

- What were the key elements of the approach?

$$f^p, f^q : timeseries$$

  - Manhattan
    - Quantifies the absolute magnitude ofthe difference between time series
    - Easy to calculate

$$D_{Man} = (\sum_{t=1}^{N} |f_t^p - f_t^q|)$$

  - Euclidean
    - Quantifies the Euclidean distance ofthe difference between time series
    - Easy to calculate
    - More sensitive to outliers, due to it's nonlinear character

$$D_E = (\sum_{t=1}^{N} |f_t^p - f_t^q|^2)^{\frac{1}{2}}$$

  - Mahalanobis
    - Quantifies the difference between time series but accounts for non-stationarity of variance and temporalcross-correlation

$$D_{Mah} = \sqrt{(f_t^p - f_t^q)^T \sum^{-1} (f_t^p - f_t^q)}$$

$$\sum : CovarianceMatrix$$

- Pearons's Correlation
  - Quantifies the degree of linear relationship between time series

$$D_{CC} = \frac{\sum_{t=0}^{N-1}\left[\left(f_t^p - \bar{f}^p\right) * \left(f_{t-s}^q - \bar{f}^q\right)\right]}{\sqrt{\sum_{t=0}^{N-1}\left(f_t^p - \bar{f}^p\right)^2} * \sqrt{\sum_{t=0}^{N-1}\left(f_{t-s}^q - \bar{f}^q\right)^2}}$$

- Principal Component
  - Quantifies the difference between times PCs that explain the majority of the variance
  - Selecting m critical

$$D_{PCA} = \sqrt{\sum_{k=1}^{m}(PC_k^p - PC_k^q)^2}$$

- 3 Fourier Based Similarities
  - Fourier Transformation of series f(t)

$$f(t) = A_0 + \sum_{k=1}^{N-1} A_k \cos(2\pi kt + \phi_k)$$

with

$$A_k = \sqrt{F_k^{c2} + F_k^{s2}}$$

and

$$\phi_k = \arctan\left(\frac{F_k^c}{F_k^s}\right)$$

Quantifies the combined effect of FT amplitude and phase differences

$$D_{FFT} = \sqrt{\sum_{k=0}^{m} (A_k^p - A_k^q)^2 + \sum_{k=1}^{m} (\phi_k^p - \phi_k^q)^2}$$

Quantifies shape similarity based onderived FT amplitude and phase differences

$$D_\xi = 3\sqrt{\sum_{k=1}^{m} \left(\alpha_k^{ref} - \alpha_k\right)^2} + \sqrt{\sum_{k=1}^{m} \left(\theta_k^{ref} - \theta_k\right)^2} \tag{9}$$

where α_k and θ_k are the relative amplitude and phase for each FT component:

$$\alpha_k = \frac{A_k}{A_1} \tag{10}$$

and

$$\theta_k = \left(\frac{A_k}{A_1}\right)_{ref} [2 + \cos(k\phi_1 - \phi_k)] \tag{11}$$

and α_k^ref and θ_k^ref are relative amplitude and phase for a reference class. D_ξ is zero if two time series, represented by their m Fourier components, have the same shape and increases as the differences between the shapes increases.

Quantifies the FT of the difference between time series

$$\begin{aligned} A_k^{p-q} &= \sqrt{\left(F_k^c(p) - F_k^c(q)\right)^2 + \left(F_k^s(p) - F_k^s(q)\right)^2} \\ &= \sqrt{F_k^c(p-q)^2 + F_k^s(p-q)^2} \end{aligned} \tag{12}$$

Based on the A_k^{p-q} of the selected m FT components, D_{Fk} can be calculated:

$$D_{Fk} = \sum_{k=0}^{N-1} w_k A_k^{p-q} \tag{13}$$

where A_k^p−q is the F_k-distance between the time series of p and q respectively, and w_k is the weight of the kth frequency FT component. Modification of the weights allows to enhance (high w_k) or diminish(low w_k) the influence of each component on D_Fk and to accentuate specific components in the similarity measure. As a result, these weights will determine the sensitivity to time series differences. For example, when w_0=0 the D_Fk will be insensitive to amplitude translations as the offset value in the y-axis is neglected in the D_Fk calculation

▼ What can you use yourself?

- Different similarity measures for time series

▼ What other references do you want to follow?

- 5.1 + 5.2 Performance of similarity measures