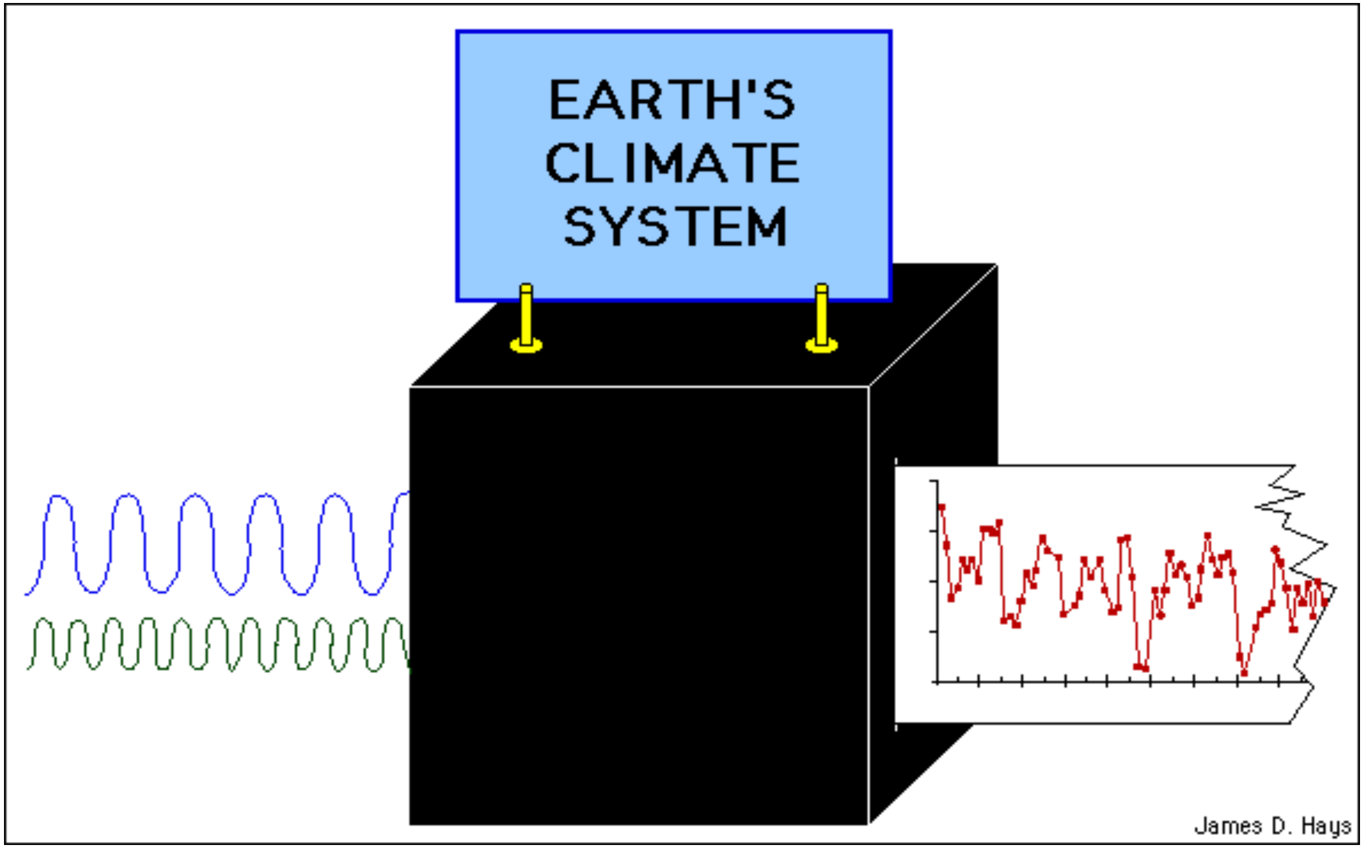
Basics of Dynamical Systems Theory

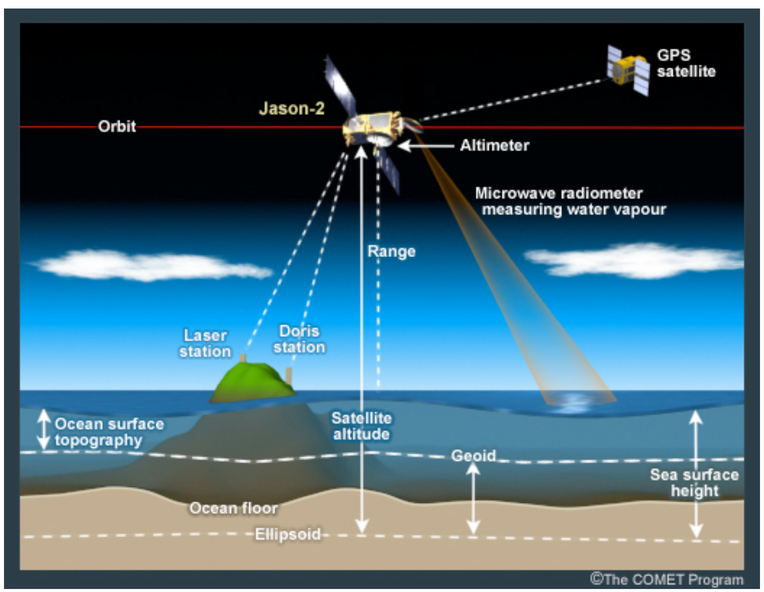


Navid & Henk, ANU, Jan 20-24, 2020

Motivation & Overview

Observations

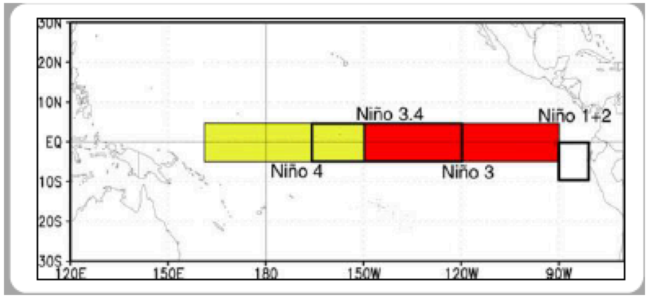
Instrumental data
(~ 1880 -)



(1992 -)

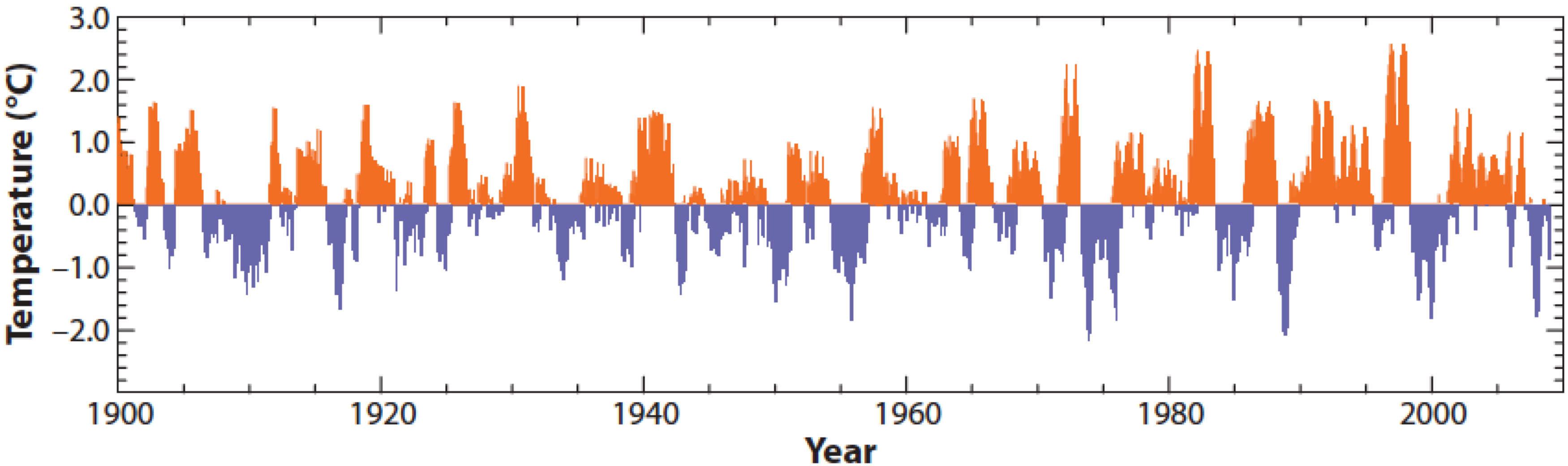
Proxy data
(geological past)

Example instrumental data: El Niño/Southern Oscillation (ENSO)



NINO3.4 index

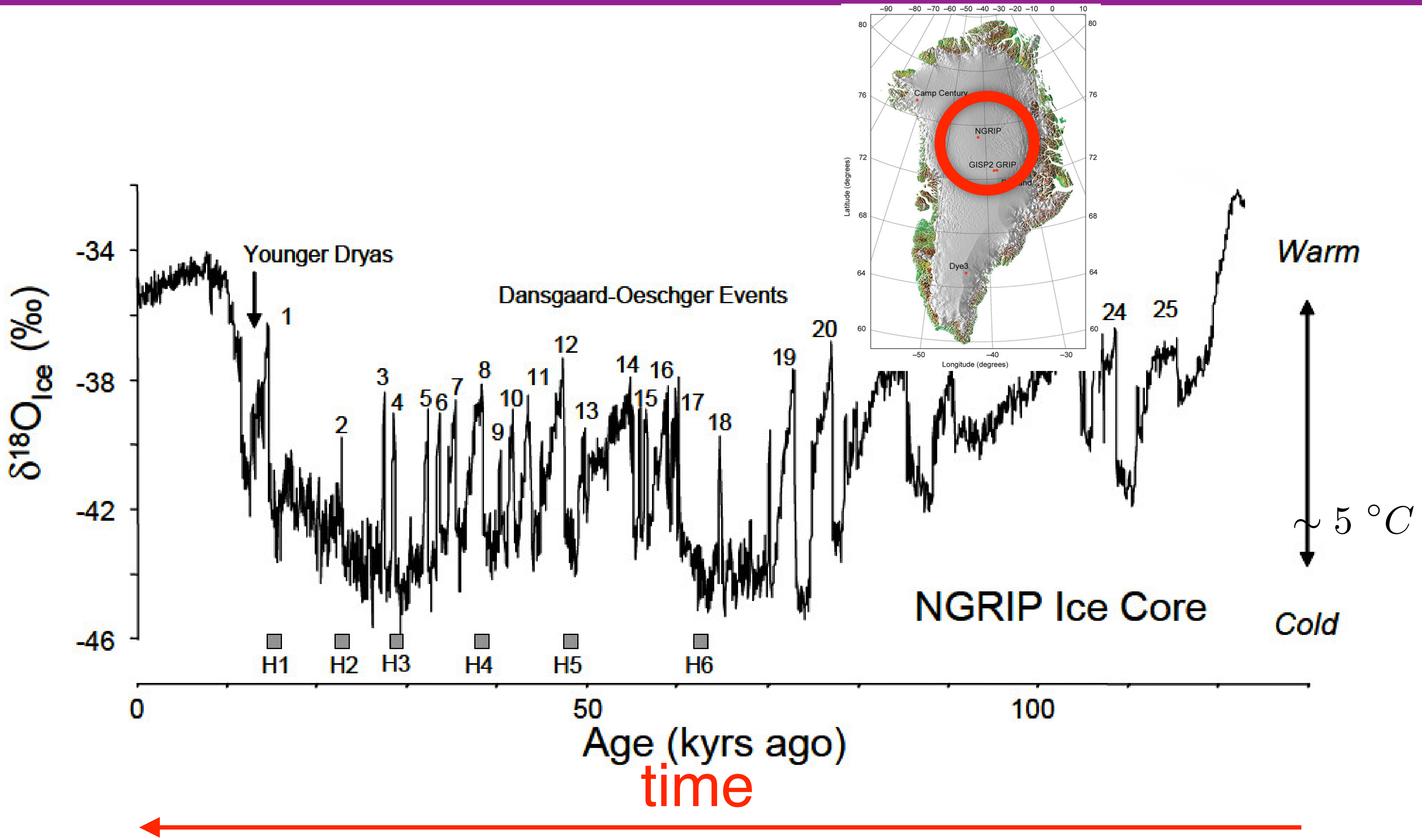
sea surface temperature anomaly equatorial Pacific



http://climexp.knmi.nl

Deser et al., Ann. Rev. Mar. Sc. (2010)

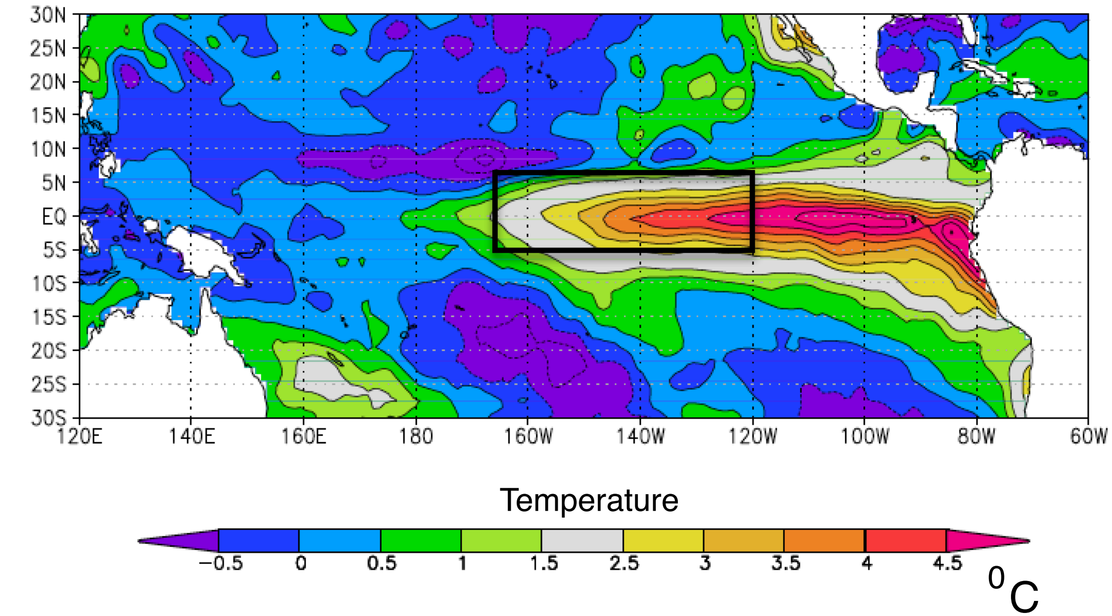
Example proxy data:
Ice core oxygen isotope record



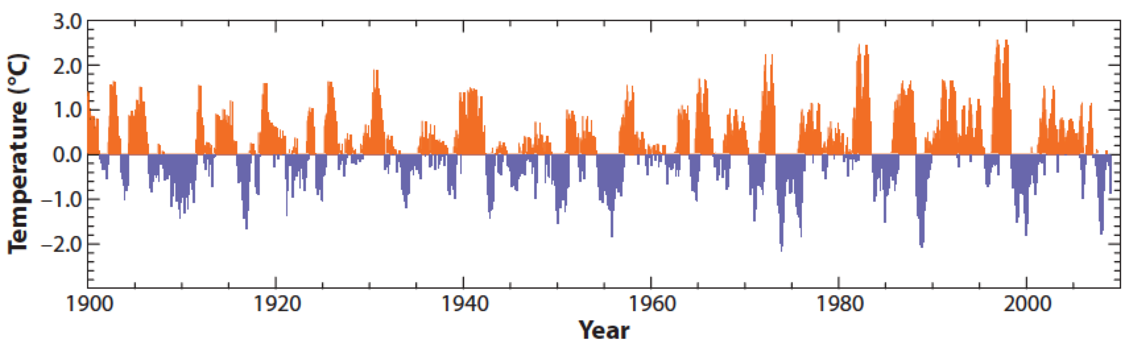
time

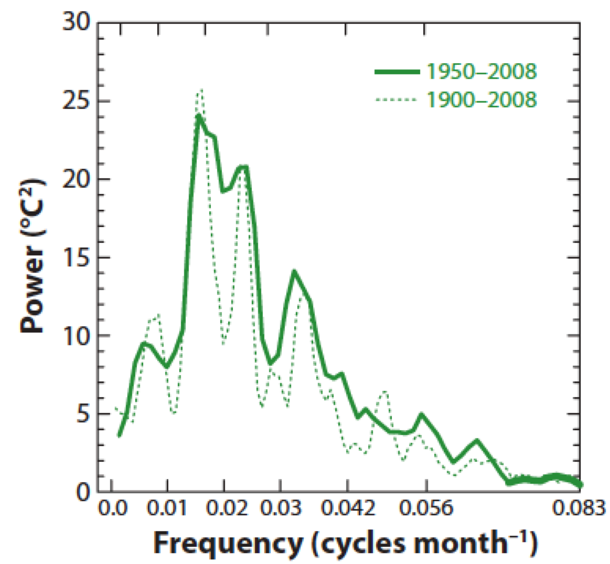
https://www.ncdc.noaa.gov/data-access/paleoclimatology-data/datasets

Patterns of variability:
El Nino/Southern Oscillation (ENSO)



SST anomaly
(1982-2010 mean)
of December 1997





NINO3.4 index

Deser et al., Ann. Rev. Mar. Sc. (2010)

Examples of questions

Which processes determine the time scales and spatial patterns of these climate variability phenomena?

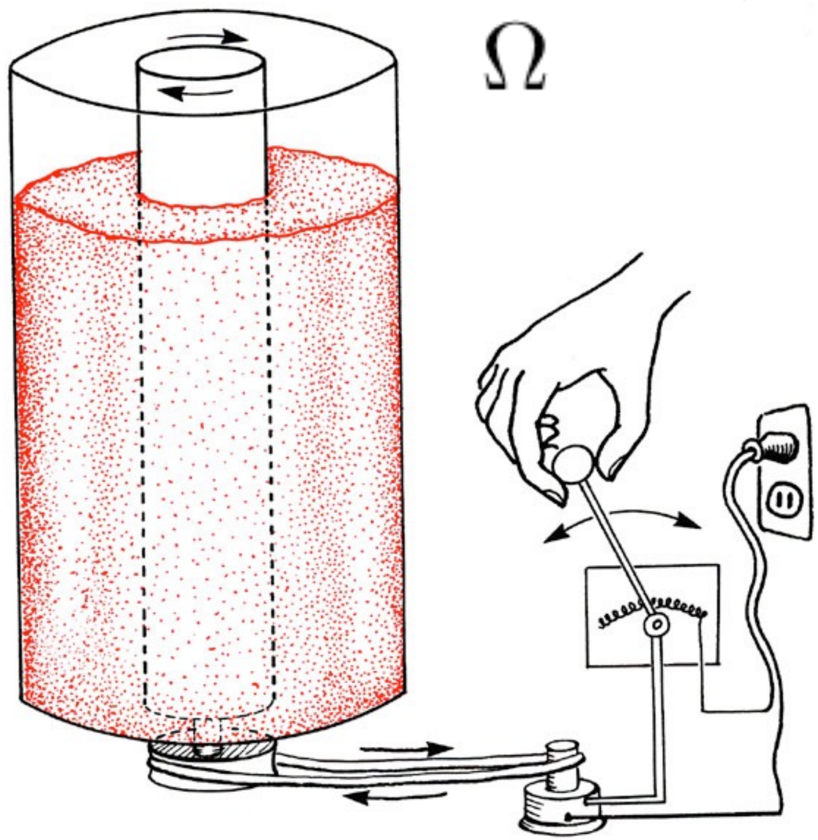
Ex: Why is the pattern of ENSO localized in the eastern Pacific and what sets its amplitude?

How do these climate variability phenomena interact with the background climate state?

Ex: How does ENSO affect the global mean surface temperature?

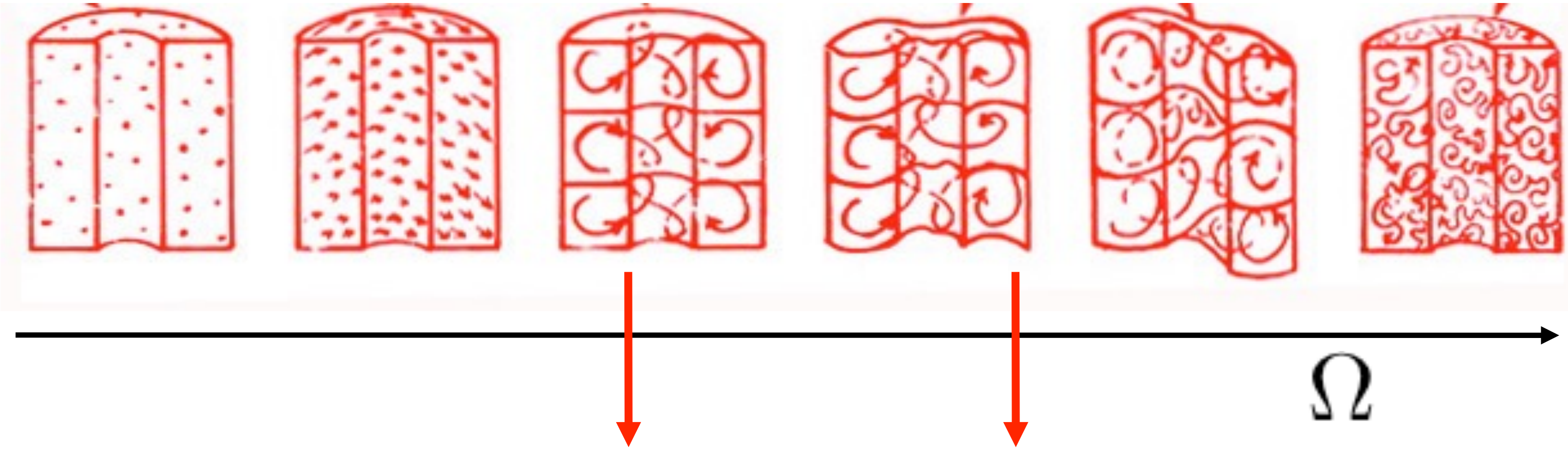
Dynamical systems approach

The Taylor-Couette Flow

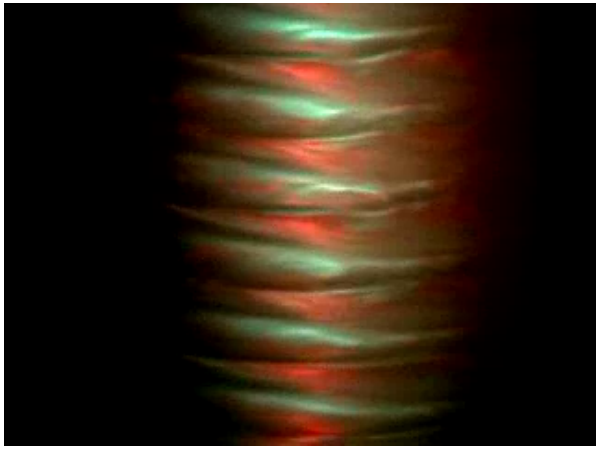


Abraham, R. H. and Shaw, C. D., Dynamics, the Geometry of Behavior, (1988)

Transition behavior

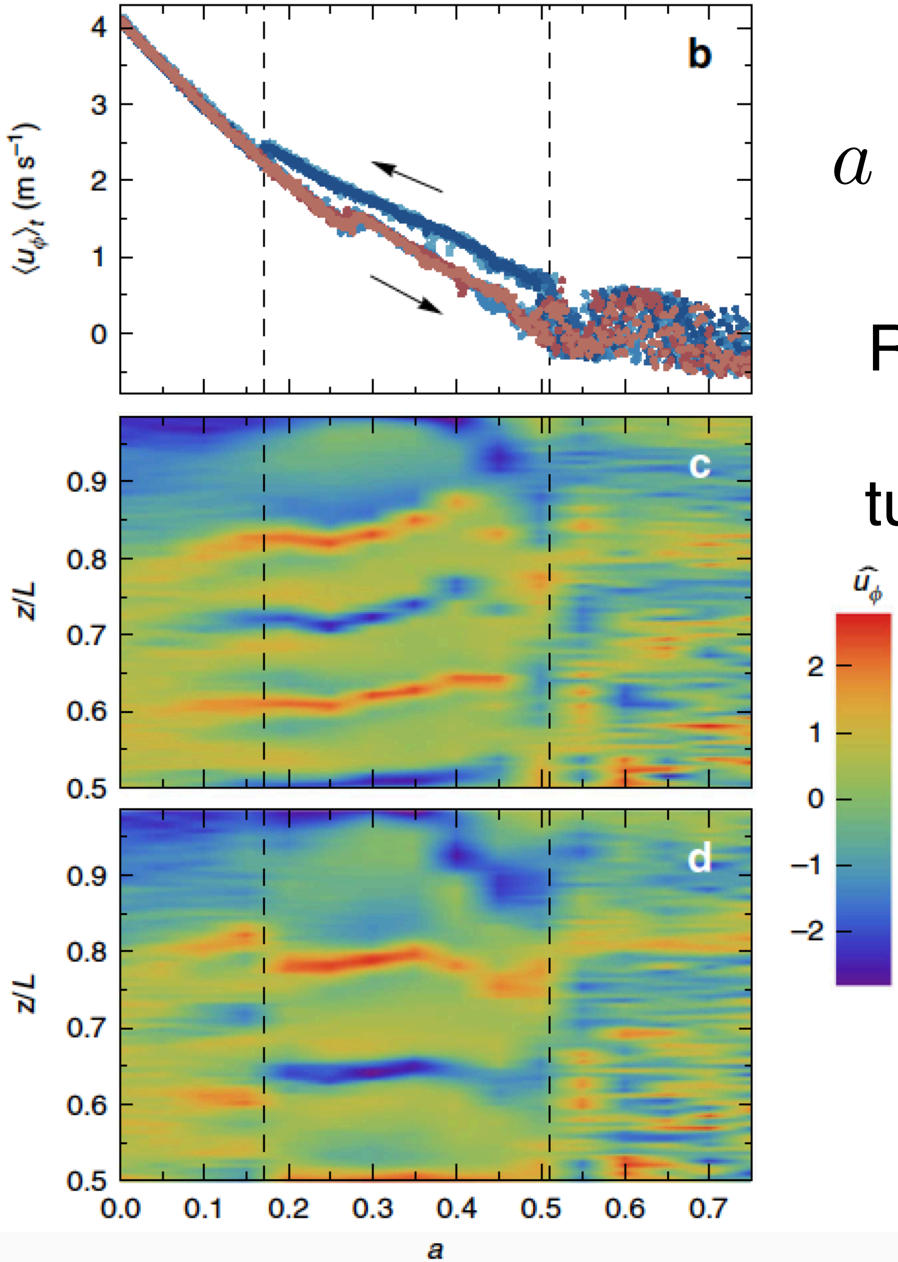


Taylor vortices



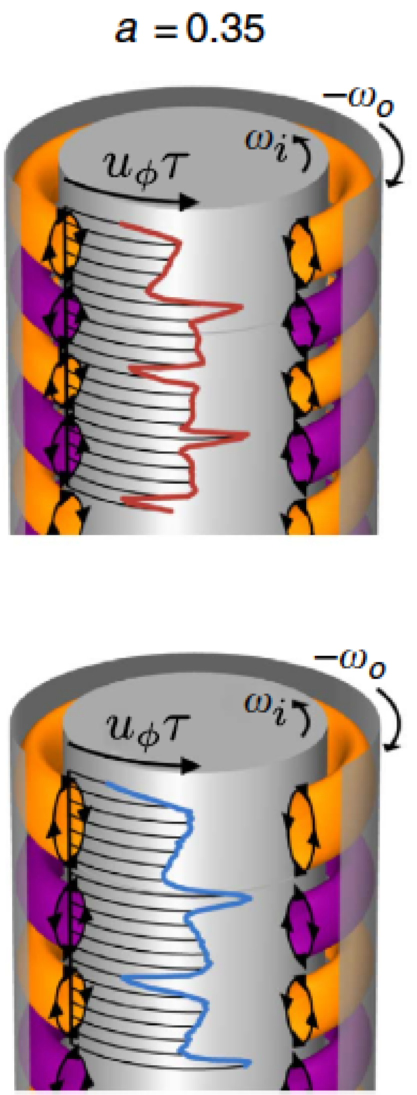
Wavy vortices

Multiple turbulent states



$$a = -\frac{\omega_0}{\omega_i}$$

Regime of ultimate turbulence

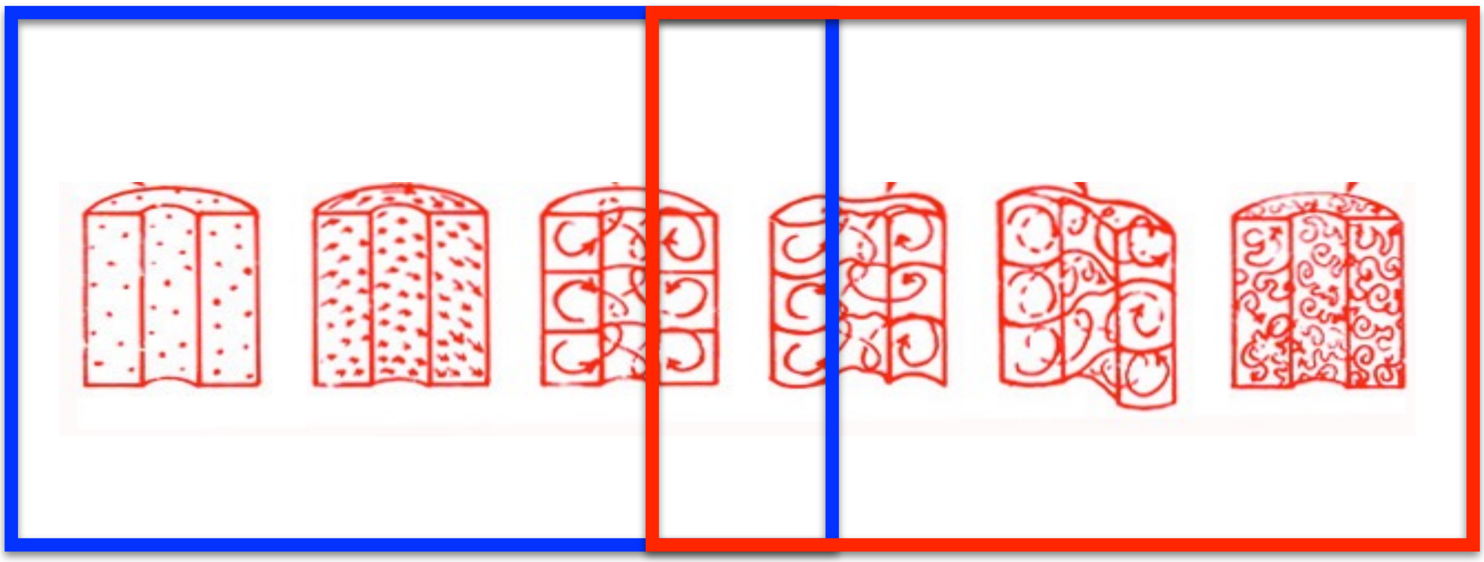


Huisman et al (2014)

Concepts/techniques

A. Bifurcation Theory

elementary transitions, normal modes (instability mechanisms)
global bifurcations transition to chaos,
inertial manifolds, synchronization



B. Ergodic Theory

long time behavior of ensembles of trajectories
almost invariant sets, transfer operators
evolution of correlations

Applications of Dynamical Systems Theory

I. (Nonlinear) time series analysis

Multi-channel singular spectrum analysis
Attractor embedding
Recurrence plots

II. Predictability & Data assimilation

Lyapunov exponents
Conditional Nonlinear Optimal Perturbations
Particle Filters

III. Particle tracking (Lagrangian)

Finite time Lyapunov coefficients
Lagrangian Coherent Structures
Mixing

Tools

Old tools

Linear stationary multivariate statistics
and (Fourier) spectral theory

New tools

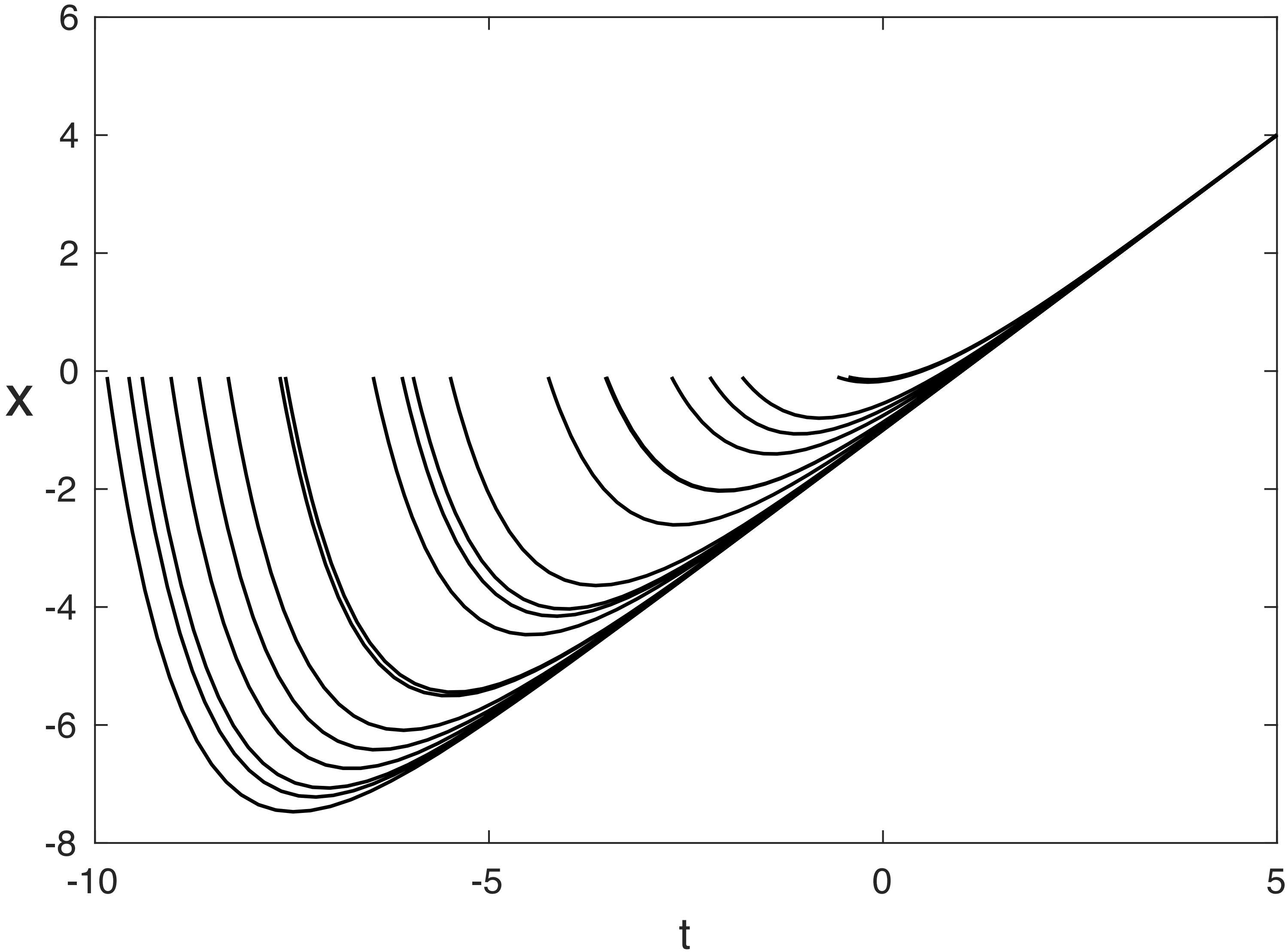
Stochastic dynamical systems theory

Aim of this short course

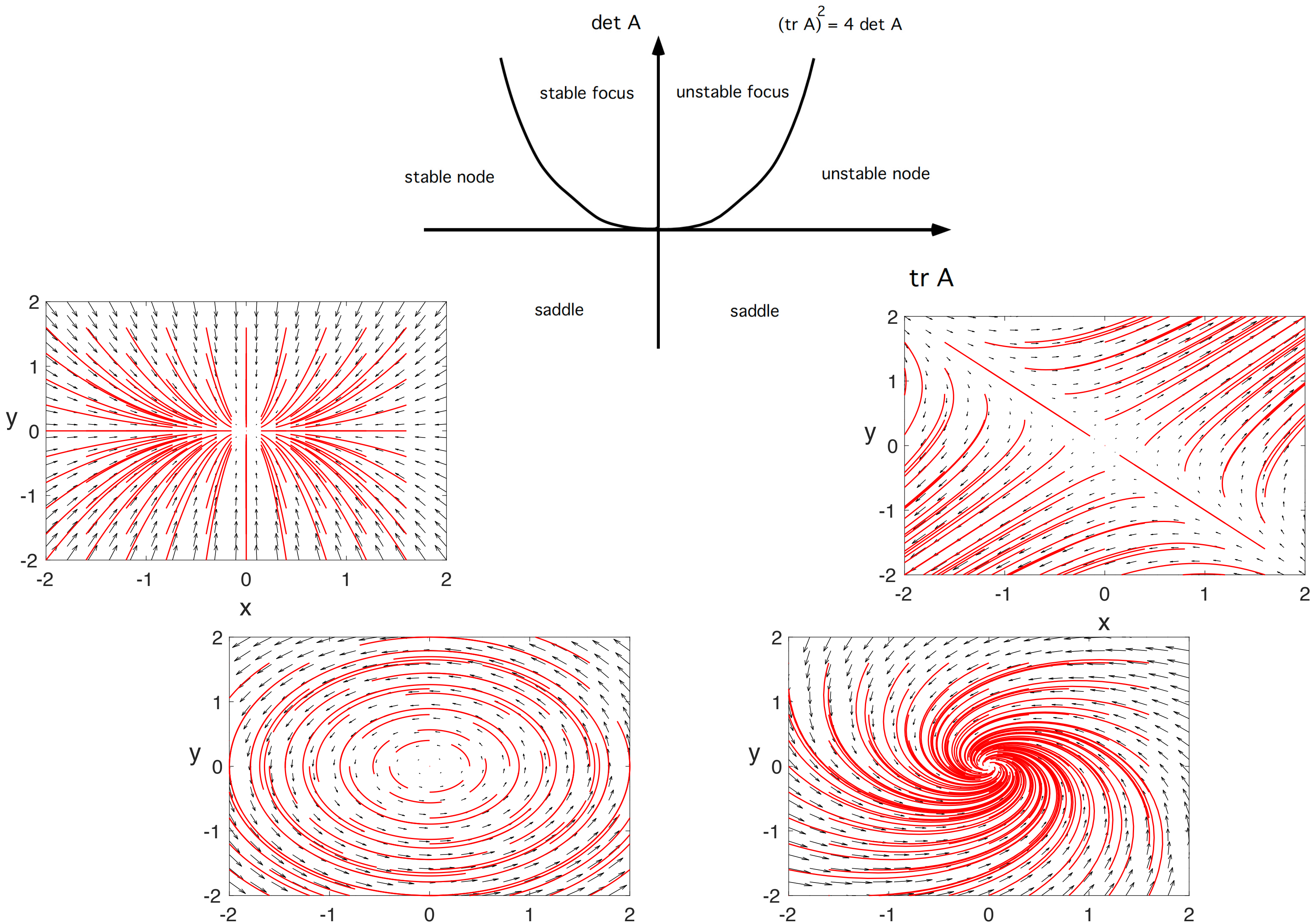
1. Learn basic concepts on dynamical systems theory
2. Be able to identify and analyse elementary (one-parameter) bifurcations
3. Be able to numerically compute bifurcations diagrams of low-dimensional dynamical systems

Illustrations basic theory

Pullback Attractor

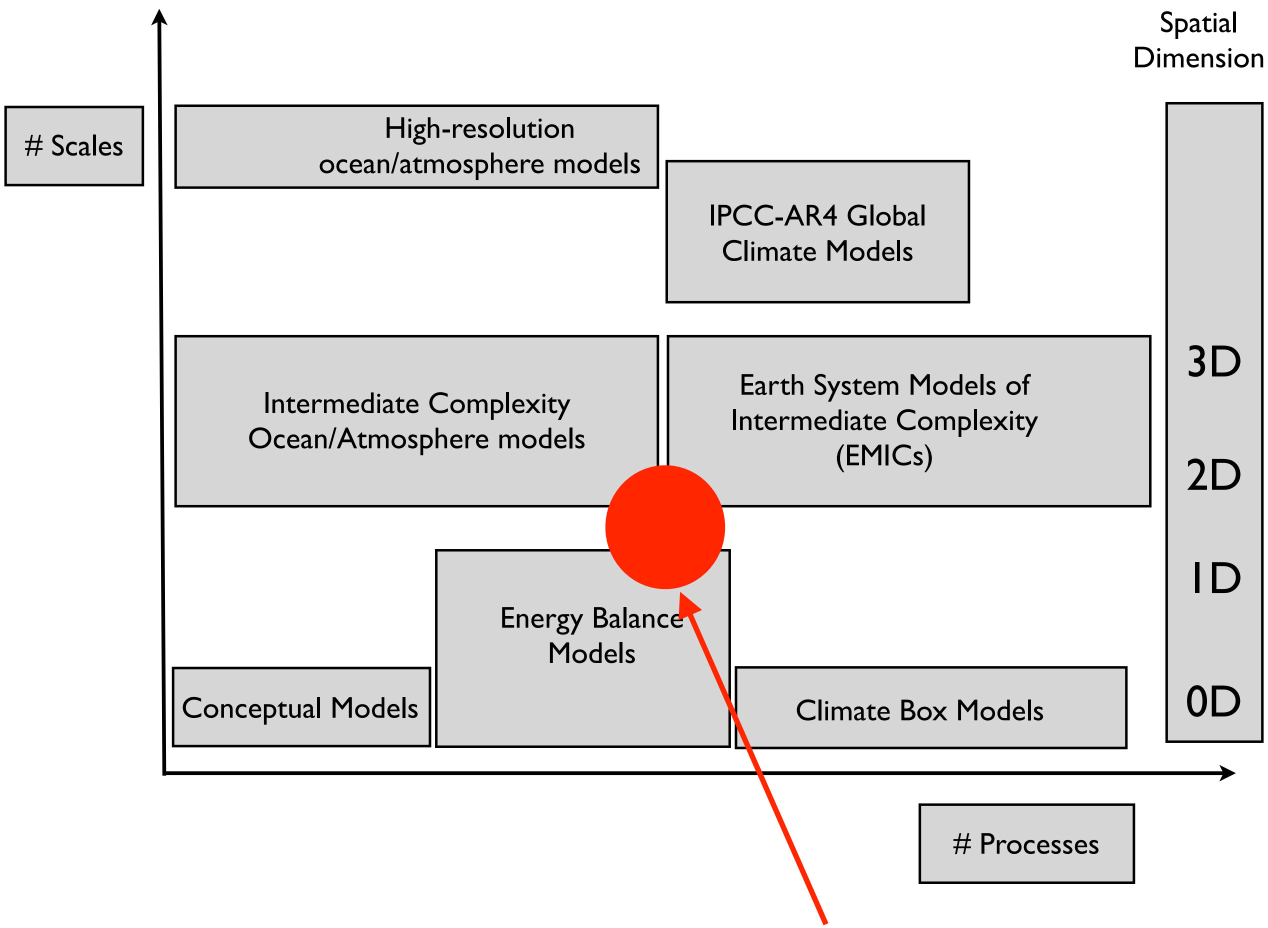


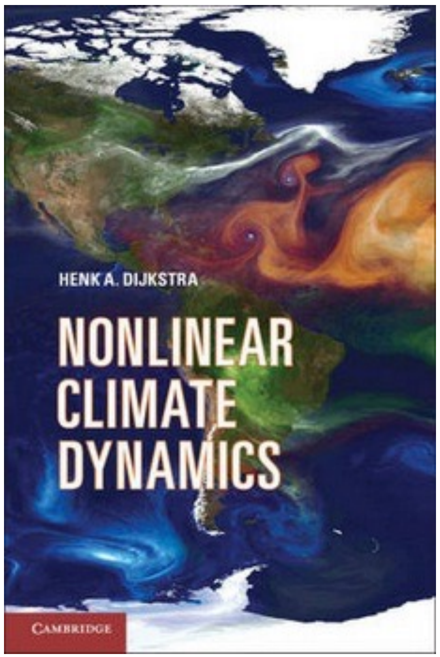
Phase portraits, 2D linear dynamical systems



Atlantic Meridional Overturning Circulation

Application to Climate Variability





chapter 6

‘Minimal’ Model:
‘just enough’ processes to capture phenomenon under study

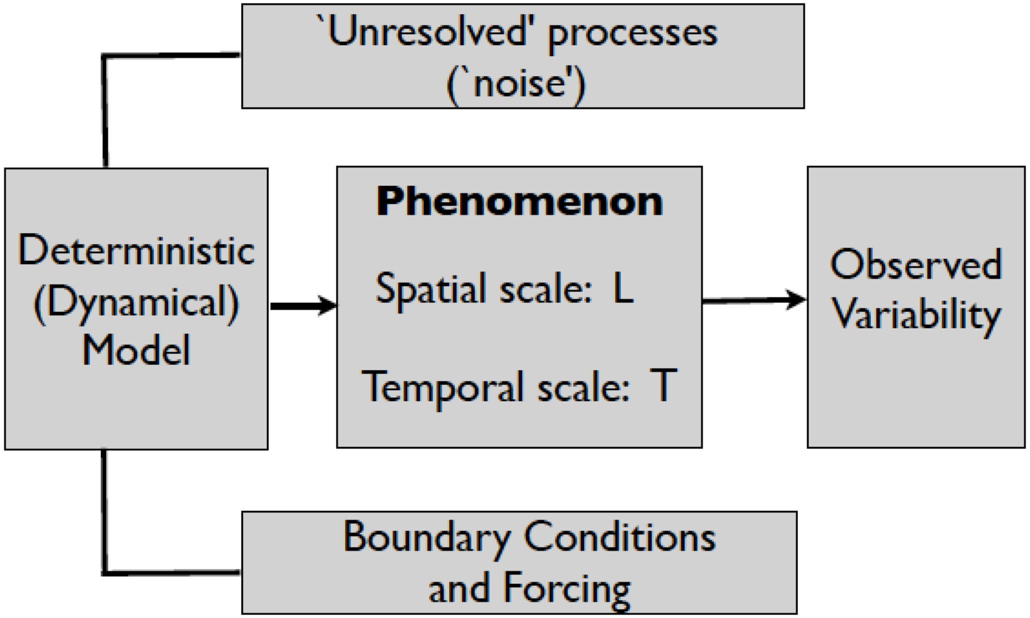
Stochastic dynamical systems approach

Hierarchy of models

Behavior (statistics) over the full parameter space

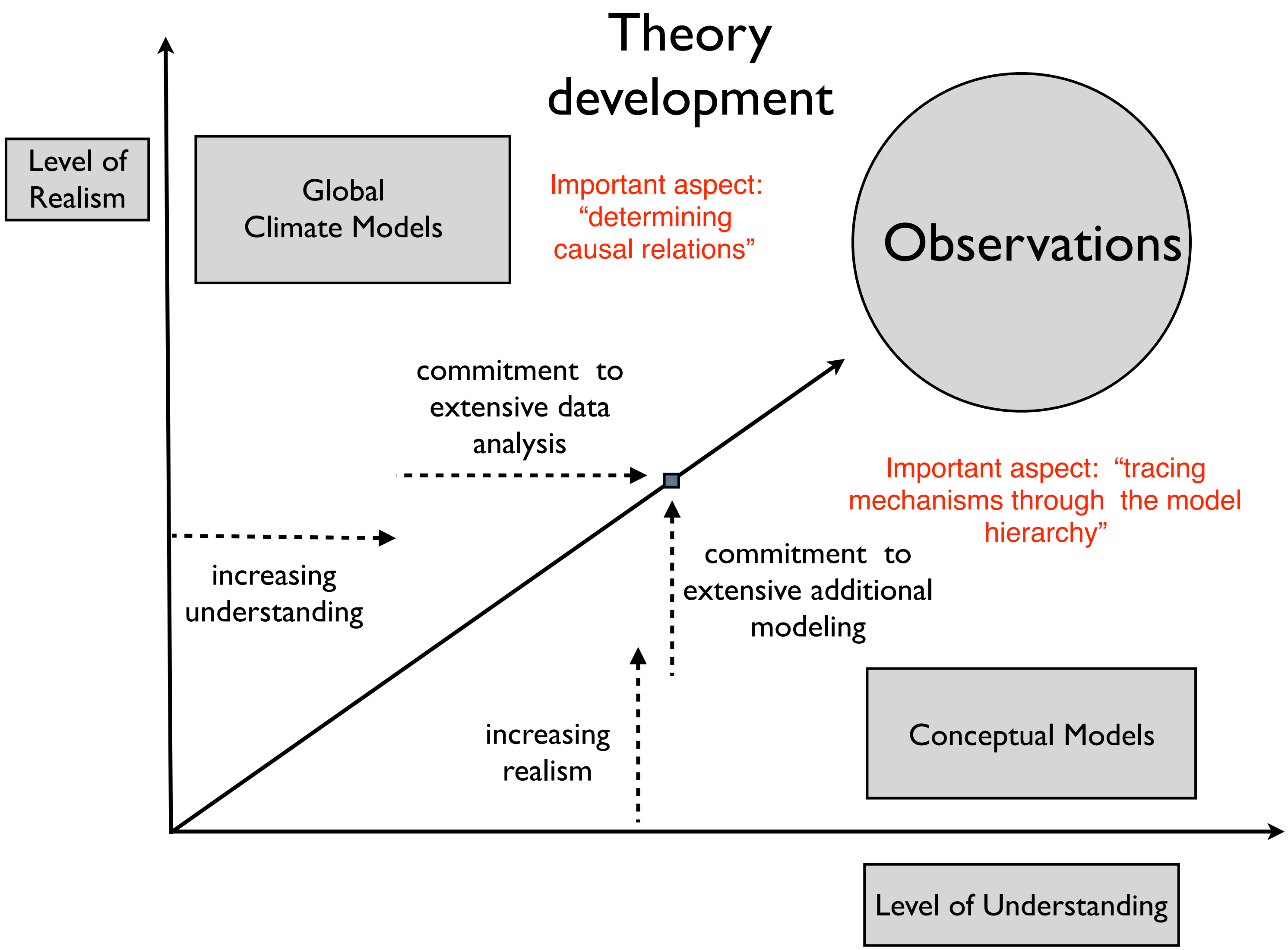
Geometrical view of motion

Dynamical system

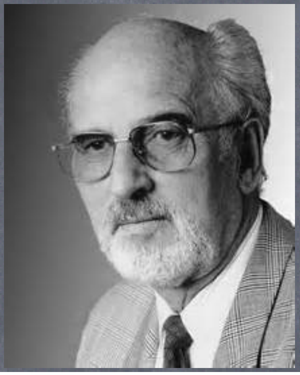


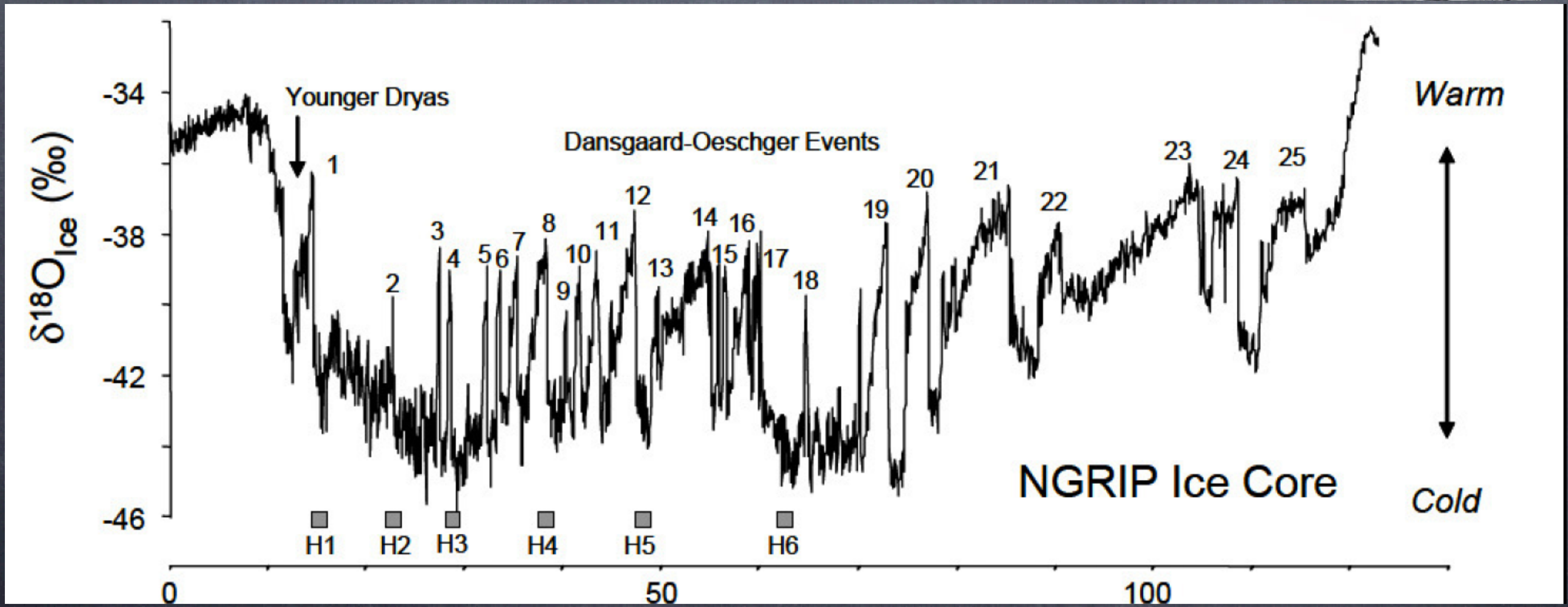
One person's signal is another person's noise

Towards understanding …



Climate variability on Greenland during the last 100,000 years

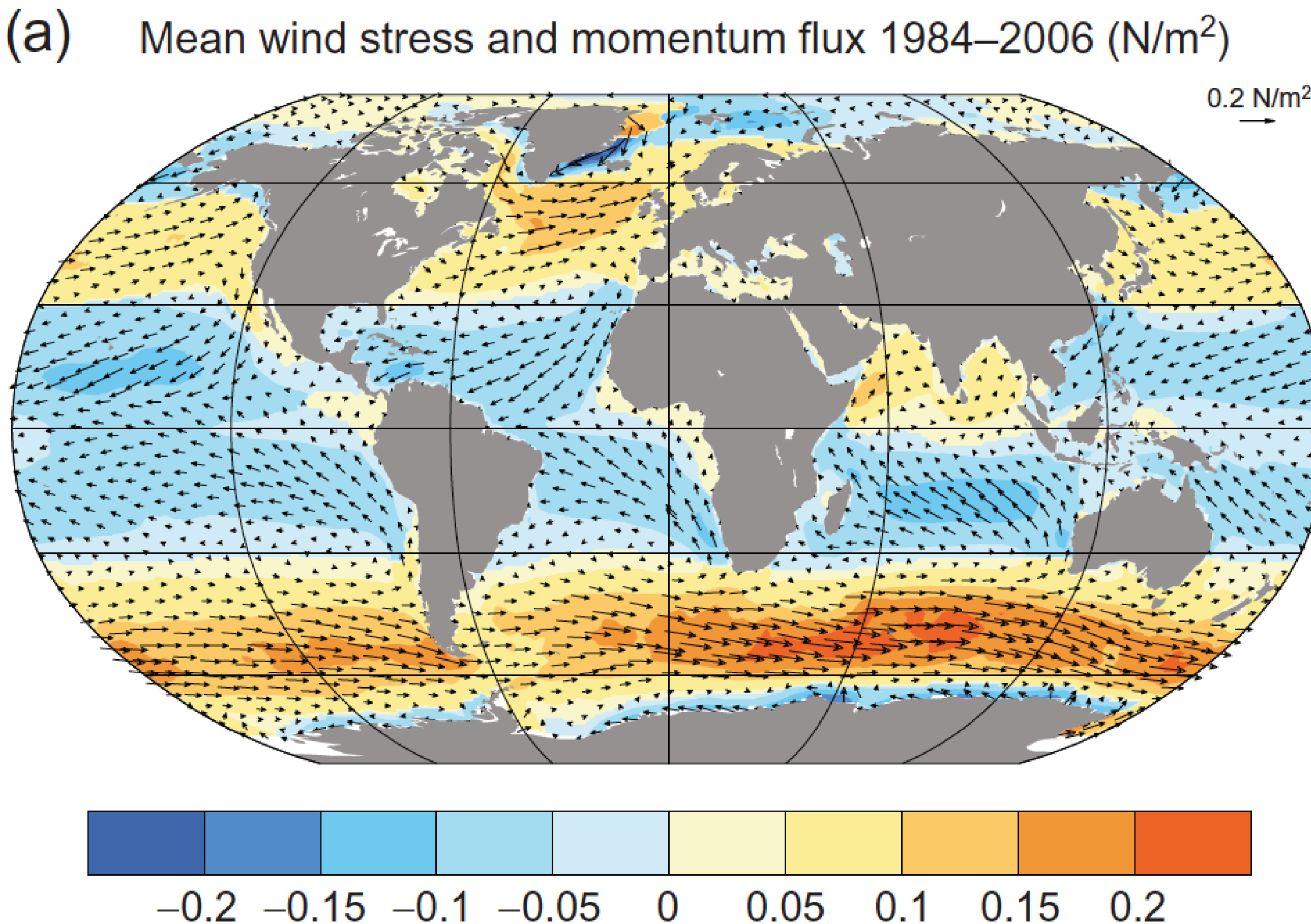




time (kyr)

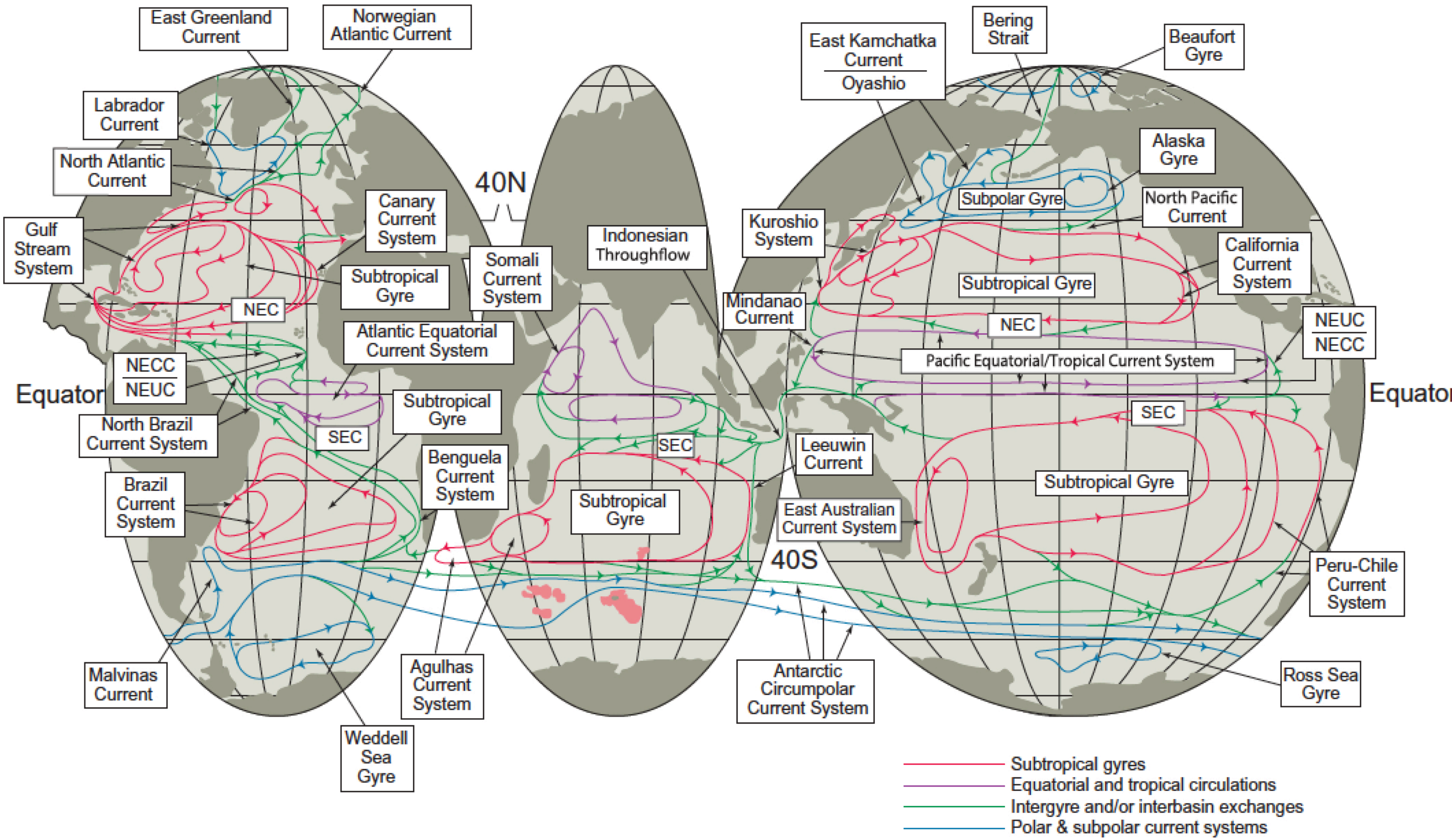
Clement et al., Rev. Geophysics (2008

Wind stress

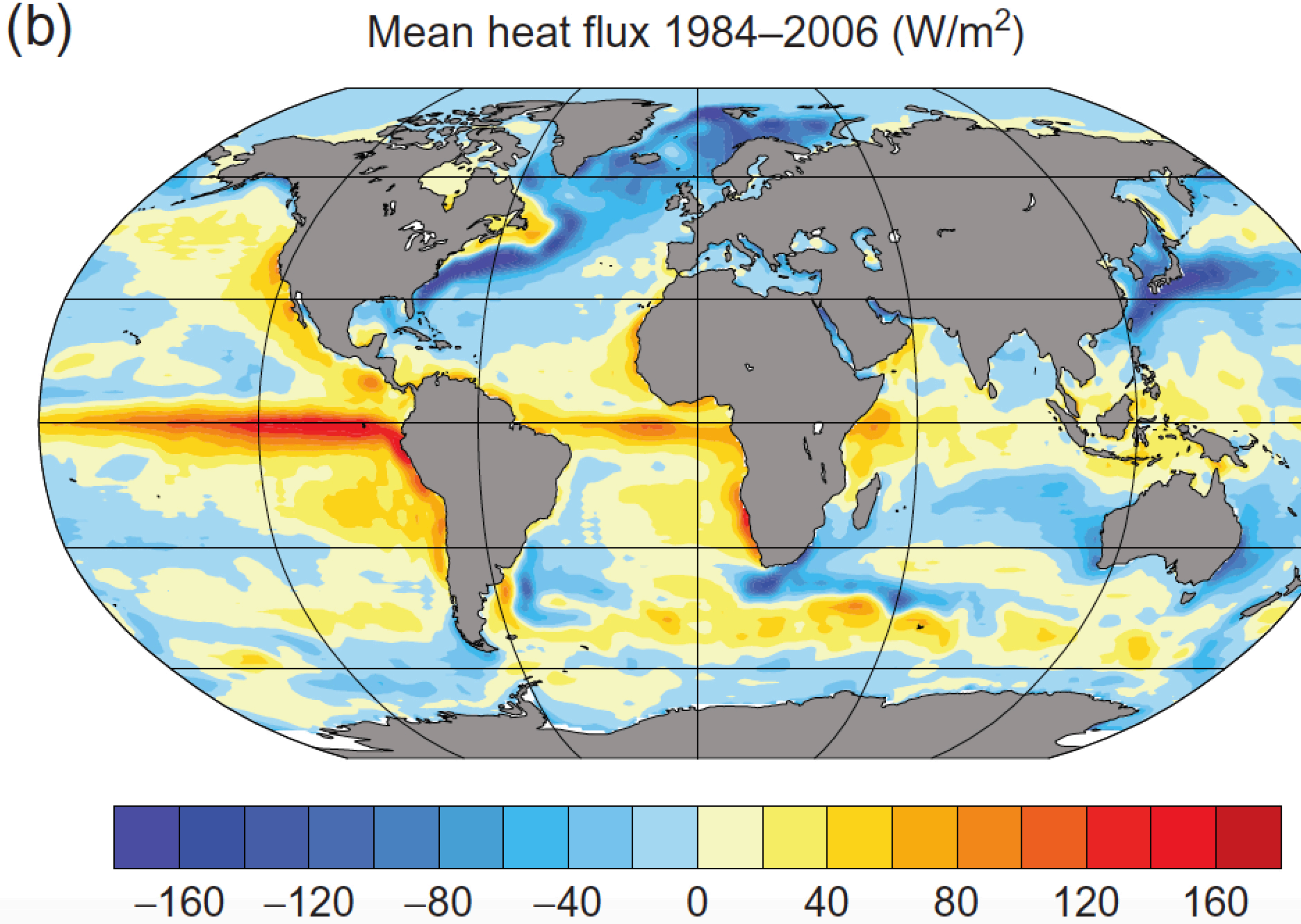


(a)
Mean wind stress and momentum flux 1984–2006 (N/m2)
0.2 N/m2
–0.2
–0.15
–0.1
–0.05
0
0.05
0.1
0.15
0.2

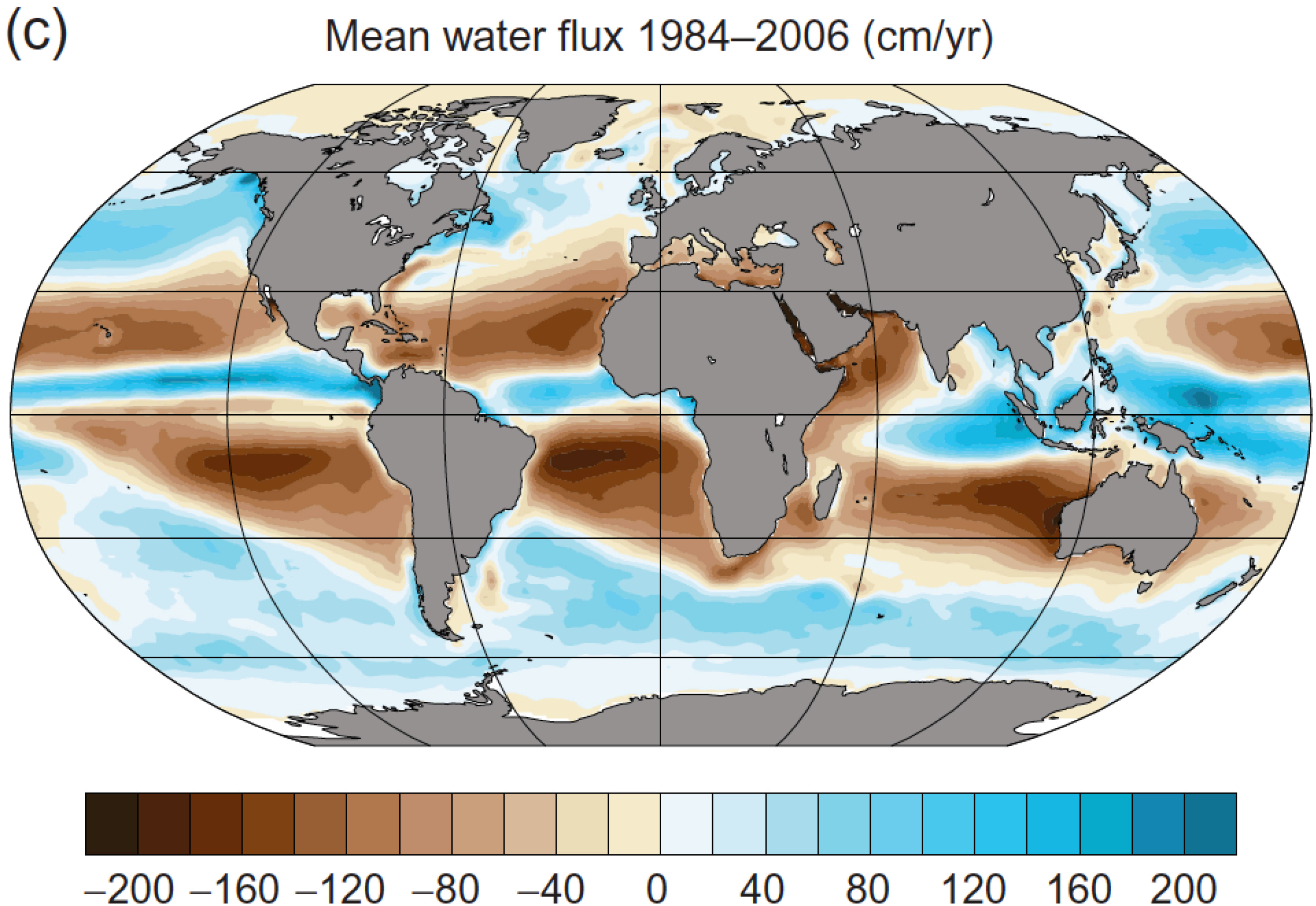
Surface Circulation



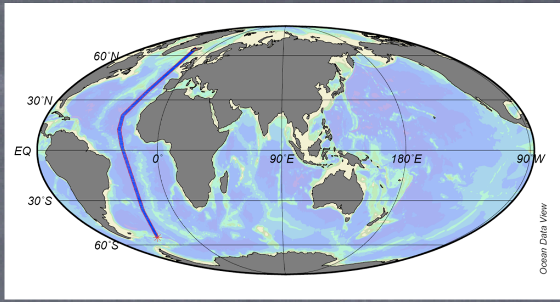
Heat Flux

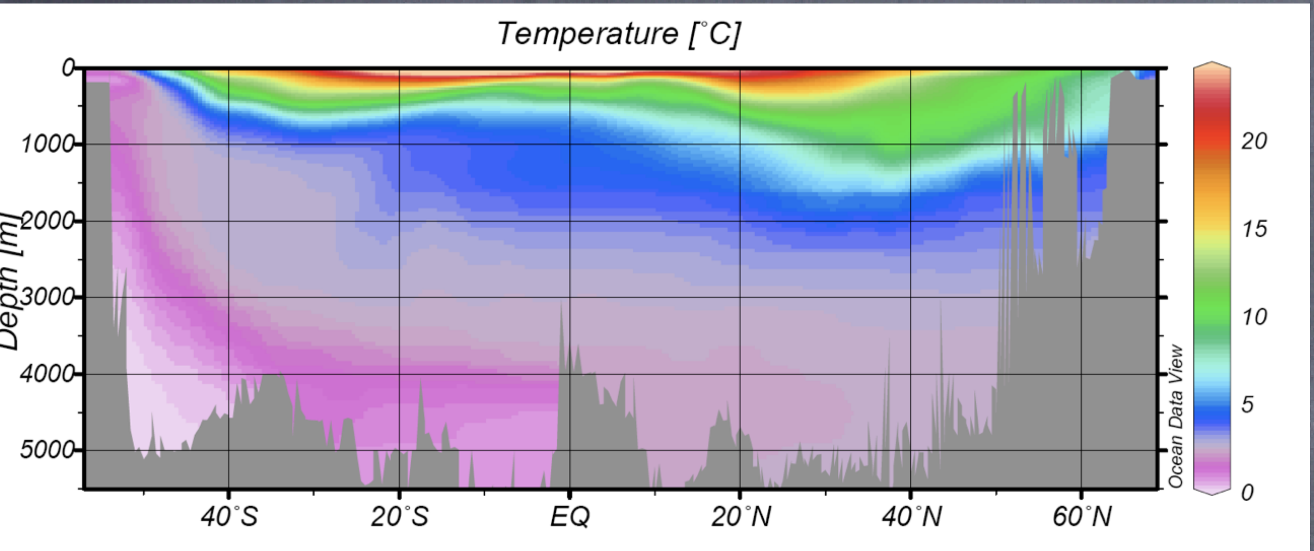


Fresh water flux

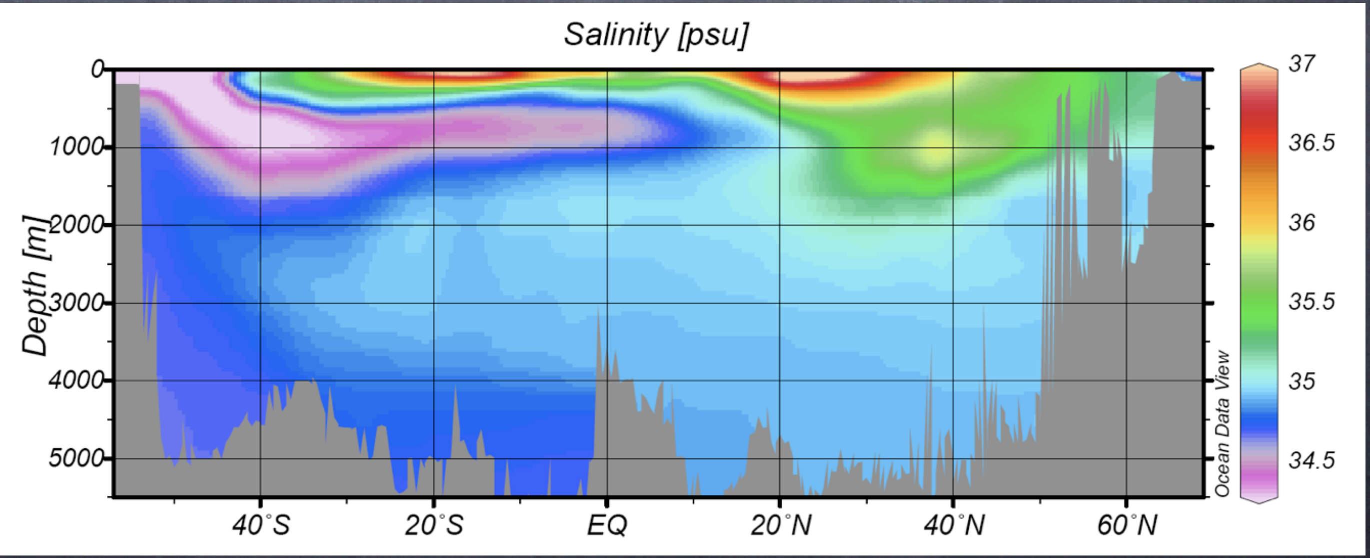


Along Atlantic N-S section:



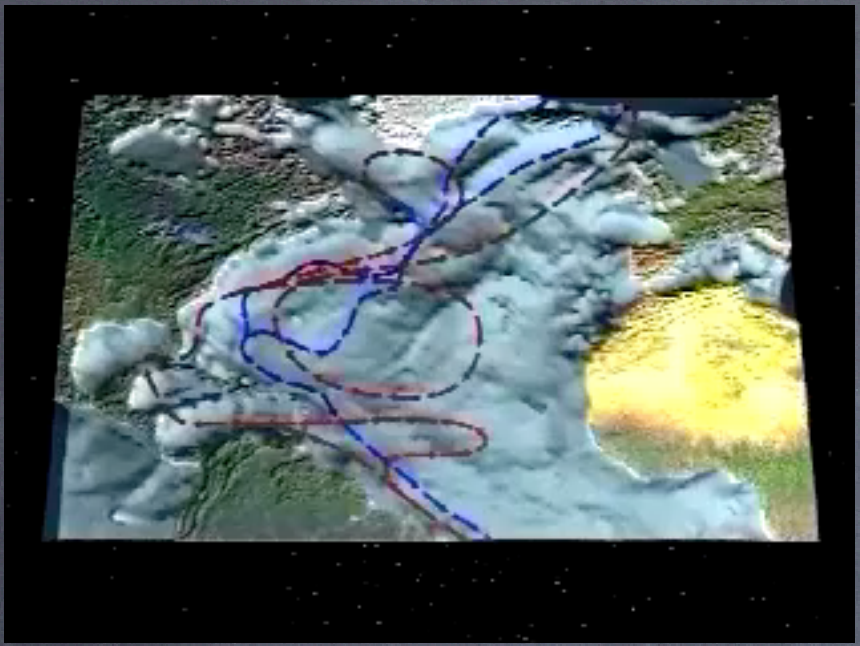


Temperature
(C)



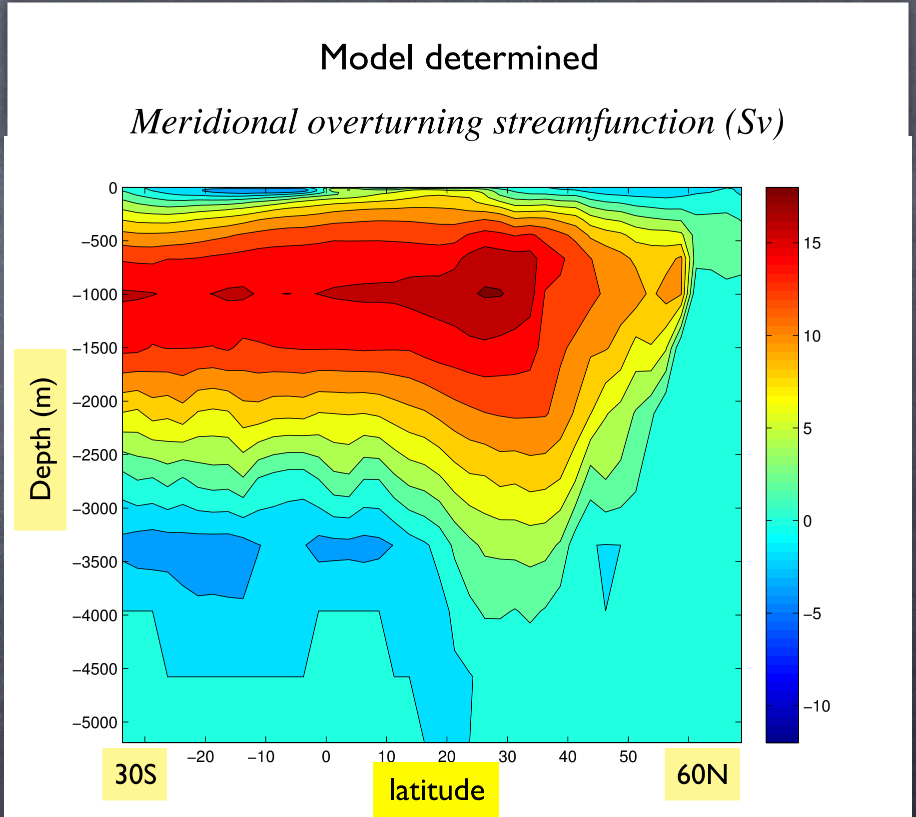
Salinity
(ppt)

Meridional Overturning Circulation (MOC)



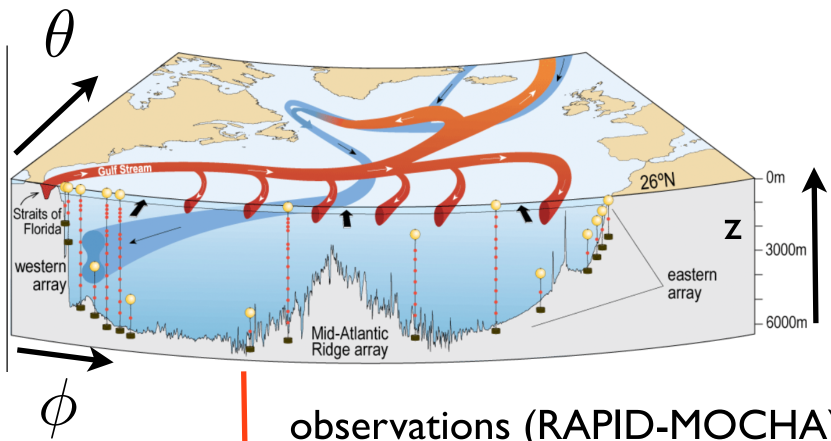
$$\Psi(y,z,t) = \int_z^0 \int_{x_w}^{x_e} v(x,y,z',t)\,dx\,dz'$$

v: meridional velocity

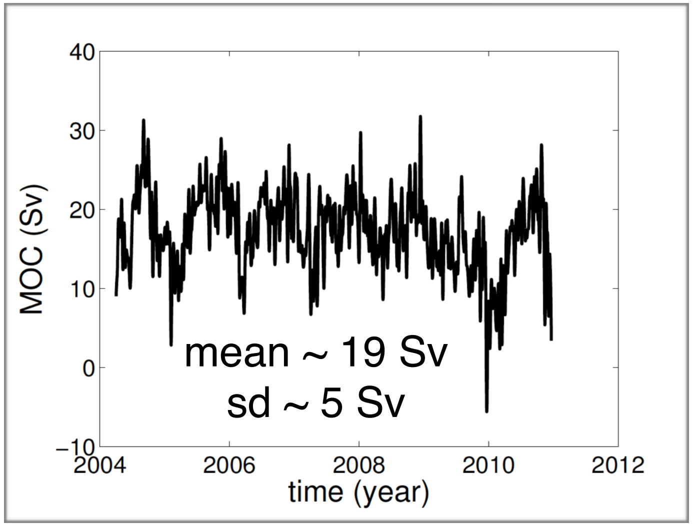


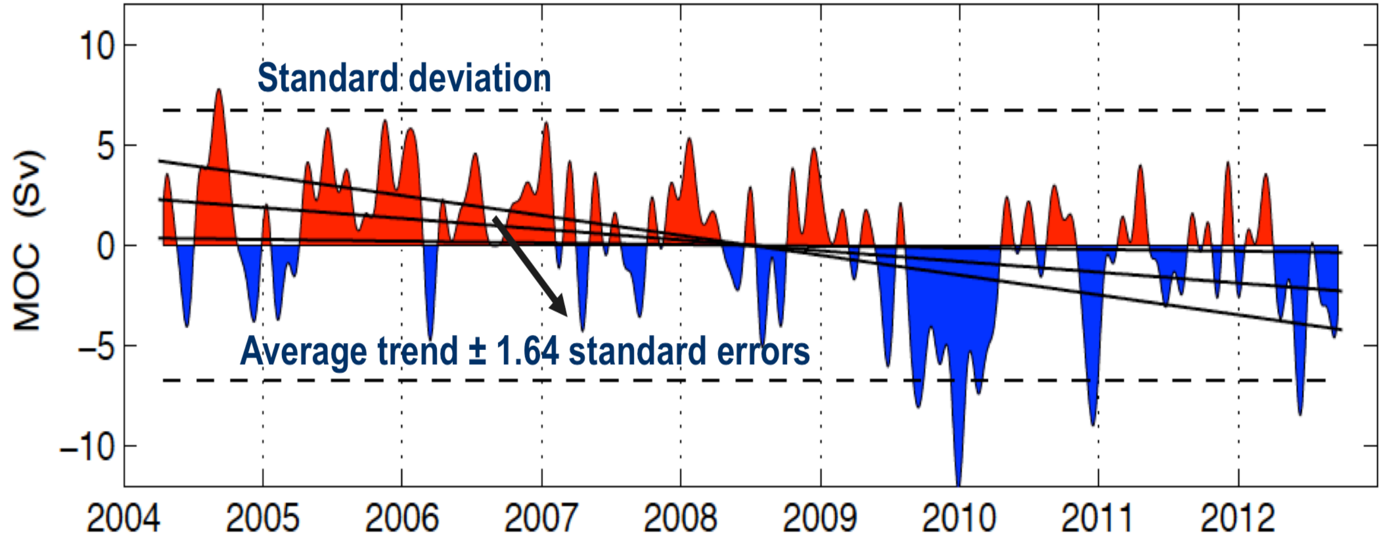
MOC: Volume transport in latitude/depth (is observable)

Changes in the MOC



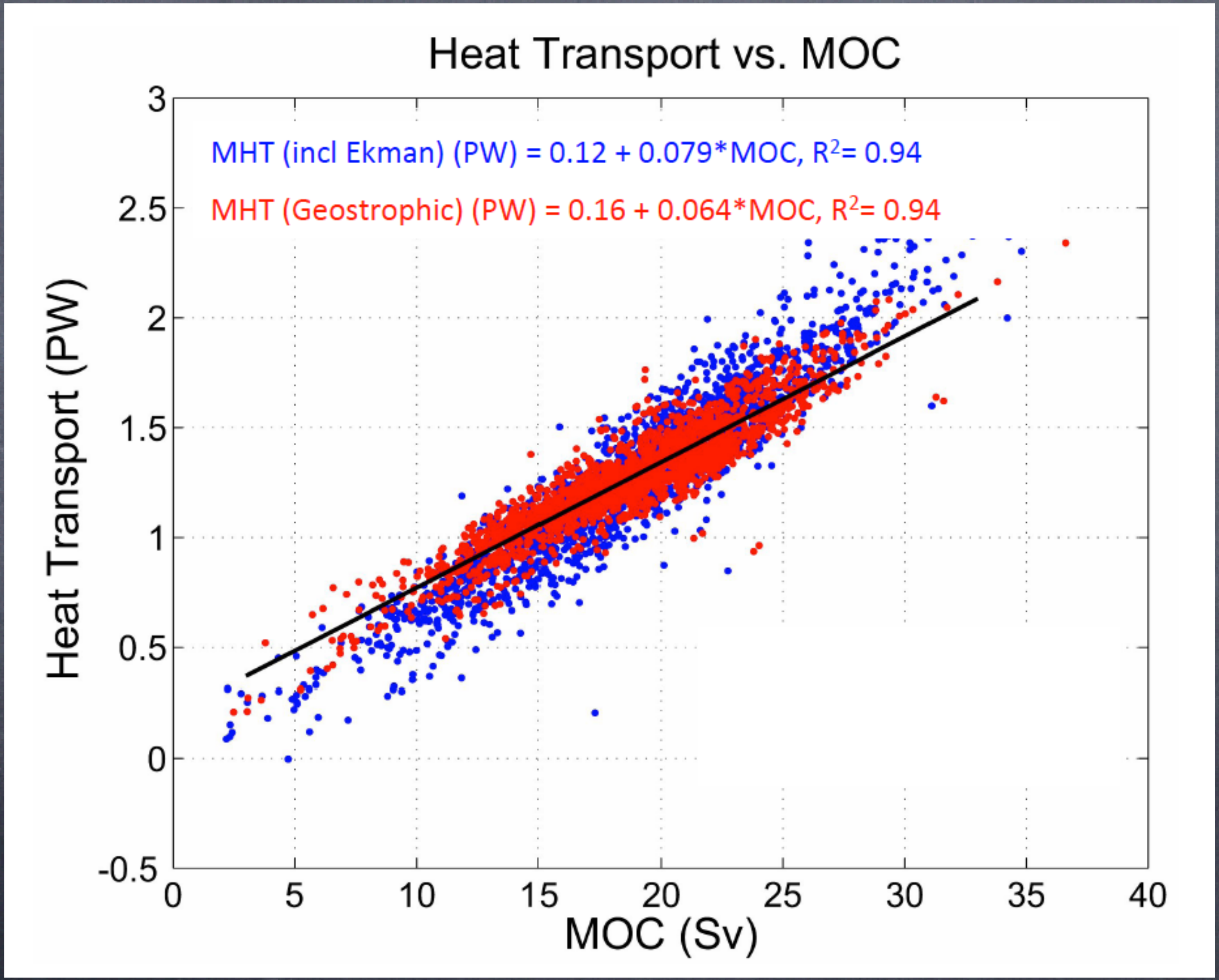
observations (RAPID-MOCHA)





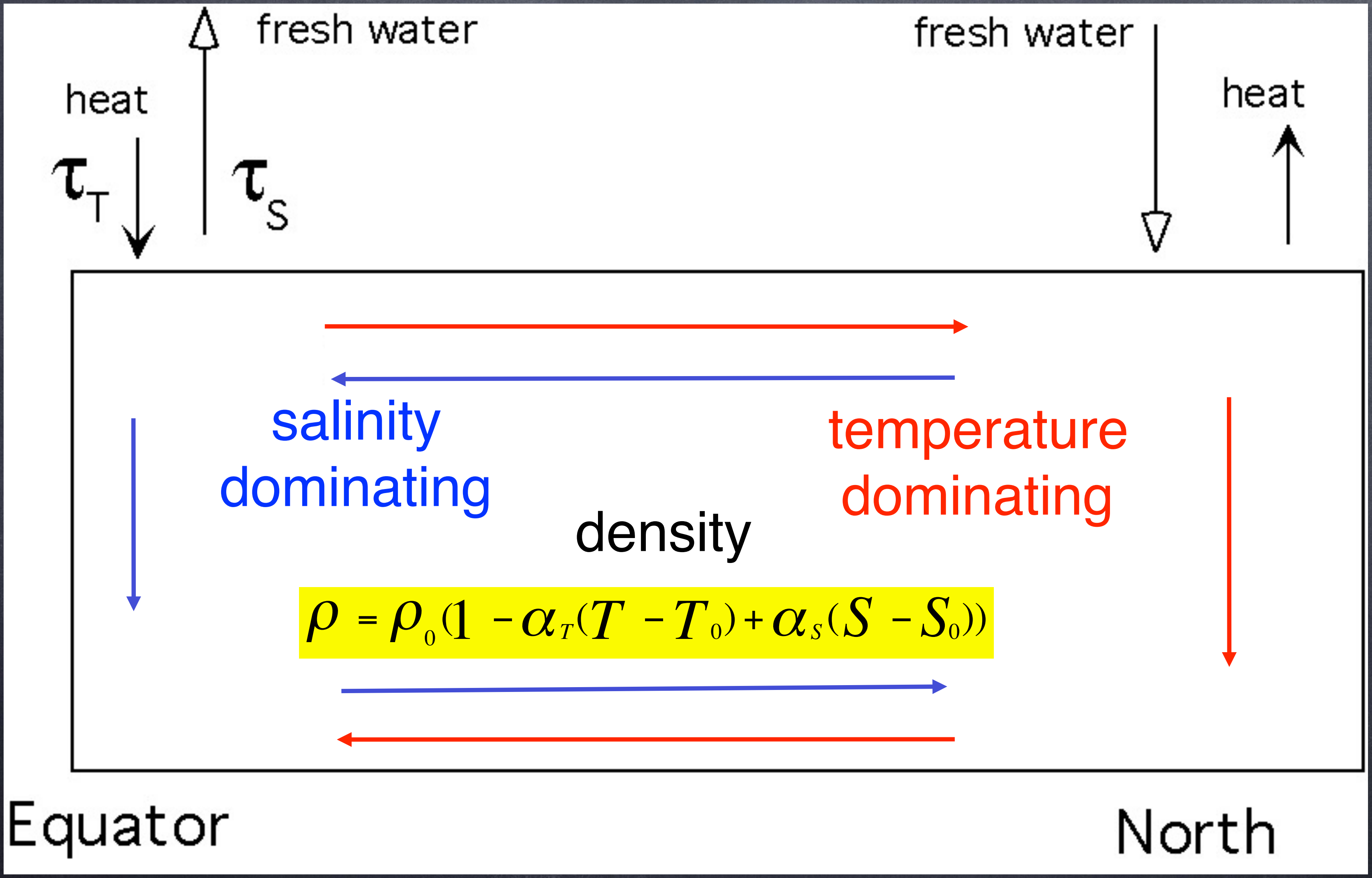
Smeet et al., Ocean Science, (2014)

Connection MOC and meridional heat transport



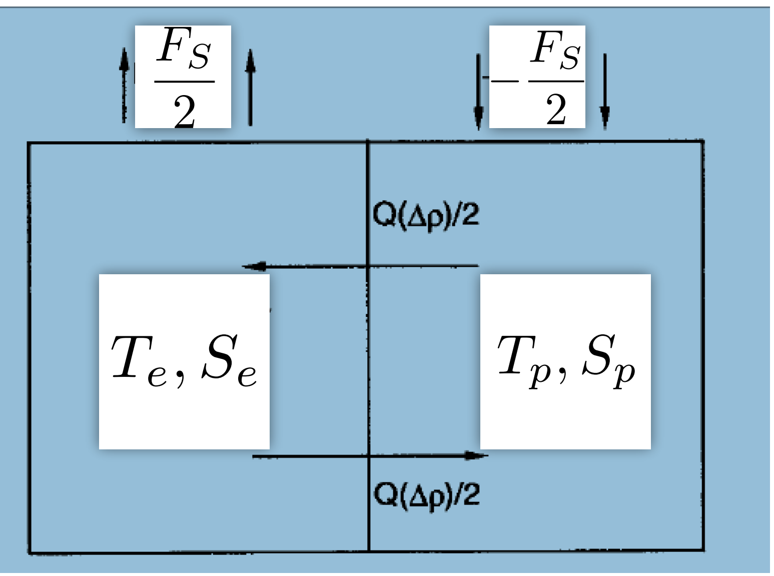
Johns et al., JClim, (2010)

Sensitivity of the MOC



The Stommel model

$$\frac{dT_e}{dt} = -\frac{1}{t_r}(T_e - (T_0 + \frac{\theta}{2})) - \frac{1}{2}Q(\Delta\rho)(T_e - T_p),$$
$$\frac{dT_p}{dt} = -\frac{1}{t_r}(T_p - (T_0 - \frac{\theta}{2})) - \frac{1}{2}Q(\Delta\rho)(T_p - T_e),$$
$$\frac{dS_e}{dt} = \frac{F_S}{2H}S_0 - \frac{1}{2}Q(\Delta\rho)(S_e - S_p),$$
$$\frac{dS_p}{dt} = -\frac{F_S}{2H}S_0 - \frac{1}{2}Q(\Delta\rho)(S_p - S_e),$$



$$\rho = \rho_0(1 - \alpha_T(T - T_0) + \alpha_S(S - S_0)),$$

$$Q(\Delta\rho) = \frac{1}{t_d} + \frac{q}{\rho_0^2 V}(\Delta\rho)^2,$$

Stommel, Tellus, (1961)

Cessi, JPO, (1994)

Dimensionless model

$$\begin{aligned}\frac{dx}{dt} &= -\alpha(x-1) - x(1+\mu^2(x-y)^2),\\ \frac{dy}{dt} &= F - y(1+\mu^2(x-y)^2),\end{aligned}$$

Parameter	Meaning	Value	Unit
t_r	temperature relaxation time scale	25	days
H	mean ocean depth	4500	m
t_d	diffusion time scale	180	years
t_a	advective time scale	29	years
q	transport coefficient	1.92×10^{12}	$m^3\ s^{-1}$
V	ocean volume	$300 \times 4.5 \times 8200$	km^3
α_T	thermal expansion coefficient	10^{-4}	K^{-1}
α_S	haline contraction coefficient	7.6 10^{-4}	-
S_0	reference salinity	35	$g\ kg^{-1}$
θ	meridional temperature difference	25	K

$$\mu^2 = \frac{q t_d (\alpha_T \theta)^2}{V}$$

$$\alpha = t_d / t_r$$

$$F = \frac{\alpha_S S_0 t_d}{\alpha_T \theta H} F_S.$$