



Koninklijk Meteorologisch Instituut  
Institut Royal Météorologique  
Königliche Meteorologische Institut  
Royal Meteorological Institute

# EDIPI predictability training with qgs

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Royal Meteorological Institute of Belgium

16-20 January 2023

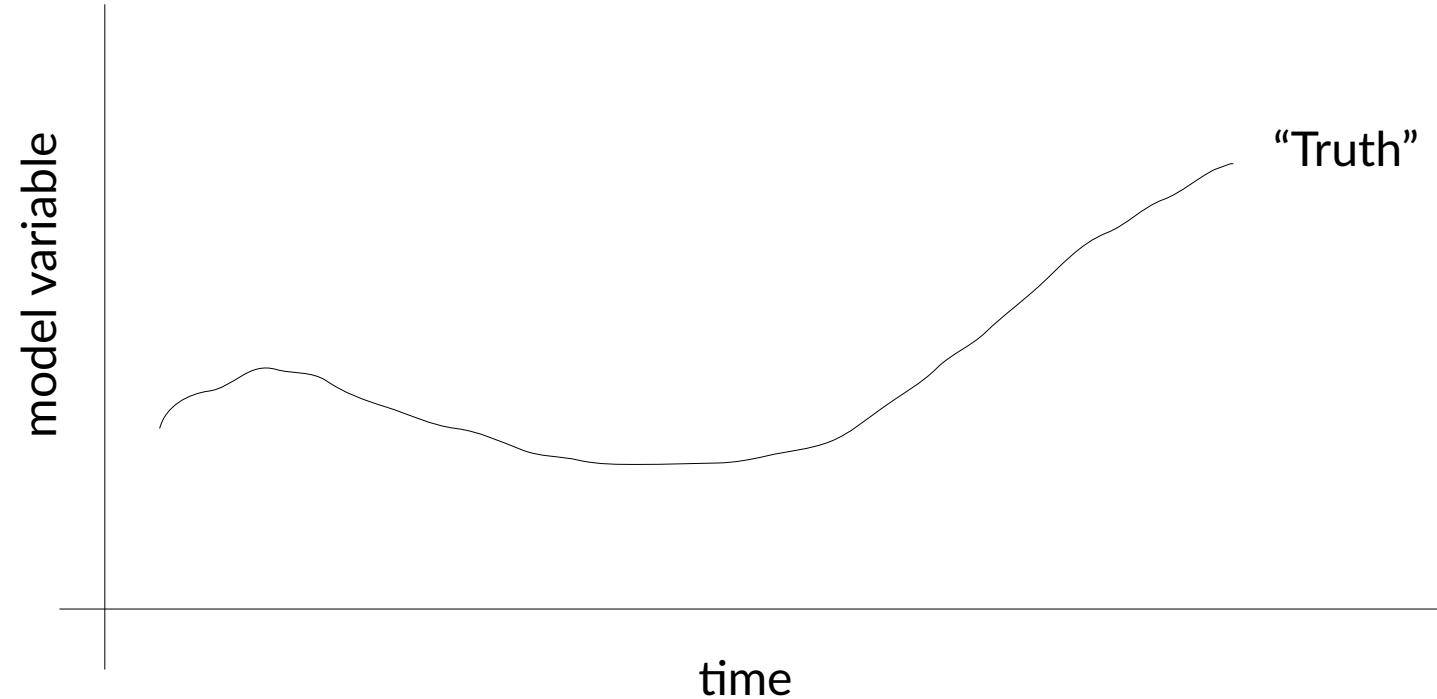


# Outline

- Introduction to forecasting
- Focus on predictability
- Introduction to qgs
- The goals of this training

# Forecasting chain

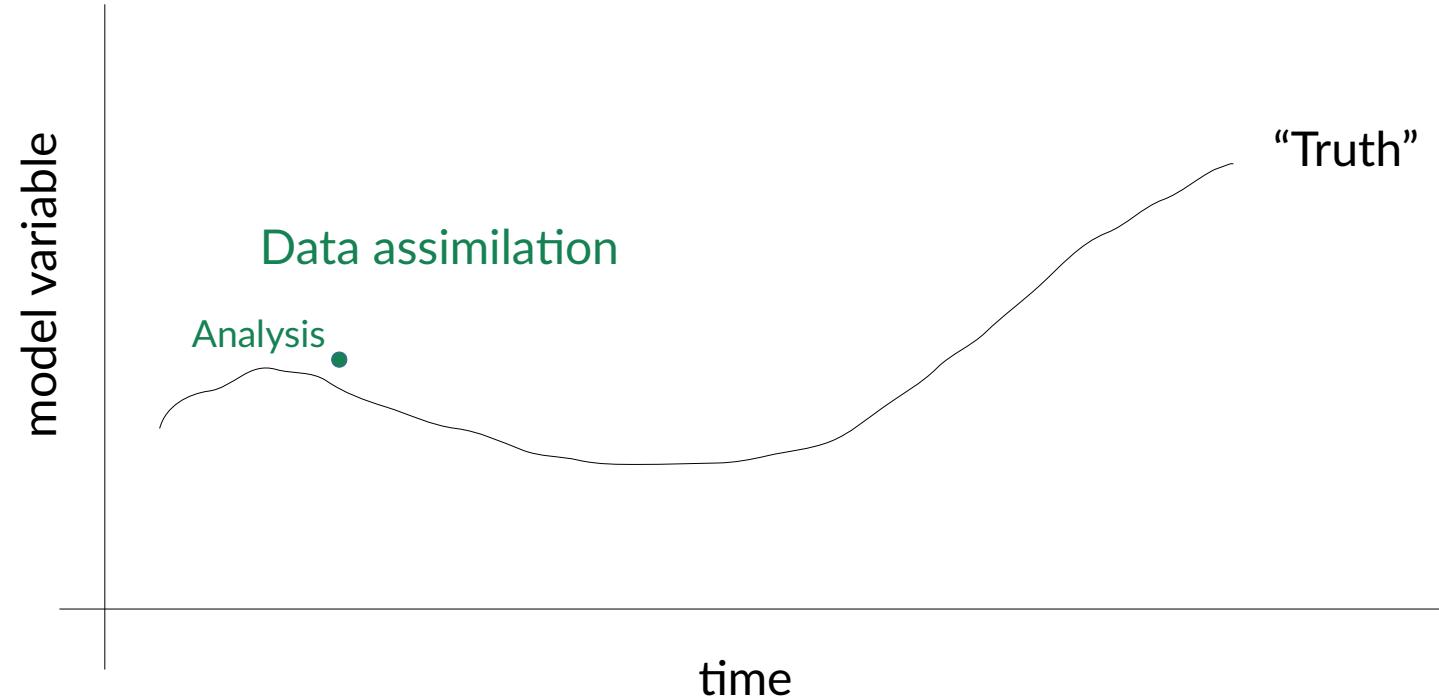
Weather forecasting is an initial value problem !



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## 1. Data assimilation:

Estimation of the best initial conditions for the model based on observations.



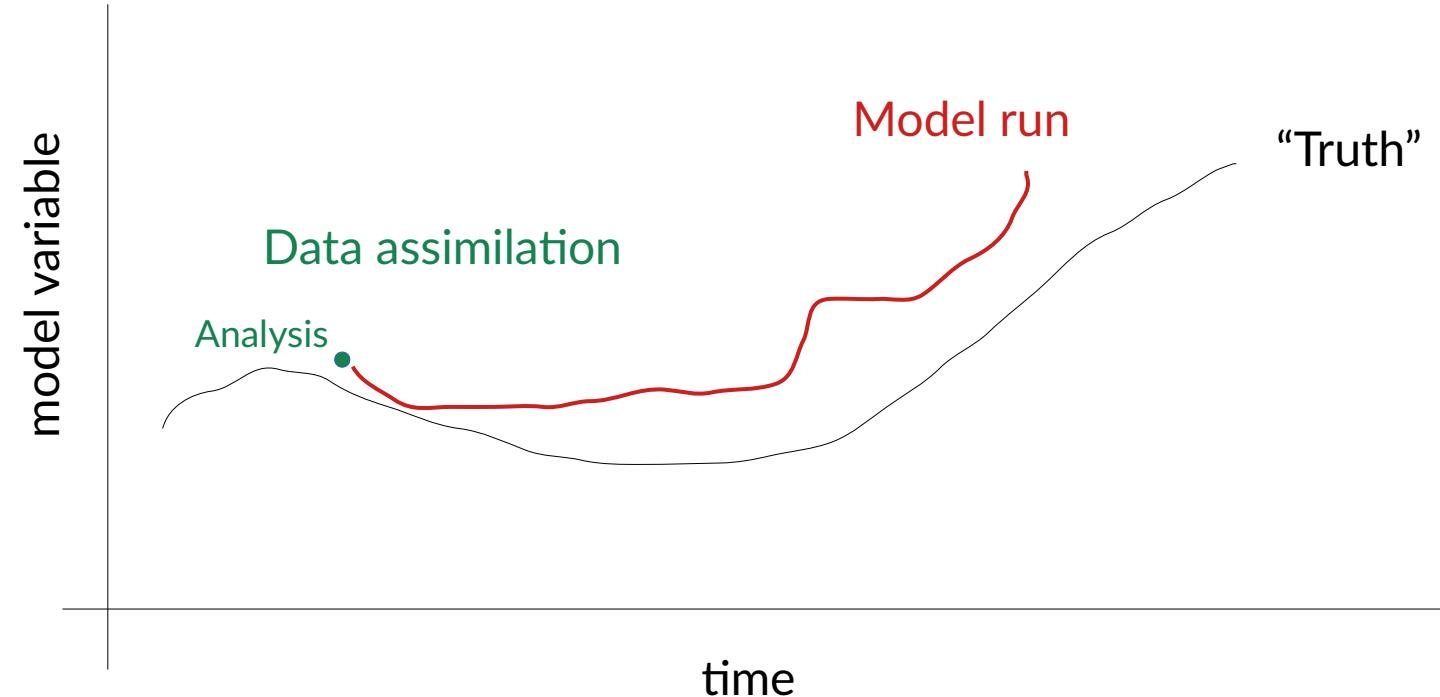
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## 2. Model run:

Computing the model time evolution.



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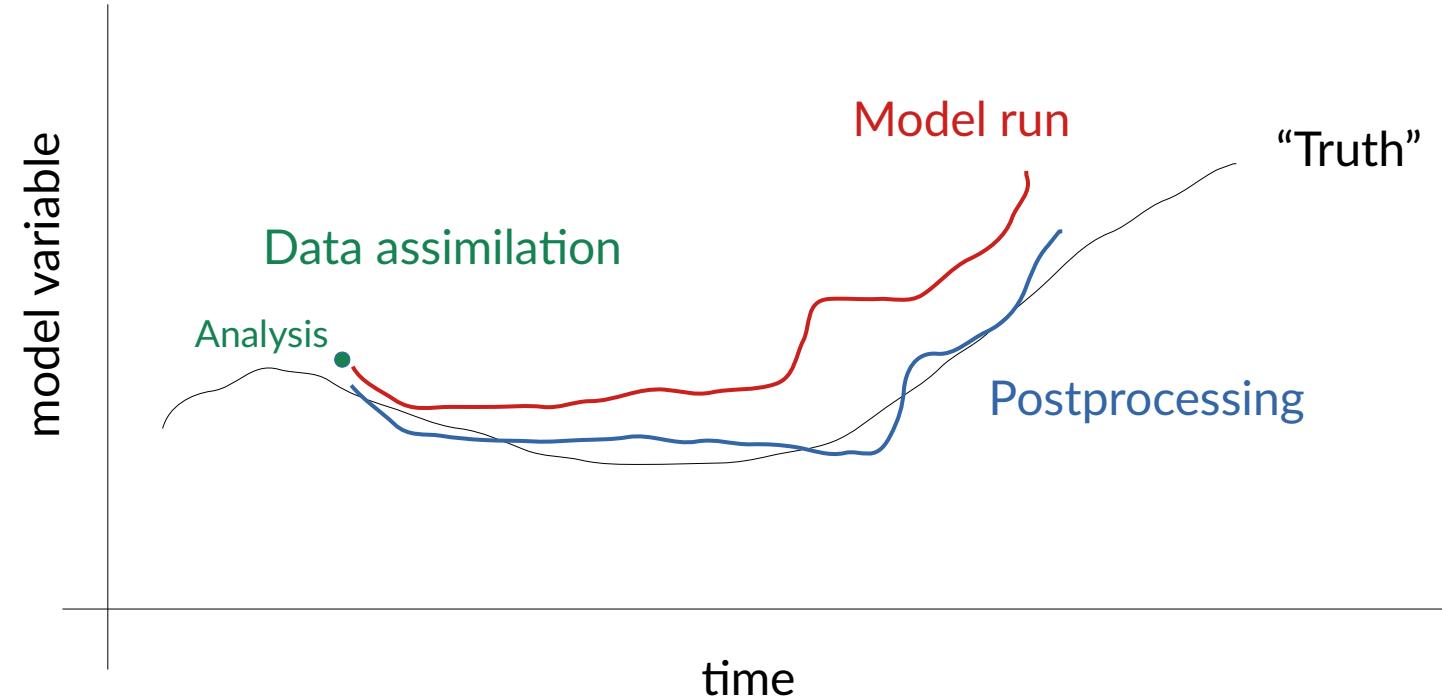
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Computing the model time evolution.

## 3. Postprocessing

Bringing back the model output to reality: bias correction, spread, ...



# Forecasting chain is a cycle !

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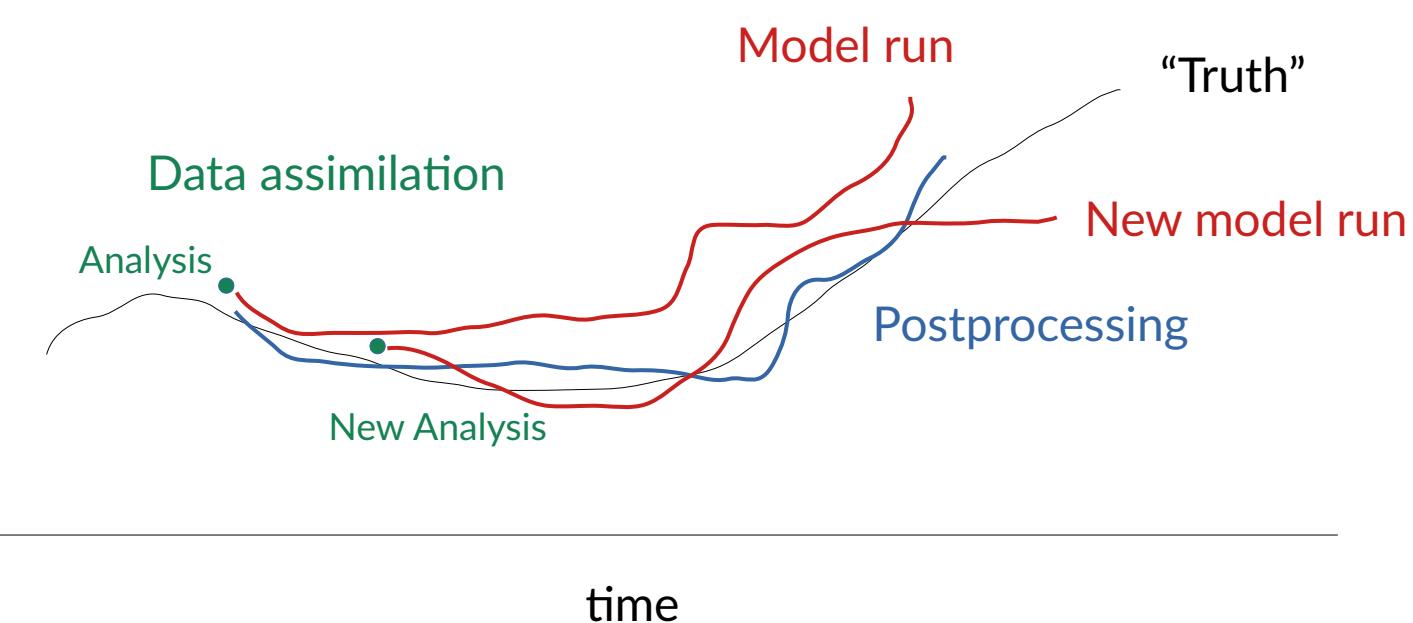
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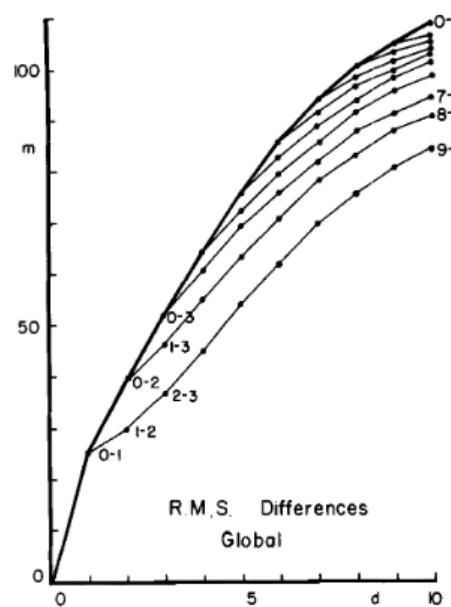
Bringing back the model output to reality: bias correction, spread, ...

Weather forecasting is an initial value problem !



The atmospheric models are chaotic systems :  
 → Sensitivity to initial conditions  
 → Forecasts become “wrong” over time

Predictability experiments in 1982:  
 global rate of the errors growth



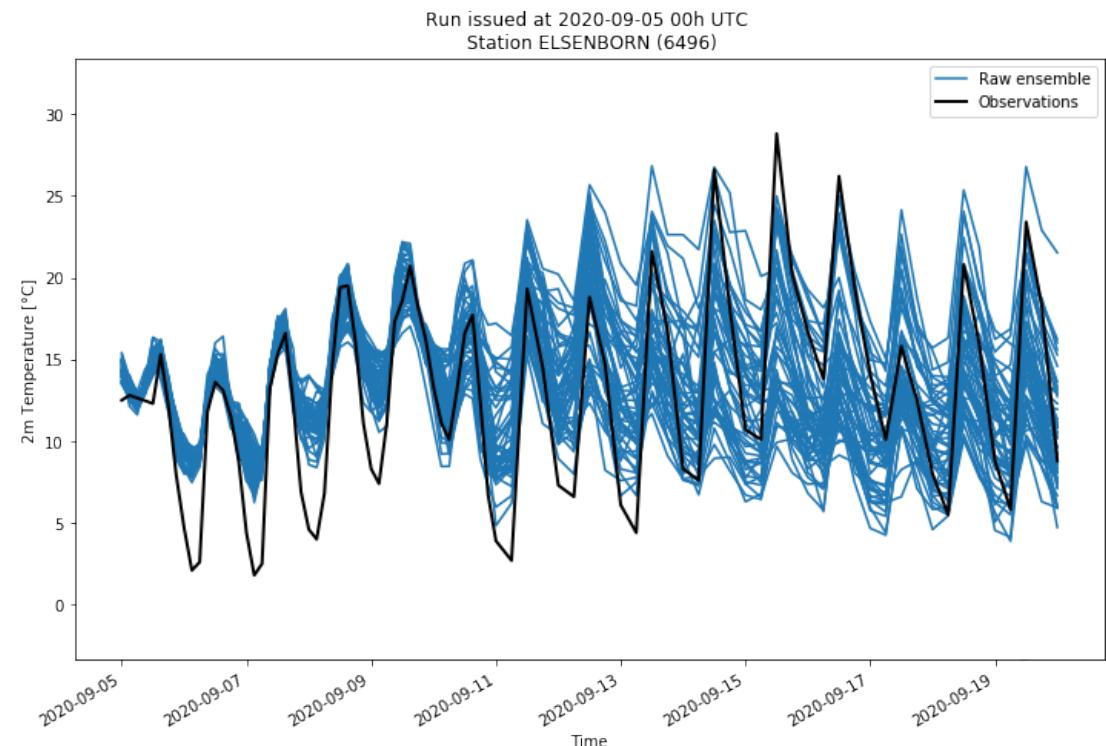
The doubling time quoted in most predictability studies is the doubling time for errors of very small amplitude. To estimate this time from Fig. 1 we should have to extrapolate the thin curves backward until they approached zero—a rather difficult task. The task becomes easy, however, if we

The rate at which separate solutions of the model diverge is supposed to approximate the rate at which separate solutions of the true atmospheric equations diverge. If it does, and if, at some time during the forecast, the model could suddenly be replaced by the true equations, the remainder of the heavy curve would follow one of the thin curves.

Lorenz 1982

Fig. 1. Global root-mean-square 500-mb height differences  $E_{jk}$  in meters, between  $j$ -day and  $k$ -day forecasts made by the ECMWF operational model for the same day, for  $j < k$ , plotted against  $k$ . Values of  $(j,k)$  are shown beside some of the points. Heavy curve connects values of  $E_{0k}$ . Thin curves connect values of  $E_{jk}$  for constant  $k - j$ .

Modern predictability experiments:  
 local properties and ensemble forecasts



How much wrong models become over time depend on the model state.  
→ In other words, how predictable is a model is a local property.



Figure 1: The scientific basis for ensemble prediction illustrated by the prototypical Lorenz model of low-order chaos, showing that in a nonlinear system, predictability is flow dependent. (a) A forecast with high probability, (b) forecast with moderate predictability, (c) forecast with low predictability.

# How to define the local predictability?

Consider a uniform ball of initial conditions around a model trajectory in the state space.

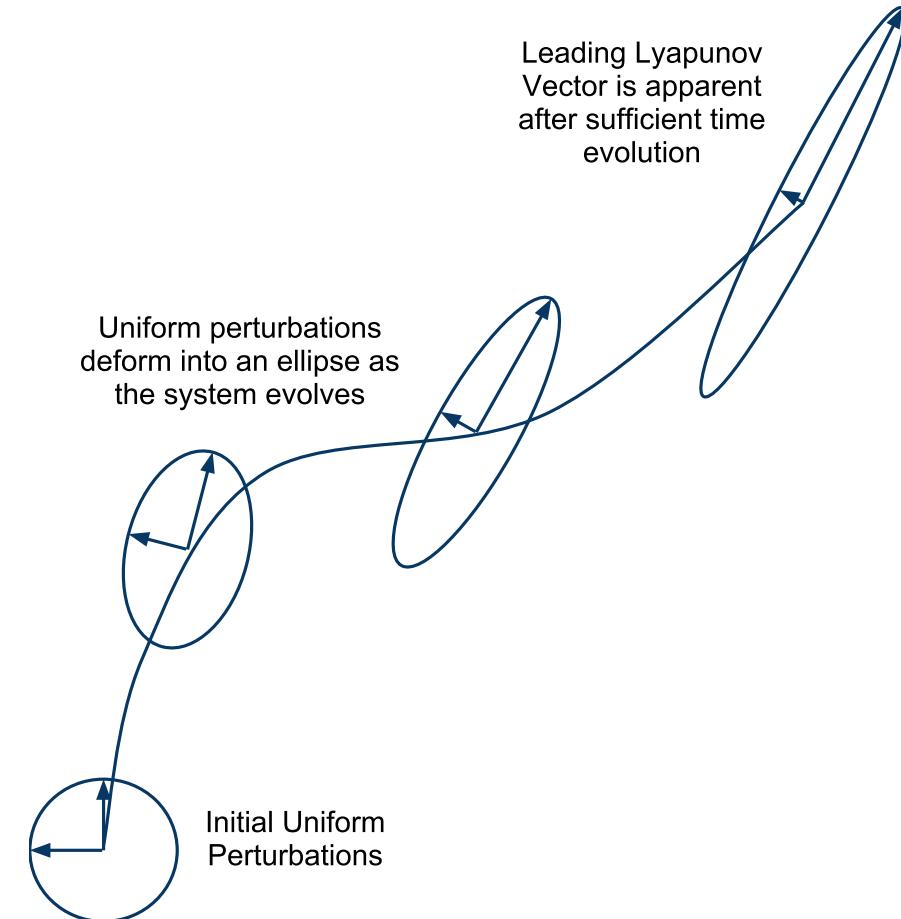
Over time:

- 1) The stable directions around the trajectory will get closer to it, and the ball “shrinks” along these directions.
- 2) The unstable directions around the trajectory will get away from it, and the ball expands along these directions.

→ Over time, the ball deforms into an ellipsoid.  
→ After a while, the most unstable direction becomes apparent.

If the length of the semiaxes of the ellipsoid are denoted  $\sigma_i$ , then the rates at which they stretch are called the Lyapunov exponents:

$$\sigma_i \sim e^{\lambda_i t} \quad \text{for long times } t$$



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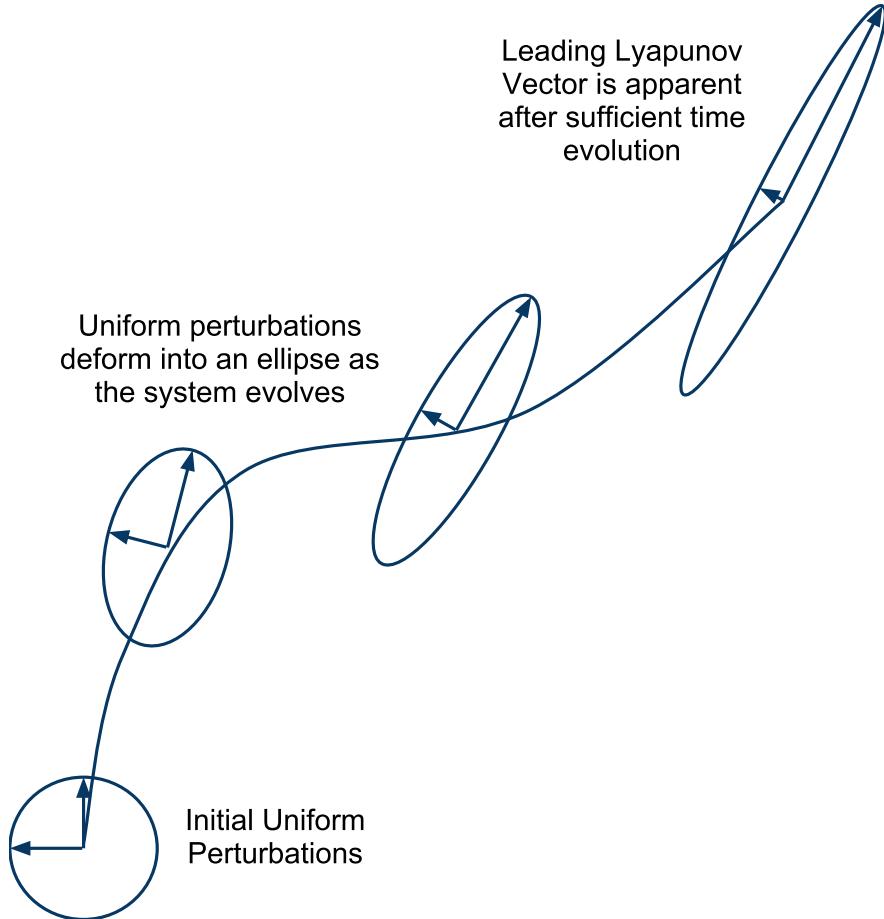
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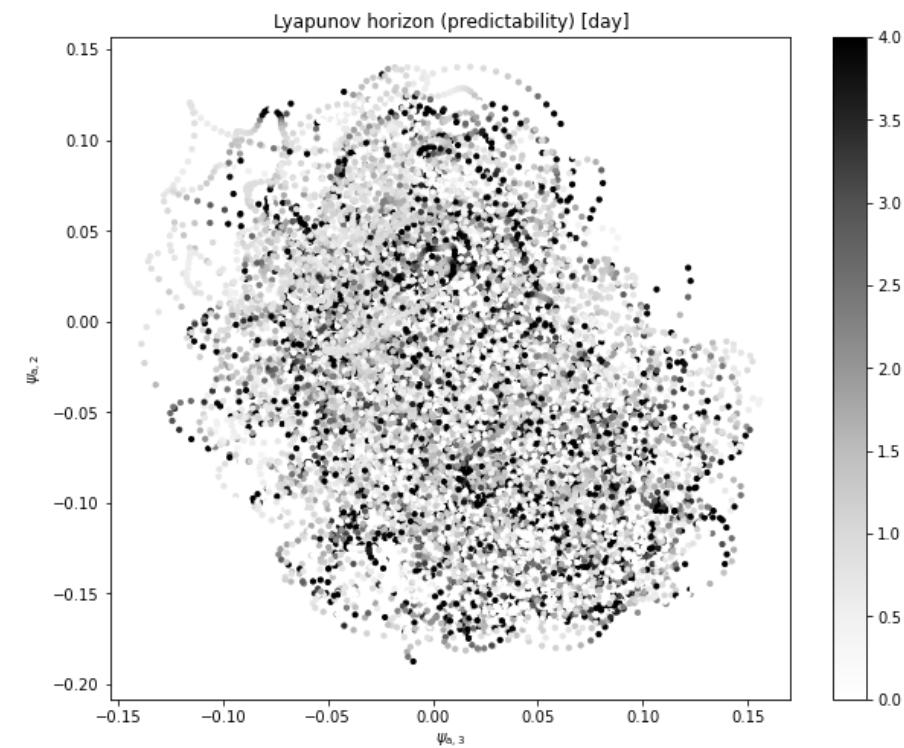
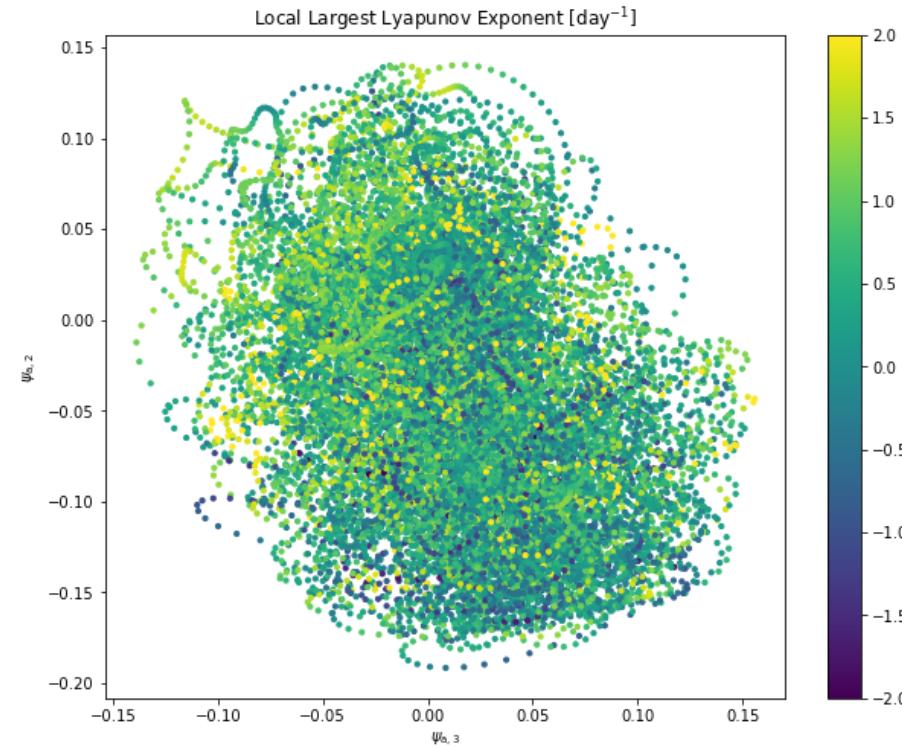
$$\sigma_i \sim e^{\lambda_i t} \quad \text{for long times } t$$



**Predictability: timescale  $1 / \lambda_1$ , where  $\lambda_1$  is the stretching rate of the most unstable direction.**

# Local predictability over a model attractor

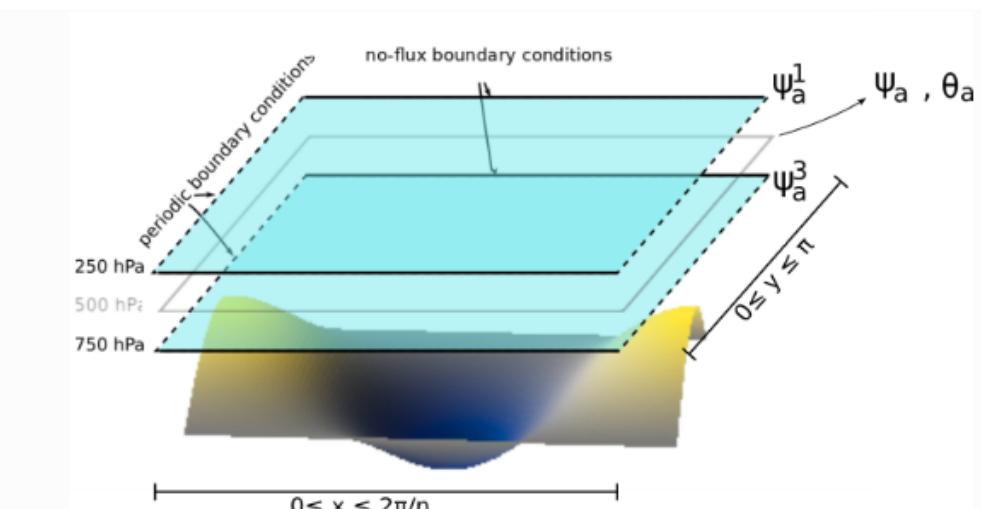
Attractor = set of states toward which a system tends to evolve  
Dissipative system !



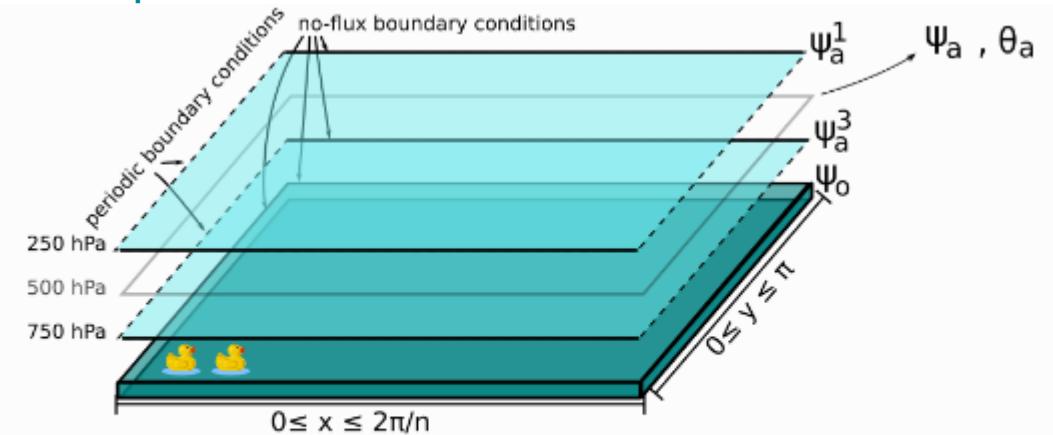
2-dimensional projection of the qgs atmospheric model attractor with the local predictability superimposed

qgs is an atmospheric model for midlatitudes.  
It models the dynamics of:

- a 2-layer quasi-geostrophic channel atmosphere
- on a beta plane (linearized Coriolis parameter)
- Coupling with a simple land (orography) or shallow-water ocean component (i.e. MAOOAM):
  - Momentum exchange (friction)
  - Heat exchange scheme or relaxation to climatological temperature



Sketch of the atmospheric model layers with a simple *orography*. The domain ( $\beta$ -plane)  
*zonal and meridional* coordinates are labeled as the  $x$  and  $y$  variables.

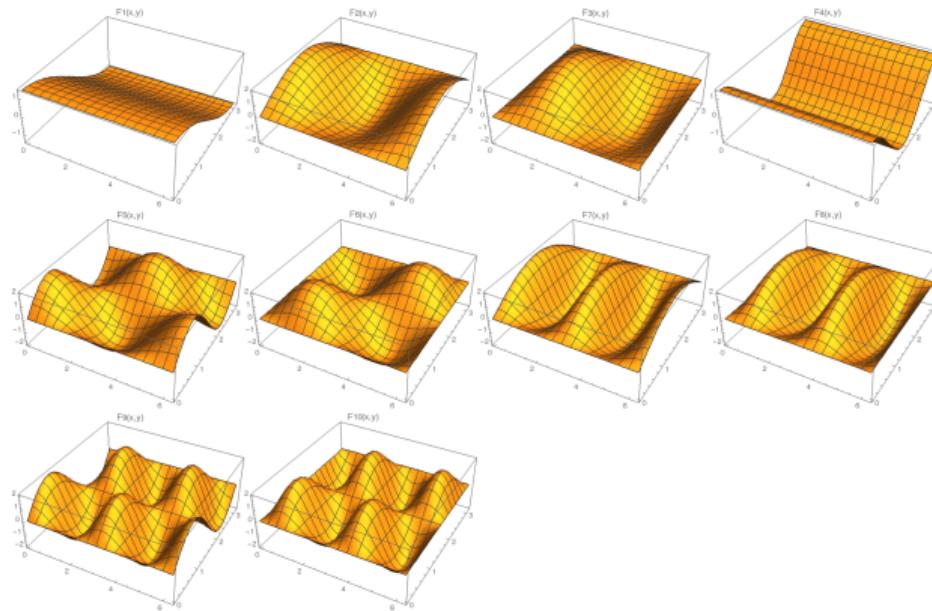


Sketch of the ocean-atmosphere coupled model layers. The domain ( $\beta$ -plane) *zonal and meridional* coordinates are labeled as the  $x$  and  $y$  variables.

## ■ Spectral approach:

Fourier expansion of the stream function fields  $\psi_a = (\psi_a^1 + \psi_a^3)/2$   
and the temperature fields  $\theta_a = (\psi_a^1 - \psi_a^3)/2$

10 atmospheric modes:  $F_i(x, y)$



Atm. streamfunction

$$\psi_a(x, y) = \sum_{i=1}^{n_a} \psi_{a,i} F_i(x, y)$$

Atm. temperature

$$\theta_a(x, y) = \sum_{i=1}^{n_a} \theta_{a,i} F_i(x, y)$$



20-dimensional system



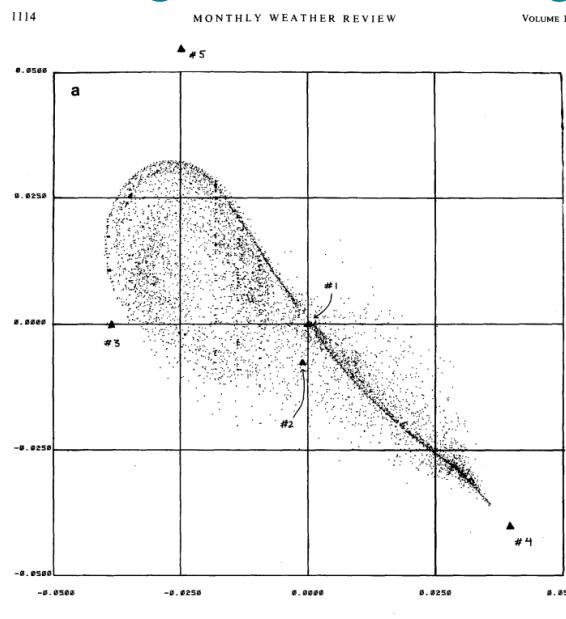
Same for the other  
components (land, ocean)



More modes can be added

## Advantage of quasi-geostrophic (QG) spectral models

- Offer a good representation of the dry atmospheric dynamics
- Allow one to identify typical features of the atmospheric circulation, such as blocked and zonal circulation regimes, and low-frequency variability
- Reduced- or intermediary-order model → understanding of the processes at play
- Fast generation of long time series



Reinhold & Pierrehumbert, MWR 1982

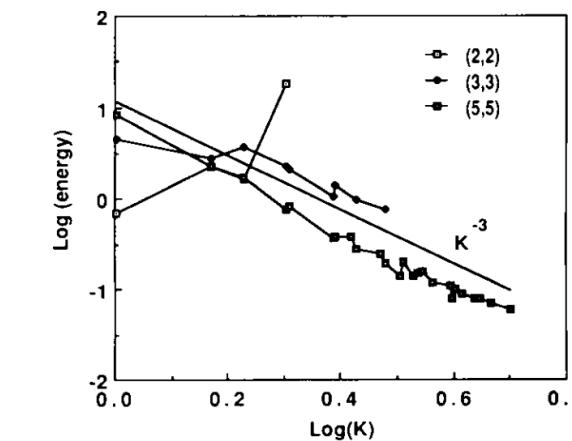


Fig. 10. Kinetic energy spectrum as a function of total wavenumber,  $K$ , for  $(2, 2)$ ,  $(3, 3)$  and  $(5, 5)$  models. A  $K^{-3}$  profile is also shown.

O'Brien & Branscome, Tellus 1988

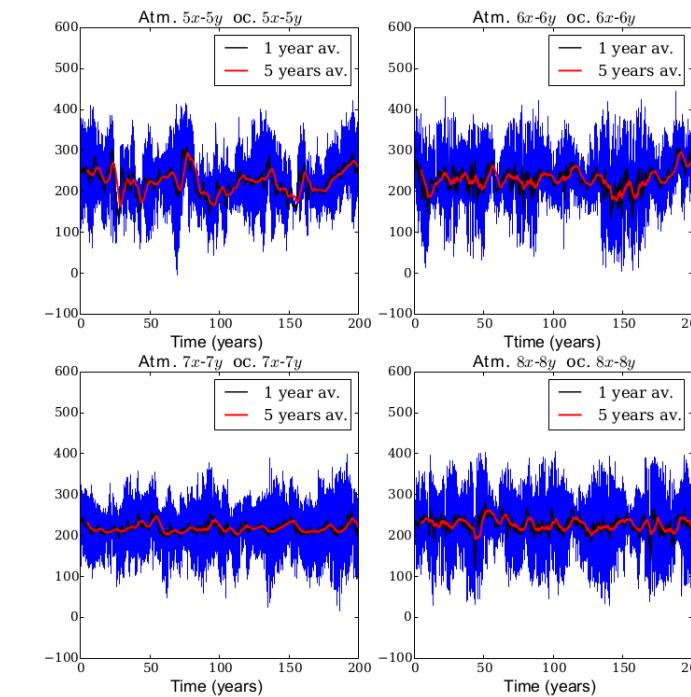
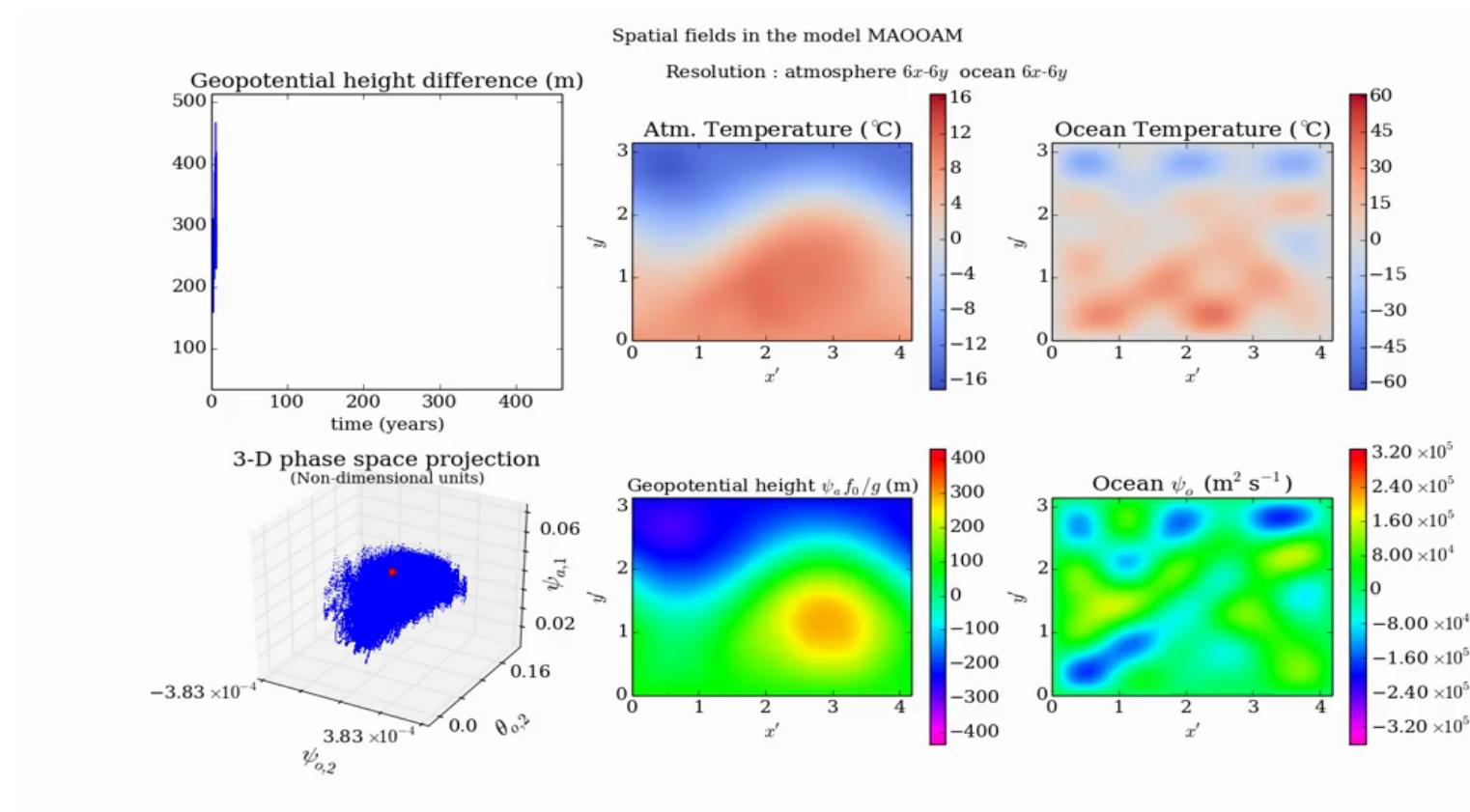


Figure 8. Time series of the geopotential height difference (continued from Fig. 7).

De Cruz et al.,  
GMD 2016.

# Example: MAOOAM with qgs

If you couple the atmosphere to a shallow-water ocean component...



Click on the image to play the video.

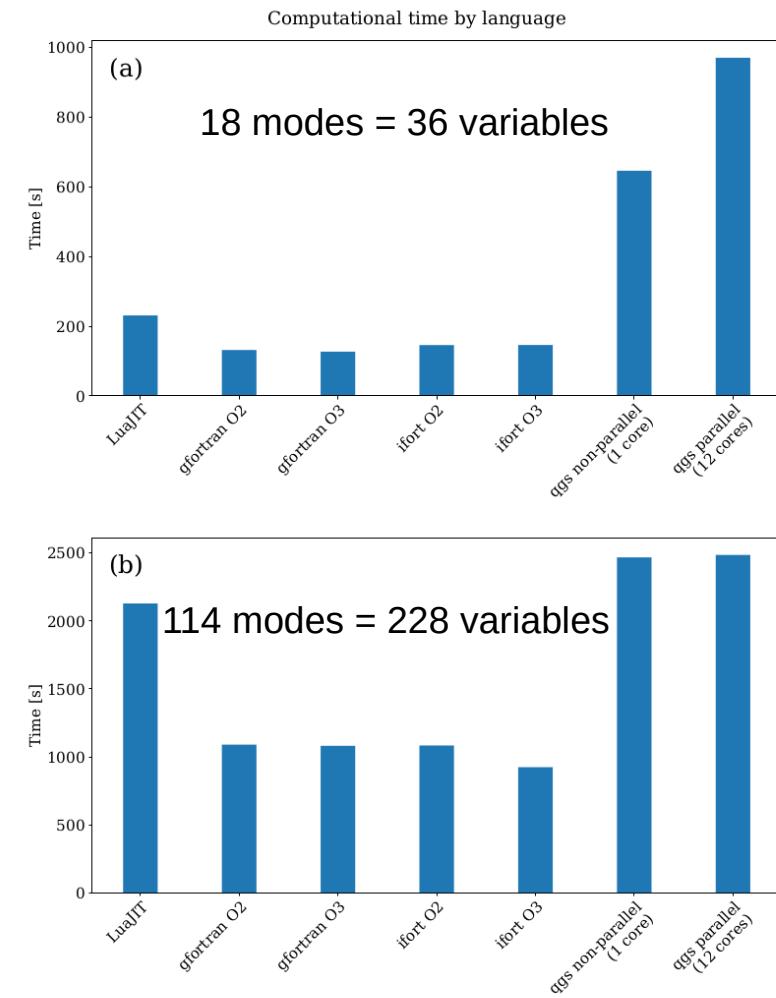
## qgs was written with some key points in mind :

- A spectral QG model where it is easy to add new features and scales to the model (modularity)
- Coupling of the atmospheric QG model to different components (ocean, land, ...)
- Straightforward configuration and usage with modern tools (Jupyter notebook)
- **A reasonably fast Python implementation** (thanks to Numba)
- Free lunch: Easily interface with amazing libraries available (e.g: SciPy, pyDMD, machine learning libraries, etc...)
- Symbolic core using the SymPy library
- Didactical and comprehensive documentation

 jupyter

 Numba

  
SymPy



**Figure 1:** Computational times in seconds of different MAOOAM implementations: (a) time to compute a  $10^7$  timeunits trajectory with a low-order model configuration (36 variables). (b) time to compute a  $10^6$  timeunits trajectory with a higher-order model configuration (228 variables).

## Simple example: An atmosphere with an orography

### Loading modules

```
Entrée [5]: from params.params import QgParams
from integrators.integrator import RungeKuttaIntegrator, RungeKuttaTglSIntegrator
from functions.tendencies import create_tendencies
from plotting.util import std_plot
```

### Setting and printing the model parameters

```
Entrée [7]: # Model parameters instantiation with some non-default specs
model_parameters = QgParams({'phi0_npi': np.deg2rad(50.)/np.pi, 'hd':0.3})
# Mode truncation at the wavenumber 2 in both x and y spatial coordinate
model_parameters.set_atmospheric_channel_fourier_modes(2, 2)

# Changing (increasing) the orography depth and the meridional temperature gradient
model_parameters.ground_params.set_orography(0.4, 1)
model_parameters.atemperature_params.set_thetas(0.2, 0)
```

```
Entrée [8]: # Printing the model's parameters
model_parameters.print_params()
```

```
Qgs parameters summary
=====

General Parameters:
'time_unit': days,
'rr': 287.058 [J][kg^-1][K^-1] (gas constant of dry air),
'sb': 5.67e-08 [J][m^-2][s^-1][K^-4] (Stefan-Boltzmann constant),

Scale Parameters:
'scale': 5000000.0 [m] (characteristic space scale (L*pi)),
'f0': 0.0001032 [s^-1] (Coriolis parameter at the middle of the domain),
'n': 1.3 (aspect ratio (n = 2 L_y / L_x)),
'rra': 6370000.0 [m] (earth radius),
'phi0_npi': 0.2777777777777778 (latitude expressed in fraction of pi),
'deltap': 50000.0 [Pa] (pressure difference between the two atmospheric layers),

Atmospheric Parameters:
'kd': 0.1 [nondim] (atmosphere bottom friction coefficient),
'kdn': 0.01 [nondim] (atmosphere internal friction coefficient).
```

## Simple example: An atmosphere with an orography

### Integrating the model

Creating the tendencies function

Entrée [9]:

```
%%time
f, Df = create_tendencies(model_parameters)
```

```
CPU times: user 4.34 s, sys: 86.6 ms, total: 4.43 s
Wall time: 2.3 s
```

### Time integration

Defining an integrator

Entrée [10]:

```
integrator = RungeKuttaIntegrator()
integrator.set_func(f)
```

Start on a random initial condition and integrate over a transient time to obtain an initial condition on the attractors

Entrée [11]:

```
%%time
ic = np.random.rand(model_parameters.ndim)*0.1
integrator.integrate(0., 200000., dt, ic=ic, write_steps=0)
time, ic = integrator.get_trajectories()
```

```
CPU times: user 13.4 ms, sys: 12.4 ms, total: 25.8 ms
Wall time: 7.83 s
```

Now integrate to obtain a trajectory on the attractor

Entrée [12]:

```
%%time
integrator.integrate(0., 100000., dt, ic=ic, write_steps=write_steps)
time, traj = integrator.get_trajectories()
```

## Simple example: An atmosphere with an orography

### Plotting the result

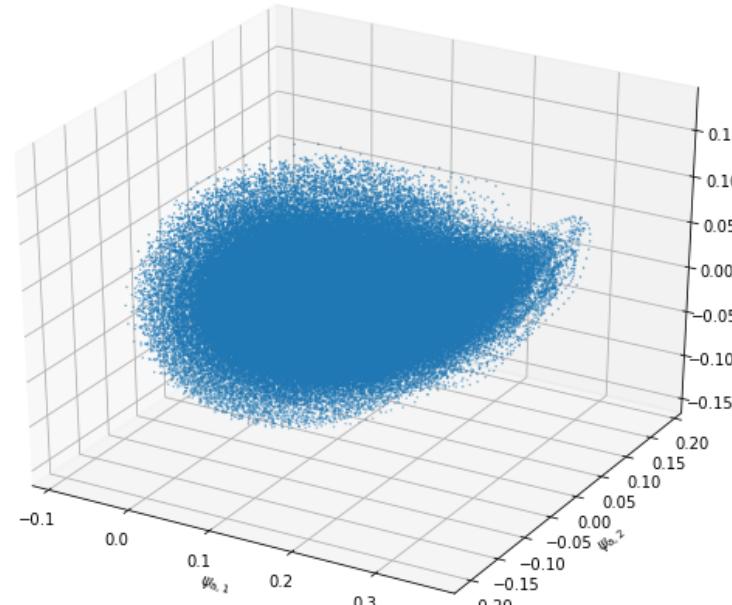
Entrée [13]:

```
varx = 0
vary = 1
varz = 2

fig = plt.figure(figsize=(10, 8))
axi = fig.gca(projection='3d')

axi.scatter(traj[varx], traj[vary], traj[varz], s=0.2);

axi.set_xlabel('$'+model_parameters.latex_var_string[varx]+'$')
axi.set_ylabel('$'+model_parameters.latex_var_string[vary]+'$')
axi.set_zlabel('$'+model_parameters.latex_var_string[varz]+'$');
```

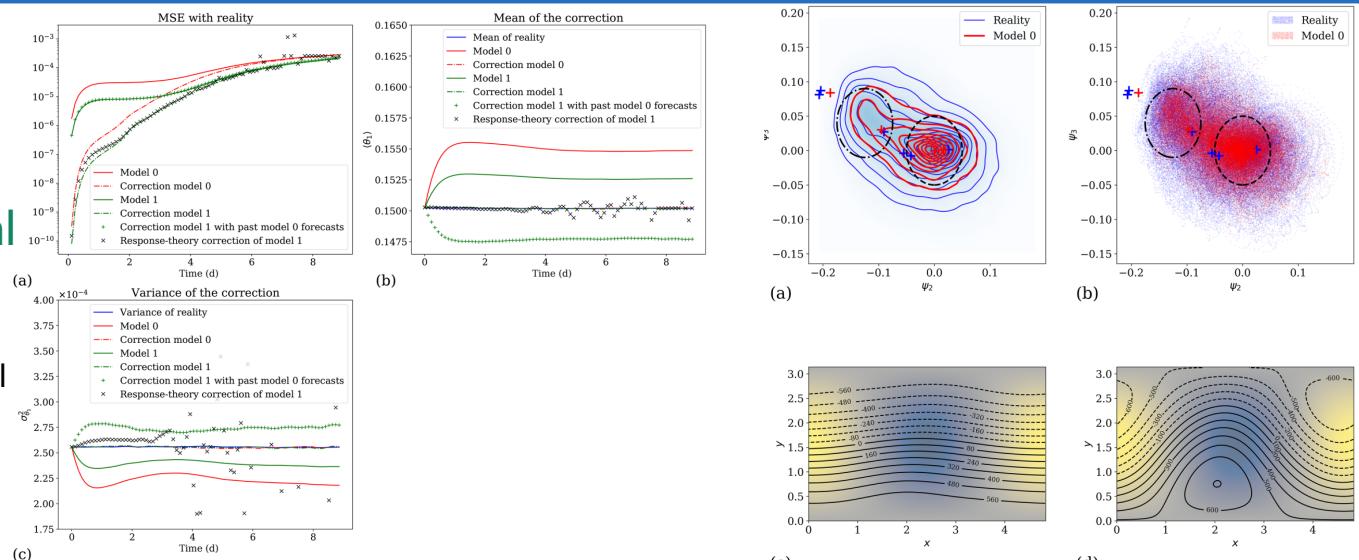


3d projection of the attractor

## Post-processing

Using qgs with an orography to study the application of response theory for statistical postprocessing

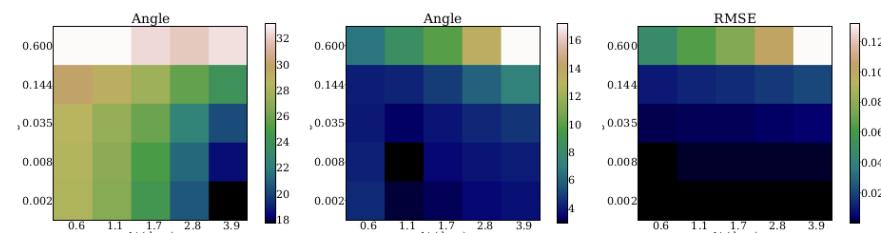
Demaeyer & Vannitsem. Correcting for model changes in statistical postprocessing – an approach based on response theory, *Nonlin. Processes Geophys.*, **27**, 307–327 (2020), <https://doi.org/10.5194/npg-27-307-2020>.



## Data assimilation

Using qgs with an ocean (MAOOAM) and an ensemble Kalman filter (EnKF)

Carrassi, et al. Data assimilation for chaotic systems. in *Data Assimilation for Atmospheric, Oceanic and Hydrological Applications* (2020). Springer Science

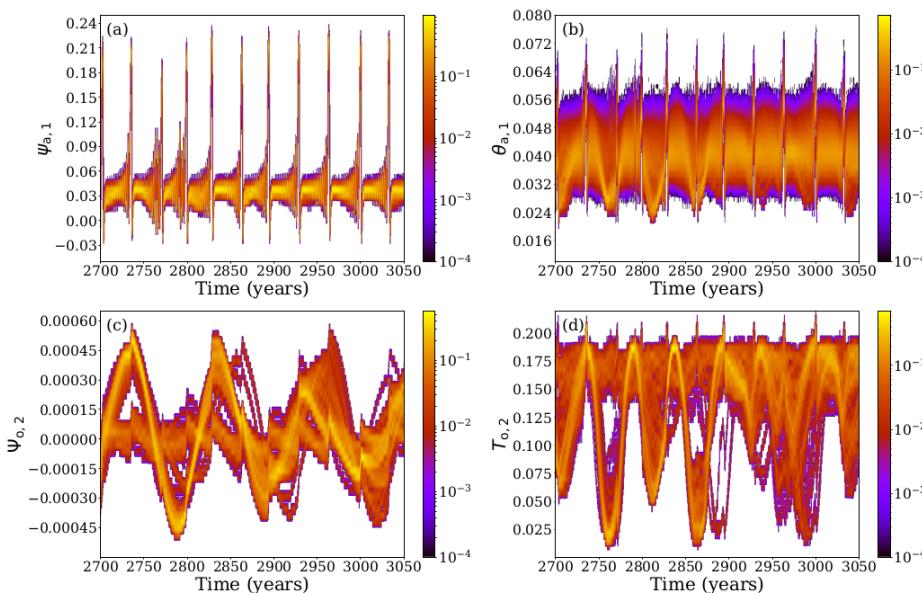


**Fig. 7** MAOOAM coupled Ocean-Atmosphere model. Time- and ensemble-averaged angle given by Eq. 37 between the anomalies of the EnKF and the unstable-neutral subspace (left panel, shadow colours, in degree) or the unstable-near-neutral subspace (mid panel, shadow colours, in degree). The time averaged normalised RMSE of the EnKF analysis is also depicted (right panel, shadow colours). All the figures are on the plane  $(x, y) = (\Delta t, \sigma^{\%})$ . The set-up is  $\mathbf{H} = \mathbf{I}_{N_x}$ ,  $\mathbf{R} = \sigma^{\%}\text{diag}(\sigma_{\text{md}}^1, \dots, \sigma_{\text{md}}^{N_x})$  and  $N = 20$ . The unstable-near-neutral subspace is defined as follows: it includes the subspace spanned by the unstable and neutral  $n_0 = 6$  directions, but also an additional  $n_1 = 10$  stable but near-neutral directions with LEs  $\lambda_i \in [-5 \times 10^{-3}, 0] \text{ day}^{-1}$ .

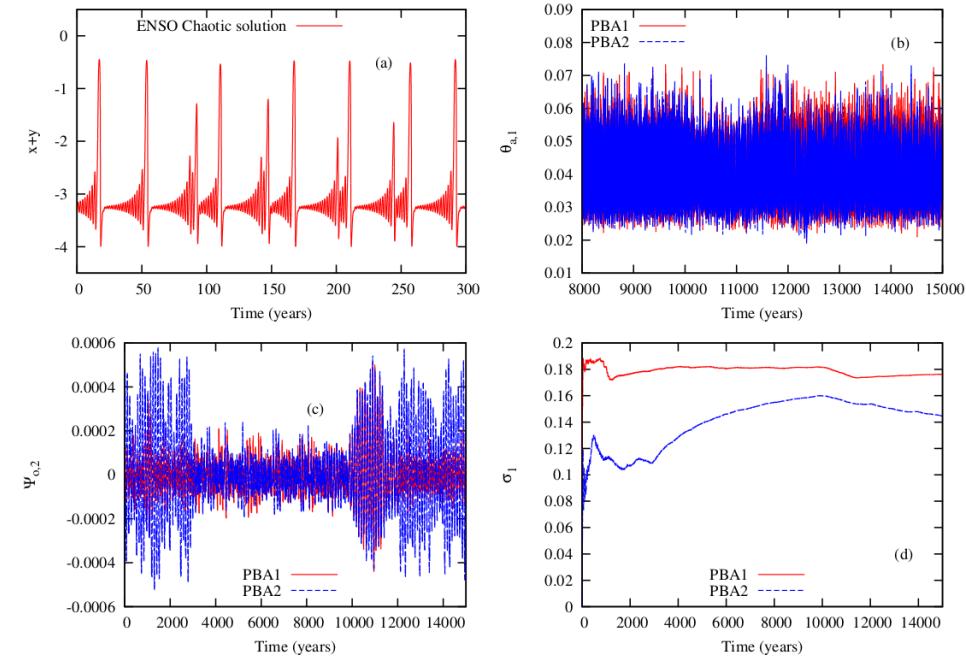
## Climate prediction

Using qgs (MAOOAM) forced with a model of ENSO to study its impact on midlatitude climate prediction, through the lens of pullback attractors.

Vannitsem, Demaeyer & Ghil (2021). Extratropical low-frequency variability with ENSO forcing: A reduced-order coupled model study. arXiv preprint arXiv:2103.00517.

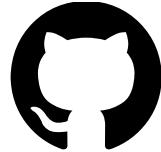


**Figure 13.** As in Fig. 12, but for PBA2.



**Figure 10.** As in Fig. 3 but for chaotic ENSO forcing, with  $g = 0.03$ . (a) Time segment of 300 yr from the chaotic forcing, identical to the one displayed in Fig. 1(b); (b,c) evolution of the

## Github repository



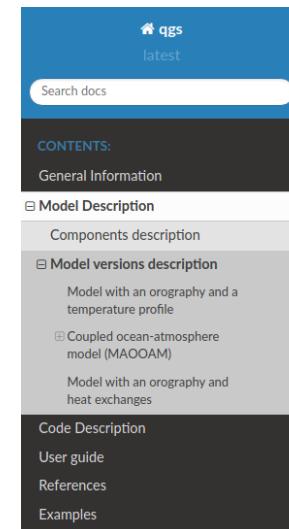
<https://github.com/Climdyn/qgs>

See also:

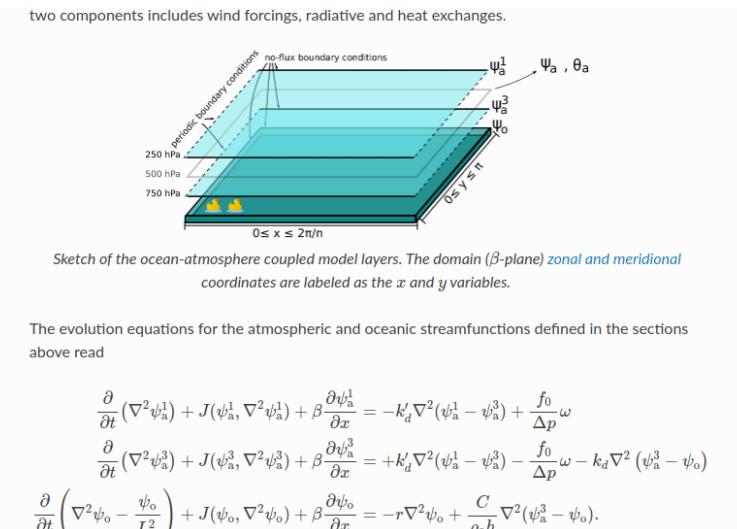
[https://qgs.readthedocs.io/en/latest/files/general\\_information.html#contributing-to-qgs](https://qgs.readthedocs.io/en/latest/files/general_information.html#contributing-to-qgs)

## Documentation

<https://qgs.readthedocs.io>



The screenshot shows the qgs documentation website. The top bar has a search bar labeled "Search docs". Below it is a "CONTENTS:" section with a "General Information" item. Under "Model Description", there are sections for "Components description", "Model versions description" (which includes "Model with an orography and a temperature profile", "Coupled ocean-atmosphere model (MAOAM)", and "Model with an orography and heat exchanges"), and "Code Description" with "User guide", "References", and "Examples".



## Description paper

Demaeyer, De Cruz & Vannitsem, (2020). qgs: A flexible Python framework of reduced-order multiscale climate models. *Journal of Open Source Software*, 5(56), 2597, <https://doi.org/10.21105/joss.02597>

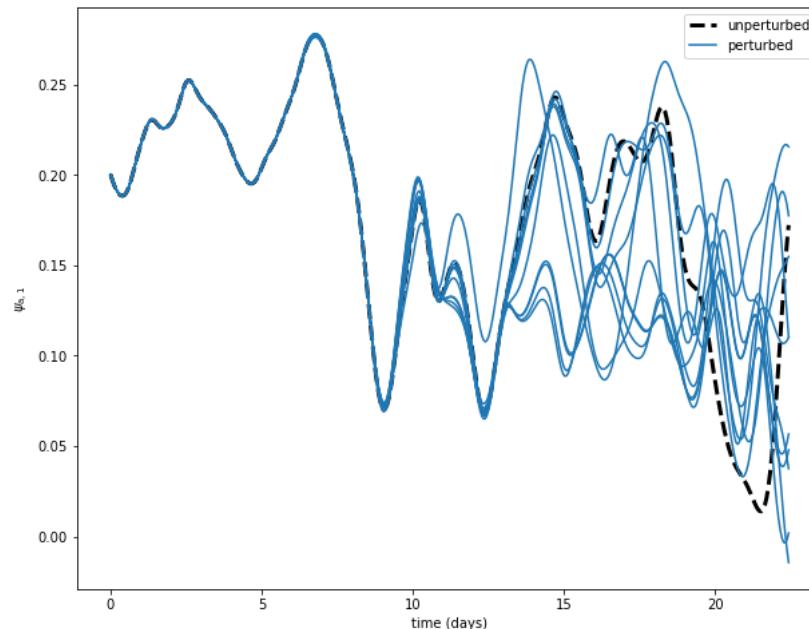
# In this tutorial about predictability

We are going to use qgs simplest atmospheric model to study its predictability...

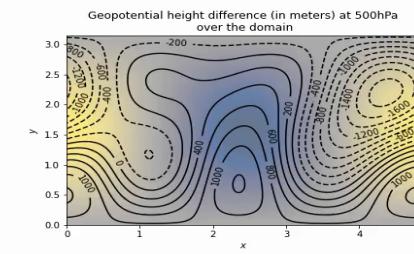
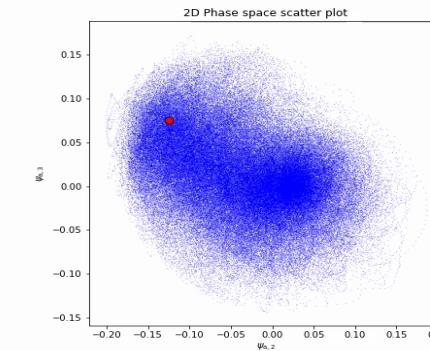
qgs can be used to make real-world looking forecast experiment:

- will allow us to get a grasp on the concept of predictability in high-dimensional forecast models
- computation of relevant quantities indicating the typical timescales at which the model is predictable (autocorrelation time, Lyapunov exponents, ...)

Simple example with an orography: ensemble forecast experiment



Simple example with an orography: animation



Click on the image to play the video.

# THANK YOU

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Het KMI verleent een betrouwbare dienstverlening aan het publiek en de overheid gebaseerd op onderzoek, innovatie en continuïteit.

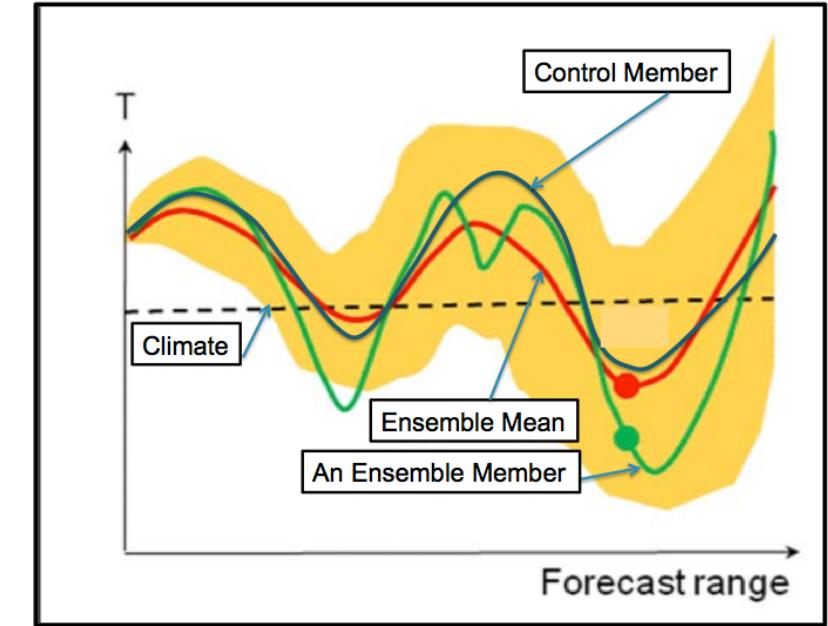
L'IRM fournit un service fiable basé sur la recherche, l'innovation et la continuité au public et aux autorités.

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## Some definitions:

- Lead time: time past since the initial time of the forecast (here called *forecast range* → )
- Ensemble mean: mean of a variable X over the M ensemble members

$$\bar{X} = \sum_{m=1}^M X_m$$



ECMWF Forecast User Guide

- Control member: Member with the best initial condition estimated from observations
- Climate: Range of observed values of the past years forecasts initialized at the same date