

BOOLEAN EXPRESSION SIMPLIFICATION

- Simplify: $C + BC$:

| <u>Expression</u> | <u>Rule(s) Used</u> |
|-------------------|--------------------------------|
| $C + BC$ | Original Expression |
| $C + (B + C)$ | DeMorgan's Law. |
| $(C + C) + B$ | Commutative, Associative Laws. |
| $T + B$ | Complement Law. |
| T | Identity Law. |

- Simplify: $AB(A + B)(B + B)$:

| <u>Expression</u> | <u>Rule(s) Used</u> |
|--------------------|---|
| $AB(A + B)(B + B)$ | Original Expression |
| $AB(A + B)$ | Complement law, Identity law. |
| $(A + B)(A + B)$ | DeMorgan's Law |
| $A + BB$ | Distributive law. This step uses the fact that or distributes over and. It can look a bit strange since addition does not distribute over multiplication. |
| A | Complement, Identity. |

- Simplify: $(A + C)(AD + AD) + AC + C$:

| <u>Expression</u> | <u>Rule(s) Used</u> |
|-------------------|---------------------|
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|-----------------------------|---------------------|
| $(A + C)(AD + AD) + AC + C$ | Original Expression |
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| $(A + C)A(D + D) + AC + C$ | Distributive. |
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|---------------------|-----------------------|
| $(A + C)A + AC + C$ | Complement, Identity. |
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|----------------------|----------------------------|
| $A((A + C) + C) + C$ | Commutative, Distributive. |
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|----------------|--------------------------|
| $A(A + C) + C$ | Associative, Idempotent. |
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|---------------|---------------|
| $AA + AC + C$ | Distributive. |
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|----------------|-------------------------------------|
| $A + (A + T)C$ | Idempotent, Identity, Distributive. |
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|---------|------------------|
| $A + C$ | Identity, twice. |
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- You can also use distribution of or over and starting from $A(A+C)+C$ to reach the same result by another route.
- Simplify: $A(A + B) + (B + AA)(A + B)$:

Expression

Rule(s) Used

| | |
|------------------------------|---------------------|
| $A(A + B) + (B + AA)(A + B)$ | Original Expression |
|------------------------------|---------------------|

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| $AA + AB + (B + A)A + (B + A)B$ | Idempotent (AA to A), then Distributive, used twice. |
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| $AB + (B + A)A + (B + A)B$ | Complement, then Identity. (Strictly speaking, we also used the Commutative Law for each of these applications.) |
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|--------------------------|---------------------------|
| $AB + BA + AA + BB + AB$ | Distributive, two places. |
|--------------------------|---------------------------|

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|--------------------|--|
| $AB + BA + A + AB$ | Idempotent (for the A's), then Complement and Identity to remove BB. |
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$$AB + AB + AT + AB$$

Commutative, Identity; setting up for the next step.

$$AB + A(B + T + B)$$

Distributive.

$$AB + A$$

Identity, twice (depending how you count it).

$$A + AB$$

Commutative.

$$(A + A)(A + B)$$

Distributive.

$$A + B$$

Complement, Identity