Temperature and Power Loss in Transformers

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Abstract

Transformer efficiency is an important topic in the modern world. Transformers are essential for the operation of modern power grids as well as many other important electrical systems. Transformer losses are quite well understood. However, a factor which is not often considered is the effects of temperature on transformer efficiency. Since transformers can become quite hot, this is an important consideration for the design of transformers. Therefore, the research question this investigation attempts to answer is “what effect does temperature have on power loss in transformers?”

To answer the research question, the effects of temperature on the power loss of a low power iron cored transformer connected to a 20.00Ω load were experimentally investigated. The power lost from the windings as well as the core were specifically considered. Two experiments were conducted to determine what impact temperature had on these losses. Firstly, the effect of temperature on the resistance of the windings as well as winding loss was found. The temperature coefficient of the windings was also found. In the second experiment the overall effect of temperature on the power loss of the transformer was found. The effects of temperature on the winding losses as well as the core losses were found from this data also.

Overall, temperature had the effect of decreasing the power lost by the transformer. Resistance was to found to increase directly with temperature. Whether winding losses increased or decreased depended on the total resistance of the circuit they were in. The core losses decreased directly with temperature by a considerable amount.

Word count: 256

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Introduction

Problem statement and goal

Transformer efficiency has always been a concern of engineers and scientists. Many aspects of transformer efficiency have been studied, and the various types of transformer losses are well understood. However, a factor which is rarely considered is the effects of temperature on the power loss of a transformer. It is often looked at as an insignificant factor. Despite this, it is important to explore how temperature affects the amount of power lost in order to better understand transformer efficiency and to build optimal transformers. Therefore, the reach question this paper will attempt to answer is “how does temperature affect power loss in transformers?” The effects of temperature on both the core losses and the winding losses will be specifically investigated.

Relevance and significance

To construct efficient transformers, it is imperative to have accurate information about expected efficiency under the conditions which they will operate under. For this reason, temperature is a very important factor to consider. The temperature at which transformers operate varies so it is an important factor to consider in design. A greater understanding of how temperature affects transformer losses would therefore allow for more efficient transformers to be designed. Also, I have always found the power grid and electricity very interesting. Performing this investigation allowed me to learn a great deal more about how transformers and electrical systems work.

Limitations of investigation

There are a few practical considerations that limit this investigation. Firstly, because of practical and safety issues, the temperature range that can be investigated is limited. It is not feasible to examine temperatures too much below room temperature or temperatures above 90° C. Therefore, only temperatures between 25° C and 70° C will be considered. All experiments will be performed with a small, low power, solid iron core transformer. For safety reasons, only relatively low amounts of power will be used in the transformer.

Description of a transformer

Transformers are electrical devices which are able to step up or down a voltage. What this means is that a transformer can receive some voltage and current, and outputs the same amount of power, but with a different voltage to current ratio. Transformers that increase the voltage and decrease the current are called step up transformers. Step down transformers do the opposite. It is important to note that transformers only work with alternating current.

Transformers are relatively simple in their construction. A transformer consists of a ferromagnetic core with two sets of wire wrapped around the core, primary and secondary windings. Typically the core forms a circle. A diagram depicting a basic transformer is shown below (figure A). The input power flows into the primary windings. The rapidly changing alternating current in the primary windings induces a magnetic field in the core[1].The magnetic field in the core induces a new voltage and current in the secondary windings[2].

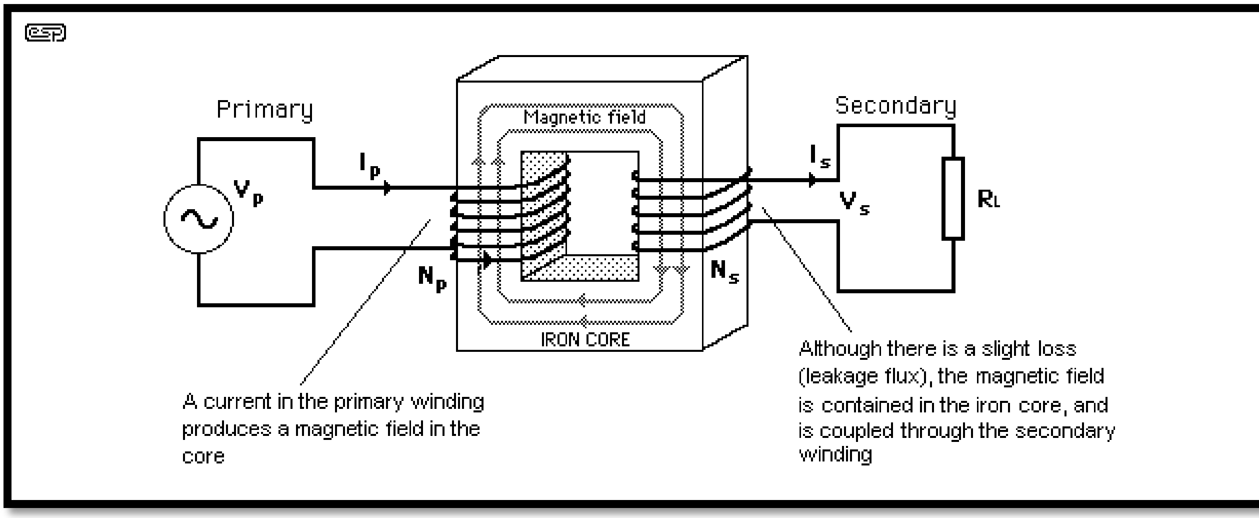

Figure A. Diagram of a transformer[3]

Theory

Energy loss in transformers

Winding losses

Energy is lost in the windings of the transformer. This is mostly because the windings have resistance. As the electrons flow through the windings, they occasionally suffer inelastic collisions with other electrons[4]. This transfers energy to the particles in the metal and causes it to heat up, wasting energy. In a circuit, the windings could be looked at as a resistor. Therefore, the power loss across each set of windings due to resistance is[5]

$$P_{loss} = I^2R$$

[1] (Nave, Transformers, 2011)
[2] (Nave, Transformers, 2011)
[3] (Elliott, 2001)
[4] (Tsokos, 2008, p. 311)
[5] (Dunlap, Siefert, & Austin, 1947, p. 18)

where I is the current in amps, and R is the resistance in ohms. It is important to point out that winding resistance losses are “load dependant”. This means that the amount of power lost is affected by how much power goes into the windings. As can be seen from the equation above, the power lost is directly proportional to the resistance, as well as the current squared. Increasing the current in considerably increases this type of loss.

Core losses

Hysteresis

One of the ways in which power is lost in the core of a transformer is hysteresis. Hysteresis loss is a type of loss related to the nature of magnetic materials. Ferromagnetic materials are said to be “soft” when they do not retain too much magnetism after the externally applied magnetic field is stopped. This is a good trait for transformers to have because this reduces what is called hysteresis losses. All ferromagnetic materials retain some of their magnetism in the absence of an applied field[6]. In transformers, the magnetic field is reversed in direction many times per second. Therefore, some energy must be wasted reversing the retained magnetism before the core can be magnetised in the other direction[7]. This amount of energy is called the coercivity of the material[8]. Figure B shows how extra energy must be applied to saturate the core. When the magnetic field is no longer present the core still retains a magnetic field of strength M. More energy must be applied to saturate the core in the other direction.

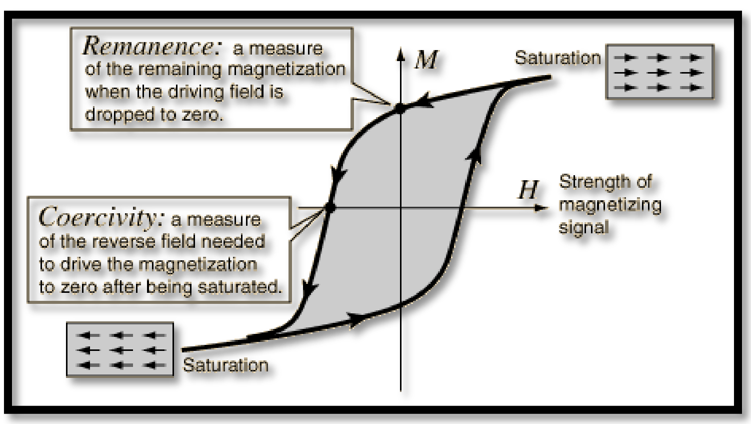

Figure B. Hysteresis of a ferromagnetic core.[9]

[6] (Kuphaldt, 2000)
[7] (Dunlap, Siefert, & Austin, 1947, p. 19)
[8] (Coercivity and Remanence in Permanent Magnets)
[9] (Coercivity and Remanence in Permanent Magnets)

Eddy Currents

Eddy currents within the core cause energy to be lost[10]. They refer to circular patterns of movement of electrons within the core[11]. Most cores are not ideal conductors, but are still capable of conducting electricity[12]. Because of this, it is possible for the magnetic flux in the core to generate a voltage in the core itself[13]. This voltage causes electrons to move in small circles. The moving electrons suffer inelastic collisions with other electrons in the core and cause energy to be lost in the form of heat[14].

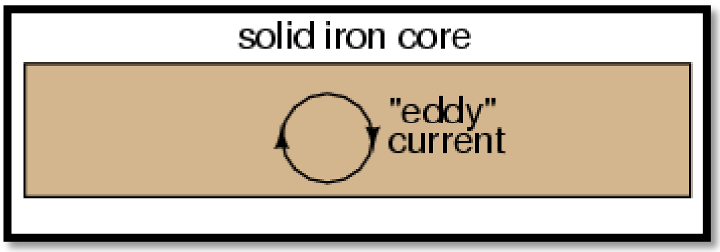

Figure C. Eddy currents in a solid iron core.[15]

Body

Hypothesis

Winding resistance losses

The temperature of the windings affects the power loss in that it increases the resistance of the windings. As the temperature of the windings increases, the random motion of the particles also increases, causing more inelastic collisions. The relationship between resistance and temperature can be approximated by the equation[16]

$$R_T = R_o(1 + \alpha T)$$

Where R_T is the resistance in ohms at a temperature T, R_o is the resistance at some reference temperature, and α is the temperature coefficient of the metal, and T is the temperature difference between the reference temperature and current temperature in °C. This equation shows that the resistance is directly proportional to the temperature.

[10] (Dunlap, Siefert, & Austin, 1947, p. 18)
[11] (Kuphaldt, 2000)
[12] (Kuphaldt, 2000)
[13] (Dunlap, Siefert, & Austin, 1947, p. 18)
[14] (Dunlap, Siefert, & Austin, 1947, p. 18)
[15] (Kuphaldt, 2000)
[16] (Simpson, 1987, p. 20)

The power lost from the resistance in one of the windings is given by

$$P_{loss} = I^2 R$$

The total power loss of the windings is therefore

$$P_{loss} = P_{p\ loss} + P_{s\ loss}$$

Where P_{loss} the total power is lost in watts, $P_{p\ loss}$ is the power lost by the primary windings in watts, and $P_{s\ loss}$ is the power lost by the secondary windings in watts. Since the resistance increases directly with temperature the total power loss of the windings also should increase directly with temperature.

Eddy current losses

Temperature should have a small effect on the eddy current losses. Since increasing temperature directly increases resistance by a small amount, higher temperatures should directly reduce the amount of power lost by eddy currents, as the resistance is higher so it is more difficult for the electrons to move in the presence of the magnetic field. This consequently reduces the power that is wasted. The increase in resistance of the core with temperature can also be predicted with the equation:

$$R_T = R_o(1 + \alpha T)$$

It is not straight forward to predict the amount of voltage and current that will be induced in the core. However, the effects of changing resistance can be considered by examining the equation for power loss.

$$P = I^2 R = \left(\frac{V}{R}\right)^2 R = \frac{V^2}{R}$$

It is clear from this that the power should be inversely proportional the resistance. Since the resistance directly proportional to the temperature, it can be predicted that the power lost to eddy currents should be inversely proportional to the temperature.

Hysteresis losses

Temperature should only have a small effect on the magnitude of the hysteresis losses of a core. Increasing the temperature of the core has the effect of increasing the random particle motion. This makes it more difficult for the magnetic domains to align[17], making the material less magnetic. The increased random particle motion should also have the effect of slightly decreasing the coercivity of the material, decreasing the losses at higher temperatures. Of course, this effect would be quite small.

The table below summarizes the predictions which have been made.

Summary of hypothesis

Type of loss	Effect of high temperature	Effect of low temperature
Resistance in windings	Increase	Decrease
Eddy currents	Decrease slightly	Increase slightly
Hysteresis	Decrease very slightly	Increase very slightly
Total winding losses	Increase	Decrease
Total core losses	Decrease slightly	Decrease slightly
Overall loss	Increase	Decrease

Methods

Two experiments are necessary to evaluate the hypothesis. Both experiments will be conducted using the same small, iron cored transformer. It consists of two solenoids of equal numbers of turns with a circular iron core that can be inserted into the two small solenoids. A picture can be seen in appendix E.

In order to evaluate each type of loss, experiments must be designed to isolate each type of loss. Therefore, experiments will be conducted to determine the effects of temperature on winding losses and core losses. An overview of the methods of each experiment will be provided here. A more detailed design for each experiment can be found in the appendices.

Experiment #1 – Winding resistance, winding loss, and temperature

The first experiment will provide data about how temperature affects the winding losses and resistance. This experiment must measure the power loss from the windings without the core loss. This is relatively simple to do. The windings will simply be directly connected in series with the load that is to receive power. In order to control the temperature the windings will be submerged in hot water. The power to the load can be measured as the temperature is changed. Figure D depicts the circuit diagram of this.

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[17] (Nave, Ferromagnetism, 2011)

This data can be used for a few things. Firstly, since the resistance of the load is known and both of the windings have the same resistance, how the resistance of the windings changes with temperature can be determined. Furthermore, the temperature coefficient of the copper wires can be found. How the power to the load changes with temperature indicates how the winding losses change with temperature. A more detailed design for the lab can be found in appendix A.

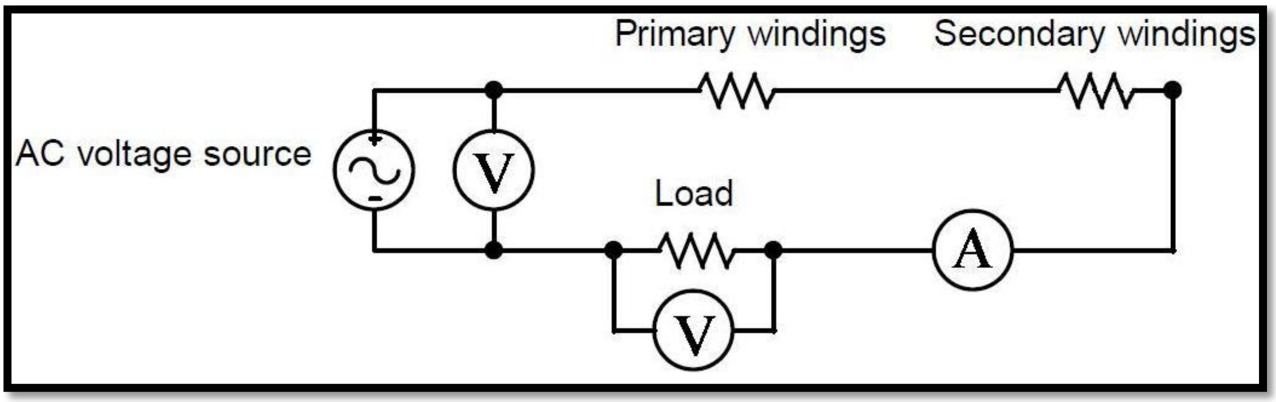


Figure D. A circuit to measure winding loss.

Experiment #2 – Overall power loss and temperature

Now only the core losses and the overall power loss of the transformer must be found. The overall power loss of the transformer will be found and the core losses can be calculated from this value. This experiment will involve constructing a circuit with the transformer connected to a load of 20.00Ω and measuring several quantities. Meters will be set up to measure the voltage and current into the primary windings. The voltage and current supplied to the load will also be measured. The transformer will be submerged in hot water during the experiment in order to vary the temperature. Figure E shows an equivalent circuit of the setup described. A detailed design of the experiment can be found in Appendix C.

A great deal of analysis can be performed using the data collected from this experiment. Firstly the primary winding and secondary winding losses can be calculated. The overall loss can be found by examining how the power to the load changes with temperature. By considering how the winding losses and overall losses change with temperature how the core losses change can be calculated also.

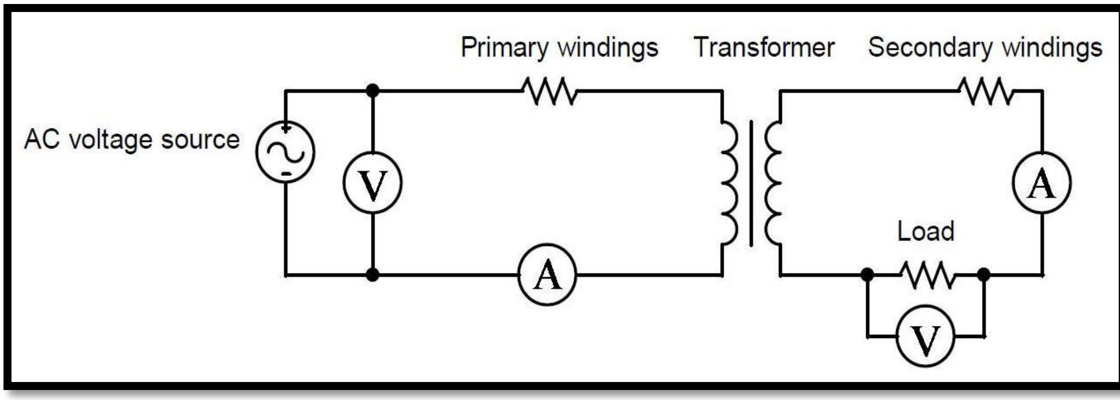


Figure E. A circuit to measure the overall power loss of a transformer.

Expectations

Experiment #1 – Winding losses and temperature

It is expected that the resistance will increase directly with temperature. If the resistance increases directly with temperature, the power to the 20.0Ω load should decrease directly with temperature. Therefore the winding losses should increase directly with temperature.

Experiment #2 – Overall power loss and temperature

It is expected that the overall transformer loss will increase with temperature. It is thought that the winding losses will directly increase with temperature in a clear and significant way. The core losses may decrease slightly, but this is not expected to be significant.

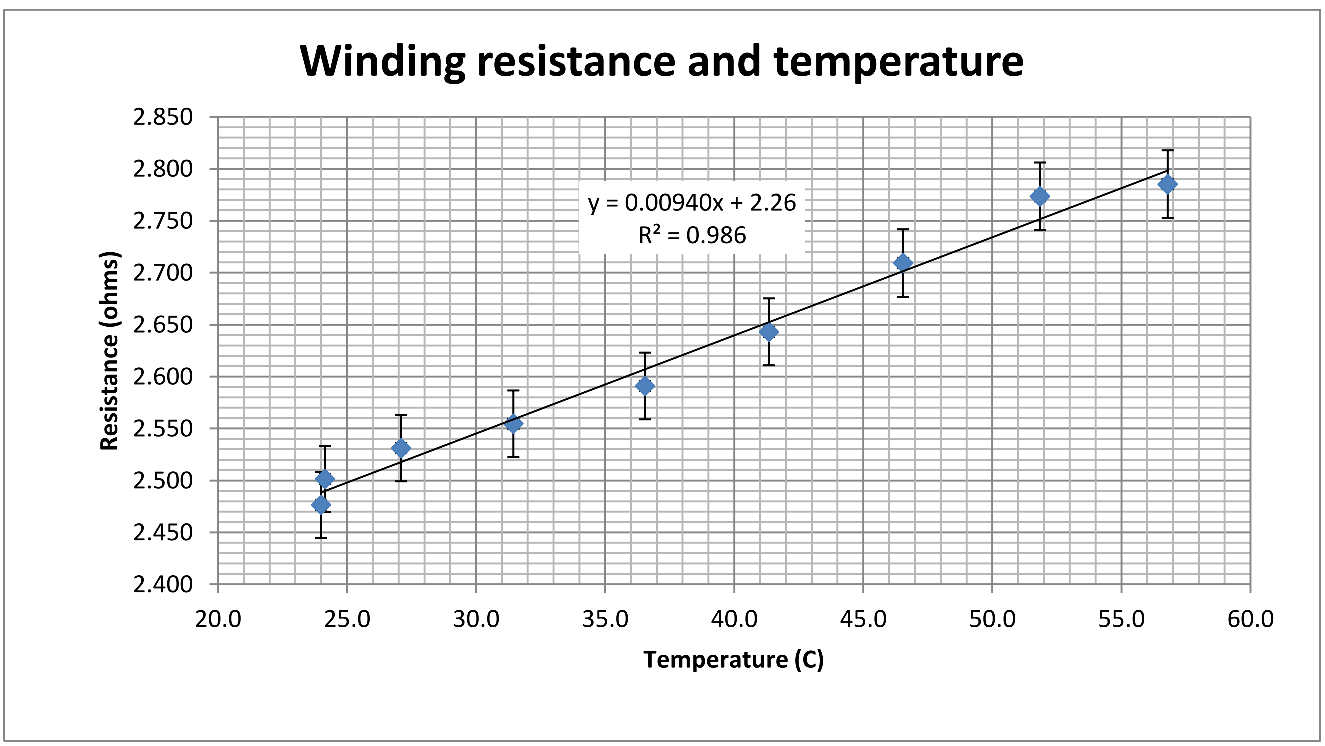
Results

Experiment #1 – Winding resistance, power loss, and temperature

The first experiment yielded the expected results. The raw data and all of the data processing is detailed in Appendix B. The resistance of the windings was directly proportional to the temperature. The table below shows the calculated values for the resistance of either set of windings at each temperature.

Average Temperature	Resistance of single set of windings
T / °C ± 0.2°C	R_w / Ω ± 0.02 Ω
56.8	2.785
51.9	2.774
46.6	2.709
41.4	2.643
36.6	2.591
31.5	2.555
27.1	2.531
24.0	2.477
24.2	2.502

In order to see the relationship between temperature and winding resistance, the winding resistance was plotted against temperature.

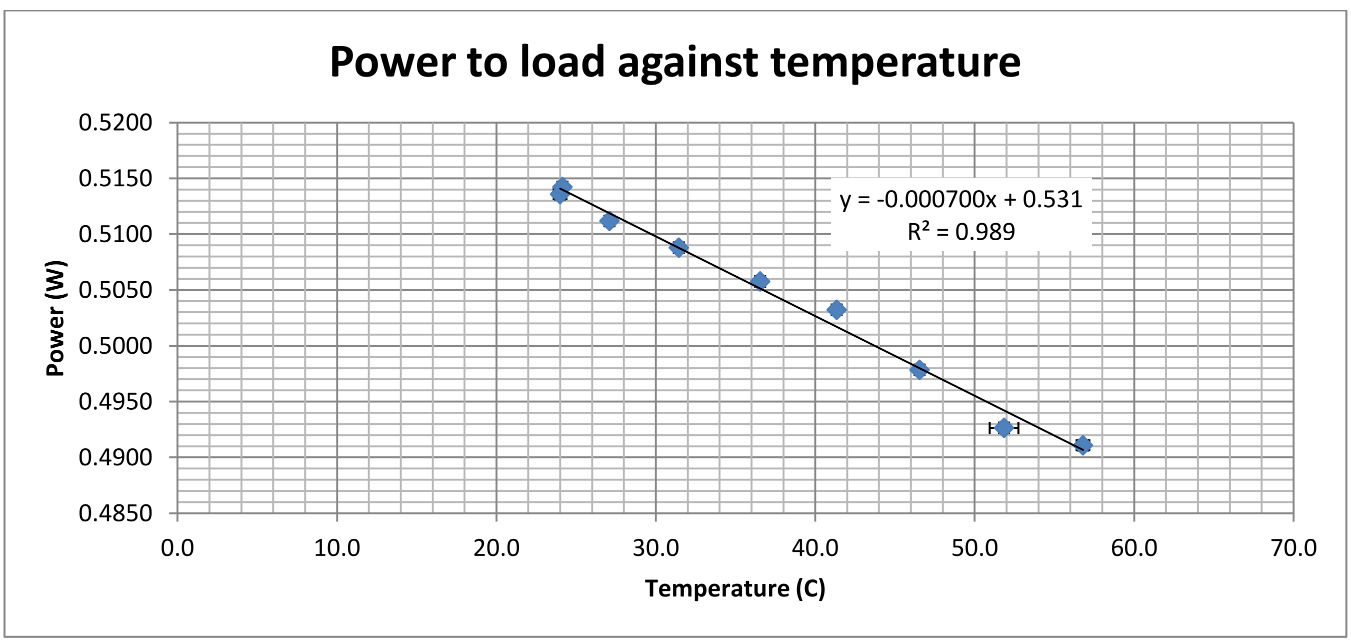


The linear line of best fit crossed through all of the points within their uncertainty and the regression had an R^2 value of 0.986, so the relationship between temperature and winding resistance and temperature is linear. The temperature coefficient was found to be 0.00332° C^{-1}. This is reasonably close to the literature value for the temperature coefficient, of 0.0039° C^{-1}.[18] This will be helpful for analysis of the second experiment.

Now power losses will be considered. The table below shows the variation of the power to the load with temperature.

Average Temperature	**Resistance** (of both windings)	**Power to load**
T /°C ± 0.2°C	R/Ω ± 0.005Ω	P_{load}/W ± 0.0005 W
56.8	5.570	0.4911
51.9	5.547	0.4927
46.6	5.419	0.4979
41.4	5.286	0.5032
36.6	5.182	0.5058
31.5	5.110	0.5088
27.1	5.062	0.5112
24.0	4.953	0.5136
24.2	5.003	0.5142

To see how the increasing temperature (and resistance) affected the winding losses, the temperature was plotted against the power to the load.



18 (Simpson, 1987, p. 20)

A linear line of best fit was drawn and a regression performed. The R^2 value was 0.989 so the relationship is seems to be linear, even though the best fit line does not cross through the uncertainty of every point. This means that the winding losses increase directly with temperature.

Experiment #2 – Overall power loss and temperature

Overall power loss

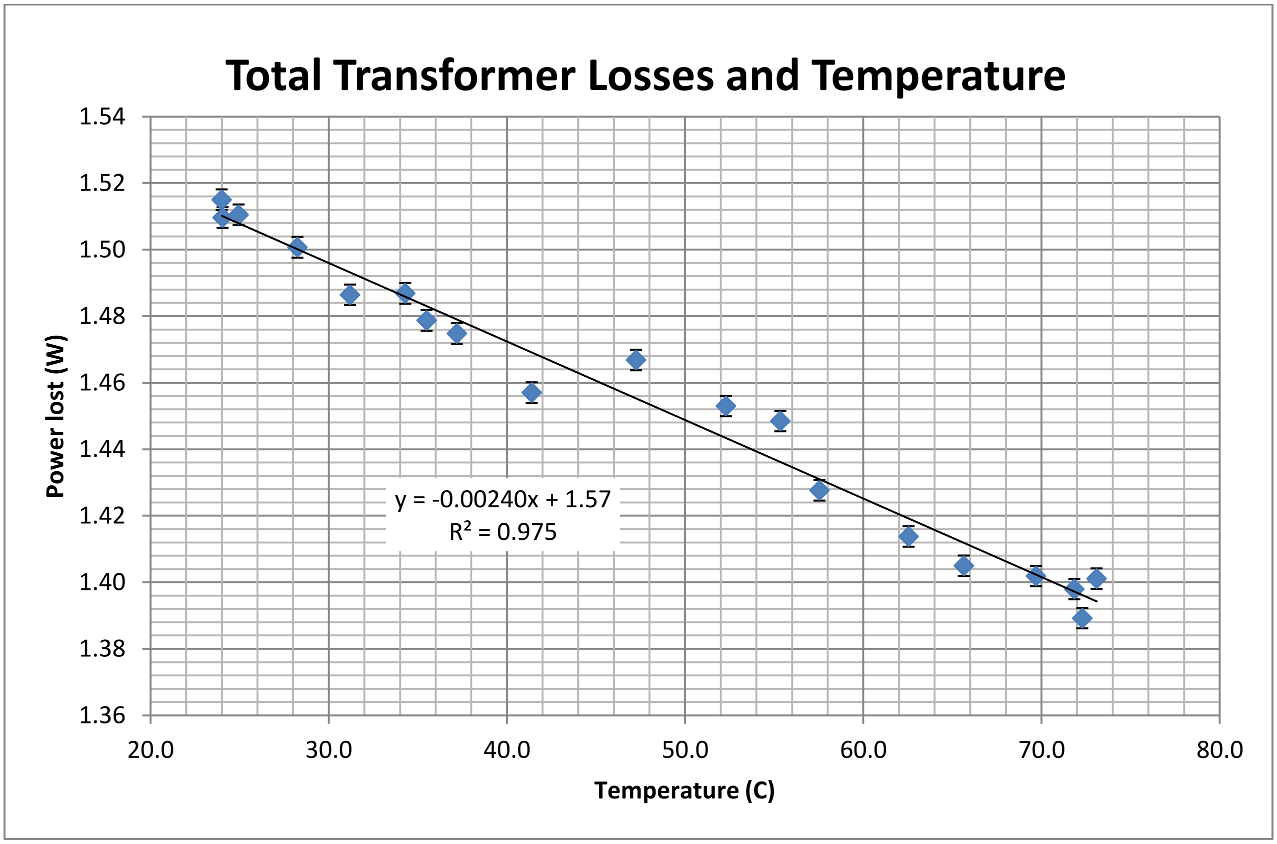
The raw data and data processing for this experiment can be found in Appendix D. The change in overall power loss can now be considered. The power loss was calculated by

$$P_{lost} = P_{in} - P_{out}$$

It was found that the total power loss seemed to decrease with temperature. The table below shows the variation of the total power loss with temperature.

Average Temperature	Total power loss
T_{avg} / °C ± 0.2°C	P_{lost} / W ± 0.002 W
73.1	1.40
72.3	1.39
71.9	1.40
69.7	1.40
65.7	1.41
62.6	1.41
57.6	1.43
55.4	1.45
52.3	1.45
47.3	1.47
41.4	1.46
37.2	1.47
35.5	1.48
34.3	1.49
31.2	1.49
28.3	1.50
25.0	1.51
24.0	1.52
24.1	1.51

Below is a graph of power loss against temperature.



It appears that the total power lost decreased directly with temperature. This is evident because the R^2 of the linear regression was 0.975. However, many of the points do not touch the line of best fit within their uncertainty. This may be due to random error.

Primary winding losses

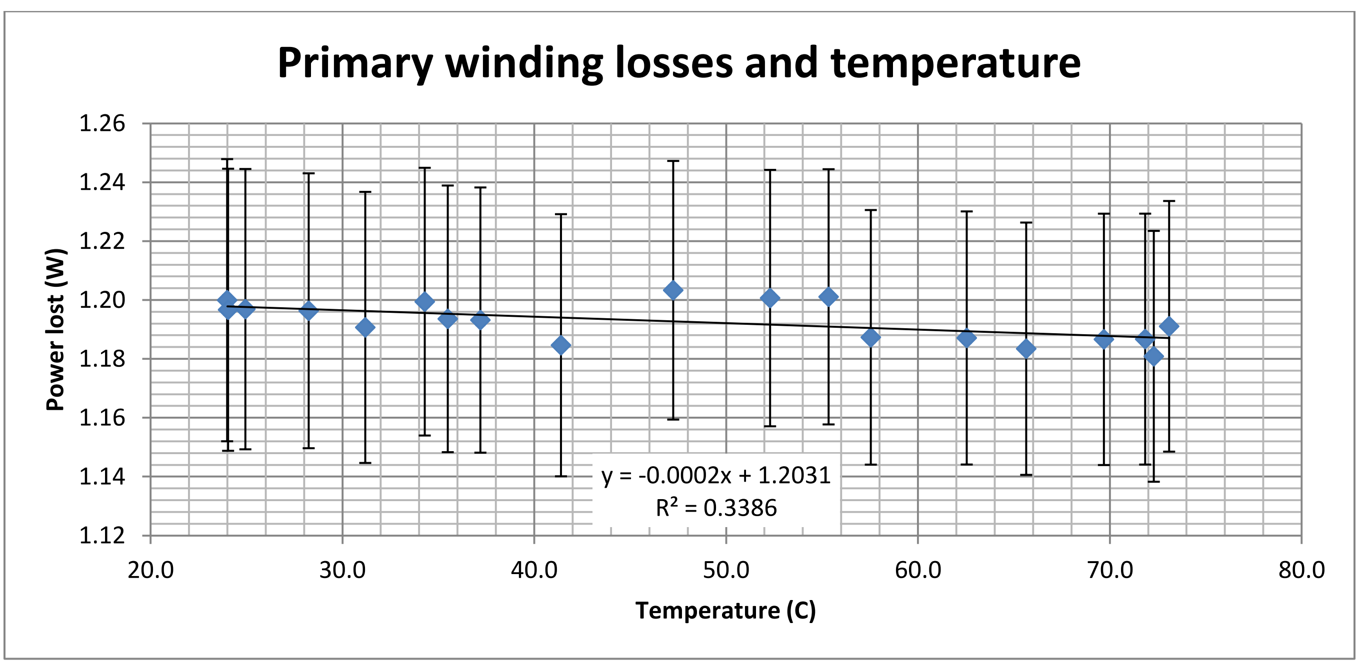
The results related to the primary winding losses will now be discussed. As the temperature increased, the primary winding resistance also increased. However, the primary winding losses did not increase. In fact, they only changed slightly. The primary losses were found with the current in and the resistance of the windings at the given temperature using the temperature coefficient found in the first experiment. The equation to find the losses is shown below.

$$P_{primary\ losses} = I_{in}^2 R_{primary}$$

The table below shows how the primary winding losses varied with temperature.

Average Temperature	Primary winding losses	
T_{avg} / °C ± 0.2 °C	P_{lost} / W	± ΔP_{lost} / W
73.1	1.19	0.04
72.3	1.18	0.04
71.9	1.19	0.04
69.7	1.19	0.04
65.7	1.18	0.04
62.6	1.19	0.04
57.6	1.19	0.04
55.4	1.20	0.04
52.3	1.20	0.04
47.3	1.20	0.04
41.4	1.18	0.04
37.2	1.19	0.05
35.5	1.19	0.05
34.3	1.20	0.05
31.2	1.19	0.05
28.3	1.20	0.05
25.0	1.20	0.05
24.0	1.20	0.05
24.1	1.20	0.05

A graph showing the actual and expected change in primary winding losses against temperature is shown below.



From the graph the change in power loss in not totally clear. The trend line indicated a slight negative trend. However, the uncertainty was considerably larger than the overall change. Consequently, the uncertainty of the slope was very high. Overall, the primary losses did not appear to change significantly with temperature. However it is hard to draw any conclusions considering the uncertainty of the slope.

Secondary winding losses

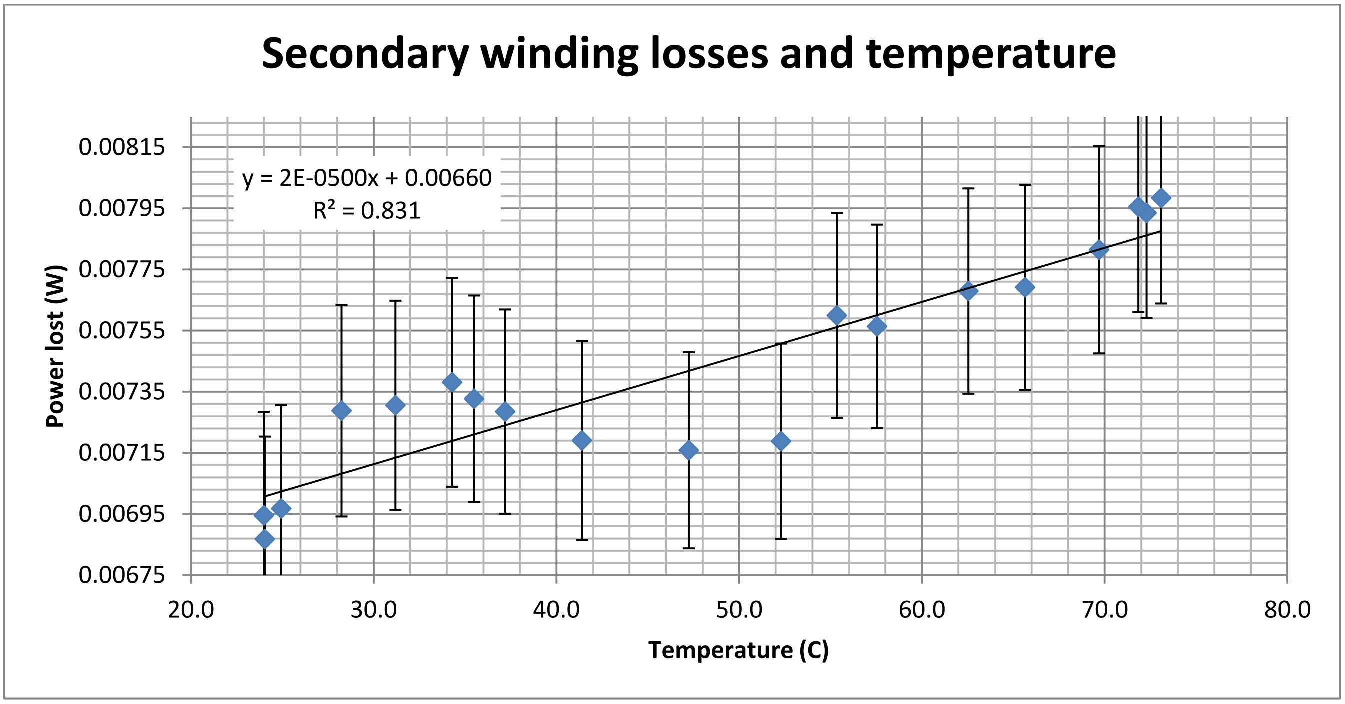
Like the primary losses, the secondary losses can be found using the measured current and the temperature coefficient found to calculate the resistance of the secondary windings. From this, the following equation could be used to find the secondary winding losses.

$$P_{secondary\ losses} = I_{out}^2\ R_{secondary}$$

The table below shows the calculated secondary winding losses for all of the measured temperatures.

Temperature	Secondary winding losses
T_{avg} / °C ± 0.2 °C	$P_{s\ loss}$ / W ± 00003 W
73.1	0.00798
72.3	0.00794
71.9	0.00796
69.7	0.00781
65.7	0.00769
62.6	0.00768
57.6	0.00756
55.4	0.00760
52.3	0.00719
47.3	0.00716
41.4	0.00719
37.2	0.00729
35.5	0.00733
34.3	0.00738
31.2	0.00731
28.3	0.00729
25.0	0.00697
24.0	0.00695
24.1	0.00687

To see the relationship between secondary winding losses and temperature, a graph of secondary winding losses and temperature was produced. It is shown below.



From the graph it appears that the secondary winding loss increased directly with temperature. The R^2 value of the regression was 0.831, so the data fits the line of best fit reasonably well. Moreover, the line of best fit was within the uncertainty of all of the points. There was an unusual curve to some of the points, perhaps from some random error.

Core losses

Since the sum of power losses must equal the total loss, it is possible to calculate the core losses of the transformer. The below equation shows this,

$$P_{total\ lost} = P_{primary\ loss} + P_{secondary\ loss} + P_{core\ loss}$$

If this expression is rearranged, the core losses are equal to

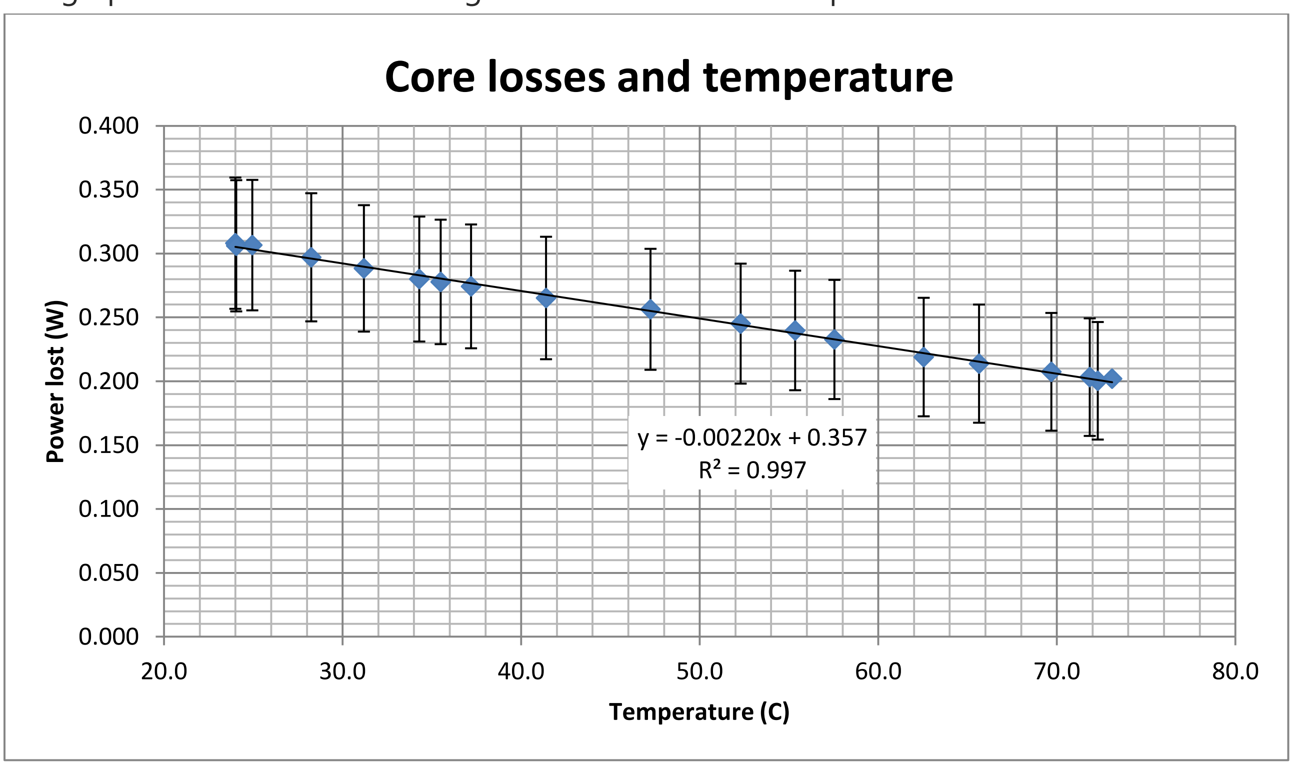
$$P_{core\ loss} = P_{total\ lost} - P_{primary\ loss} - P_{secondary\ loss}$$

Using this equation the core losses were calculated.

The table below shows the core losses at each temperature.

Temperature	Core losses
T_{avg} / °C ± 0.2 °C	P_c / W ± 0.05 W
73.1	0.20
72.3	0.20
71.9	0.20
69.7	0.21
65.7	0.21
62.6	0.22
57.6	0.23
55.4	0.24
52.3	0.25
47.3	0.26
41.4	0.27
37.2	0.27
35.5	0.28
34.3	0.28
31.2	0.29
28.3	0.30
25.0	0.31
24.0	0.31
24.1	0.31

The graph below shows the change in core losses with temperature.



It seems from the graph that the core loss did decrease directly with temperature. The line of best fit crosses through the uncertainty of all of the points, and the R^2 value was 0.977.

Conclusion

Evaluation of the hypothesis

The hypothesis was not totally accurate. The first experiment showed that as expected, the resistance of the windings was directly proportional to the temperature. The power lost in the windings was also found to be directly proportional to temperature. This was also consistent with what was expected.

Some of the results of the second experiment were more surprising. Firstly, the overall power loss actually decreased with temperature, rather than increasing. To explain this, the individual types of losses have to be considered. Contrary to expectations, the power lost to the primary windings did not significantly change[19] with temperature. If anything, they decreased slightly. This can be explained in terms of how the experiment was set up. Since the primary windings were connected directly to the power source, as the temperature increased the winding resistance also increased, decreasing the power into the circuit, therefore decreasing the power dissipated by the primary windings. Had there been another resistor in series with the voltage source, the primary winding losses likely would have increased. The winding losses are dependant on the total resistance of the circuit, although this was not anticipated. The secondary winding losses did increase with temperature as expected. The core losses decreased with temperature as was anticipated. However, they decreased considerably more than was expected. This was likely due to the temperature increasing the resistance of the core, consequently decreasing the eddy currents induced in the core. This effect was clearly more significant than was thought. This may have been because the eddy currents were the most significant source of loss in the core since it was not laminated[20]. The significant decrease in the core losses was the main cause of the decrease in overall transformer loss.

[19] The change was within the uncertainty of the measurements.
[20] Laminated cores are considerably less prone to eddy current losses.

The table below summarizes how the results compare to the hypothesis.

Comparison of results to hypothesis

Type of loss	Expected effect of higher temperatures	Actual effect of higher temperatures
Resistance in windings	Increase	No significant change in primary windings; slight increase in secondary windings
Eddy currents	Decrease slightly	Not known
Hysteresis	Decrease very slightly	Not known
Total winding losses	Increase	Increased slightly
Total core losses	Decrease slightly	Decreased significantly
Overall loss	Increase	Decreased

Evaluation of error

There were significant sources of error in these experiments which skewed some of the results. First a few errors which apply to all of the experiments conducted will be considered. Then, additional errors will be considered for the second experiment.

The largest error related to both of the experiments in this investigation was the difficulty to make accurate temperature readings. Both experiments involved submersing the windings or the whole transformer in hot water to attempt to control the temperature. There were a few errors that arose from this. Firstly, the temperature of the water was never the same in all places because of convection and cooling. This made it impossible to get a totally accurate temperature reading of the water. Also, when the water temperature changed the temperature of the objects submersed in the water did not instantly change. Depending on their thermal resistivity, they all took different amounts of time to reach equilibrium. During the experiments, more effort should have been taken to control the temperature and let the system come to thermal equilibrium.

Another error common to all of the experiments was the resistance of the connecting cables. Since the multi-meter used measured the resistance as 0.0Ω it was ignored. However, it would have some impact on the results.

While the second experiment produced good data, there seemed to be quite a bit of error for some of the measurements. One of the largest errors for this experiment was random and caused by the core shifting slightly. Since the iron core was actually two separate pieces of iron set together (see pictures in Appendix E) the top piece easily slid even with slight movements. As the core moved out of position the core efficiency was considerably affected.

This generally had an effect on the transformer loss similar to the actual effect of the temperature, making it a significant error.

Another error was that not all of the instruments could all be read and recorded at once since there were four multi-meters and two temperature probes. This meant that some of the values recorded did not quite correspond perfectly to each other since the power in and temperature continued to fluctuate.

It is also worth noting that any error in finding the temperature coefficient considerably increased the error in the second experiment, since the estimated resistance was needed to estimate the winding and core losses.

A final significant issue with the analysis done in this essay is that some of the transformer loss calculations were simplified, and did not fully take into account some of the more complex effects which occur in transformers. For example it was assumed that the measured power in was the actual power in during the second experiment. However this is not fully the case because of the inductance of the core. Other issues which were ignored include AC resistive effects, winding inductance, and magnetic leakage. Analysis of these elements was considerably beyond the scope of this investigation, but would need to be considered for a fully accurate assessment of the losses of the transformer.

Evaluation of sources

The sources used in this essay can reasonably be considered quite reliable. Several textbooks were used which were written by reputable authors in the field of electrical engineering. A few online websites were also used. These can also be considered reliable since the ones used were respected and credible. Moreover, the information on the websites was checked against other websites, as well as the textbooks, and always agreed.

Possible improvements

Improvements

Many of the difficulties faced in this investigation could be reduced with apparatus that is precise to a larger number of decimal places. This would allow for less uncertainty in the loss calculations, most notably the primary winding losses. This would be very helpful since the change of power loss with temperature is quite small and difficult to accurately measure with imprecise instruments. The more precise instruments could also be used to measure the resistance of the connecting wires in the circuits to a higher degree of accuracy, allowing them to be more correctly factored in.

Another useful change would be to use multiple temperature probes attached to each part of the transformer. It would be more complicated, but the temperature of each component could be considered individually. At the very least, more temperature probes could be used to verify that the transformer is roughly at thermal equilibrium with the water.

Another possible improvement would be to use a more professional transformer with a core that is mounted in place and does not come apart. This would prevent the core from moving around during the experiment.

Lastly, the use of data-logging multi-meters would make data collection much easier, as they could automatically all collect data at once. Considering the large number of measurements, this would be a significant help.

Additional variables to consider

Hysteresis and eddy currents were not directly considered in this investigation. The coercively of the core could be investigated at different temperatures to see what affects it has. Also, the resistance of the core and temperature could be experimentally considered. Experiments to determine the magnitude of eddy currents could be conducted. A wider range of temperatures could be considered since some of these effects may not be linear over all temperatures. Moreover, other factors such as frequency, the load being powered, the external resistance of the cables connecting to the transformer, as well as the type of core should be considered. There are a number of different types of transformer cores whose properties vary much differently with temperature[21] compared to iron. If enough variables were investigated, a general expression for transformer efficiency could potentially be found.

[21] (Orenchak, 2004)

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Appendices

Appendix A – Winding resistance and loss (experiment #1) – Design

Aim: To determine the effects of temperature on the total winding loss of a transformer

Variables:

Independent:

The temperature of the windings

Dependent:

The voltage and current received by a 20.0Ω load connected in series with the windings.

Controlled:

The load (20.0 Ω)

The windings

The voltage in (~4.0 V)

The position of the temperature probes in the water

The position of the windings in the water

Assumptions:

Both windings are always at thermal equilibrium with the water.

Materials:

1. Primary and secondary transformer windings (two solenoids)
2. AC voltage source
3. GLX data logger
4. Two GLX temperature probes (Sample rate: 5 Hz. Precision: ± 0.1 °C)
5. AC ammeters (Precision: ± 0.001 A)
6. Two AC voltmeters (Precision: ± 0.001 V)
7. A container
8. A kettle or hotplate
9. Tongs
10. 20.00 Ω resistor
11. Low resistance connecting cables
12. Tape

Procedure:

Setup

1. The GLX was turned on.
2. The voltage source, primary windings, secondary windings, load, and ammeter were connected in series. Voltmeters were connected parallel to the voltage source and to the load. The appropriate arrangement of these parts is shown in the circuit diagram in figure F.
3. A temperature probe was attached to each set of windings with tape.
4. The voltage source was plugged in.
5. The kettle was filled with water and plugged in.

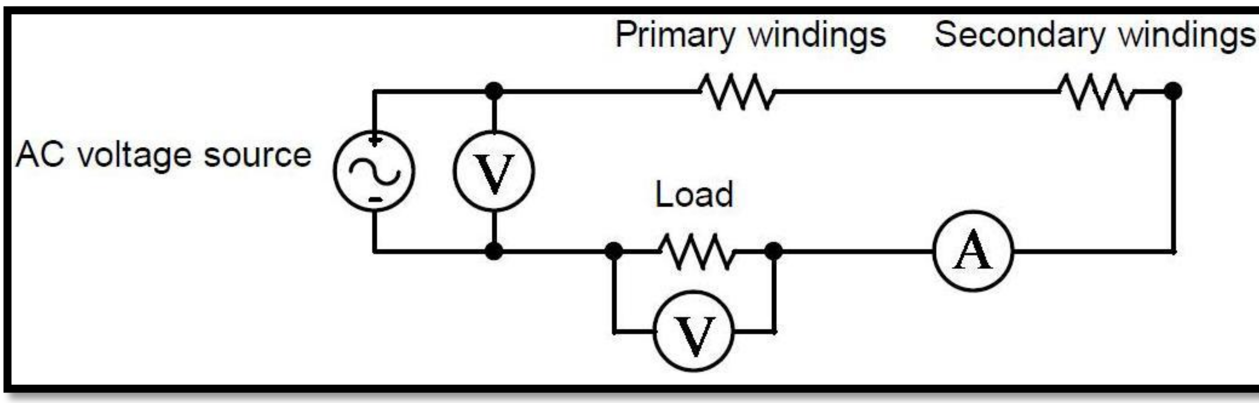


Figure F. A circuit to measure winding loss

Experiment

1. The water was heated until near boiling.
2. The hot water was carefully poured into the container.
3. Using tongs, both windings were placed into the hot water.
4. The voltage source was set to 4.0V.
5. The temperature of the windings, as well as the voltage in, current, and the voltage across the load were recorded.
6. Data was recorded as the water cooled to room temperature. At least seven data points were recorded.

Appendix B – Winding resistance and loss (experiment #1) – Data collection and processing

Raw data

Temperature (primary windings) T_p / °C ± 0.1 °C	Temperature (secondary windings) T_s / °C ± 0.1 °C	Voltage in V_{in} / volt ± 0.001 V	Current I / A ± 0.0001 A	Voltage across load V_{load} / volt ± 0.001 V
57.0	56.6	4.012	0.1569	3.130
52.3	51.4	4.016	0.1572	3.134
46.5	46.6	4.006	0.1576	3.159
41.4	41.3	4.018	0.1589	3.167
36.5	36.6	4.009	0.1592	3.177
31.5	31.4	4.010	0.1597	3.186
27.2	27.0	4.015	0.1602	3.191
24.2	23.8	4.005	0.1605	3.200
24.3	24.0	4.018	0.1607	3.200

Average temperature and resistance

R_{load} = 20.00Ω ±0.01Ω. The resistance of both of the windings were the same, as they both had the same number of turns of wire.

Average Temperature T/°C ± 0.2 °C	Total resistance R_T/Ω ± 0.02 Ω	Resistance of both windings R_{both}/Ω ± 0.03 Ω	Resistance a single winding R_w/Ω ± 0.016 Ω
56.8	25.57	5.570	2.785
51.9	25.55	5.547	2.774
46.6	25.42	5.419	2.709
41.4	25.29	5.286	2.643
36.6	25.18	5.182	2.591
31.5	25.11	5.110	2.555
27.1	25.06	5.062	2.531
24.0	24.95	4.953	2.477
24.2	25.00	5.003	2.502

Sample calculations

$$T_{avg} = \frac{T_1+T_2}{2} = \frac{57.0+56.6}{2} = 56.8$$

$$R_T = \frac{V}{I} = \frac{4.012}{0.1569} = 25.57\ \Omega$$

$$R_{both} = R_T - 20.00 = 25.57 - 20.00 = 5.570\ \Omega$$

$$R_w = \frac{1}{2} R_{both} = \frac{1}{2}(5.570) = 2.785\ \Omega$$

$$\Delta T_{avg} = \Delta T_1 + \Delta T_2 = 0.1 + 0.1 = 0.2^{\circ} C$$

$$\Delta R_{total} = \left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right) R_{total} = \left(\frac{0.001}{4.012} + \frac{0.001}{0.1569}\right) 25.57 = 0.02267 = 0.02\Omega$$
$$R_{both} = \Delta R_{total} + \Delta R_{load} = 0.02 + 0.01 = 0.03\Omega$$
$$\Delta R_w = \left(\frac{\Delta R_{both}}{R_{both}}\right) R_w = \frac{0.03}{5.570} 2.785 = 0.016\Omega$$

Now that the resistance of the windings for a range of temperatures is known, the temperature coefficient of the metal can be found. Since the resistance at 31.5° C appears to be close to the line of best fit, it will be used for the resistance temperature coefficient calculations. The table below shows the temperature coefficient calculated for each resistance, as well as the average.

R_o = 2.555 ± 0.016 Ω (at 31.5° C ± 0.2° C)

Average Temperature		**Resistance a single winding**	**Temperature change**	**Temperature coefficient**	
T /°C	ΔT /°C	R_w/Ω ± 0.016 Ω	T /°C ± 0.4°C	α / °C^{-1}	±Δ α / °C^{-1}
56.8	0.2	2.785	25.4	0.00356	0.00008
51.9	0.2	2.774	20.4	0.00420	0.00009
46.6	0.2	2.709	15.1	0.00401	0.00009
41.4	0.2	2.643	9.9	0.00349	0.00008
36.6	0.2	2.591	5.1	0.00279	0.00007
31.5	**0.2**	**2.555**	**0.0**	/	/
27.1	0.2	2.531	-4.4	0.00212	0.00006
24.0	0.2	2.477	-7.5	0.00411	0.00011
24.2	0.2	2.502	-7.3	0.00285	0.00008
Average					
				0.00332	0.00008

From this, the temperature coefficient of the wires is 0.00332° C^{-1}. This can be used to predict their resistance at any temperature, and will be helpful in the second experiment.

Sample calculations

$$R_T = R_o(1 + \alpha\Delta T)$$
$$\alpha = \frac{\left(\frac{R_T}{R_o} - 1\right)}{\Delta T} = \frac{\left(\frac{2.785}{2.555} - 1\right)}{56.8 - 31.5} = 0.0035568 = 0.00356\ °\mathrm{C}^{-1}$$
$$\Delta\alpha = \left(\frac{\Delta R_T}{R_T} + \frac{\Delta R_o}{R_o} + \frac{\Delta T_f}{T_f} + \frac{\Delta T_o}{T_o}\right)\alpha = \left(\frac{0.016}{2.785} + \frac{0.016}{2.555} + \frac{0.2}{25.4} + \frac{0.2}{31.5}\right) 0.00356 = 0.00008°\mathrm{C}$$

Now the relationship between the temperature and the power lost in the windings will be considered. The table below shows the power in, power to the load, and power lost.

Average temperature and power loss

Average Temperature	Resistance (of both windings)	Power in	Power to load	Power lost
T / °C ± 0.2° C	R / Ω ± 0.005 Ω	P_{in} / W ± 0.0006 W	P_{load} / W ± 0.0005 W	P_{loss} / W ± 0.001
56.8	5.570	0.6295	0.4911	0.1384
51.9	5.547	0.6313	0.4927	0.1387
46.6	5.419	0.6313	0.4979	0.1335
41.4	5.286	0.6385	0.5032	0.1352
36.6	5.182	0.6382	0.5058	0.1325
31.5	5.110	0.6404	0.5088	0.1316
27.1	5.062	0.6432	0.5112	0.1320
24.0	4.953	0.6428	0.5136	0.1292
24.2	5.003	0.6457	0.5142	0.1315

Sample calculations

$$P_{in} = IV_{in} = (0.1569)4.012 = 0.6295\ W$$

$$P_{load} = IV_{load} = 0.1569(3.130) = 0.4911\ W$$

$$P_{lost} = P_{in} - P_{load} = 0.6295 - 0.4911 = 0.1384$$

$$\Delta P_{in} = \left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right) P_{in} = \left(\frac{0.001}{4.012} + \frac{0.0001}{0.1569}\right) 0.6295 = 0.000558 = 0.0005\ W$$

$$\Delta P_{load} = \left(\frac{\Delta V}{V} + \frac{\Delta I}{I}\right) P_{load} = \left(\frac{0.001}{3.130} + \frac{0.0001}{0.1569}\right) 0.4911 = 0.0004699 = 0.0005\ W$$

$\Delta P_{loss} = \Delta P_{in} + \Delta P_{load} = 0.0005 + 0.0006 = 0.0010\ W$ (without rounding)

Appendix C – Overall transformer loss (experiment #2) – Design

Aim: To determine the effects of temperature on the overall power loss of a transformer and consequently also find the effect of temperature on core loss.

Variables:

Independent:

The temperature of the transformer

Dependent:

The voltage and current in and out of the transformer

Controlled:

The load (20.00 Ω)

The windings

The core

The voltage in

The position of the core

The position of the temperature probes in the water

Assumptions:

Every part of the transformer is always at thermal equilibrium with the water. Also, every part of the transformer is the same temperature.

Materials:

1. Transformer [or two solenoids and an iron core]
2. GLX data logger
3. Two GLX temperature probes (Sample rate: 5 Hz. Precision: ± 0.1 °C)
4. Two AC ammeters (Precision: ± 0.001 A)
5. Two AC voltmeters (Precision: ± 0.001 V)
6. A container
7. A kettle or hotplate
8. Tongs
9. 20.0 Ω resistor or some other load
10. Tape
11. Connecting cables

Procedure:

Transformer setup

1. The GLX was turned on.
2. The bottom piece of the iron core of the transformer was slid into both solenoids. The top part of the core was set on top of the bottom part.
3. The primary windings were connected to an AC voltage source, ammeter, and voltmeter. The secondary windings were connected to the load resistance, an ammeter, and a voltmeter. The appropriate arrangement of these parts is shown in the circuit diagram below in figure G.
4. One temperature probe was taped securely to the middle of the core.
5. The other temperature probe was set in the middle on the container and taped in place.
6. The transformer was placed into the container.
7. The voltage source was plugged in.
8. The kettle was filled with water and plugged in.

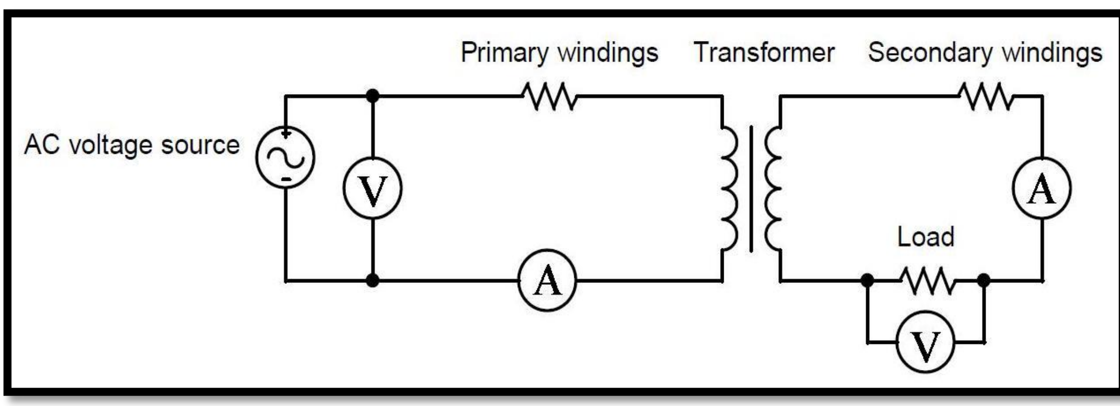


Figure G. A circuit to measure overall transformer loss.

Experiment

1. The water was heated until near boiling.
2. The hot water was carefully poured into the container.
3. If core moved while the water was being poured in, the thongs were used to reposition it to its original position.
4. The voltage source was set to 5.0V.
5. The temperature of the core and water, as well as the voltage in, current in, voltage out, and current out were recorded.
6. Data was recorded as the water cooled to room temperature.
7. At least five data points were recorded.

Appendix D –Overall transformer loss (experiment #2) – Data collection and processing

Raw data

Temperature		Electricity In		Electricity Out	
Temperature #1 T_1 / °C ± 0.1 °C	Temperature #2 T_2 / °C ± 0.1 °C	Voltage V_{in} / volt ± 0.001 V	Current I_{in} / A ± 0.001 A	Voltage V_{load} / volt ± 0.001 V	Current I_{out} / A ± 0.0001 A
73.9	72.3	2.275	0.640	1.047	0.0524
72.8	71.8	2.263	0.638	1.043	0.0523
72.0	71.7	2.270	0.640	1.046	0.0524
69.7	69.7	2.268	0.642	1.039	0.0521
65.4	65.9	2.262	0.645	1.038	0.0520
62.2	62.9	2.262	0.649	1.039	0.0522
57.4	57.7	2.266	0.654	1.040	0.0522
55.0	55.7	2.278	0.660	1.047	0.0525
52.1	52.5	2.273	0.663	1.052	0.0513
46.7	47.8	2.274	0.669	1.055	0.0516
41.3	41.5	2.256	0.670	1.043	0.0522
37.3	37.1	2.261	0.677	1.056	0.0529
35.0	36.0	2.261	0.679	1.061	0.0532
33.2	35.4	2.264	0.682	1.068	0.0535
31.1	31.3	2.260	0.683	1.068	0.0535
28.2	28.3	2.265	0.688	1.072	0.0537
24.9	25.0	2.264	0.692	1.064	0.0528
24.0	24.0	2.264	0.694	1.064	0.0528
24.0	24.1	2.259	0.693	1.063	0.0525

Processed data

Average temperature, and power in, and power out.

Average Temperature	Power in	Power to load
T_{avg} / °C ± 0.2°C	P_{in} / W ± 0.003 W	P_{load} / W ± 0.0002 W
73.1	1.46	0.0549
72.3	1.44	0.0545
71.9	1.45	0.0548
69.7	1.46	0.0541
65.7	1.46	0.0540
62.6	1.47	0.0542
57.6	1.48	0.0543
55.4	1.50	0.0550
52.3	1.51	0.0540
47.3	1.52	0.0544
41.4	1.51	0.0544
37.2	1.53	0.0559
35.5	1.54	0.0564
34.3	1.54	0.0571
31.2	1.54	0.0571
28.3	1.56	0.0576
25.0	1.57	0.0562
24.0	1.57	0.0562
24.1	1.57	0.0558

Sample Calculations

$$T_{avg} = \frac{T_1+T_2}{2} = \frac{73.9+72.3}{2} = 73.1°\ C$$
$$P_{in} = I_{in}V_{in} = 0.640(2.275) = 1.456 = 1.46\ W$$
$$P_{load} = I_{out}V_{load} = 0.0524(1.047) = 0.05486 = 0.0549\ W$$
$$\Delta T_{avg} = 2\Delta T = 0.2°\ C$$
$$\Delta P_{in} = \left(\frac{\Delta I}{I} + \frac{\Delta V}{V}\right) P_{in} = \left(\frac{0.0001}{0.640} + \frac{0.001}{2.275}\right) 1.46 = 0.00292 = 0.003\ W$$
$$\Delta P_{load} = \left(\frac{\Delta I}{I} + \frac{\Delta V}{V}\right) P_{load} = \left(\frac{0.0001}{0.0524} + \frac{0.001}{1.047}\right) 0.0549 = 0.000157 = 0.0002\ W$$

Total power loss and temperature

Average Temperature	Total power lost
T_{avg} / °C ± 0.2°C	P_{in} / W ± 0.003 W
73.1	1.40
72.3	1.39
71.9	1.40
69.7	1.40
65.7	1.41
62.6	1.41
57.6	1.43
55.4	1.45
52.3	1.45
47.3	1.47
41.4	1.46
37.2	1.47
35.5	1.48
34.3	1.49
31.2	1.49
28.3	1.50
25.0	1.51
24.0	1.52
24.1	1.51

Sample calculations

$$P_{lost} = P_{in} - P_{load} = 1.46 - 0.0549 = 1.4051 = 1.40\ W$$
$$\Delta P_{lost} = P_{in} + P_{load} = 0.0002 + 0.003 = 0.0032 = 0.003\ W$$

Predicted winding resistances for temperatures (from experiment #1)

Temperature	**Estimated primary resistance**	**Estimated secondary resistance**
T_{avg} / °C ± 0.2°C	R_p / Ω ± 0.1 Ω	R_s / Ω ± 0.1 Ω
73.1	2.91	2.91
72.3	2.91	2.91
71.9	2.90	2.90
69.7	2.88	2.88
65.7	2.84	2.84
62.6	2.82	2.82
57.6	2.77	2.77
55.4	2.75	2.75
52.3	2.73	2.73
47.3	2.68	2.68
41.4	2.64	2.64
37.2	2.60	2.60
35.5	2.58	2.58
34.3	2.57	2.57
31.2	2.55	2.55
28.3	2.53	2.53
25.0	2.50	2.50
24.0	2.49	2.49
24.1	2.49	2.49

Sample calculations

$$R_T = R_o(1 + \alpha\Delta T) = 2.555[1 + 0.00332(73.1 - 31.5)] = 2.91\ \Omega$$

$$\Delta R_T = \left(\frac{\Delta R_o}{R_o} + \frac{\Delta\alpha}{\alpha} + \frac{\Delta T_f}{T_f} + \frac{\Delta T_o}{T_o}\right) R_T = \left(\frac{0.016}{2.555} + \frac{0.00008}{0.00332} + \frac{0.2}{73.1} + \frac{0.2}{31.5}\right) 2.91 = 0.1\ \Omega$$

Primary winding losses

The primary winding losses were calculated using the expected resistance of the primary windings and the current flowing into the circuit at each temperature. The results are shown below.

Temperature	**Estimated resistance**	**Primary winding losses**	
T_{avg} / °C ± 0.2°C	R_p / Ω ± 0.1 Ω	P_p / W	± ΔP_p / W
73.1	2.91	1.19	0.04
72.3	2.90	1.18	0.04
71.9	2.90	1.19	0.04
69.7	2.88	1.19	0.04
65.7	2.84	1.18	0.04
62.6	2.82	1.19	0.04
57.6	2.78	1.19	0.04
55.4	2.76	1.20	0.04
52.3	2.73	1.20	0.04
47.3	2.69	1.20	0.04
41.4	2.64	1.18	0.04
37.2	2.60	1.19	0.05
35.5	2.59	1.19	0.05
34.3	2.58	1.20	0.05
31.2	2.55	1.19	0.05
28.3	2.53	1.20	0.05
25.0	2.50	1.20	0.05
24.0	2.49	1.20	0.05
24.1	2.49	1.20	0.05

Sample calculations

$$P_{primary\ loss} = I_{in}^2\, R_T = 0.640^2(2.91) = 1.19\ W$$

$$\Delta P_{primary\ loss} = \left(\frac{2\Delta I}{I} + \frac{\Delta R}{R}\right) P_{primary\ loss} = \left(\frac{2(0.001)}{0.640} + \frac{0.1}{2.91}\right) 1.19 = 0.0446 = 0.04\ W$$

Secondary winding losses

The power lost in the secondary windings can be worked out using the expected resistance of the secondary windings and the current out, similarly to how the primary winding losses were found. The table below shows the calculated values for the secondary losses.

Temperature	**Estimated resistance**	**Secondary winding losses**
T_{avg} / °C ± 0.2°C	R_s / Ω ± 0.1 Ω	P_s / W ± 0.0003 W
73.1	2.91	0.00798
72.3	2.90	0.00794
71.9	2.90	0.00796
69.7	2.88	0.00781
65.7	2.84	0.00769
62.6	2.82	0.00768
57.6	2.78	0.00756
55.4	2.76	0.00760
52.3	2.73	0.00719
47.3	2.69	0.00716
41.4	2.64	0.00719
37.2	2.60	0.00729
35.5	2.59	0.00733
34.3	2.58	0.00738
31.2	2.55	0.00731
28.3	2.53	0.00729
25.0	2.50	0.00697
24.0	2.49	0.00695
24.1	2.49	0.00687

Sample calculations

$$P_{secondary\ loss} = I_{out}^2\ R_T = 0.0524^2(2.91) = 0.00798\ W$$

$$\Delta P_{secondary\ loss} = \left(\frac{2\Delta I}{I} + \frac{\Delta R}{R}\right) P_{primary\ loss} = \left(\frac{2(0.0001)}{0.0524} + \frac{0.1}{2.91}\right) 0.00798 = 0.0446 = 0.0003\ W$$

Core losses

The core losses can now be calculated by considering the total power lost as well as the winding losses. The total loss must equal the sum of winding and core losses. Therefore the core losses can be found. The equation to find the core losses is shown in the sample calculations below. The table below shows the result the core losses at each temperature.

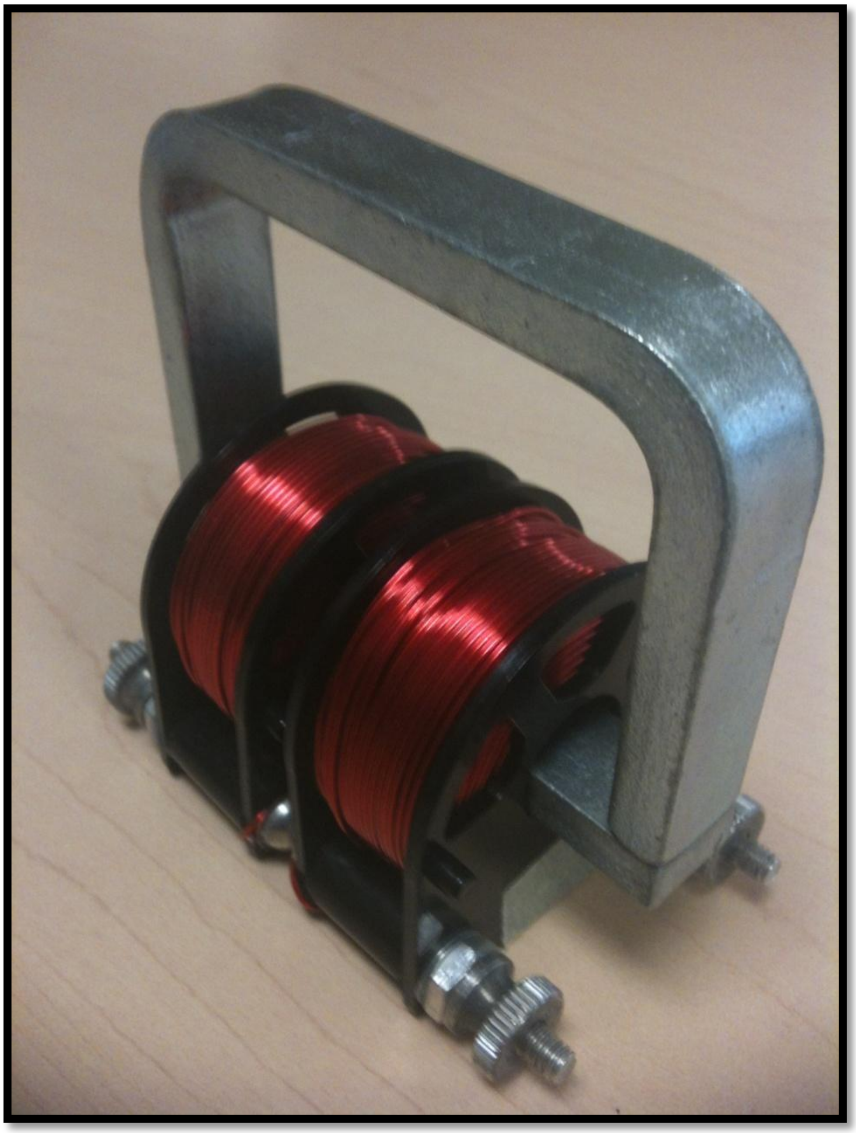
Temperature	**Core losses**
T_{avg} / °C ± 0.2°C	P_s / W ± 0.05 W
73.1	0.202
72.3	0.200
71.9	0.203
69.7	0.207
65.7	0.214
62.6	0.219
57.6	0.233
55.4	0.240
52.3	0.245
47.3	0.256
41.4	0.265
37.2	0.274
35.5	0.278
34.3	0.280
31.2	0.288
28.3	0.297
25.0	0.307
24.0	0.308
24.1	0.306

Sample Calculations

$P_{core\ loss} = P_{total\ loss} - P_{primary\ loss} - P_{secondary\ loss} = 1.40 - 1.19 - 0.00798 = 0.202\ W$

$\Delta P_{core\ loss} = \Delta P_{total\ loss} + \Delta P_{primary\ loss} + \Delta P_{secondary\ loss} = 0.003 + 0.04 + 0.0003 =$
$0.05\ W$ (without rounding)

Appendix E - Pictures



A picture of the transformer used for all experiments. The solenoid on the right was used as the primary windings, and the solenoid on the left was used as the secondary windings.