NAME ________________________________

1. For the surface given by $z = e^{x \cos y} + 1/(x^2 + 1)$,

(10) (i) find a normal vector to the surface at (0,0,2),

Answer:

(10) (ii) find the equation of the tangent plane at (0,0,2).

Answer:

2. For the integral

$$\int_C \left(\frac{2xy}{1+x^2} + z\right) dx + \ln(1+x^2)\, dy + x\, dz,$$

(15) (i) show that the form under the integral sign is exact,

(10) (ii) evaluate the integral for a curve C which goes from (0,1,2) to (1,0,3).

Answer:

3. Let $\vec{F} = 3x\vec{i} + 3y\vec{j} + z\vec{k}$ and T be the solid region bounded above by the surface of $z = 9 - x^2 - y^2$, and below by the xy plane.

(15) (i) Without using the divergence theorem, compute the surface integral $\int\int_S \vec{F} \cdot \vec{n} dA$, where S is the boundary of T, and $\vec{n}$ is the outward unit normal.

Answer:

(15) (ii) Use the divergence theorem to recompute the integral in (i) as a volume integral.

Answer:

4. Let S be the surface given by the portion of the graph of $z = 4 - y^2$ cut off by the planes $x = 0$, $z = 0$, and $y = x$. Let $\mathbf{F} = xz\mathbf{j}$.

(10) (i) Set up, but do not evaluate, the integral $\int\int_S curl\mathbf{F} \cdot \mathbf{n}\, dA$, where $\mathbf{n}$ is the unit upward pointing normal to S.

$$\int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} \boxed{}\; du\, dv$$

(10) (ii) Using Stokes's Theorem, evaluate the integral in (i) by means of a line integral.

Answer: