HW 7 solutions

13.6 p.636

#1) $\cosh z = \dfrac{e^{z}+e^{-z}}{2} =$

$\dfrac{e^{x}(\cos y + i\sin y) + e^{-x}(\cos y - i\sin y)}{2} =$

$\dfrac{e^{x}\cos y + e^{-x}\cos y}{2} + i\,\dfrac{e^{x}\sin y - e^{-x}\sin y}{2} =$

$\cosh x \cos y + i \sinh x \sin y$

$\sinh z = \dfrac{e^{z}-e^{-z}}{2} =$

$\dfrac{e^{x}(\cos y + i\sin y) - e^{-x}(\cos y - i\sin y)}{2} =$

$\dfrac{e^{x}\cos y - e^{-x}\cos y}{2} + i\,\dfrac{e^{x}\sin y + e^{-x}\sin y}{2}$

$= \sinh x \cos y + i \cosh x \sin y$

#3) $\cosh^2 z - \sinh^2 z = \dfrac{(\cancel{e^{2z}} + 2 + \cancel{e^{-2z}}) - (\cancel{e^{2z}} - 2 + \cancel{e^{-2z}})}{4}$

$= \dfrac{2+2}{4} = 1$

$\cosh^2 z + \sinh^2 z = \dfrac{(e^{2z} + \cancel{2} + e^{-2z}) + (e^{2z} - \cancel{2} + e^{-2z})}{4}$

$= \dfrac{2e^{2z} + 2e^{-2z}}{4} = \cosh 2z$

#7) $\cos i = \dfrac{e^{i^2}+e^{-i^2}}{2} = \dfrac{e^{-1}+e}{2} = \cosh 1$

$\sin i = \dfrac{e^{i^2}-e^{-i^2}}{2i} = \dfrac{e^{-1}-e}{2i} = i \sinh 1$

#13) $\cos(-z) = \dfrac{e^{-iz} + e^{iz}}{2} = \cos z$

$\sin(-z) = \dfrac{e^{-iz} - e^{iz}}{2i} = -\sin z$

#16) $\sin z = \dfrac{e^{iz} - e^{-iz}}{2i} = 100$

$e^{iz} - e^{-iz} = 200i$, $(e^{iz})^2 - 200i\, e^{iz} - 1 = 0$

$e^{iz} = 100i \pm \sqrt{-100^2 + 1} = i(100 \pm \sqrt{99 \cdot 101})$

$= i(100 \pm 3\sqrt{11 \cdot 101}) = i(100 \pm 3\sqrt{1111})$

$iz = \ln(i(100 \pm 3\sqrt{1111})) =$

$\ln i + \ln(100 \pm 3\sqrt{1111}) =$

$\dfrac{\pi i}{2} + \ln(100 \pm 3\sqrt{1111}) + 2k\pi i$

$k = 0, \pm 1, \pm 2, \ldots$

Note that both numbers under ln are real positive, so ln is understood as in calculus (or as Ln).

<u>Answer</u>: $z = \dfrac{\pi}{2} + 2k\pi - i \ln(100 \pm 3\sqrt{1111})$

$= \dfrac{\pi}{2} + 2k\pi + \dfrac{1}{i} \ln(100 \pm 3\sqrt{1111})$

$= \dfrac{\pi}{2} + 2k\pi - i \ln(100 \pm \sqrt{9999})$

$= \dfrac{\pi}{2} + 2k\pi + \dfrac{1}{i} \ln(100 \pm \sqrt{9999})$.

Any of these answers is OK.

13.7 p. 640

#5) $\mathrm{Ln}(-11) = \mathrm{Ln}(-1) + \ln(11) = \ln(11) + \pi i$

Here $\ln(11)$ is understood as in calculus

#7) $\mathrm{Ln}(4-4i) = \ln(4\sqrt{2}) + \mathrm{Ln}\left(\frac{1-i}{\sqrt{2}}\right)$

$= \frac{5}{2}\ln 2 - \frac{\pi}{4} i$

#8) $\mathrm{Ln}(1 \pm i) = \pm \frac{\pi}{4} i + \frac{\ln 2}{2}$

#15) $\ln(e^{i}) = i + 2k\pi i, \quad k = 0, \pm 1, \pm 2, \ldots$

#19) $\ln z = 4 - 3i, \quad z = e^{4-3i} = e^{4}(\cos 3 - i \sin 3)$

#23) $(1+i)^{1-i} = e^{(1-i)\ln(1+i)}$

Principal value is

$$e^{(1-i)\mathrm{Ln}(1+i)} = e^{(1-i)\left(\frac{\ln 2}{2} + \frac{\pi i}{4}\right)} =$$

$$e^{\frac{\ln 2}{2} + \frac{\pi}{4} + i\left(\frac{\pi}{4} - \frac{\ln 2}{2}\right)} = e^{\frac{\ln 2}{2} + \frac{\pi}{4}}\left(\cos\left(\frac{\pi}{4} - \frac{\ln 2}{2}\right) + i \sin\left(\frac{\pi}{4} - \frac{\ln 2}{2}\right)\right)$$

14.1 p. 651

#1) $z(t) = \left(1 + \frac{i}{2}\right)t, \qquad 2 \le t \le 5$

A segment from $z(2) = 2+i$ to $z(5) = 5 + \frac{5}{2}i$

#11) Segment from $(-1,1)$ to $(1,3)$ can be parameterized as $z = t + (2+t)i$, $-1 \le t \le 3$. Alternatively, as $(1,3)-(-1,1) = (2,2)$, the same segment is

$z = (1+3i) + (2+2i)t = (1+2t) + i(3+2t), \quad 0 \le t \le 1.$

#19) $y = 1 - \frac{1}{4}x^2, \qquad -2 \le x \le 2$

$z = t + \left(1 - \frac{t^2}{4}\right)i, \qquad -2 \le t \le 2$

#21) C is the segment from $1+i$ to $3+3i$,

$z = (1+i)t, \quad 1 \le t \le 3$

$$\int_C \operatorname{Re} z \, dz = \int_1^3 t(1+i)\,dt = (1+i)\frac{t^2}{2}\Big|_1^3 = 4+4i$$

The first method of integration (using antiderivative) does not apply since $\operatorname{Re} z$ is not analytic.

#25) C is from 1 to i along the axes.

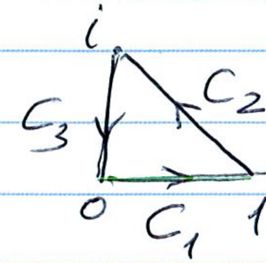
$$\int_C z \exp(z^2)\,dz = \frac{1}{2}\exp(z^2)\Big|_1^i =$$

$$\frac{1}{2}\exp(-1) - \frac{1}{2}\exp(1) = \frac{e^{-1} - e}{2} = -\sinh 1.$$

#29) $\int_C \mathrm{Im}(z^2)\,dz$ counterclockwise

around the triangle with vertices

0, 1, i



The first method of integration (using antiderivative) does not apply since $\mathrm{Im}(z^2) = 2xy$ is not analytic

$C_1 = \{ z = t, \ 0 \le t \le 1 \}$

$C_2 = \{ z = (1-t) + it, \quad 0 \le t \le 1 \}$

$C_3 = \{ z = -it, \quad -1 \le t \le 0 \}$

$$\int_C \mathrm{Im}(z^2)dz = \int_{C_1} \mathrm{Im}(z^2)dz + \int_{C_2} \mathrm{Im}(z^2)dz + \int_{C_3} \mathrm{Im}(z^2)dz$$

The integrals over C_1 and C_3 are 0.

$$\int_{C_2} \mathrm{Im}(z^2)dz = \int_0^1 2t(1-t)\cdot(-1+i)\,dt =$$

$$(1-i)\left(\frac{2t^3}{3} - t^2\right)\Big|_0^1 = (1-i)\left(\frac{2}{3} - 1\right) = -\frac{1}{3} + \frac{i}{3}$$