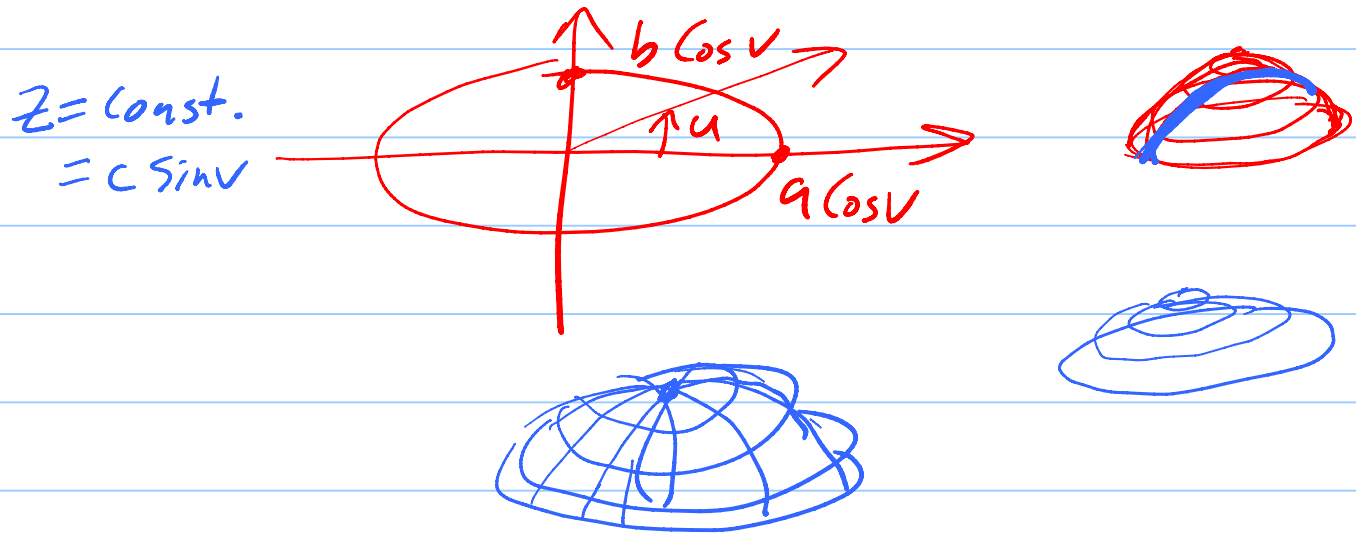
10.5 ; 7. $\vec{r}(u,v)$

$= a\cos v \cos u\, \hat{i}$

$+ b\cos v \sin u\, \hat{j}$

$+ c \sin v\, \hat{k}$ ← $v = \text{Const.}$, $z = c$

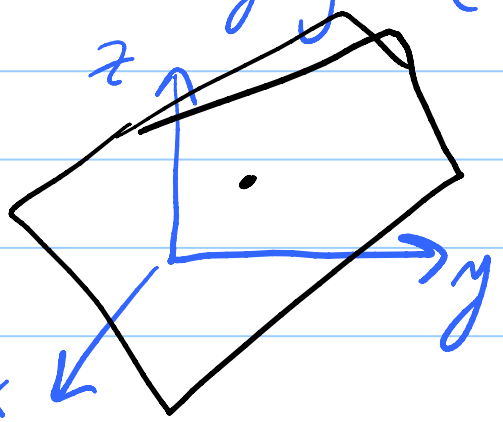



10.5 ; 14. $4x + 3y + 2z = 12$

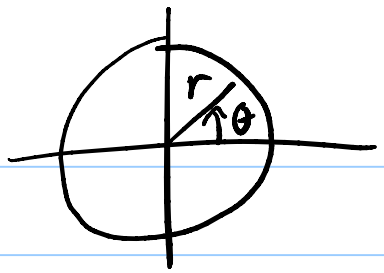
$z = 6 - 2x - \frac{3}{2}y$

$\vec{r}(x,y) = x\,\hat{i} + y\,\hat{j} + \left(6 - 2x - \frac{3}{2}y\right)\hat{k}$

$u \quad v$



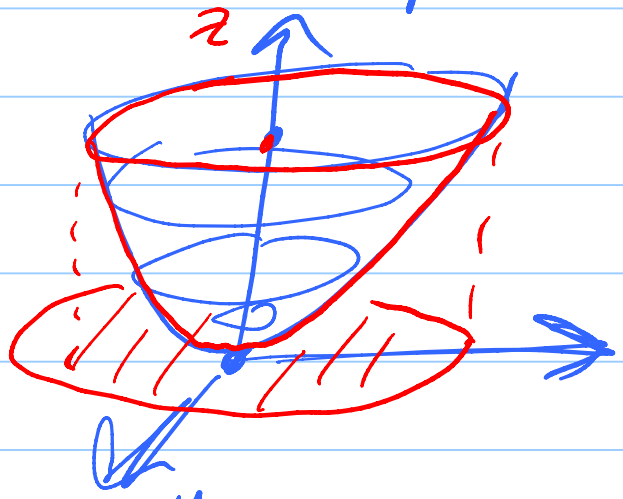

$z = f(x,y)$




$$\vec{r} = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j} + z\hat{k}$$

$$z = f(x,y)$$

10.5; 18. $z = \sqrt{x^2 + 4y^2}$



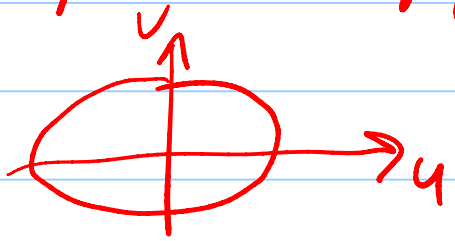

$$\frac{x^2}{z^2} + \frac{4y^2}{z^2} = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a = z \qquad b = \frac{z}{2}$

$u$ $v$

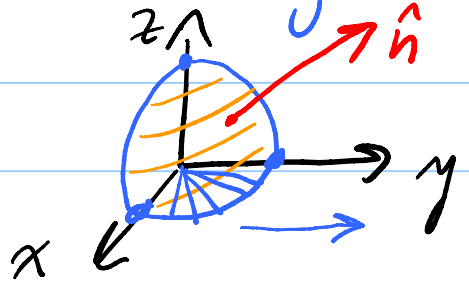
$$\vec{r}(x,y) = x\hat{i} + y\hat{j} + \sqrt{x^2+4y^2}\,\hat{k}$$




$u$ $v$

$r, \theta \qquad u, v$

10.6; 3. $\vec{F} = x\hat{j}$ $S$: $x^2 + y^2 + z^2 = 1$




$$0 \le \theta \le \frac{\pi}{2}$$

$$0 \le \psi \le \frac{\pi}{2}$$

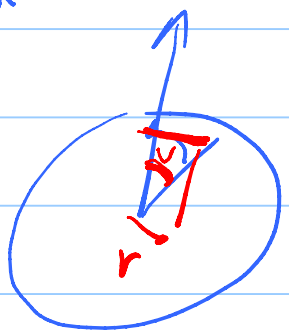
$$\iint_S \vec{F}\cdot\hat{n}\,dA$$

$\hat{n} = \langle a, b, c\rangle$ $c>0$ up, $c<0$ down

$z = f(x,y)$

$\vec{N} = -f_x\hat{i} - f_y\hat{j} + \hat{k}$

1




$$\begin{cases} x = \sin v\cos u \\ y = \sin v \sin u \\ z = \cos v \end{cases}$$

$$\vec{r} = \sin v\cos u\,\hat{i} + \sin v\cos u\,\hat{j} + \cos v\,\hat{k}$$

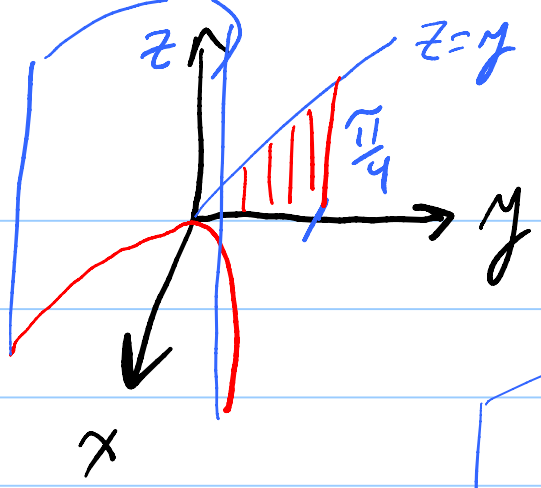
$$\vec{N} = \vec{r}_u \times \vec{r}_v$$

$$\iint_S \vec{F}\cdot\hat{n}\,dA = \iint_R \vec{F}(\vec{r})\cdot\frac{\vec{r}_u\times\vec{r}_v}{\|\vec{r}_u\times\vec{r}_v\|}\underbrace{\|\vec{r}_u\times\vec{r}_v\|\,du\,dv}_{dA}$$

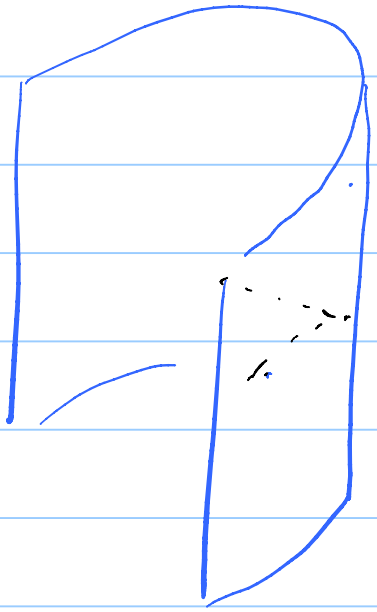
$0\le u\le \frac{\pi}{2}$, $0\le v\le \frac{\pi}{2}$

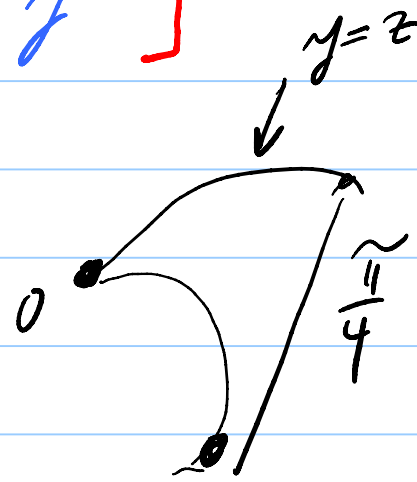
$$= \int_0^{\pi/2}\int_0^{\pi/2}\left[(\sin v\cos u)\hat{j}\right]\cdot(\vec{r}_u\times\vec{r}_v)\,du\,dv$$

10.6 : 7. $\vec{F} = \sin y\,\hat{j} + \cos z\,\hat{k}$ $S: x = y^2$




$$\begin{bmatrix} 0 \le y \le \frac{\pi}{4} \\ 0 \le z \le y \end{bmatrix} \leftarrow y, z$$





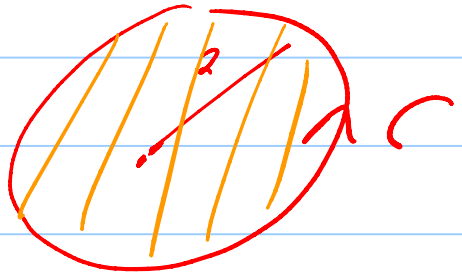

10.4: 16. $W = x^2 + y^2$

(a) $\iint_R \Delta w \, dx\,dy = \oint_C \frac{\partial w}{\partial n} \, ds$

$R$ $4 \cdot \pi 2^2$

$ds = 2d\theta$

$W = x^2 + y^2$

$R$ = 


$\Delta w = 4$

$C \cdot 2\pi \cdot 2$

$\frac{\partial w}{\partial n} = \nabla w \cdot \hat{n}$

$$z = f(x,y)$$

$$\vec{r} = x\hat{i} + y\hat{j} + f(x,y)\hat{k}$$

$$\vec{N} = \vec{r}_x \times \vec{r}_y = -\frac{\partial f}{\partial x}\hat{i} - \frac{\partial f}{\partial y}\hat{j} + \hat{k}$$ ← upward normal

$$\iint_S \vec{F}\cdot\hat{n}\, dA = \iint_R \vec{F}(x,y,f(x,y)) \cdot \frac{\left[-f_x\hat{i} - f_y\hat{j} + \hat{k}\right]}{\|\vec{r}_x \times \vec{r}_y\|} \underbrace{\|\vec{r}_x \times \vec{r}_y\|}_{dA} \cdot dx\,dy$$