Section 13.1 p. 612

#1) From the multiplication formula

$$(x+iy)(x+iy) = (x^2-y^2) + 2xy\,i$$

applied to $i = 0 + i1$, we get $i^2 = -1$.
(In fact, multiplication formula was derived from the identity $i^2=-1$.)
Then, $i^3 = i^2 \cdot i = -1 \cdot i = -i$,
$i^4 = (i^2)^2 = (-1)^2 = 1$, etc.

#9) $z_1 = -2 + 11i$, $z_2 = 2 - i$
$\mathrm{Re}\, z_1 = -2$, $(\mathrm{Re}\, z_1)^2 = 4$
$z_1^2 = (-2)^2 - 11^2 + 2(-2)\cdot 11\, i = -117 - 44i$, $\mathrm{Re}(z_1^2) = -117$

#11) $z_1 - z_2 = \underbrace{-4+12i}_{} = 4(-1+3i)$, $(z_1-z_2)^2 = 16(-1+3i)^2 = 16(-8-6i)$

$(z_1-z_2)^2/16 = -8-6i$

#13) $(z_1+z_2)(z_1-z_2) = 10i(-4+12i) = -120-40i$
$z_1^2 - z_2^2 = (z_1+z_2)(z_1-z_2) = -120 - 40i$

#17) $z^4 = (z^2)^2 = (x^2-y^2+2xyi)^2 =$
$(x^2-y^2)^2 - (2xy)^2 + 2(x^2-y^2)\,2xyi =$
$(x^4 - 6x^2y^2 + y^4) + 4xy(x^2-y^2)\,i$
$\mathrm{Re}\, z^4 = x^4 - 6x^2y^2 + y^4$
$\mathrm{Re}\, z^2 = x^2-y^2$, $(\mathrm{Re}\, z^2)^2 = x^4 - 2x^2y^2 + y^4$
$\mathrm{Re}\, z^4 - (\mathrm{Re}\, z^2)^2 = -4x^2y^2$

#19) $\dfrac{z}{\bar z} = \dfrac{z^2}{z\bar z} = \dfrac{x^2-y^2+2xyi}{x^2+y^2}$

$\mathrm{Re}\,\dfrac{z}{\bar z} = \dfrac{x^2-y^2}{x^2+y^2}$, $\mathrm{Im}\,\dfrac{z}{\bar z} = \dfrac{2xy}{x^2+y^2}$

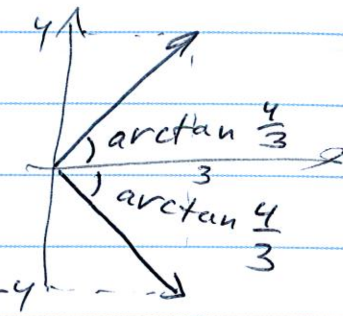
HW 5-6 Solutions

13.2 p 618

#1) $1+i = \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

#5) $\dfrac{\sqrt{2}+\frac{i}{3}}{-\sqrt{8}-2i/3} = \dfrac{\sqrt{2}+\frac{i}{3}}{-2\left(\sqrt{2}+\frac{i}{3}\right)} = -\frac{1}{2} = \frac{1}{2}(\cos\pi + i\sin\pi)$

#11) $3 \pm 4i = 5\left(\frac{3}{5} \pm \frac{4}{5}i\right)$, $\quad \operatorname{Arg}(3 \pm 4i) = \pm\arctan\frac{4}{3}$



#13) $(1+i)^{20} = (\sqrt{2})^{20}\left(\cos\frac{20\pi}{4} + i\sin\frac{20\pi}{4}\right)$

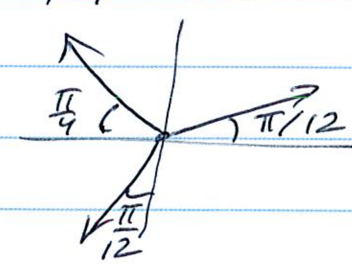
$= 2^{10}(\cos 5\pi + i\sin 5\pi)$

$= 1024(\cos\pi + i\sin\pi) = -1024$

#16) $6\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right) = 6\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = 3 + i3\sqrt{3}$

#21) $\sqrt[3]{1+i} = \sqrt[6]{2}\left(\cos\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right) + i\sin\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right)\right)$

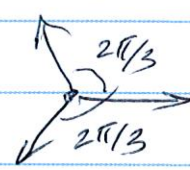
$k = 0, 1, 2$



#23) $\sqrt[3]{216} = 6\sqrt[3]{1} =$

$6\left(\cos\frac{2k\pi}{3} + i\sin\frac{2k\pi}{3}\right)$

$k = 0, 1, 2$



#28) $z^2 - (6-2i)z + 17 - 6i = 0$

$z_{1,2} = (3-i) \pm \sqrt{(3-i)^2 - (17-6i)}$

$= (3-i) \pm \sqrt{(8-6i) - (17-6i)}$

$= (3-i) \pm 3i$

$z_1 = 3+2i, \quad z_2 = 3-4i$