13.3 p. 624

#1) $|z+1-5i| \le \frac{3}{2}$ closed disk, radius $\frac{3}{2}$, center $-1+5i$

#3) $\pi < |z-4+2i| < 3\pi$ open annulus, $r_1=\pi$, $r_2=3\pi$ center $4-2i$

#5) $|\arg z| < \frac{\pi}{4}$ a sector $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$

#7) $\operatorname{Re} z \ge -1$ a half-plane to the right of the vertical line $z=-1+iy$, including the line

#11) $f(z) = \dfrac{1}{1-z} = \dfrac{1-x+iy}{(1-x)^2+y^2}$

$$u = \frac{1-x}{(1-x)^2+y^2}, \quad v = \frac{y}{(1-x)^2+y^2}$$

$$f(1-i) = \frac{1}{i} = -i.$$

#21) $\left(i(1-z)^n\right)' = -in(1-z)^{n-1}$ equals $-in$ at $z=0$
(the answer in the book is for $i(1-z)^{-n}$)

#23) $\left(\dfrac{z^3}{(z+i)^3}\right)' = \dfrac{3z^2}{(z+i)^3} - \dfrac{3z^3}{(z+i)^4}$

Alternatively: $\left(\dfrac{1}{\left(1+\frac{i}{z}\right)^3}\right)' = -\dfrac{3}{\left(1+\frac{i}{z}\right)^4} \cdot \left(-\dfrac{i}{z^2}\right)$

equals $-\dfrac{3}{2^4} \cdot \left(-\dfrac{1}{i}\right) = -\dfrac{3i}{16}$ at $z=i$

13.4 p. 629

#3) $f(z) = e^{-2x}(\cos 2y - i \sin 2y)$ $\left[= e^{-2z} \right]$

is analytic.

$u = e^{-2x} \cos 2y \quad , \quad v = -e^{-2x} \sin 2y$

$$\left.\begin{aligned} \frac{\partial u}{\partial x} &= -2e^{-2x} \cos 2y = \frac{\partial v}{\partial y} \\ \frac{\partial u}{\partial y} &= -2e^{-2x} \sin 2y = -\frac{\partial v}{\partial y} \end{aligned}\right\} \text{C.-R. equations satisfied}$$

#5) $f(z) = \mathrm{Re}(z^2) - i\, \mathrm{Im}(z^2)$ $\left[= \overline{z^2} = (\bar{z})^2 \right]$

is not analytic

$u = x^2 - y^2 \quad , \quad v = -2xy$

$$\frac{\partial u}{\partial x} = 2x \neq \frac{\partial v}{\partial y} = -2x$$ C.-R. equations not satisfied

(In general, if $f(z)$ is analytic then $\overline{f(z)}$ and $f(\bar{z})$ are not analytic unless f is a constant.)

#7) $f(z) = i/z^8 = \dfrac{i}{r^8}(\cos 8\theta - i \sin 8\theta)$

$u = \dfrac{\sin 8\theta}{r^8} \quad , \quad v = \dfrac{\cos 8\theta}{r^8}$ is analytic

$u_r = -\dfrac{8 \sin 8\theta}{r^9} = \dfrac{1}{r} v_\theta , \quad v_r = -\dfrac{8 \cos 8\theta}{r^9} = -\dfrac{1}{r} u_\theta$

C.-R. equations in polar form satisfied.

#13) $u = xy \left[= \mathrm{Im}\left(\frac{z^2}{2}\right)\right]$ is <u>harmonic</u>

$$\nabla^2 u = \frac{\partial^2}{\partial x^2} xy + \frac{\partial^2}{\partial y^2} xy = 0$$

$$v_y = u_x = y \Rightarrow v = \frac{y^2}{2} + h(x)$$

$$v_x = -u_y = -x \Rightarrow h'(x) = -x \Rightarrow h(x) = -\frac{x^2}{2} + \text{const}$$

$$f(z) = xy + i\left(\frac{y^2 - x^2}{2}\right) \quad \left[= -i\frac{z^2}{2}\right]$$

is analytic

#17) $v = (2x+1)y \quad \left[= \mathrm{Im}(z^2 + z)\right]$

is <u>harmonic</u>

$$\nabla^2 v = \frac{\partial^2}{\partial x^2}(2xy + y) + \frac{\partial^2}{\partial y^2}(2xy + y) = 0$$

$f(z) = z^2 + z = u + iv$, where $u = x^2 - y^2 + x$

#19) $v = e^x \sin 2y$ is <u>not harmonic</u>

$$\nabla^2 v = \frac{\partial^2}{\partial x^2}(e^x \sin 2y) + \frac{\partial^2}{\partial y^2}(e^x \sin 2y) =$$

$$e^x \sin 2y - 4e^x \sin 2y = -3e^x \sin 2y \neq 0$$

13.5 p. 632

#3) $z = 2\pi i(1+i)$

$$e^z = e^{-2\pi + 2\pi i} = e^{-2\pi} \quad \underline{\text{real}}$$

$$|e^z| = e^{-2\pi}$$

#9) $z = 4+3i, \quad |z| = 5, \quad \text{Arg } z = \arctan\frac{3}{4}$

$$z = 5e^{i\arctan\frac{3}{4}}$$

#15) $w = \exp(z^2) = \exp[(x^2 - y^2) + i\,2xy]$

$$= e^{x^2-y^2}(\cos(2xy) + i\sin(2xy)).$$

$$\text{Re } w = e^{x^2-y^2}\cos(2xy)$$

$$\underline{\text{Im}}\ w = e^{x^2-y^2}\sin(2xy)$$

#17) $w = \exp(z^3) = \exp(x^3 + 3ix^2y - 3xy^2 - iy^3)$

$$= \exp[(x^3 - 3xy^2) + i(3x^2y - y^3)]$$

$$= e^{x^3-3xy^2}(\cos(3x^2y - y^3) + i\sin(3x^2y - y^3))$$

$\text{Re } w = e^{x^3-3xy^2}\cos(3x^2y - y^3)$, $\text{Im } w = e^{x^3-3xy^2}\sin(3x^2y - y^3)$

#19) $e^z = 1 \Rightarrow z = 2\pi i k,$

$$k = 0, \pm 1, \pm 2, \ldots$$