$$\begin{cases} z = f(x,y) \quad \Leftarrow \varphi = 0 \\ \varphi(x,y,z) = z - f(x,y) \\ \vec{N} = \nabla \varphi \end{cases}$$

$$\vec{r}(x,y) = x\,\hat{\imath} + y\,\hat{\jmath} + f(x,y)\,\hat{k}$$

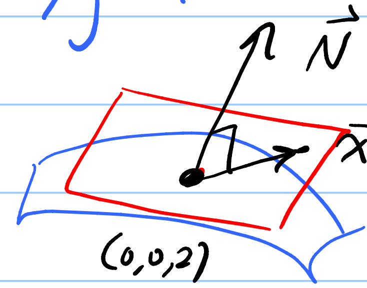
$$\vec{N} = \vec{r}_x \times \vec{r}_y$$

1. ii) $$\vec{N} = \nabla\left(z - e^{x\cos y} - \frac{1}{x^2+1}\right)$$

$$= \left(-\cos y\, e^{x\cos y} + \frac{2x}{(x^2+1)^2}\right)\hat{\imath}$$

$$+ \left(\sin y\, e^{x\cos y}\right)\hat{\jmath} + \hat{k}$$

$$\vec{N}\Big|_{(0,0,2)} = -\hat{\imath} + \hat{k}$$



Tang. Plane $\vec{N}\cdot(\vec{x} - (0,0,2)) = 0$

$$(-1)(x-0) + 0(y-0) + 1(z-2) = 0$$

$$-x + z = 2$$

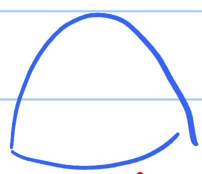
$$z = f(x,y)$$

$$\vec{r}(x,y) = x\,\hat{\imath} + y\,\hat{\jmath} + f(x,y)\,\hat{k}$$

$$\|\vec{r}_x \times \vec{r}_y\|$$

Cylindrical coords.

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$



$$z = \sqrt{R^2 - x^2 - y^2}$$

$$\frac{2xy}{1+x^2} + z \qquad \ln(1+x^2) \qquad x$$

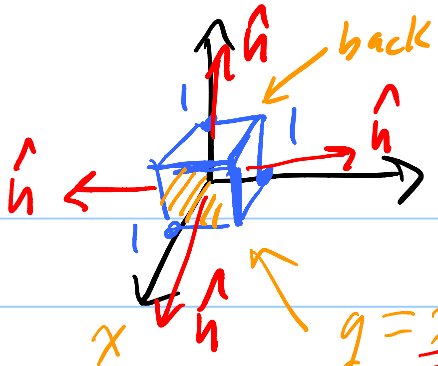
2. $F_1\,dx + F_2\,dy + F_3\,dz$

$$\vec{F} = F_1\,\hat{\imath} + F_2\,\hat{\jmath} + F_3\,\hat{k}$$

Exact if Curl $\vec{F} = 0$

$$\frac{\partial F_1}{\partial y} \overset{?}{=} \frac{\partial F_2}{\partial x}, \quad \frac{\partial F_1}{\partial z} \overset{?}{=} \frac{\partial F_3}{\partial x}, \quad \frac{\partial F_2}{\partial z} \overset{?}{=} \frac{\partial F_3}{\partial y}$$

462: 3. $f = 4y^2, \quad g = x^2$



$g = \underline{\underline{x^2}}$

$\frac{\partial g}{\partial n} = \frac{\partial g}{\partial x} = 2x \Big|_{x=1} = \underline{\underline{2}}$

$$\iiint_{\text{Cube}} \underbrace{f \Delta g + \nabla f \cdot \nabla g}_{8y^2} \, dV = \iint_S f \frac{\partial g}{\partial n} \, dA$$

$= 0$ on ~~4~~ 5 out of 6 squares.

$$\int_{z=0}^{1} \int_{y=0}^{1} (4y^2) \cdot 2 \, dy \, dz$$

10.8:

7. $\displaystyle \iint_{S'} x \, dy \, dz = \iint_S [x,0,0] \cdot \hat{n} \, dA$

p.444. (5) $\displaystyle = \iiint_\Omega \mathrm{Div}\,[x,0,0] \, dV$

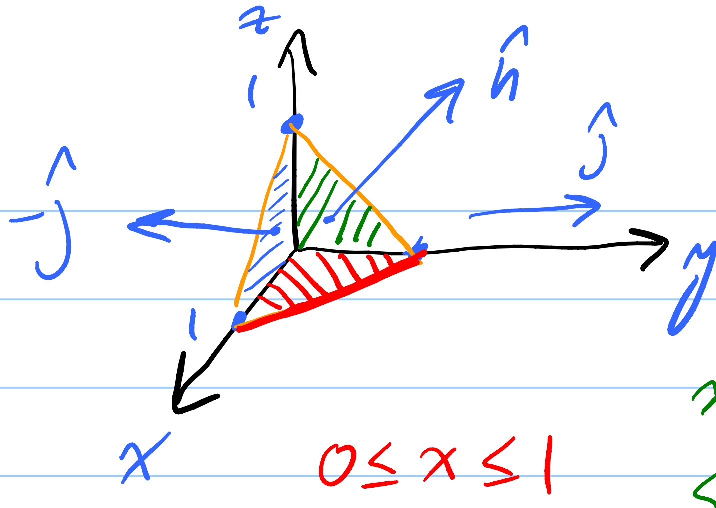
$$\iint_{S'} x \, dy \, dz + y \, dz \, dx + z \, dx \, dy$$

$$= \iint_{S'} \underbrace{[x, y, z]}_{F} \cdot \hat{n} \, dA$$

$$= \iiint_\Omega \underbrace{\mathrm{Div}\,\vec{F}}_{\frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z} = 3} \, dV = 3\,\mathrm{Vol}(\Omega)$$

450: 2. $\vec{F} = [e^y, e^x, 1]$ $\quad S: x+y+z=1$

$x \geq 0, y \geq 0, z \geq 0$



$$\iint_S \vec F\cdot\hat n\, dA = \iiint_\Omega \underbrace{Div\,\vec F}_{=0}\, dV$$

$0\le x\le 1$

$0\le y\le 1-x$

$z=1-x-y$

$x=0,\ y,\ z$

$$\begin{cases} 0\le y\le 1 \\ 0\le z\le 1-y \end{cases}$$

$\vec F=[e^y,1,1]\cdot(-\hat i)=-e^y$

$$\int_{y=0}^{1}\int_{z=0}^{1-y} -e^y\,dz\,dy$$

$$\vec r(x,y)=x\,\hat i+y\,\hat j+(1-x-y)\,\hat k$$

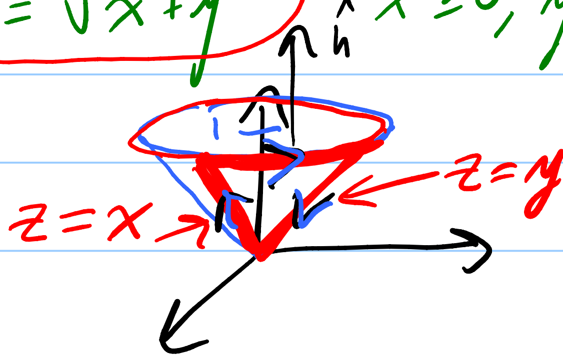
$$\iint_\triangle \vec F\cdot\hat n\, dA$$

$$=\int_{x=0}^{1}\int_{y=0}^{1-x}[e^y,e^x,1]\cdot(\vec r_x\times\vec r_y)\,dx\,dy$$

$$\vec r_x\times\vec r_y=\det\begin{bmatrix}\hat i & \hat j & \hat k\\ & & \\ & & \end{bmatrix}$$

10.9: 19. $\vec F=[z,e^z,0]$

$z=\sqrt{x^2+y^2}$ $\quad x\ge 0,\ y\ge 0\quad 0\le z\le 1$



$$\int_C \vec F\cdot d\vec r=\iint_S Curl\,\vec F\cdot\hat n\, dA$$

$$\boxed{\vec{r}(x,y) = x\,\hat{i} + y\,\hat{j} + \sqrt{x^2+y^2}\,\hat{k}}$$

$$\pm\vec{N} = \vec{r}_x \times \vec{r}_y = (\quad)\hat{i} + (\quad)\hat{j} + (\sim)\hat{k}$$

>0

upward

$$\iint_S \vec{F}\cdot\hat{n}\,dA \qquad \hat{n}\,dA = \frac{\vec{r}_x\times\vec{r}_y}{\|\vec{r}_x\times\vec{r}_y\|}\,\|\vec{r}_x\times\vec{r}_y\|\,dx\,dy$$

$$\left[\begin{array}{l} \vec{r}(r,\theta) = r\cos\theta\,\hat{i} + r\sin\theta\,\hat{j} + r\,\hat{k} \\ \qquad 0\le\theta\le\frac{\pi}{2} \qquad\qquad z = r \\ \qquad 0\le r\le 1 \end{array}\right.$$