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1. For the surface given by $z = e^{x \cos y} + 1/(x^2 + 1)$,
(10) (i) find a normal vector to the surface at $(0,0,2)$,

Answer:

- (10) (ii) find the equation of the tangent plane at $(0,0,2)$.

Answer:

2. For the integral

$$\int_C \left(\frac{2xy}{1+x^2} + z \right) dx + \ln(1+x^2) dy + x dz,$$

(15) (i) show that the form under the integral sign is exact,

(10) (ii) evaluate the integral for a curve C which goes from $(0,1,2)$ to $(1,0,3)$.

Answer:

3. Let $\vec{F} = 3x\vec{i} + 3y\vec{j} + z\vec{k}$ and T be the solid region bounded above by the surface of $z = 9 - x^2 - y^2$, and below by the xy plane.
- (15) (i) Without using the divergence theorem, compute the surface integral $\int \int_S \vec{F} \cdot \vec{n} dA$, where S is the boundary of T , and \vec{n} is the outward unit normal.

Answer:

- (15) (ii) Use the divergence theorem to recompute the integral in (i) as a volume integral.

Answer:

4. Let S be the surface given by the portion of the graph of $z = 4 - y^2$ cut off by the planes $x = 0$, $z = 0$, and $y = x$. Let $\mathbf{F} = xz\mathbf{j}$.
- (10) (i) Set up, but do not evaluate, the integral $\iint_S \text{curl}\mathbf{F} \cdot \mathbf{n} \, dA$, where \mathbf{n} is the unit upward pointing normal to S .

Diagram illustrating a region in the $u-v$ plane. The region is a large rectangle. A smaller rectangle is attached to the left side of the large rectangle. This smaller rectangle is divided into four sub-rectangles by a horizontal and a vertical line. The top-left and bottom-right sub-rectangles are shaded. To the left of the smaller rectangle are two integral symbols, each with a shaded sub-rectangle below it. To the right of the large rectangle is the text $du dv$.

- (10) (ii) Using Stokes's Theorem, evaluate the integral in (i) by means of a line integral.

Answer: