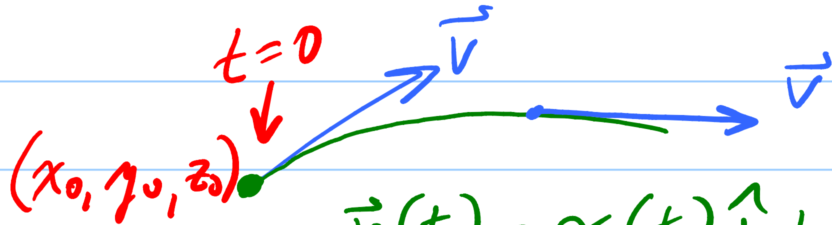
409:9. Streamlines

$$\vec{V} = 0\cdot\hat{\imath} + 3z^2\hat{\jmath} + 0\cdot\hat{k}$$



$$\vec{r}(t) = x(t)\hat{\imath} + y(t)\hat{\jmath} + z(t)\hat{k}$$

$$\vec{r}\,'(t) = \vec{V}$$

$$\dot{x}\hat{\imath} + \dot{y}\hat{\jmath} + \dot{z}\hat{k} = 3z^2\hat{\jmath}$$

$\dot{x} = 0 \leftarrow x(t) = \underbrace{\text{const.}}_{x_0}$ $\quad x(0) = x_0$

$\dfrac{dy}{dt} = 3z^2 \leftarrow 3z_0^2$

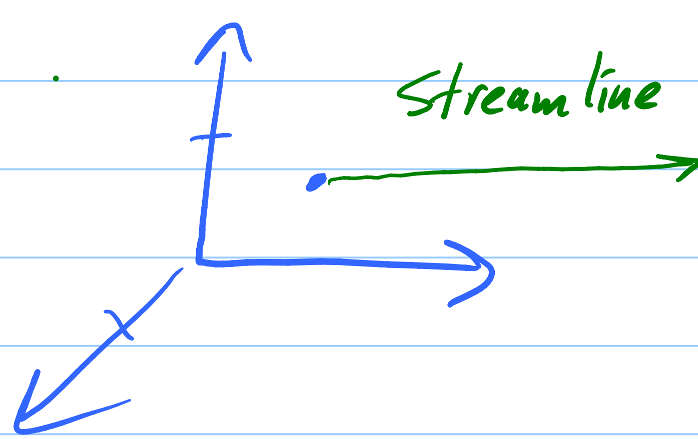
$\dot{z} = 0 \leftarrow z(t) = \underbrace{\text{const.}}_{z_0}$ $\quad z(0) = z_0$

$y(t) = 3z_0^2 t + C$ $\quad y(0) = 0 + C = y_0$

$$\boxed{y(t) = 3z_0^2 t + y_0}$$

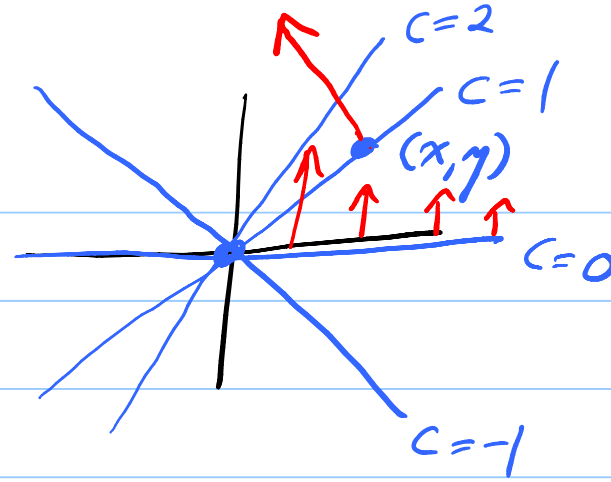
$x(t) = x_0$

$z(t) = z_0$



402:3. $f = \frac{y}{x} = c$

$y = cx$

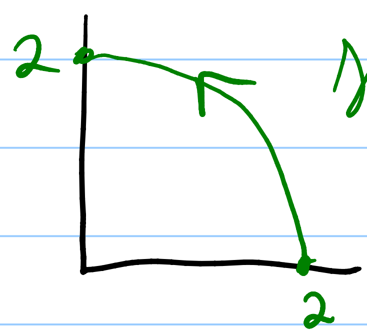


$$\nabla f = \frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath}$$

$$= \frac{-y}{x^2}\hat{\imath} + \frac{1}{x}\hat{\jmath} = \frac{-y}{x^2}\hat{\imath} + \frac{x}{x^2}\hat{\jmath} = \frac{1}{x^2}\left(-y\hat{\imath} + x\hat{\jmath}\right)$$

$$\| \ \| = \frac{1}{x^2}\|x + iy\|$$

10.1: 5. $\vec{F} = xy\,\hat{\imath} + xy^2\hat{\jmath}$



γ: $\vec{r}(t) = 2\cos t\,\hat{\imath} + 2\sin t\,\hat{\jmath}$

$\vec{r}\,'(t) = -2\sin t\,\hat{\imath} + 2\cos t\,\hat{\jmath}$

$$\int_\gamma F\cdot d\vec{r} = \int_0^{\pi/2}\left[2\cos t\sin t\,\hat{\imath} + 4\cos^2 t\sin^2 t\,\hat{\jmath}\right] \cdot \left[-2\sin t\,\hat{\imath} + 2\cos t\,\hat{\jmath}\right]dt$$

$$= \int_0^{\pi/2} 2\underbrace{\sin^2 t}_{u=\sin t}\ \underbrace{\cos t\,dt}_{-du} + \int_0^{\pi/2} 8\underbrace{\cos^3 t}_{\cos t\ \underbrace{\cos^2 t}_{(1-\sin^2 t)}}\sin^2 t\,dt$$

$$\int 8 \cos^3 t \sin^2 t \, dt$$

$$= \int 8 \underline{\cos t} \, (1 - \sin^2 t) \sin^2 t \, \underline{dt}$$

$u = \sin t$

$du = \cos t \, dt$

$$\int \cos^7 t \, \underbrace{\sin t \, dt}_{-du}$$

$u = \cos t$