14.2: 13. $f(z) = \dfrac{1}{(z^4 - 1.1)}$

Denom $= 0$: $z^4 - 1.1 = 0$

$z^4 = 1.1$

$z = re^{i\theta}$

$|z^4| = 1.1$

$|z|^4 = 1.1$

$|z| = (1.1)^{1/4}$

$(re^{i\theta})^4 = 1.1$

$r^4 e^{i4\theta} = 1.1$

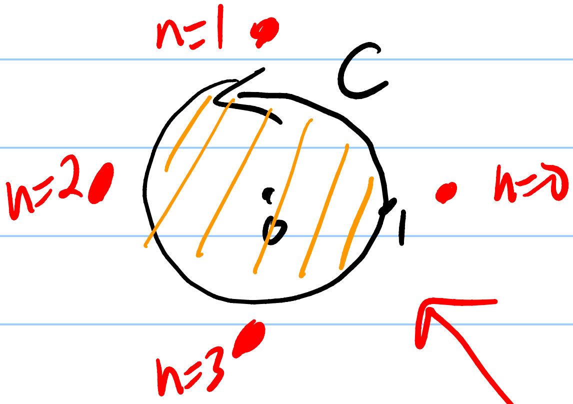
$= 1.1 e^{i \cdot 0}$

$r^4 = 1.1$

$r = |z| = (1.1)^{1/4}$

$4\theta = 0 + 2\pi n$

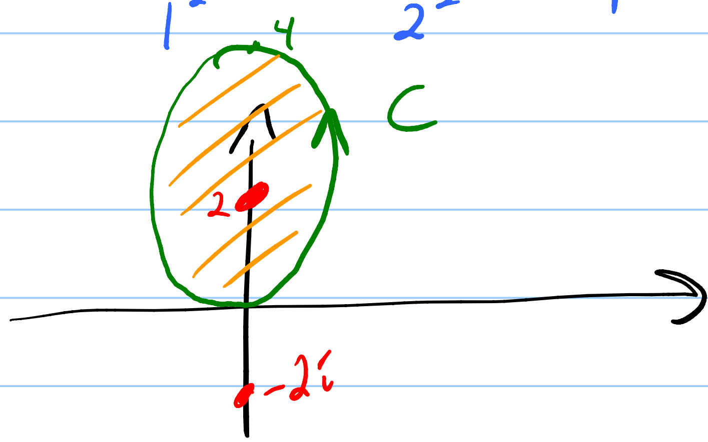
$\theta = n\frac{\pi}{2}$



$\dfrac{f}{g}$ $g \neq 0$.

14.3: 11. $4x^2 + (y-2)^2 = 4$

$\dfrac{x^2}{1^2} + \dfrac{(y-2)^2}{2^2} = 1$ ← ellipse



$f(z)$

$$\int_C \frac{\left(\frac{1}{(z+2i)}\right)}{(z-2i)}\,dz$$

$z^2 + 4 = (z+2i)\underline{(z-2i)}$

Basic Ineq. $\left|\int_\gamma f\,dz\right| \le \left(\max_\gamma |f|\right) \text{Length}(\gamma)$

$$\approx \left|\sum f(z_i)\Delta z\right| \le \sum \underbrace{|f(z_i)|}_{\le M} |\Delta z|$$

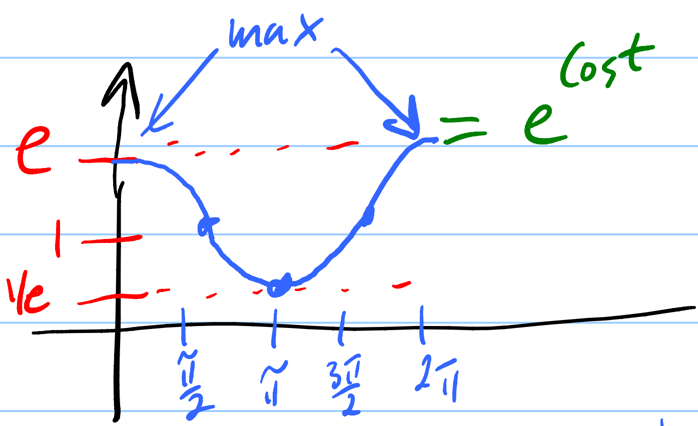
$$\le M \sum |\Delta z| \le M\,\text{Length}(\gamma)$$

Ex: $\left|\int_{C_1} e^z\,dz\right| \le \left(\max_{C_1} |e^z|\right) 2\pi \cdot 1 \le e \cdot 2\pi$

$C: z(t) = e^{it} = \cos t + i \sin t$

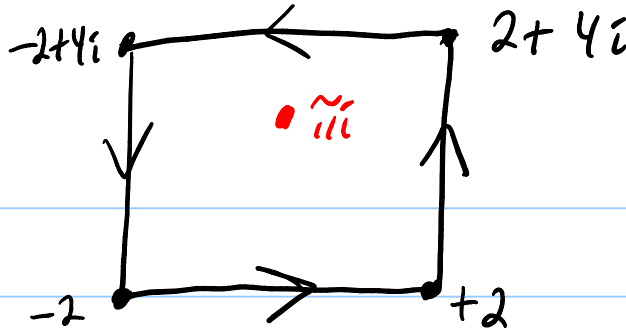
on C: $\left|e^{z(t)}\right| = \left|e^{\cos t + i \sin t}\right|$

$$= \left|e^{\cos t} e^{i \sin t}\right| = e^{\cos t} \underbrace{\left|e^{i \sin t}\right|}_{1}$$



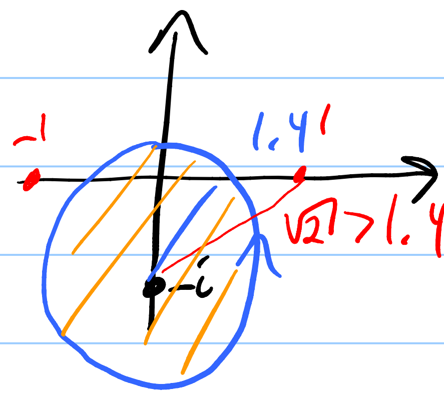
$$\left|\int_{C_R} \frac{1}{z^3+1}\,dz\right| \le \left(\frac{1}{R^3-1}\right) \cdot 2\pi R$$

14.3 : 15.



14.3 : 3. $\dfrac{z^2}{\underbrace{z^2-1}_{f(z)}}$ $\qquad |z-(-i)| = 1.4$

Cauchy Thm.

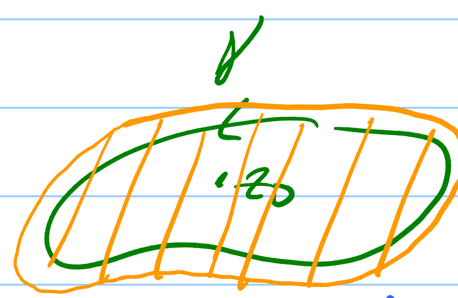


$\dfrac{z^2}{z^2-1}$ ← Step 1. Long division.

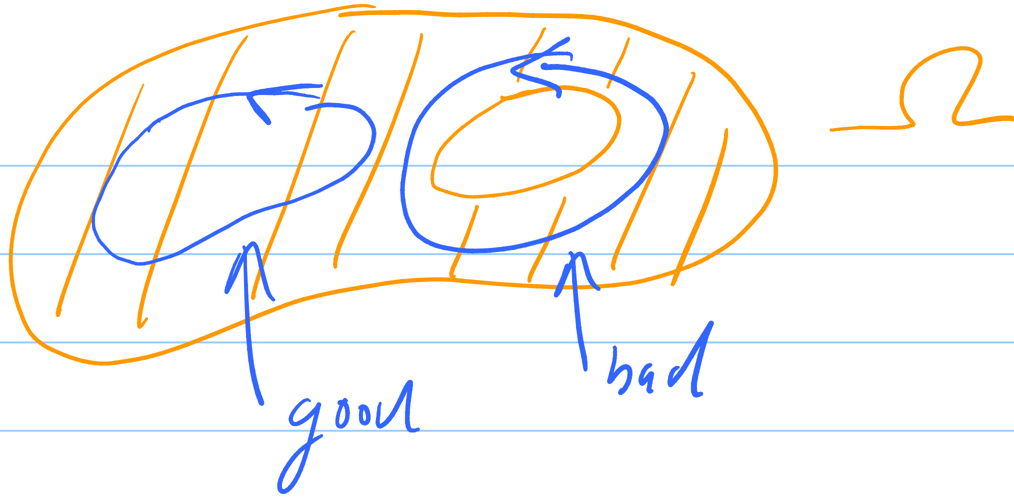
or $= z^2\left(\dfrac{1}{z^2-1}\right) = z^2\left(\dfrac{A}{z-1} + \dfrac{B}{z+1}\right)$

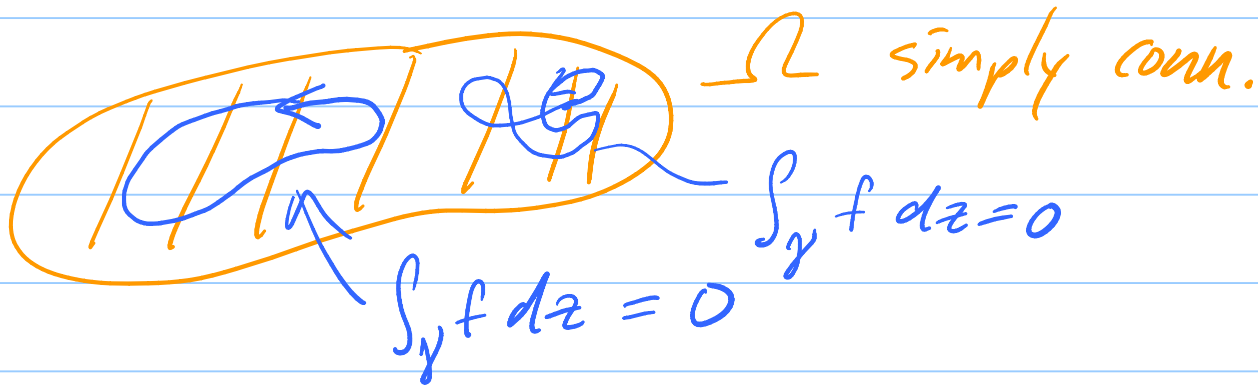
$f(z)$

$$\frac{1}{2\pi i}\int \frac{f(z)}{z-z_0}\,dz = f(z_0)$$



f analytic inside and on a simple closed curve γ


Ω
good
bad


Ω simply conn.
$\int_\gamma f\,dz = 0$
$\int_\gamma f\,dz = 0$