Assignment 1

Deadline: Friday, 17th of April

1. **(0.5 points)** Consider $\mathcal{H} = \{h_{\theta_1}: \mathbb{R} \to \{0,1\}, h_{\theta_1}(x) = \mathbf{1}_{[x \geq \theta_1]}(x) = \mathbf{1}_{[\theta_1, +\infty)}(x), \theta_1 \in \mathbb{R}\} \cup \{h_{\theta_2}(x) = \mathbf{1}_{[x < \theta_2]}(x) = \mathbf{1}_{(-\infty, \theta_2)}(x), \theta_2 \in \mathbb{R}\}$. Compute VCdim($\mathcal{H}$).

2. **(0.75 points)** Consider $\mathcal{H}$ to be the class of all centered in origin sphere classifiers in the 3D space. A centered in origin sphere classifier in the 3D space is a classifier h_r that assigns the value 1 to a point if and only if it is inside the sphere with radius $r > 0$ and center given by the origin **O**(0,0,0).
   a. show that the class $\mathcal{H}$ can be (ϵ, δ) − PAC learned by giving the algorithm A and determining the sample complexity $m_H(\epsilon, \delta)$ such that the definition of PAC-learnability is satisfied. **(0.5 points)**
   b. compute VCdim($\mathcal{H}$). **(0.25 points)**

3. **(1 point)** What is the VC-dimension of the set of subsets I_θ of the real line parameterized by a single parameter θ where $I_\theta = [\,\theta, \theta + 1] \cup [\theta + 2, \infty)$?

4. **(1.25 points)** An axis aligned square classifier in the plane is a classifier that assigns the value 1 to a point if and only if it is inside a certain square. Formally, given the real numbers $a_1, a_2, r > 0 \in \mathbb{R}$ we define the classifier $h_{(a_1,a_2,r)}$ by

$$h_{(a_1,a_2,r)}(x_1, x_2) = \begin{cases} 1, \text{ if } a_1 \leq x_1 \leq a_1 + r,\ a_2 \leq x_2 \leq a_2 + r \\ 0, \text{ otherwise} \end{cases}$$

   The class of all axis aligned squares in the plane is defined as

$$\mathcal{H} = \{h_{(a_1,a_2,r)}: \mathbb{R}^2 \to \{0,1\} \mid a_1, a_2, r \in \mathbb{R}, r > 0\}$$

   Consider the realizability assumption.
   a. give a learning algorithm A that return a hypothesis h_S from $\mathcal{H}$, h_S = A(S) consistent with the training set S (h_S has empirical risk 0 on S); **(0.5 points)**
   b. find the sample complexity $m_H(\varepsilon, \delta)$ in order to show that $\mathcal{H}$ is PAC – learnable. **(0.5 points)**
   c. compute VCdim($\mathcal{H}$). **(0.25 points)**

Bonus Problem (1 point)

Compute the VC-dimension of the class of convex d-gons (convex polygons with exactly d sides) in the plane. Provide a detailed proof of your result.