

# Lecture 3 | Locally Weighted & Logistic Regression

🕒 Date de création	@December 19, 2024 4:46 PM
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## Outline

- Locally Weighted Regression
- Probabilistic Interpretation
- Logistic Regression
- Newton's Method

## Recap

$(x^i, y^i) \rightarrow i^{th}$  exemple

$x^{(\cdot)} \in \mathbb{R}^{n+1}, y^i \in \mathbb{R}, x_0 = 1$

$m$  = exemples,  $n$  = features

$h_\theta(x) = \sum_{j=0}^n \theta_j x_j = \theta^T x$

$J(\theta) = \frac{1}{2} \sum_{i=1}^m ((h_\theta(x^i) - y^i))^2$

## Intro

## Parametric Learning

implement weighted regression

Let's say. For Linear Regression is fit  $\Theta$  to your data and return  $\Theta^t x$ . You should minimize Loss with this  $\Theta$ .

## Locally weighted regression (non-parametric algorithm)

Fit  $\Theta$  to minimize a modified loss function:  $\sum w^i (y^i - \Theta^t x)^2$ , where  $w^i = \exp\left(-\frac{(x^i - x)^2}{2\tau^2}\right)$

property 1: if  $|x^i - x|$  is small,  $w^i \sim 1$

property 2: if  $|x^i - x|$  is large,  $w^i \sim 0$

Suppose we compute the house price, like that:  $y^i = \Theta^t x^i + \epsilon^i$ , where this  $\epsilon^i$  is some error.

This simply means that the value of the house is our hypothesis but some "force" of nature or noise, as you wish.

i.i.d.  $\rightarrow$  The error of one house doesn't affect another.

## Logit

One reason we use Linear regression is that, we want our  $h(x) \in [0, 1]$ .

$h(x) = g(\theta^t x) = \frac{1}{1 + e^{-\theta^t x}}$ , we call  $g(z) = \frac{1}{1 + e^{-z}}$ , 'sigmoid function' or logistic function.

Binary Problem

$$P(y|x; \theta) = h(x)^y (1 - h(x))^{1-y}$$

"Log Likelihood"

$$l(\theta) = \log L(\Theta)$$

$$= \sum_{i=1}^n y^i \log h\theta(x^i) + (1 - y^i) \log(1 - h\theta(x^i))$$

Suppose you have a data  $(x, y)$ . You compute the Log of  $L(\theta)$ , then you choose  $\theta$  to maximize  $L(\theta)$  using gradient ascent. Ascent means we want the maximum value:

$$\Theta^j := \Theta^j + \alpha \frac{\partial}{\partial \Theta^j} * l(\Theta)$$

## Newton's Method

In comparison with Gradient Ascent or Descent you can give high steps using Newton's method.

Ex: Have  $F$ , want to find  $\Theta$  such that  $F(\Theta) = 0$

Newton's method uses the concept of quadratic iterations, which make the convergence much more faster.

When  $\Theta$  is a **vector**:

$$\Theta^{(t+i)} := \Theta^{(t)} + H^{-1} \nabla \theta_l, \text{ where } H \text{ is The Hessian Matrix.}$$

Newton's method is faster than Gradient Ascent or Descent, but if you have high dimensional data the vector increases the cost of each iteration.

# Questions

▼ When use Linear weighted regression?

a few features, lots of data and don't care about what features to select.

▼ Why specifically choose the Sigmoid Function?

▼ Why we use squared error?

▼ if that data we want to fit isn't in the format  $\Theta_0 + \Theta x$ ? but  $\Theta_0 + \Theta x_i + \Theta_{ii}x^2$ ?

▼ What's Locally weighted regression? And why does it matter?

It consist in select some area to make predictions based on that area (K-Neighbor)

▼ What's the difference between parametric algorithms and non-parametric algorithms.

Parametric: Fit fixed set of parameters  $\Theta_i$ , to data.

Non-Parametric: The number of parameters grows with data size.

▼ What's the Hessian Matrix? What does it matter?

A vector of **partial derivatives**:

$$\frac{\partial^2 l}{\partial \Theta_i, \partial \Theta_j}$$

## Futher Reding

### ML | Locally weighted Linear Regression - GeeksforGeeks

A Computer Science portal for geeks. It contains well written, well thought and well explained computer science and programming articles, quizzes and practice/competitive

➤ <https://www.geeksforgeeks.org/ml-locally-weighted-linear-regression/>

