FIT5097 Assignment 1

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Semester 2

Question 1 – Many products, many components [6 + 6 + 4 + 2 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 45 marks]

Consider several types of product: P_1 (product 1), …, P_i (product i), and P_3 (product 3), with the production of products P_1, ..., P_i, …, P_6 in quantities X_1, ..., X_i ..., X_3. These are made up of 4 components (or resources or things) : $Component_1$, …, $Component_j$, …, $Component_4$. (We will possibly sometimes use the terms L_i and X_i interchangeably; we will possibly use the terms component and resource interchangeably.) The profit for each of X_1, ..., X_i, …, X_3 is c_1, ..., c_i..., c_3 shown below. Unless stated otherwise, you should not assume that the X_i are integers. We also show below the amounts of $Component_1$, …, $Component_j$, ..., $Component_4$ required to make each of P_1, ..., P_3; and, for each of $Component_1$, ..., $Component_4$, we show how much of that component (or resource) is available.

a)Formulate a Linear Programming (an LP) formulation for this problem.

<u>Answer1a</u>

Linear Programming (an LP) formulation

MAX: $350X_1$+$300X_2$+$550X_3$

Constraints:

X_1+X_2+$2X_3 \leq 1577$

$9X_1$+$6X_2$+$11X_3 \leq 1990$

$12X_1$+$15X_2$+$23X_3 \leq 1321$

$24X_1$+$30X_2$+$40X_3 \leq 1920$

X_1,X_2,$X_3 \geq 0$, non-negativity constraint

d) What is the optimal production plan (X_1, X_2, X_3) and the associated profit? Refer to your answers to any of a), b) and/or c) above as appropriate.

Answer1d

The optimal production plan is to produce 80 of P1, 0 of P2 and 0 of P3. The maximum profit is $28000.

The allowable increase for P1 is 1E+30 and the allowable decrease is 20. Thus, this is the range in which the coefficient of X1 in the objective function may be increased/decreased without changing the optimal solution. Hence if the profit goes down by more than $20, we will see change in the optimal solution, same case if the profit goes above 1E+30. The allowable increase and decrease ranges for X2 and X3 can be found in the sensitivity report as well.

<u>Associated profits from each product:</u>

P1 - $28000, P2 - $0, P3 - $0

e) Continuing from the above (the end of part (d)), and now setting $X_1 = X_3$, what is the optimal production plan and the associated profit? Clearly explain why, showing the optimal production plan.

Answer1e

The optimal production plan is to produce 30 of P1, 0 of P2 and 30 of P3. The maximum profit is $27000. Associated profits: P1 = $350*30 = $10500, P2 = $300*0 = 0, P3 = $550*30 = $16500.

f) In part (e), which constraints – if any - are binding? Explain clearly.

Answer1f

A binding constraint is a constraint where some optimal solution is on the line for the constraint. Therefore, if this constraint were to be changed slightly, this optimal solution would no longer be feasible. We can see for the sensitivity report of q2.e that the binding constraint here related to the **component C4**. Another way to identify is that non-binding constraints have a shadow price of 0, whereas binding constraints have a shadow price. Binding constraints related to the resources which are all used up, and we can clearly see that component 4 is all used up after the excel solver executes.

g) Returning to the end of part (d), add the requirement that $X_1 = X_2 = X_3$. What is the optimal production plan and the associated profit? Clearly explain.

Answer1g

The optimal production plan is to produce 20.42553 of P1, 20.42553 of P2 and 20.42553 of P3. The maximum profit is $24510.64. Associated profits: P1 = $350*20.42553 = $7148.9355, P2 = $300*20.42553 = 6127.659, P3 = $550*20.42553 = $11234.0415.

h) For the problem in part (d), what is the effect on the optimal solution (i.e., on the objective function) if we increase the amount of Resource1 available by 3?
Explain clearly.

Answer1h

Increasing the amount of resource 1 available by 3 won't have any effect on the optimal solution, as resource 1 is a non-binding constraint which we can clearly see from the sensitivity report as well as the used column in the excel spreadsheet. To get the optimal solution we the solver isn't even utilizing the full amount of resource 1 available, thus by increasing it by 3 won't influence the optimal solution.

i) For the problem in part (d), suppose we change the profitability of each product as follows: we decrease the profitability of P_1 by 25, we decrease the profitability of P_2 by 5, and we increase the profitability of P_3 by 3.
What effect – if any - will this have on (X_1, X_2, X_3)? Clearly explain why.

Answer1i

By manipulating the profitability of the products, we can also notice that the optimal solutions for the products have changed. After changing the profits and executing the solver, we find that the optimal solutions has changed. The new optimal solution indicated that we should make 0 of P1, 0 of P2 and 48 of P3 which puts the maximum profit at $26544.

P1 had an allowable decrease of 20, **thus changing the coefficient (profit) of P1 made the optimal solution to change completely (this makes sense as the solver reduced the production of P1 to 0 from 80 as its not profitable anymore).** Also increasing the profit of P3 helped in making P3 more profitable overall which means the solver will now try to put more importance on P3.

j) The company has committed to filling an order of (150, 100, 50) of (P_1, P_2, P_3).
For the various quantities required, goods can be made in house by the company (given the constraints) or goods can be purchased at prices of ($370, $320, $540) each for ($P_1$, P_2, P_3) respectively.
Show the optimal production plan and the optimal value of the objective function, explaining your working.
Would this result in an overall profit or an overall loss? And what would be this overall profit or overall loss? Make sure to show all working.

Answer1j

Let M1, M2, M3 be the 3 decision variables for the profits from the 3 products and B1,B2,B3 be the variables for buying them from a third party.

LP problem formulation:

We want to see if we are getting an overall profit or loss from the make vs buy decision.

Max overall total profit – loss (here I'm counting every unit that must be bought as a loss thus the cost means the total cost of buying the product from a third party):

Objective function:

MAX: 350M1+300M2+550M3-370B1-320B2-540B3

Demand constraints:

M1+B1 = 150

M2+B2 = 100

M3+B3 = 50

Resource constraints:

M_1+M_2+$2M_3 \leq 1577$

$9M_1$+$6M_2$+$11M_3 \leq 1990$

$12M_1$+$15M_2$+$23M_3 \leq 1321$

$24M_1$+$30M_2$+$40M_3 \leq 1920$

Non-negativity constraint:

M1, M2, M3, B1, B2, B3 >= 0

Solved in the excel spreadsheet, the optimal production plan is as follows:

	P1	P2	P3
Make	80	0	0
Buy	70	100	50

Optimal value of the objective function is -56900. This means that we would make an overall loss of $56900 from this endeavour. If we had the component cost given for this problem, we would be able to calculate the selling price of the product. Then we can calculate the overall profit in a better sense as then we would also know the profit, we are making by selling the units that we bought from the third-party seller.

l) Let us continue from the end of part 1(d) above. Assume now that we have start-up (or set-up) costs of ($8000, $5000, $100) respectively. Show the optimal production plan and the optimal value of the objective function, clearly explaining your working.

Answer1l:

The optimal production plan is 0 of P1, 0 of P2, and 48 of P3. The optimal value of the objective function is 26300, Which means that we are making a profit of $26300 after taking into consideration the fixed costs. Linking constraints were used to solve this problem as seen in the excel spreadsheet q1.l

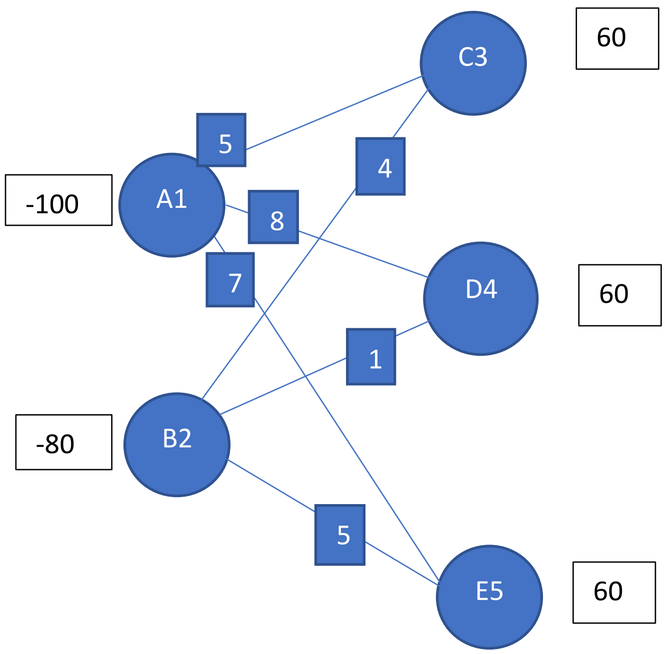
Question 2 – Trans-shipment [7 + 7 + 6 + 6 + 5 + 5 + 4 + 4 + 3 + 3 = 50 marks]

Note: In all the network figures I have drawn for question 2, the nodes are named A1, B1 etc. A1 just means Node A or Node 1. As in the excel spreadsheet I referred the node A1 as 1 and in the pdf an edge which goes from node A1 to C3 is referred to as X_{13}. Xij = Quantity shipped from node i to node j. Square text boxes along the edges are edge unit costs.

2a) In the first problem, there is demand of 60 at C, 60 at D and 60 at E. Solve for the flow along all edges giving rise to the minimum total cost. Clearly show and explain your working.

Answer 2a

Here, inflow-outflow = supply/Demand



Linear model:

Objective function:

MINIMIZE TOTAL COST: $5X_{13}+8X_{14}+7X_{15}+4X_{23}+X_{24}+5X_{25}$

Subject to constraints:

$-X_{13}-X_{14}-X_{15}$ = -100

$-X_{23}-X_{24}-X_{25}$ = -80

$X_{13}+\ \ X_{23}$ = 60

$X_{14}+\ \ X_{24}$ = 60

$X_{15}+\ \ X_{25}$ = 60

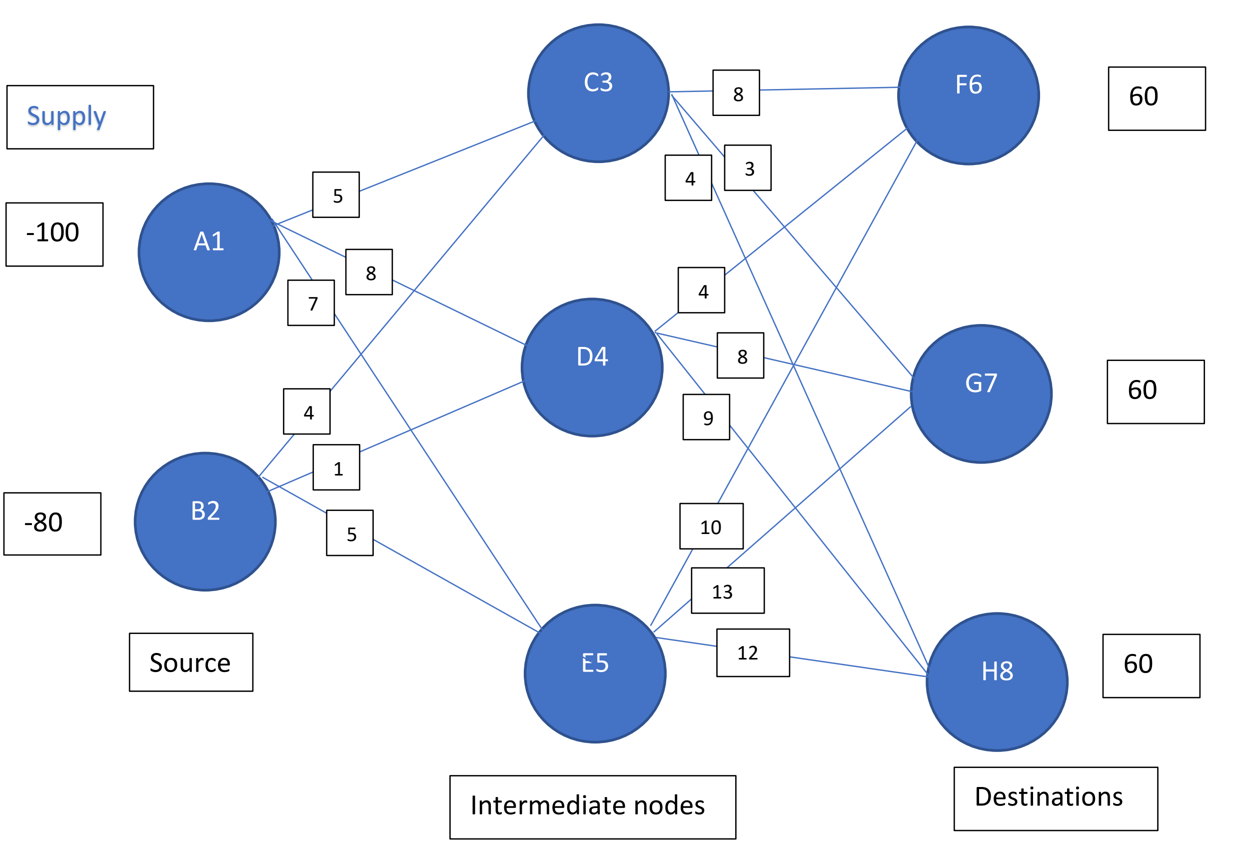
Non-negativity of Variables: xij > 0, for all i and j.

Minimum total cost calculation done in the excel spread sheet (q2.a)

Minimum total cost = **$740**

2b) The problem from 2a above is now modified. In the new problem, C, D and E serve only as intermediate nodes. There is demand of 60 at F, 60 at G and 60 at H. Solve for the flow along all edges giving rise to the minimum total cost.
As in earlier instructions and through at least this question, show your answer clearly both in your .pdf and a corresponding cross-referenced spreadsheet tab – and clearly explain your working.

Answer 2b



Linear model:

Objective function:

MINIMIZE TOTAL COST:
$5X_{13}+8X_{14}+7X_{15}+4X_{23}+X_{24}+5X_{25}+8X_{36}+3X_{37}+4X_{38}+4X_{46}+8X_{47}+9X_{48}+10X_{56}+13X_{57}+12X_{58}$

Subject to constraints:

Amount out of A1: $X_{13}+X_{14}+X_{15}$ = 100

Amount out of B2: $X_{23}+X_{24}+X_{25}$ = 80

Amount through C3: $X_{13}+X_{23}-X_{36}-X_{37}-X_{38}$ = 0

Amount through D4: $X_{14}+X_{24}-X_{46}-X_{3477}-X_{48}$ = 0

Amount through E5: $X_{15}+X_{25}-X_{56}-X_{57}-X_{57}$ = 0

Amount into F6: $X_{36}+X_{46}+X_{56}$ = 60

Amount into G7: $X_{37}+X_{47}+X_{57}$ = 60

Amount into H8: $X_{38}+X_{48}+X_{58}$ = 60

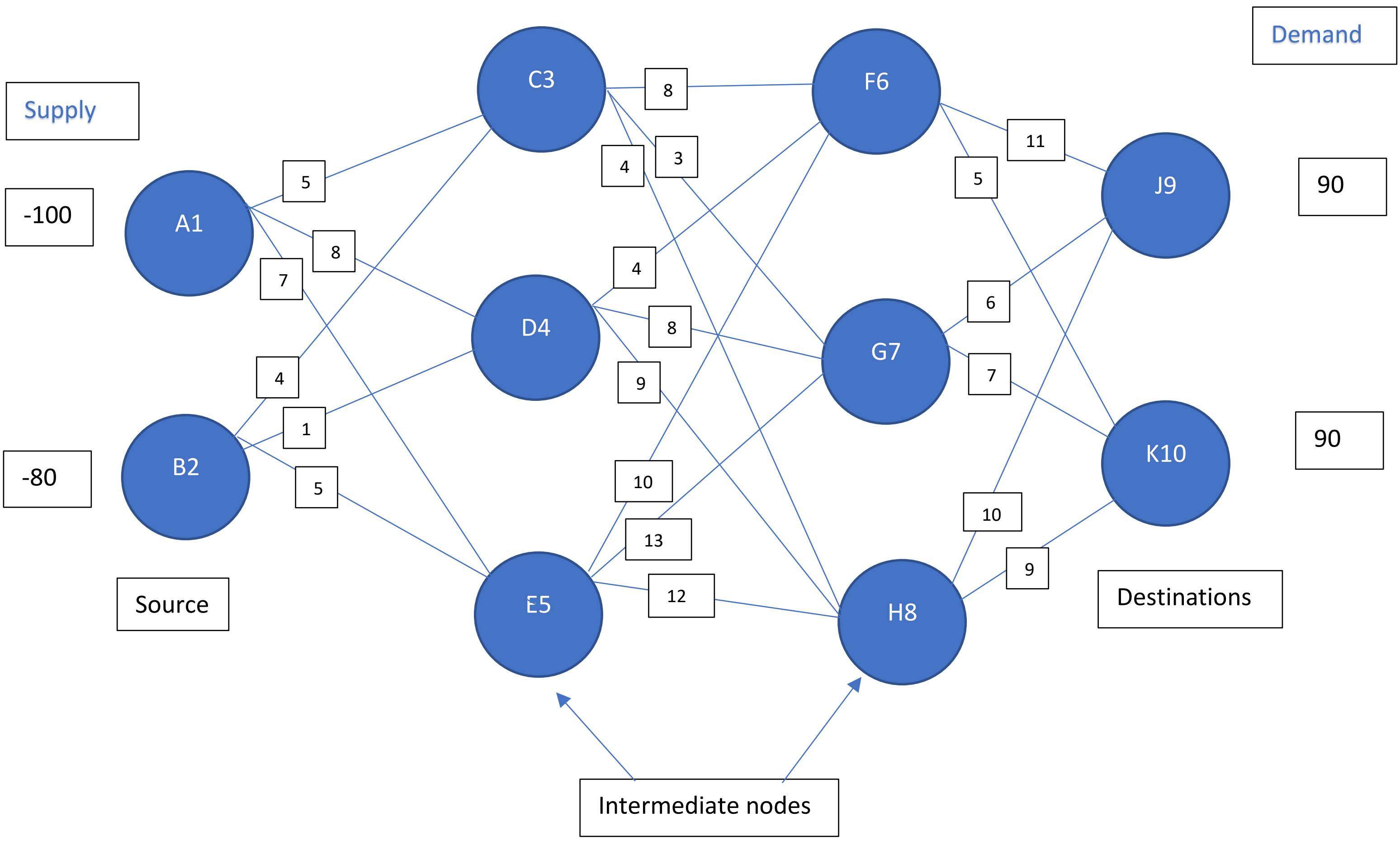
Non-negativity of Variables: xij > 0, for all i and j.

Minimum total cost calculation done in the excel spread sheet (q2.b)

Minimum total cost = **$1300**

2c) The problem from 2a and 2b above is now modified. In the new problem, C, D, E, F, G and H serve only as intermediate nodes. There is demand of 90 at J and 90 at K. Solve for the flow along all edges giving rise to the minimum total cost. As in earlier instructions and through at least this question, show your answer clearly both in your .pdf and a corresponding cross-referenced spreadsheet tab – and clearly explain your working.

Answe2c



Linear model:

Objective function:

MINIMIZE TOTAL COST:
5X_{13}+8X_{14}+7X_{15}+4X_{23}+X_{24}+5X_{25}+8X_{36}+3X_{37}+4X_{38}+4X_{46}+8X_{47}+9X_{48}+10X_{56}+13X_{57}+12X_{58}+11X_{69} +5X_{610}+6X_{79}+7X_{710}+10X_{89}+9X_{810}

Subject to constraints:

Amount out of A1: X_{13}+X_{14}+X_{15} = 100

Amount out of B2: X_{23}+X_{24}+X_{25} = 80

Amount through C3: X_{13}+X_{23}-X_{36}-X_{37}-X_{38} = 0

Amount through D4: X_{14}+X_{24}-X_{46}-X_{3477}-X_{48} = 0

Amount through E5: X_{15}+X_{25}-X_{56}-X_{57}-X_{57} = 0

Amount through F6: X_{36}+X_{46}+X_{56}-X_{69}-X_{610} = 0

Amount through G7: X_{37}+X_{47}+X_{57}-X_{79}-X_{710} = 0

Amount through H8: X_{38}+X_{48}+X_{58}-X_{89}-X_{810} = 0

Amount into J9: X_{69}+X_{79}+X_{89} = 90

Amount into K10: X_{610}+X_{710}+X_{810} = 90

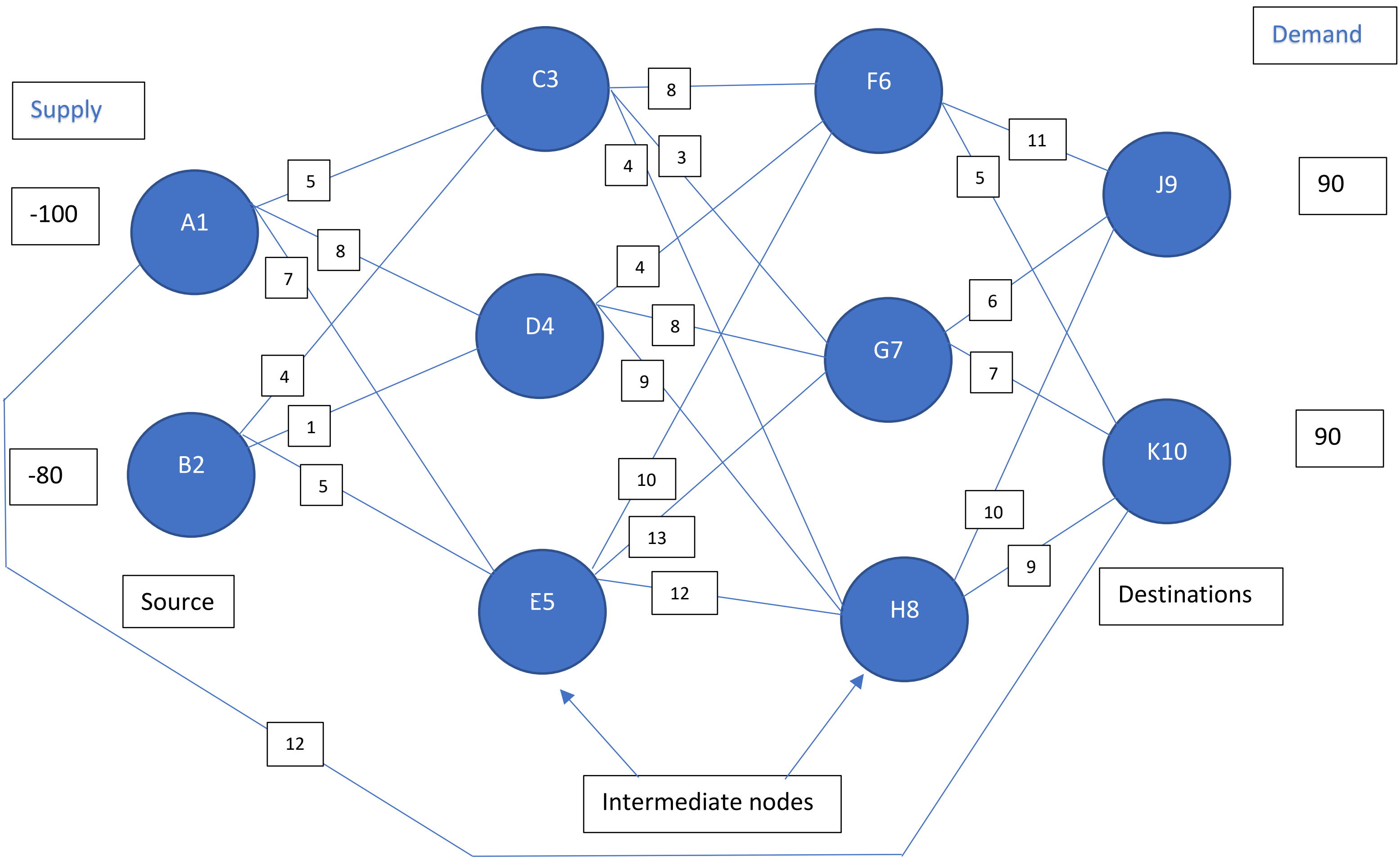
Non-negativity of Variables: xij > 0, for all i and j.

Minimum total cost calculation done in the excel spread sheet (q2.c)

Minimum total cost = **$2210**

2d) Following on from the problem at 2c), we introduce a new edge, directly from A to K, with cost $12/unit. Solve for the flow along all edges giving rise to the minimum total cost. As in earlier instructions and through at least this question, show your answer clearly both in your .pdf and a corresponding cross-referenced spreadsheet tab – and clearly explain your working.

Answe2d



Linear model:

Objective function:

MINIMIZE TOTAL COST:

$5X_{13}$+$8X_{14}$+$7X_{15}$+$12X_{110}$+$4X_{23}$+X_{24}+$5X_{25}$+$8X_{36}$+$3X_{37}$+$4X_{38}$+$4X_{46}$+$8X_{47}$+$9X_{48}$+$10X_{56}$+$13X_{57}$+

$12X_{58}$+$11X_{69}$+$5X_{610}$+$6X_{79}$+$7X_{710}$+$10X_{89}$+$9X_{810}$

Subject to constraints:

Amount out of A1: X_{13}+X_{14}+X_{15} +X_{110}=100

Amount out of B2: X_{23}+X_{24}+X_{25} = 80

Amount through C3: X_{13}+X_{23}-X_{36}-X_{37}-X_{38} = 0

Amount through D4: X_{14}+X_{24}-X_{46}-X_{47}-X_{48} = 0

Amount through E5: X_{15}+X_{25}-X_{56}-X_{57}-X_{57} = 0

Amount through F6: X_{36}+X_{46}+X_{56}-X_{69}-X_{610} = 0

Amount through G7: X_{37}+X_{47}+X_{57}-X_{79}-X_{710} = 0

Amount through H8: X_{38}+X_{48}+X_{58}-X_{89}-X_{810} = 0

Amount into J9: X_{69}+X_{79}+X_{89} = 90

Amount into K10: X_{610}+X_{710}+X_{810} + X_{110}= 90

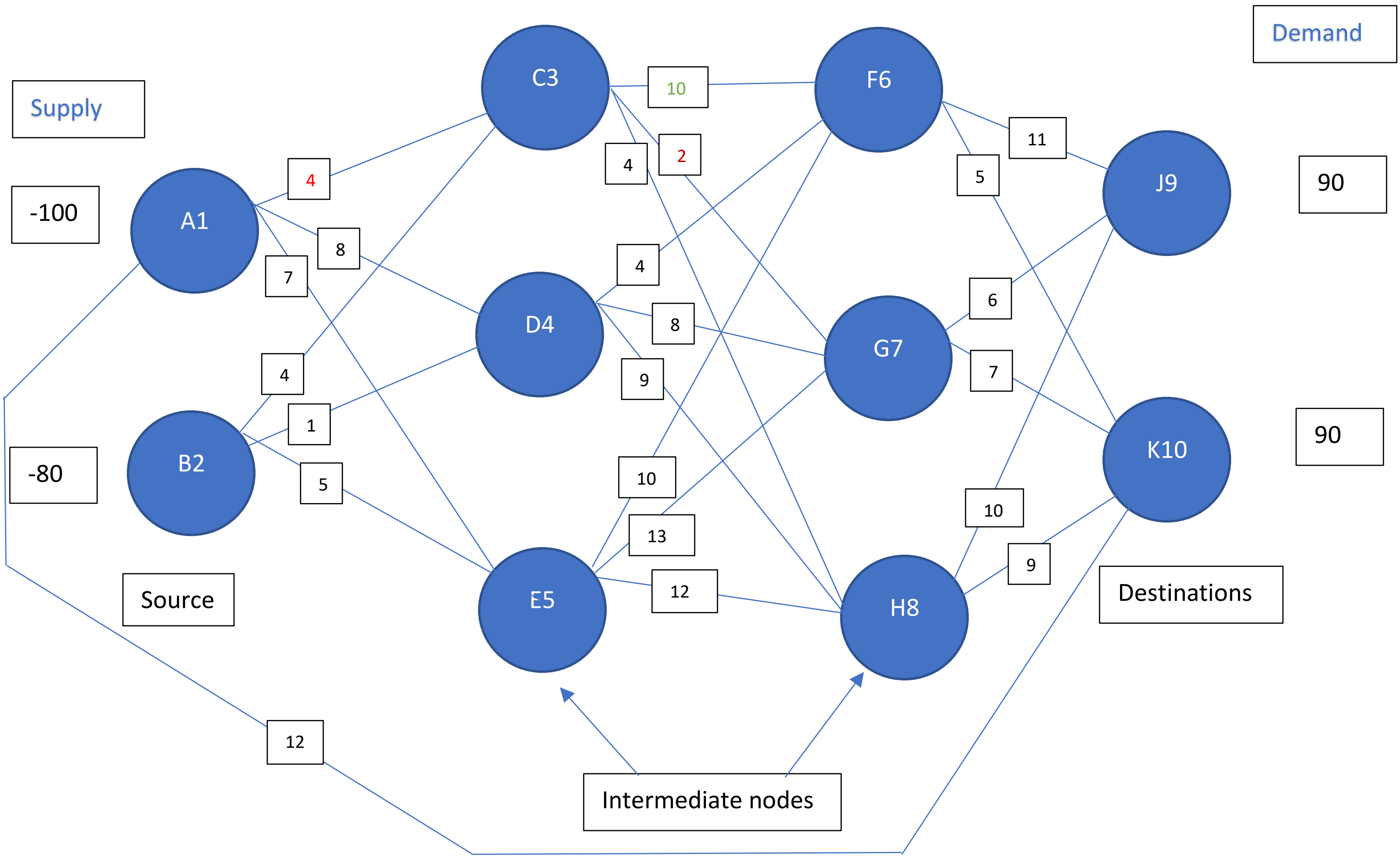
Non-negativity of Variables: xij > 0, for all i and j.

Minimum total cost calculation done in the excel spread sheet (q2.d)

Minimum total cost = **$2180**

2e) Following on from your answer at 2d), suppose the cost along AC were to decrease by $1, the cost along CF were to increase by $2 and the cost along CG were to decrease by $1. Providing the most elegant solution that you can, what would happen to the amounts of flow and what would happen to the objective function? In other words, answering as elegantly and clearly as possible, how much would the amounts of flow and the objective function change by – and why?

Answe2e



Here, cost along A1C3 decreased by $1, so new cost is 4. Cost along C3F6 increased by $2, so new cost is $10. Cost along C3G7 decreased by $1, so new cost is $2.

Linear model:

Objective function:

MINIMIZE TOTAL COST:

$4X_{13}$+$8X_{14}$+$7X_{15}$+$12X_{110}$+$4X_{23}$+X_{24}+$5X_{25}$+$10X_{36}$+$2X_{37}$+$4X_{38}$+$4X_{46}$+$8X_{47}$+$9X_{48}$+$10X_{56}$+$13X_{57}$+

$12X_{58}$+$11X_{69}$+$5X_{610}$+$6X_{79}$+$7X_{710}$+$10X_{89}$+$9X_{810}$

Subject to constraints:

Amount out of A1: X_{13}+X_{14}+X_{15} +X_{110}=100

Amount out of B2: X_{23}+X_{24}+X_{25} = 80

Amount through C3: X_{13}+X_{23}-X_{36}-X_{37}-X_{38} = 0

Amount through D4: X_{14}+X_{24}-X_{46}-X_{47}-X_{48} = 0

Amount through E5: X_{15}+X_{25}-X_{56}-X_{57}-X_{57} = 0

Amount through F6: X_{36}+X_{46}+X_{56}-X_{69}-X_{610} = 0

Amount through G7: X_{37}+X_{47}+X_{57}-X_{79}-X_{710} = 0

Amount through H8: X_{38}+X_{48}+X_{58}-X_{89}-X_{810} = 0

Amount into J9: X_{69}+X_{79}+X_{89} = 90

Amount into K10: X_{610}+X_{710}+X_{810} + X_{110}= 90

Non-negativity of Variables: xij > 0, for all i and j.

Minimum total cost calculation done in the excel spread sheet (q2.e)

Minimum total cost = **$4970**

After making these changes to the unit cost and implementing it on excel solver in sheet q2.e on my excel file, we can notice some interesting findings.

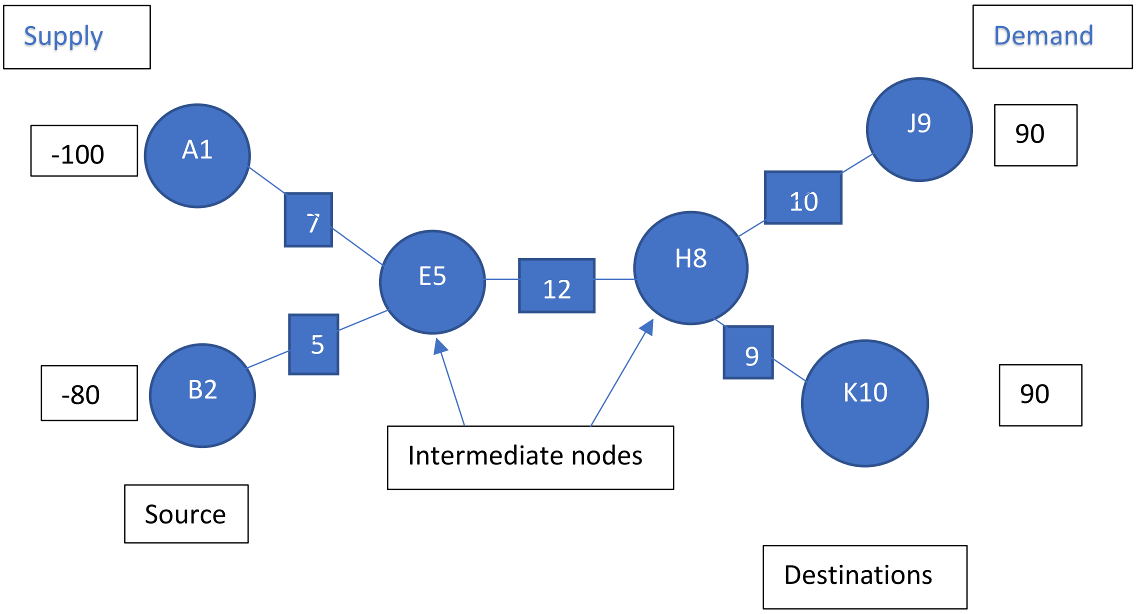
Findings:

The minimum total cost has increased to $4970.

Amounts of flow changes: we can also notice that now we ship all the 100 units from A1 directly to E5 and ignore C3 and D4. We also ship all the units from B2 straight to E5. Thus, we are not shipping any units from A1/B2 to C3/D4.

Now we have the 180 units which are in E5 move to H8 (skipping F6 and G7 completely). Then finally moving 90 units each to J9 and K10 respectively.

The solution network becomes:



Total minimum cost = 100*7+80*5+180*12+90*10+90*9 = $4970

By not sending to the nodes of C3 and D4 from A1, the solver has eliminated the cheap edges from C3 and D4. We also see that now the costlier edge out of B2 is utilized which is B2E5. The algorithm also chooses the costlier route along edge E5H8, and as there isn't any direct linkage between J9, K10 and E5 the units have to go through H which increases the cost further. All these factors have influence in the total minimum cost which increases due to inefficient routing.

2f) Following on from the problems at 2d) and 2e), we introduce a penalty for each edge with positive flow (i.e., for each edge where the flow along the edge is greater than 0). The penalty is $10 for each edge with positive flow. Solve for the flow along all edges giving rise to the minimum total cost. As in earlier instructions and through at least this question, show your answer clearly both in your .pdf and a corresponding cross-referenced spreadsheet tab – and clearly explain your working.

Answer2f

From part 2.e, we notice that the edges where the flow is greater than 0 are, AE, BE, EH, HJ and HK.

The positive flow (1) else 0 column determines if the ship column is greater than 0 or not which help in applying the penalty term for the edges with flow greater than 0.

in the objective function we just update it by applying the additional penalty for the particular edges (SUMPRODUCT(B6:B27,C6:C27)*10).

We can see that the total minimum cost goes up from the solution of 2.e which makes sense as the penalty is increasing the cost.

The minimum total cost is now **$5760**.

2g) Following on from the problem at 2f), if there is any flow from G to J then we restrict this to be one of 5, 25, 50, 86, 91. Solve for the flow along all edges giving rise to the minimum total cost. As in earlier instructions and through at least this question, show your answer clearly both in your .pdf and a corresponding cross-referenced spreadsheet tab – and clearly explain your working.

Answer2g

Question 3 – Economic Order Quantity (EOQ) [4 + 4 + 2 = 10 marks]

This question is based on the economic order quantity (EOQ) theme. OurNon-ExistentWidgetCompany needs 1200 widgets every month. Widgets cost $500 each, but every time that it orders one or more widgets, there is an order (or delivery) cost of $20. The annual holding cost of a widget is 20% of the purchase cost of a widget.

a) Based on the Economic Order Quantity (EOQ) model, what is the optimal number of widgets to order at a time? Show clear calculations to get your result, with clearly documented cross-reference to any spreadsheet tab that you might use. Such a spreadsheet tab should also be clearly laid out. Clearly explain your working.

Answer 3a

Using the EOQ model, here we must find the optimal order quantity (optimal number of widgets to order at a time) to answer this question. This is because the EOQ model minimizes the total variable cost over a specific period.

We need 1200 widgets every month, so we need 1200*12 = 14400 widgets every year/annually.

So, Annual demand (A) = 14400
Order cost (k) = 20
Widget unit cost (c) = $500
Annual holding cost (h) = 20% (0.20)

Optimal order quantity = Q* = $\sqrt{\frac{2Ak}{ch}} = \sqrt{\frac{2*14400*20}{500*0.20}}$ = 75.89

So, the optimal number of widgets to order at a time is **75.89 widgets.**

b) Following on your answer to a) immediately above, how many orders should be placed per annum? Again, show clear calculations to get your result, with clearly documented cross-reference to any spreadsheet tab that you might use. Such a spreadsheet tab should also be clearly laid out. Clearly explain your working.

Answer 3b

Here we want to know how many orders should be placed per annum. For that we need to find out number of orders placed per year which is found by dividing annual demand by the optimal order quantity.

So, orders placed per year = Annual demand/optimal order quantity = $\frac{A}{Q*} = \frac{1200*12}{75.89}$ = **189.74 orders per year.**

We need to place approximately 190 orders per year to satisfy the demand.

c) Re-visiting (a) above but now require that the number of widgets that must be ordered is an integer. What is the optimal number of widgets to order at a time? Again, show clear calculations to get your result, with clearly documented cross reference to any spreadsheet tab that you might use. Such a spreadsheet tab should also be clearly laid out. Clearly explain your working.

Answer 3c

If the number of widgets to be ordered must be and integer, then the optimal number of widgets needs to be **76 units**. EOQ is the quantity that should be ordered to meet the requirements of a period with the cheapest price. If we order let's say 75 instead of 76 (rounding up), then we might have to order another lot of the widgets or worse, placing the next order faster which might increase costs. Ordering more is safer as excess widgets can be sold on the next cycle. Excess inventory can be used to clear back-orders.