FIT5201 (Machine learning) Assessment 1

Question 4:

Recall the simple example from Appendix A of Module 1. Suppose we have one red, one blue, and one yellow box. In the red box we have 3 apples and 1 orange, in the blue box we have 4 apples and 4 orange, and in the yellow box we have 5 apples and 3 oranges. Now suppose we randomly selected one of the boxes and picked a fruit. If the picked fruit is an orange, what is the probability that it was picked from the yellow box?

Note that the chances of picking the red, blue, and yellow boxes are 50%, 30%, and 20% respectively and the selection chance for any of the pieces from a box is equal for all the pieces in that box. Please show your work in your PDF report.

ANSWER:

Let X be the random variable which represents the boxes (Red, Blue, and Yellow).

$P\{X = ''Red''\} = \ 0.5\ (given\ in\ the\ question\ above)$

$P\{X = ''Blue''\} = 0.3\ (given\ in\ the\ question\ above)$

$P\{X = ''Yellow''\} = 0.2\ (given\ in\ the\ question\ above)$

Let Y be the random variable which represents the choice of fruits (Orange and apple).

$P\{Y = ''Apple''\} = 12/20 = 0.6$

$P\{Y = ''Orange\} = 8/20 = 0.4$

From the information given, we know that red box contains 3 apples and 1 orange, blue box contains 4 apples and 4 oranges, and yellow box contains 5 apples and 3 oranges. Also, every fruit in a box has the same probability of getting selected.

Thus, in the red box – conditional probabilities are:

$P\{Y = Apple |\ X = Red\} = 3\ /\ 4 = 0.75$

$P\{Y = Orange |\ X = Red\} = 1/\ 4 = 0.25$

Same for Blue box,

$P\{Y = Apple |\ X = Blue\} = 4\ /\ 8 = 0.5$

$P\{Y = Orange |\ X = Blue\} = 4\ /\ 8 = 0.5$

And Yellow,

$P\{Y = Apple|\ X = Yellow\} \ = \ 5/\ 8 \ = \ 0.625$

$P\{Y = Orange|\ X = Yellow\} \ = \ 3\ /\ 8 \ = \ 0.375$

The question requires us to find the probability of Box is yellow given the fruit is orange.

We must calculate marginal probability of fruit being Orange as,

$P\{Y = Orange\} \ =$

$= P\{Y = Orange|\ X = Red\}\ P\{X = "Red"\} + P\{Y = Orange|\ X = Blue\}P\{X = "Blue"\} + P\{Y = Orange|\ X = Yellow\}\ P\{X = "Yellow"\}$

$= \ 0.25 * \ 0.5 \ + \ 0.5 \ * \ 0.3 \ + \ 0.375 \ * \ 0.2$

$= \ 0.35$

Now, using the Bayes' theorem we can calculate the required probability

$P\{X = Yellow|\ Y = Orange\} \ =$

$= \ \frac{P\{Y=Orange|\ X=Yellow\} \times P\{X="Yellow"\}}{P\{Y=Orange\}}$

$= \frac{0.375 \times 0.2}{0.35} = \frac{3}{14} = 0.2143\ (4\ d.p)$

Question 5.1:

With L2- Regularization (Ridge regularization) in our model our error function is as follows,

$$E = \frac{1}{2} \sum_{n=1}^{N} (t_n - w.\phi(x))^2 + \frac{\lambda}{2}\ w^T.w$$

For Stochastic gradient algorithm (SGD) we update weights accordingly,

$W^{t+1} = W^t - \ \eta\ \nabla E$

To define equation for weight update first we will differentiate the error function with respect to W (to calculate the derivative) as:

$$\nabla E = \ \frac{\partial}{\partial w}\ \frac{1}{2} \sum_{n=1}^{N} \left(t_n - w.\phi(x)\right)^2 + \frac{\lambda}{2}\ w^T.w$$

$$= \lambda w - \phi(x)(t - w\phi(x))$$

Therefore, our final equation to update the weight is as follows:

$$W^{t+1} = W^{t} - \eta\,(\lambda w - \phi(x)(t - w\phi(x)))$$