Forecasting the Yield Curve: An Econometric Study

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Abstract

This research study employs econometric analysis techniques to investigate the forecasting of the yield curve, analyze impulse response functions (IRFs), and detect structural breaks. Accurate forecasting of the yield curve is crucial for investors, policymakers, and risk managers in making informed decisions. The analysis of IRFs provides insights into the dynamic response of the yield curve to shocks in macroeconomic variables, allowing for a deeper understanding of the transmission mechanisms. Additionally, the study examines structural breaks in the yield curve associated with unpredictable events, providing valuable insights into shifts in market dynamics. By combining these three components, this research contributes to a broader understanding of the yield curve's behavior and its implications for financial markets and economic policies. The repository of the research is available on GitHub.

JEL Classification: C53, G17, E47

Introduction

The yield curve, depicting the relationship between time to maturity and yields on zero-coupon bonds, serves as a vital indicator of market expectations, economic conditions, and future monetary policy. Accurate forecasting of the yield curve has tremendous implications for investors, policymakers, and risk managers. Simultaneously, understanding the dynamic response of the yield curve to shocks and structural breaks enables a deeper comprehension of the underlying economic factors and their impact on financial markets.

This paper aims to contribute to the existing literature by conducting a comprehensive econometric analysis that encompasses yield curve forecasting, impulse response analysis, and the investigation of structural breaks. By incorporating these three key components, this research seeks to shed light on the interplay between economic variables, forecast future yield curve movements, and detect shifts in the yield curve structure associated with unpredictable events.

The first component of this study focuses on yield curve forecasting using econometric techniques, such as Vector Autoregression (VAR) models or Dynamic Nelson-Siegel (DNS) models. By leveraging historical data on zero-coupon yields and potential explanatory variables, the chosen model will provide forecasts for the yield curve over a specified time horizon. The accuracy of these forecasts will be rigorously evaluated using statistical measures such as mean absolute error (MAE) or root mean squared error (RMSE), allowing for an assessment of the model's predictive capabilities.

In addition to forecasting, this research incorporates the analysis of impulse response functions (IRFs). Through estimating the dynamic response of the yield curve to shocks in relevant macroeconomic variables such as GDP growth, inflation, or monetary policy indicators, the IRFs provide insight into the transmission channels and the lagged effects of these shocks on the yield curve. This analysis will enhance our understanding of the interactions between the yield curve and important economic factors, contributing to the wider field of monetary policy and financial markets.

Furthermore, we address the critical aspect of detecting and studying structural breaks in the yield curve. Unforeseen events, whether political, social, or economic in nature, can lead to significant shifts in the yield curve's structure. By employing robust econometric techniques, such as Chow tests, Bai-Perron tests, or Markov-switching models, this study will identify and examine these structural breaks. The timing, magnitude, and nature of the breaks will be analyzed, providing valuable insights into the factors driving the shifts and their implications for market dynamics.

In summary, this research aims to offer a comprehensive analysis of the yield curve, incorporating yield curve forecasting, impulse response analysis, and the study of structural breaks. By combining these three aspects, we provide insights into the future movements of the yield curve, the dynamic response to macroeconomic shocks, and the detection of structural changes associated with unpredictable events. These findings hold significant implications for investors, policymakers, and market participants, ultimately contributing to a deeper understanding of macroeconomic dynamics and aiding in informed decision-making in financial markets.

1 Data Collection and Preprocessing

We use the Russian Government Bond Zero Coupon Yield Curve, provided by the Moscow Exchange (MOEX), for our research. It is calculated from the curve basis, and the detailed methodology for defining the curve can be found in Moscow Exchange 2021.

For our analysis, we collected the daily yield curve data spanning from February 1, 2008, to January 30, 2014. The train-test split date we applied is November 15, 2012, to ensure accurate modeling and evaluation. Additionally, to examine a more concentrated time frame, we created a smaller dataset containing monthly data from February 1, 2008, to November 1, 2013. For Sections 3 to 5 we collected the daily yield curve data from January 4, 2003 to December 1, 2023.

In Fig. 1, you can observe the typical behavior of yield curves. In Fig. 2, which visualizes the dynamic changes in bond yields, you can observe the typical behavior of short-term, mid-term, and long-term yields based on our analysis.

2 Yield Curve Forecasting

2.1 Stationarity and cointegration research of the bond yields

Stationarity and absence of cointegration are the key assumption for the correctness of many models. We shall test the stationarity of the bond yields and the cointegration of the yield curve and the macroeconomic variables.

2.1.1 Stationarity

In order to test the stationarity of the bond yields, we shall use the *Augmented Dickey-Fuller test* (ADF test). The null hypothesis of the test is that the time series is non-stationary. The ADF test is a unit root test, which means that it determines how strongly a time series is defined by a trend. The test is based on the assumption that the time series is a random walk with drift. The test is performed using the `ur.df` function from the `urca` package in R. The results of the test are presented in Table 1 (yields) and Table 2 (first difference).

2.1.2 Cointegration

In order to test the cointegration of the yield curve and the macroeconomic variables, we shall use the *Johansen test*. The null hypothesis of the test is that the time series are not cointegrated.

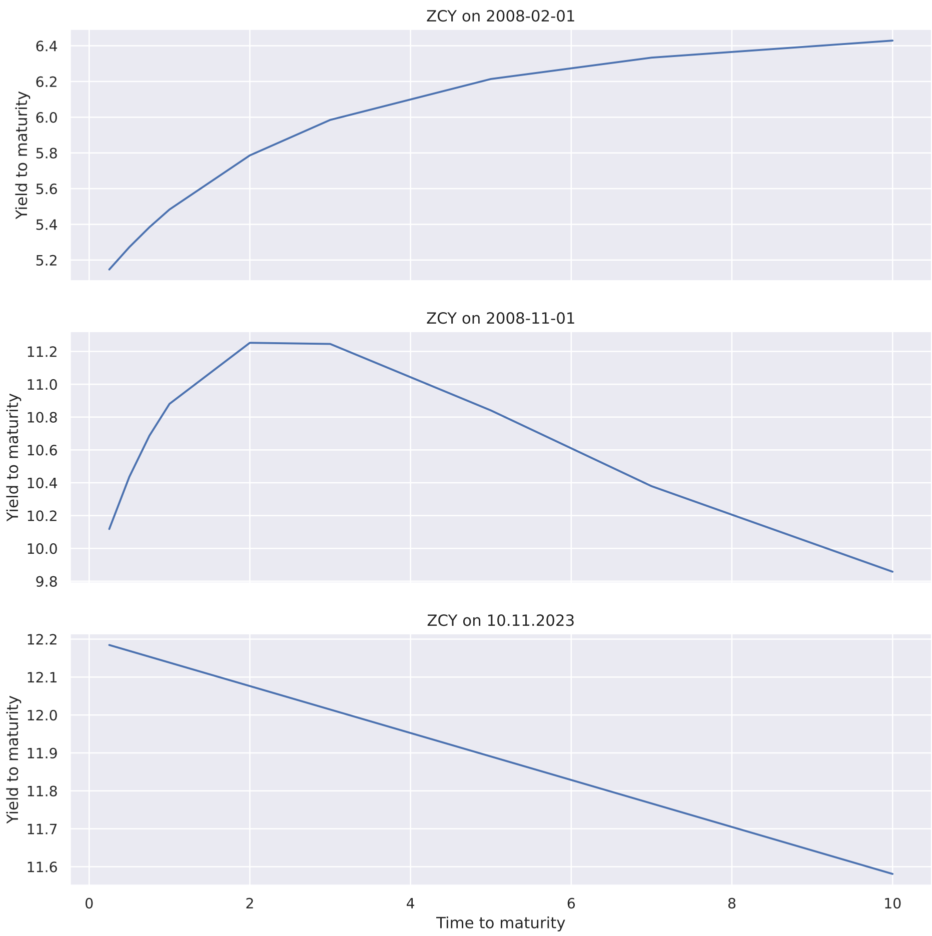

Figure 1: Russian government bond yield curves

Statistic	Parameter	Alternative	p.value	Value name
−1.424	10	stationary	0.822	`bonds$month3`
−1.428	10	stationary	0.820	`bonds$month6`
−1.461	10	stationary	0.806	`bonds$month9`
−1.504	10	stationary	0.788	`bonds$year1`
−1.658	10	stationary	0.722	`bonds$year2`
−1.763	10	stationary	0.678	`bonds$year3`
−1.850	10	stationary	0.641	`bonds$year5`
−1.983	10	stationary	0.585	`bonds$year10`
−2.090	10	stationary	0.540	`bonds$year15`

Table 1: ADF test for the values of the zero-bond yields

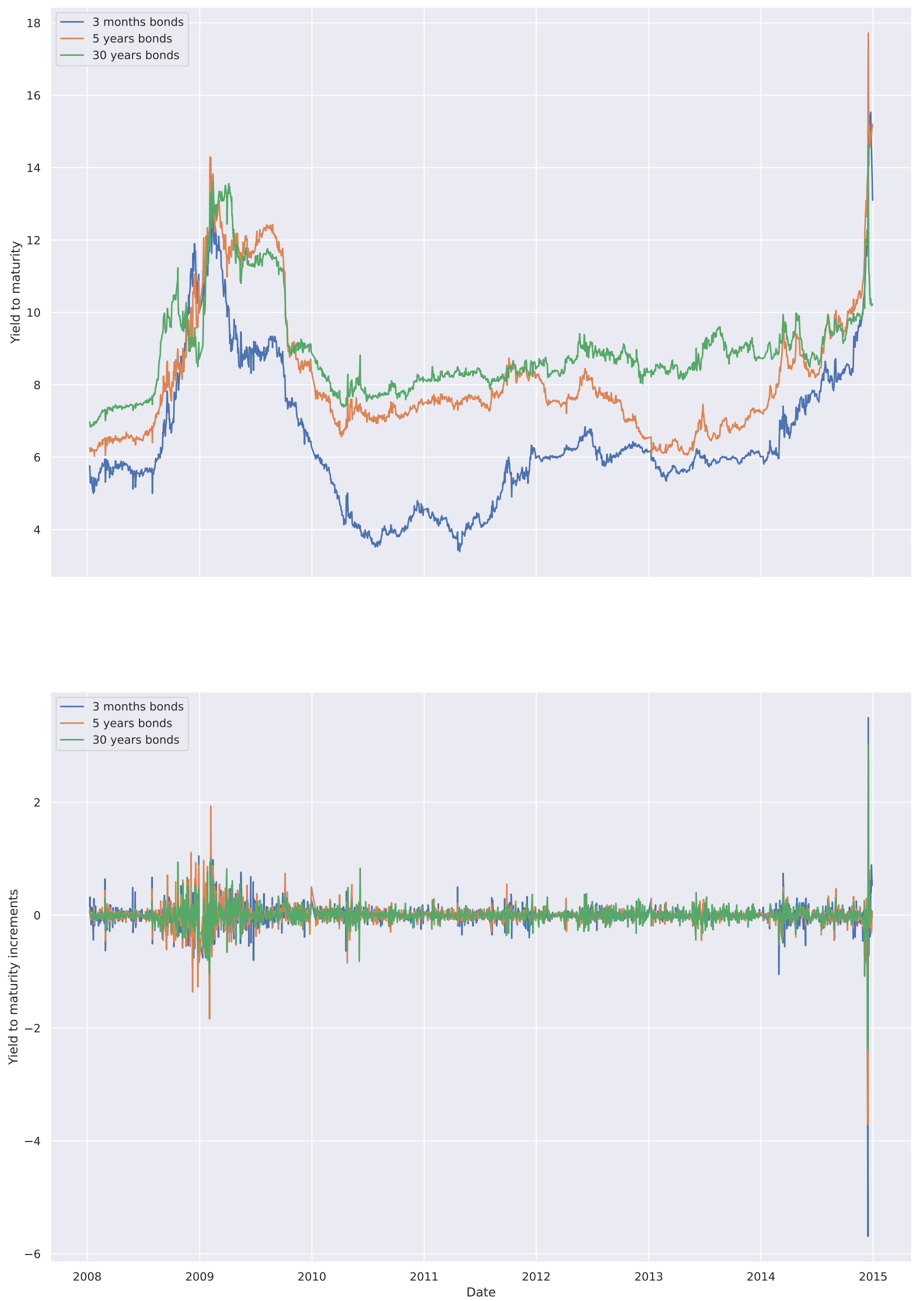

Figure 2: Russian government bond yield to maturity dynamics

Statistic	Parameter	Alternative	p.value	Value name
-9.283	10	stationary	0.01	`exp(diff(log(bonds$month3))) - 1`
-8.957	10	stationary	0.01	`exp(diff(log(bonds$month6))) - 1`
-8.850	10	stationary	0.01	`exp(diff(log(bonds$month9))) - 1`
-8.843	10	stationary	0.01	`exp(diff(log(bonds$year1))) - 1`
-9.108	10	stationary	0.01	`exp(diff(log(bonds$year2))) - 1`
-9.170	10	stationary	0.01	`exp(diff(log(bonds$year3))) - 1`
-9.288	10	stationary	0.01	`exp(diff(log(bonds$year5))) - 1`
-9.649	10	stationary	0.01	`exp(diff(log(bonds$year10))) - 1`
-9.449	10	stationary	0.01	`exp(diff(log(bonds$year15))) - 1`

Table 2: ADF test for the increments of the zero-bond yields

The test is performed using the `ca.jo` function from the `urca` package in R. The results of the test are presented in Table 3.

m3	m6	m9	m12	m24	m36	m60	m84	m120
1.000	-2.238	1.240	0	0	0	0	0	0
0	0	0	1.000	-2.787	1.869	0	0	0
0	0	0	0	0	0	1.000	0.094	-1.245

Table 3: Johansen test for zero-bond yields

We found out that the bond yields with 3m, 6m, and 9m time to maturity are cointegrated. The obtained results could be explained with the so-called *market segmentation hypothesis*.

The market segmentation hypothesis states that the yield curve is segmented into different maturity sectors, and the yields in each sector are determined by the supply and demand for bonds in that sector, since the investors could be differentiated by their investment purpose (buying short-term bonds to obtain small but guaranteed revenue, or long-term bonds to hedge against the drop in the interest rate). It implies that the yields with similar maturities can be cointegrated.

2.2 Naïve approach for the bond yield forecasting

2.2.1 Random walk

We assumed the random walk model for the bond yields. The model is defined as follows:

$$y_t = y_{t-1} + \epsilon_t. \tag{1}$$

It is used as a baseline model for the comparison with other models.

2.2.2 Auto-ARIMA

Later, we assumed the ARIMA model set for the bond yields. The model is defined as follows:

$$\Delta^d y_t = \phi_0 + \phi_1 \Delta^d y_{t-1} + \ldots + \phi_p \Delta^d y_{t-p} + \epsilon_t + \theta_1 \epsilon_{t-1} + \ldots + \theta_q \epsilon_{t-q}, \tag{2}$$

where ϵ_t is the white noise process with zero mean and variance σ^2, and Δ is a first difference operator. The model is estimated using the `auto.arima` function from the `forecast` package in R.

2.2.3 Vector Error Correction

Later, we assumed the VEC model for the bond yields. The model is defined as follows:

$$\Delta y_t = \Pi y_{t-1} + A_0 + A_1 \Delta y_{t-1} + \cdots + A_p \Delta y_{t-p} + \epsilon_t. \tag{3}$$

where ϵ_t is the white noise process with zero mean and variance σ^2. The model is estimated using the `ca.jo` function from the `urca` package in R.

2.2.4 GARCH

Later, we assumed the GARCH model for the bond yields. The model for volatility is defined as follows:

$$s_t^2 = \omega + \alpha \epsilon_{t-1}^2 + \beta s_{t-1}^2, \tag{4}$$

where ϵ_t is the white noise process with zero mean and variance σ^2. The model is estimated using the `ugarchspec` and `ugarchfit` functions from the `rugarch` package in R.

2.2.5 Results

[1]

Maturity	autoARIMA	ARIMA order	ARIMA(0, 0, 0)	RW	VECM(2)	GARCH
3 months	0.0045	$(1, 0, 0)$	0.0047	0.0109	0.0193	0.6115
6 months	0.0039	$(0, 0, 1)$	0.0041	0.0100	0.0182	0.4658
9 months	0.0035**	$(0, 0, 1)$	0.0038	0.0095	0.0178	0.5676
12 months	0.0038**	$(0, 0, 1)$	0.0039	0.0069	0.0194	0.7794
5 years	0.0052	$(2, 1, 3)$	0.0053	0.0072	0.0182	1.2742
15 years	0.0059	$(2, 1, 3)$	0.0061	0.0076	0.0174	1.9276

Table 4: Forecasting results with 1 month horizon and L^1 loss (MAE)

2.3 Nelson-Siegel parametric model

2.3.1 Theoretical description

Let us now remind the *Nelson-Siegel model* introduced in Nelson and Siegel 1987 for the yield curve estimation. The static NS model is defined as follows:

$$G(T) = \beta_0 + (\beta_1 + \beta_2)\frac{\tau}{T}\left(1 - e^{-\frac{T}{\tau}}\right) - \beta_2 e^{-\frac{T}{\tau}}, \tag{5}$$

where T is the time to maturity, $G(T)$ is the yield estimator of the government bonds from the curve basis, and the parameters to be estimated are

1. τ is the 'typical' time to maturity,
2. β_0 is the long-run of zero-bond yields,
3. β_1 is the mid-run of zero-bond yields,
4. β_2 is the short-run of zero-bond yields.

[1] ** means the significance at 5% confidence level that auto-ARIMA is better then ARIMA(0,0,0). All the other differences are significant at 1% confidence level. We used the Diebold-Mariano test.

The Nelson-Siegel model offers significant advantages for yield curve estimation and interest rate forecasting. Its simplicity, flexibility, stability, and market insights make it a highly valuable tool.

The model's simplicity is evident through its use of only a few parameters, making it easy to understand and interpret. Its flexibility allows for capturing both short-term fluctuations and long-term trends in interest rates accurately. The stability of the model is tested and validated across different markets and economic conditions, instilling confidence in its reliability.

Additionally, the Nelson-Siegel model provides valuable insights into market expectations by decomposing the term structure of interest rates. This information is crucial for decision-making and portfolio management. The model's versatility allows for customization and extensions to meet specific needs, further enhancing its applicability.

We shall suggest a modification of the *Dynamic Nelson-Siegel model* introduced in Diebold and Li 2006. First of all, we shall discuss the original DNS model. The functional dependency of the yield curve on the time to maturity T is given by

$$G(t,T) = \beta_0(t) + \beta_1(t)\frac{\tau(t)}{T}\left(1 - e^{-\frac{T}{\tau(t)}}\right) - \beta_2(t)e^{-\frac{T}{\tau(t)}}, \tag{6}$$

where t is the current time, T is the time to maturity at time t, $\beta_0(t)$, $\beta_1(t)$, $\beta_2(t)$, and $\tau(t)$ are time-varying parameters. The factors are modeled as AR(1) processes:

$$\begin{aligned} \beta_0(t) &= \beta_0 + \phi_{0,1}(\beta_0(t-1) - \beta_0) + \epsilon_{0,t}, \\ \beta_1(t) &= \beta_1 + \phi_{1,1}(\beta_1(t-1) - \beta_1) + \epsilon_{1,t}, \\ \beta_2(t) &= \beta_2 + \phi_{2,1}(\beta_2(t-1) - \beta_2) + \epsilon_{2,t}. \end{aligned} \tag{7}$$

2.3.2 Forecasting results

To calibrate the suggested model we followed the steps described below:

1. Using the non-linear least squares method, we estimated the parameters of the static Nelson-Siegel for each day of the sample period.
2. Given a set of estimated factors, we estimated the parameters of the chosen model using the standard OLS method. The models we used for the factor forecasting:
   - auto-ARIMA,
   - VAR(1),
   - Random Walk as a baseline.

After the non-linear least squares optimization, we obtained the factor dynamics found in Figure 5.

It is possible to use the standard OLS method to estimate the parameters of the DNS model since there is weak stationarity of the factor increments.

Statistic	Parameter	Alternative	p_{value}	Variable name
-3.374	3	stationary	0.068	`train_b0`
-5.041	3	stationary	0.01	`train_b1`
-3.848	3	stationary	0.022	`train_b2`
-5.257	3	stationary	0.01	`train_tau`

Table 5: ADF test for the increments of the NS factors

It turns out that the best forecast of the NS factors is the constant forecast. The results of the model calibration on a train dataset are presented in Table 7[2].

[2] Auto-ARIMA is not significantly better then RW. All the other differences are significant at 1% confidence level. We used Diebold-Mariano test.

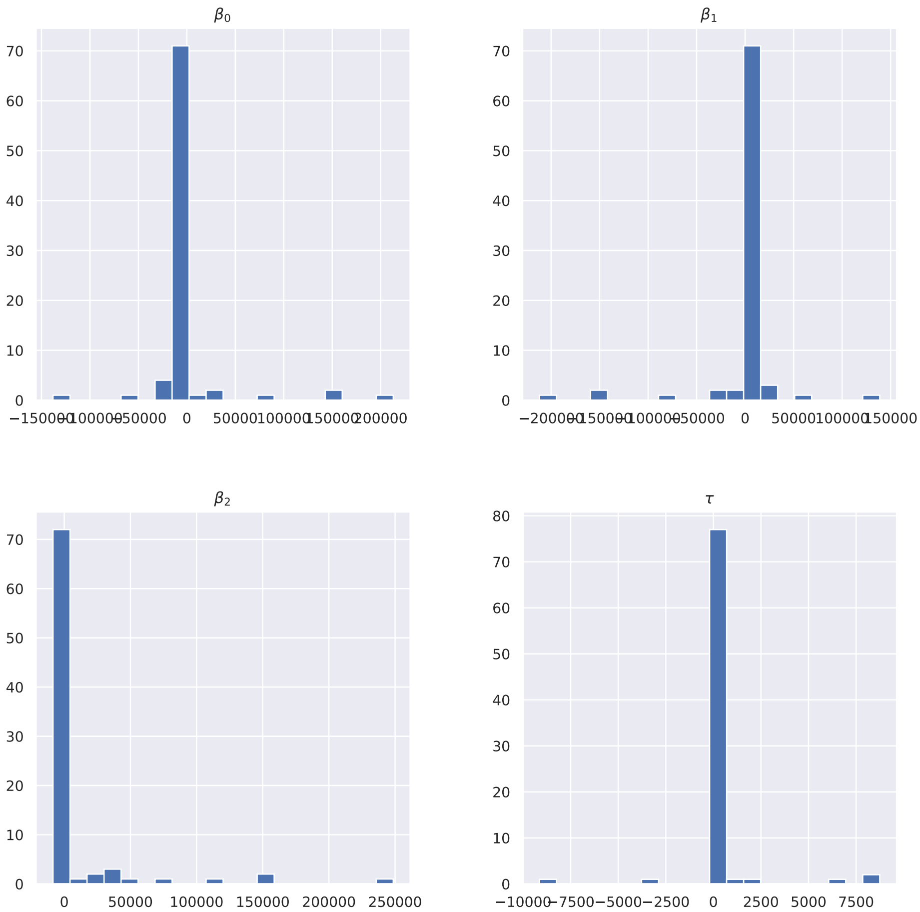

Figure 3: Nelson-Siegel factor distribution (with outliers).

Statistic	Parameter	Alternative	p_{value}	Variable name
-2.132	3	stationary	0.521	`train_b0`
-2.562	3	stationary	0.347	`train_b1`
-2.304	3	stationary	0.451	`train_b2`
-2.223	3	stationary	0.484	`train_tau`

Table 6: ADF test for the values of the NS factors

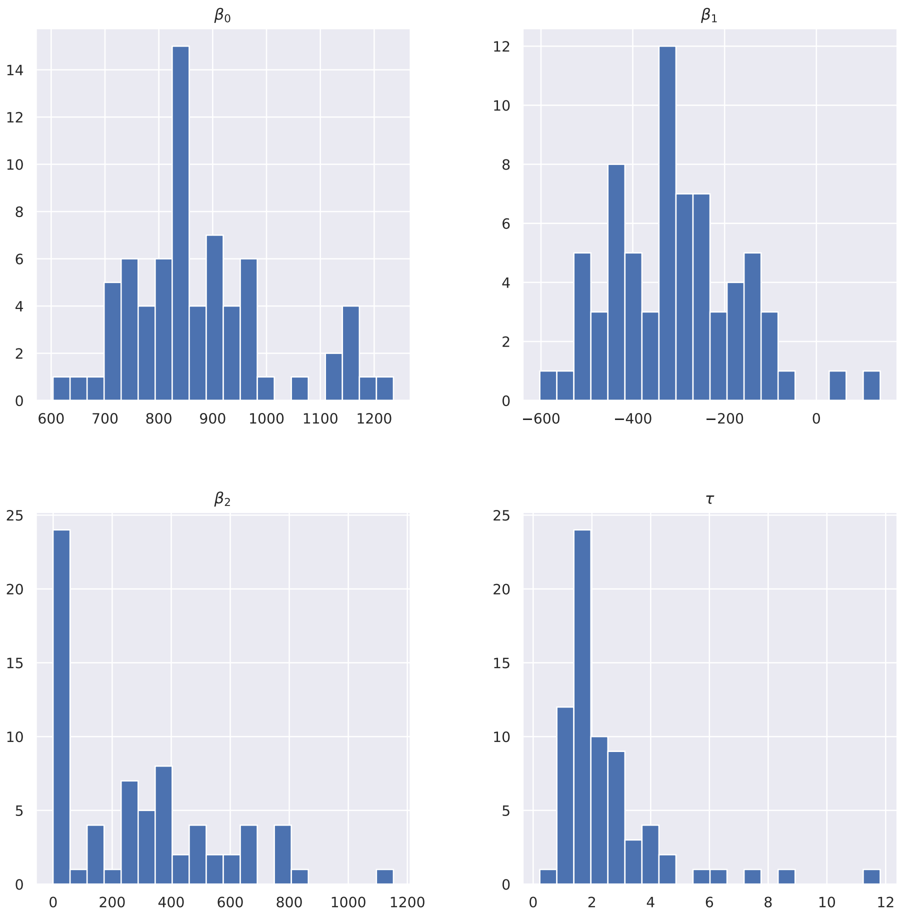

Figure 4: Nelson-Siegel factor distribution (without outliers)

Coefficient	auto-ARIMA	VAR(1)	RW
β_0	53.78356	131.1459	66.3105
β_1	63.31042	143.9235	66.25878
β_2	133.9688	388.3436	177.1525
τ	1.083687	2.569167	1.328986

Table 7: Parameters of the DNS model for 3 different forecasting models.

Figure 5: Nelson-Siegel factor dynamics

2.3.3 Summary

Due to inadequacy of the pointwise bond yield forecasting results, we concluded that the Nelson-Siegel model is more suitable for our needs. Therefore, the rest of the research will be devoted to the deeper understanding of the chosen model and its generalization.

3 Impulse Response Analysis

We calculated the orthogonal impulse response functions from Nelson-Siegel factors and they turned out to be insignificant (see Fig. 6). This implies that the market segmentation hypothesis holds for the MOEX Russian government bond market.

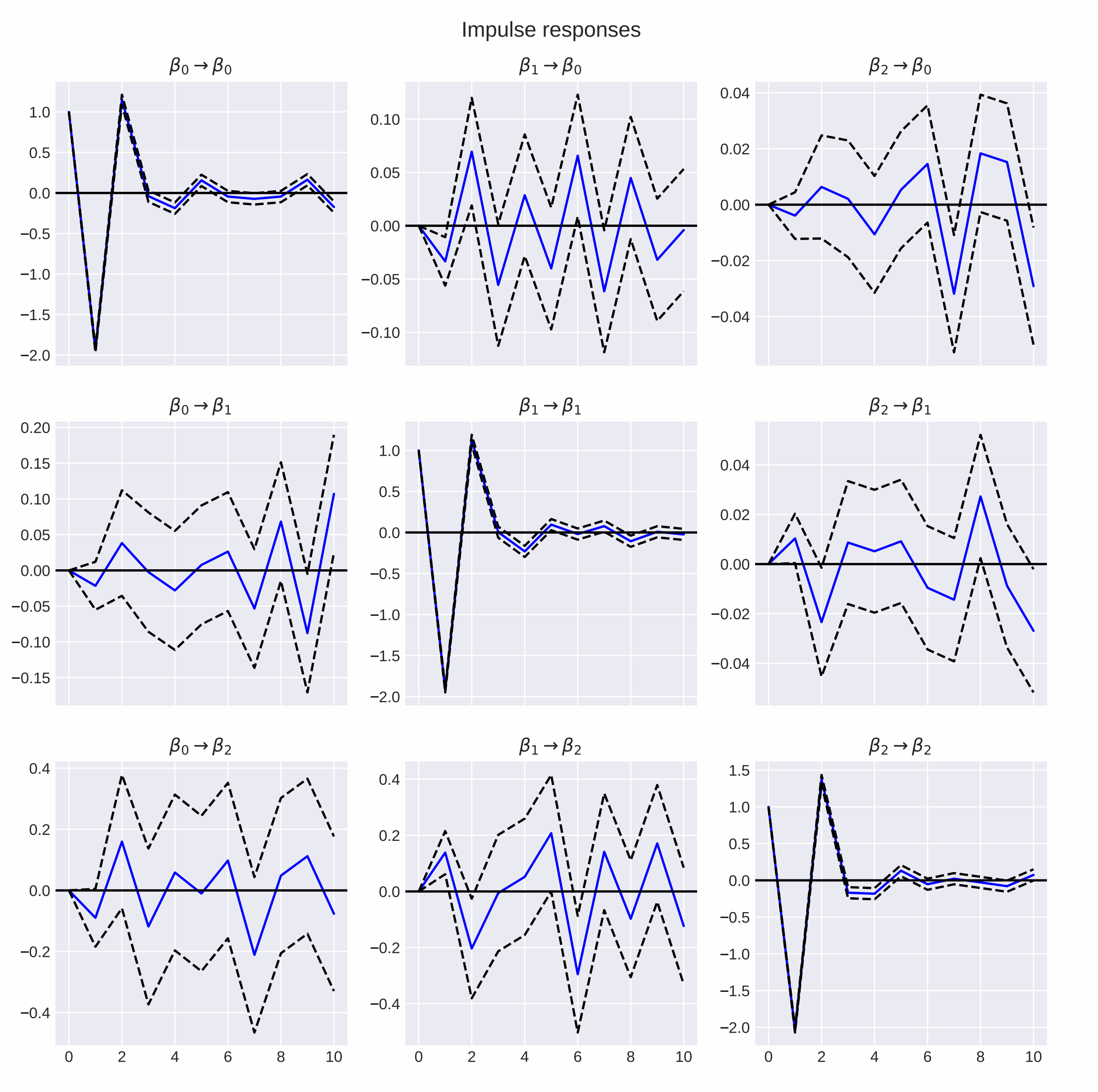

Figure 6: Nelson-Siegel impulse response functions.

4 Structural Breaks Analysis

The analysis of the structural breaks in the Nelson-Siegel factor dynamics allows for the identification of periods of volatility or instability in the bond market, which is crucial for investors and policymakers to make informed decisions. This section aims to detect structural breaks in the Nelson-Siegel factor dynamics and examine the timing, magnitude, and nature of these breaks.

Now let us examine the events that happened around the detected structural breaks. We will fix $m = 4$ (corresponds to top-4 shocks for 20 years). The detected structural breaks can be found in Tables 8 and 9. In Tables 10 to 12, you can find the full list of optimal $(m+1)$-segment partition for β_0, β_1, and β_2 dynamics, respectively.

TTM Scale	First shock	Second shock	Third shock	Fourth shock
Long-run (β_0)	657	1384	3943	4502
Medium-run (β_1)	938	1495	2832	3524
Short-run (β_2)	1589	2252	2869	3115

Table 8: Detected structural breaks, numbers of observation detected as breakpoints.

TTM Scale	First shock	Second shock	Third shock	Fourth shock
Long-run (β_0)	2005-08-30	2008-08-08	2018-06-21	2020-09-09
Medium-run (β_1)	2006-10-17	2009-01-22	2014-01-21	2016-10-21
Short-run (β_2)	2009-06-09	2012-02-02	2014-03-14	2015-03-10

Table 9: Detected structural breaks, dates.

m																			
1																			4619
2						1390													4619
3			657			1380													4619
4			657			1384										3943		4502	
5			657			1384										3943		4376	4622
6			657			1391	1654		2277							3868		4502	
7			657			1391	1654		2277							3943		4376	4622
8			657			1391	1654		2279			2966				3945		4376	4622
9			657			1391	1654		2279			2966		3525		3867		4376	4622
10			657			1391	1654		2279			2966		3370	3616	3945		4376	4622
11			659		1146	1392	1654		2279			2966		3370	3616	3945		4376	4622
12			657			1391	1654		2210	2465	2714	2966		3370	3616	3945		4376	4622
13			659		1146	1392	1654		2210	2465	2714	2966		3370	3616	3945		4376	4622
14	322		657		1146	1392	1654		2210	2465	2714	2966		3370	3616	3945		4376	4622
15	322		654	900	1146	1392	1654		2210	2465	2714	2966		3370	3616	3945		4376	4622
16	322		654	900	1146	1392	1654	1936	2210	2465	2714	2966		3370	3616	3945		4376	4622
17	322		654	900	1146	1392	1654	1936	2210	2465	2714	2966		3370	3616	3870	4116	4376	4622
18	322		654	900	1146	1392	1654	1936	2210	2465	2711	2958	3204	3450	3696	3942	4188	4434	4680
19	246	492	738	984	1230	1476	1722	1968	2214	2465	2711	2958	3204	3450	3696	3942	4188	4434	4680

Table 10: Optimal $(m+1)$-segment partition for β_0 dynamics.

m																			
1											2814								
2			672								2829								
3			672									2832		3524					
4				938		1495						2832		3524					
5				938		1495						2832		3521					4619
6				938		1495						2832		3524				4361	4607
7				917			1772		2115			2832		3524				4361	4607
8	246		672				1773		2115			2832		3524				4361	4607
9	246		660	959			1772		2115			2832		3524				4361	4607
10	246		660	992		1489	1775		2115			2832		3524				4361	4607
11	246		660	992		1489	1775		2115		2751	2997		3521				4361	4607
12	246		660	992		1489	1775		2115		2751	2997		3521			4080	4359	4605
13	246		660	992		1489	1775		2115		2751	2997		3521	3767		4080	4359	4605
14	246		660	992		1489	1775		2115	2503	2751	2997		3521	3767		4080	4359	4605
15	246		660	992		1489	1775		2115	2503	2751	2997	3256	3521	3767		4080	4359	4605
16	246		660	995	1243	1489	1775		2115	2503	2751	2997	3256	3521	3767		4080	4359	4605
17	246	492	738	995	1243	1489	1775		2115	2503	2751	2997	3256	3521	3767		4080	4359	4605
18	246	492	738	995	1243	1489	1765	2011	2257	2503	2751	2997	3256	3521	3767		4080	4359	4605
19	246	492	738	984	1230	1476	1722	1968	2214	2460	2706	2952	3198	3444	3690	3936	4182	4428	4674

Table 11: Optimal $(m+1)$-segment partition for β_1 dynamics.

m																			
1									2252										
2						1589			2223										
3									2252			2869	3115						
4						1589			2252			2869	3115						
5	326					1589			2252			2869	3115						
6	326					1589			2252			2869	3115						4605
7	326			907		1451			2252			2869	3115						4605
8	326					1589			2252			2868	3114	3374	3620	3866			
9	326			907		1451			2252			2868	3114	3374	3620	3866			
10	326					1589			2252			2868	3114	3374	3620	3911	4266		4605
11	326			907		1451			2252			2868	3114	3374	3620	3911	4266		4605
12	326			907		1451		2005	2252			2868	3114	3374	3620	3911	4266		4605
13	326			907		1451		1977	2223	2503		2868	3114	3374	3620	3911	4266		4605
14	326			907		1451	1697	1977	2223	2503		2868	3114	3374	3620	3911	4266		4605
15	326		725	971		1451	1697	1977	2223	2503		2868	3114	3374	3620	3911	4266		4605
16	326	572		908	1200	1451	1697	1977	2223	2503		2868	3114	3374	3620	3911	4266		4605
17	326	572		908	1200	1451	1697	1977	2223	2503		2868	3114	3374	3620	3866	4112	4359	4605
18	284	530	776	1022	1268	1514	1760	2006	2252	2503		2868	3114	3374	3620	3866	4112	4359	4605
19	246	492	738	984	1230	1476	1722	1968	2214	2460	2706	2952	3198	3444	3690	3936	4182	4428	4675

Table 12: Optimal $(m+1)$-segment partition for β_2 dynamics.

Now we shall begin the research of the historical events that happened around the detected structural breaks. Note that the 'date' of the break is estimated approximately, therefore, there could be some discrepancies between the actual date of the suggested event and the estimated date of the detected break.

4.1 Long-run yields

August 30, 2005 The detected structural break could be associated with three events:

1. The complete stabilization of the Russian economy after the 1998 crisis. In January 2005, free budget balances in the amount of 218.4 billion rubles were transferred to the Stabfond. For the period January-November of this year, the fund was replenished with revenues of the first quarter - in the amount of 210.8 billion rubles, the second quarter - in the amount of 294.4 billion rubles, the third quarter - in the amount of 373.5 billion rubles, October - by 137.7 billion rubles, November - 154.4 billion rubles. The Stabilization Fund began to be formed in Russia on January 1, 2004 in order to reduce the risks associated with unfavorable foreign economic conditions, as well as a tool for sterilizing excess money supply in circulation. It receives huge income of the budget from high oil prices. See RBK 2006. This event could have led to the decrease in the future supply of the government bonds due to the possible government budget proficit.
2. The war in Iraq as an indirect cause due to the drastic increase in both spot and futures prices of oil, see Berkmen, Ouliaris, and Samiei 2005. At the time, the government budget in Russia was calculated using the international oil prices. The increase in oil prices led to the proficit of the government budget, which, in fact, means that the government had more money to spend on the economy (i.e. increase the foreign exchange reserves, reduce government debt and spend more for unforeseen circumstances). Consequently, the emission of the government bonds was reduced, which led to the change in the future supply of the bonds.
3. United Nations Security Council resolution 1615, adopted unanimously on 29 July 2005, after reaffirming all resolutions on Abkhazia and Georgia, particularly Resolution 1582 (2005), the council extended the mandate of the United Nations Observer Mission in Georgia (UNOMIG) until 31 January 2006.

August 08, 2008 The detected structural break could be (and probably is) associated with the start of the Russo-Georgian Conflict and the beginning of the world finanical crisis. Both of these events had a negative impact on the Russian economy. The conflict could have led to the increase in the government spending, which, in turn, led to the increase in the government debt. The world financial crisis led to the decrease in the oil prices, which, in turn, led to the decrease in the government budget proficit. The decrease in the proficit led to the increase in the government debt. Both of these events led to the increase in the future supply of the government bonds.

June 21, 2018 The detected structural break could be associated with two events:

1. The Russian Federation experienced a prolonged period of widespread protests from March 2017 to the end of 2018, with a focus being on fighting corruption within the government and opposing the increase in the retirement age. The aftermath of these protests could potentially include economic instability due to the disruption of normal business operations, increased government spending to address the demands of the protesters, and a loss of investor confidence in the Russian economy.

2. 2018 FIFA World Cup. The economic aftermaths of the 2018 FIFA World Cup in Russia included a boost in tourism and hospitality sectors, leading to increased consumer spending and infrastructure development. Additionally, the tournament provided opportunities for small businesses and entrepreneurs, creating a positive impact on the local economy. However, there were also concerns about the long-term utilization of the newly built infrastructure and the potential impact on public finances due to the high costs of hosting the event.

September 09, 2020 The detected structural break could be associated with first and second waves of COVID-19 pandemic. There was a sharp decline in global oil prices, a key source of revenue for Russia, leading to budgetary challenges. Additionally, the pandemic-induced lockdowns and travel restrictions led to a contraction in economic activity, particularly in sectors such as tourism and hospitality. The Russian government implemented several measures, including financial support to businesses and individuals, to reduce the economic impact of the pandemic. The pandemic caused a recession and a range of challenges for businesses and the workforce.

4.2 Medium-run yields

October 17, 2006 The detected structural break could be associated with two events:

1. The end of the Chechen War and the elimination of a number of militants in Chechnya: Basayev, the main representative of Al-Qaeda in the North Caucasus, the Jordanian Sheikh Abu Omar Al-Seif, Maskhadov's successor, the so-called president of Ichkeria Abdul Halim Saidulayev. The announcement of an amnesty, which, according to the latest data, was used by about five hundred members of illegal armed groups. This could lead to the increase of the nationalistic sentiment, investor confidence, international authority and stability of the economy.
2. A series of high-profile murders - the first deputy chairman of the Central Bank of Russia, Andrei Kozlov (September 13), the columnist of Novaya Gazeta, Anna Politkovskaya (October 7). This possibly led to the negative aftermaths.

January 22, 2009 The detected structural break could be associated with the 2009 Russia-Ukraine gas dispute. In 2009, a dispute arose between Russian gas company Gazprom and Ukrainian gas company Naftogaz over accumulating debts for previous gas supplies. The conflict led to a cutoff of Russian gas supplies to Ukraine, which in turn disrupted gas flows to Southeastern Europe and parts of other European countries for 13 days. Despite attempts by the European Union to intervene, the crisis was not resolved until January 18, when Russian Prime Minister Vladimir Putin and Ukrainian Prime Minister Yulia Tymoshenko negotiated a new contract. Following the resolution, gas flows to Europe resumed, but both Russia and Ukraine suffered economic losses, and their reputations as energy supplier and transit country were negatively impacted.

January 21, 2014 The detected structural break could be associated with two major events:

1. 2014 Winter Olympics. The economic aftermaths of the 2014 Winter Olympics in Russia included a boost in tourism and hospitality sectors, leading to increased consumer spending and infrastructure development.
2. The joining of Crimea into the Russian Federation. The event was followed by several economic sanctions imposed by the United States and the European Union, which led to a decline in investor confidence and a reduction in foreign direct investment. Additionally,

the Russian economy was negatively impacted by the decline in oil prices and the depreciation of the ruble. Furthermore, the 2014-2016 Russian economic crisis started.

October 21, 2016 The detected structural break could be associated with two events:

1. Russian military operation in Syria. It had several impacts on the Russian economy. The defense budget increased significantly to support military operations, diverting funds away from other sectors. However, the operation also led to an increase in nationalistic sentiment, which potentially bolstered the mood of the investors. Overall, the operation had mixed economic effects on the Russian economy.
2. Allegations of institutionalized doping use by Russian athletes. The Russian doping scandal refers to a series of allegations made against Russia by the World Anti-Doping Agency (WADA) concerning doping violations in multiple sports in the country. The scandal had a negative impact on the reputation of Russian athletes and the country's sports industry, leading to a decline in sponsorship deals and investments.

4.3 Short-run yields

We shall not describe the structural breaks for the short-run yields due to the volatile nature of the subject. However, we shall note that the structural breaks for the short-run yields are mostly associated with the events that caused the structural breaks for the medium-run yields.

4.4 Forecasting results

We decided to divide the dataset into 4 segments which we took from structural analysis of the long-run yields. The results of ARIMA and VAR models restricted to each sector could be found in Tables 13 and 14, respectively.

Segment	Factor	MAPE	ME	MAE	MPE	RMSE
0	β_0	0.0083	6.6921	6.6921	0.0083	7.7627
	β_1	0.0155	5.8167	7.4981	-0.012	7.9825
	β_2	0.1538	27.8315	30.2092	0.1455	47.6721
	τ	0.1499	0.2511	0.2511	0.1499	0.2786
1	β_0	0.0076	-5.9872	5.9872	0.0076	6.2486
	β_1	0.0275	-5.15	6.8624	0.0209	7.2744
	β_2	21312.0946	-5.3942	55.4278	-21312.0946	55.9851
	τ	0.7482	-1.5317	1.5317	-0.7482	1.7142
2	β_0	0.0578	48.5869	48.5869	0.0578	57.57
	β_1	0.0972	-15.7947	19.6762	0.0816	27.1274
	β_2	0.1214	7.6326	27.0065	-0.0172	29.0144
	τ	0.0916	0.3636	0.5129	0.069	0.5959
3	β_0	0.006	-2.0011	5.5102	-0.0021	6.0729
	β_1	0.3471	23.9805	29.3697	0.2982	32.2907
	β_2	0.5223	-93.8611	93.8611	-0.5223	95.6724
	τ	0.9588	1.8988	1.8988	0.9588	1.987

Table 13: ARIMA forecasting results for the structural NS factors.

Segment	Factor	MAPE	ME	MAE	MPE	RMSE
0	β_0	0.0038	0.6743	3.053	0.0009	4.3991
	β_1	0.017	5.8745	8.1959	-0.0121	8.7872
	β_2	0.1034	15.4491	20.2472	0.0863	32.5685
	τ	0.1096	0.1865	0.1865	0.1096	0.196
1	β_0	0.0058	-0.3276	4.6186	-0.0004	5.1622
	β_1	0.0355	-8.4288	8.8186	0.0339	10.2788
	β_2	26187.8876	-40.8754	47.3524	-26187.8563	57.5888
	τ	0.1294	0.0056	0.2309	0.0279	0.2566
2	β_0	2.4086	2062.9983	2062.9983	2.4086	2109.0292
	β_1	8.8918	-1986.5645	1986.5645	8.8918	2009.4069
	β_2	30.0551	-6835.3624	6835.3624	30.0551	7388.3095
	τ	4.9541	-28.5051	28.5051	-4.9541	30.4631
3	β_0	0.0139	-12.8457	12.8457	-0.0139	14.9255
	β_1	0.134	-9.6703	12.1766	-0.1002	15.9566
	β_2	0.3394	-60.4861	60.4861	-0.3394	61.5831
	τ	0.3582	0.68	0.68	0.3582	0.7972

Table 14: VAR forecasting results for the structural NS factors.

5 Markov Chain State Modelling

Let us suppose that there are two states of market volatility: high (1) and moderate (0). We consider the MS-ARIMA (ARIMAX with economy state as an exogenous variable) model for the dynamics of the bonds YTM. We tested everything on stationarity of values and first difference and the Granger causality of the factors. The details of the methodology could be found in the repository. You can see the calibration and forecasting results in Table 15 and Figs. 7 to 9.

6 Conclusion

During project 2 we researched the following:

1. Impulse response functions for the Nelson-Siegel factors;
2. Structural breaks of the factors and their possible causes;
3. MS-ARIMA forecasting of the factor time series.

We plan to improve our research in this are and contribute greatly to the field.

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Figure 7: A bond yield dynamics with 3 moths to maturity.

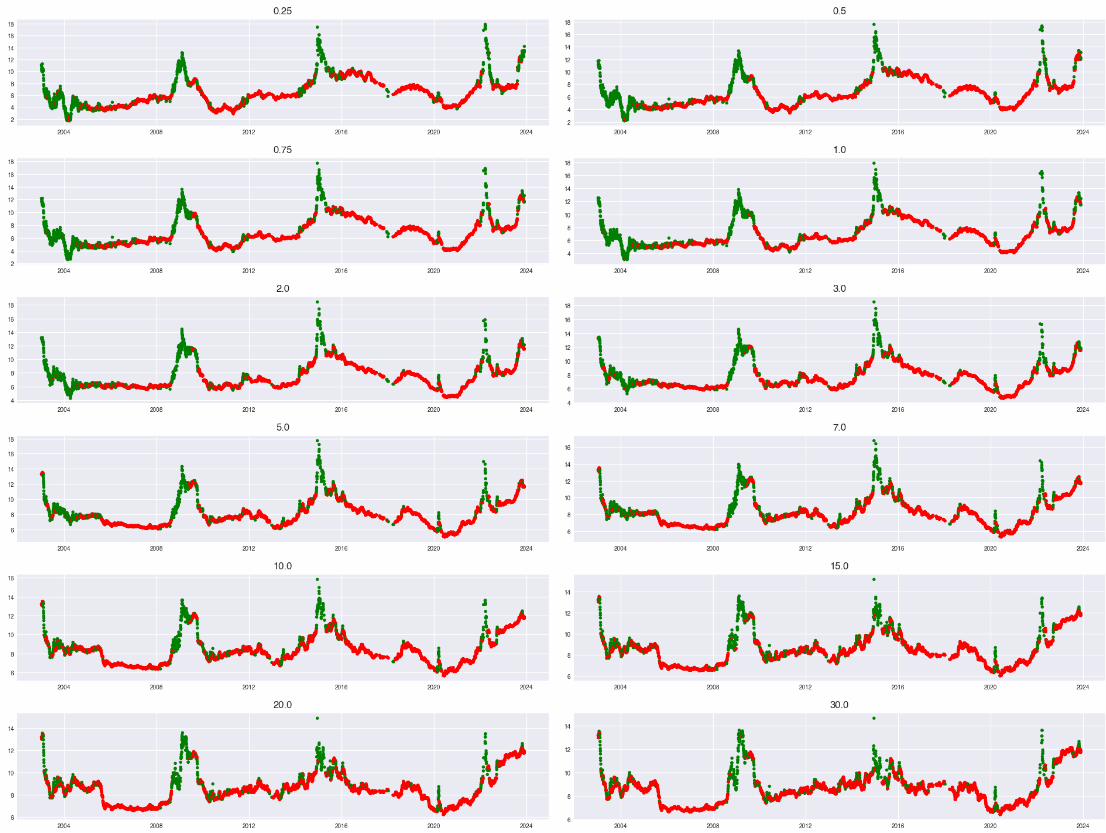

Figure 8: Bond-wise yield curve dynamics.

Segment	TTM	MAPE	ME	MAE	MPE	RMSE
0	0.25	0.009	0.0092	0.0335	0.0026	0.045
	0.5	0.0084	0.0027	0.0352	0.0008	0.0455
	0.75	0.0114	-0.0202	0.0523	-0.0043	0.0572
	1.0	0.0118	-0.0321	0.0583	-0.0064	0.06
	2.0	0.0123	-0.0695	0.0733	-0.0116	0.0846
	3.0	0.0094	-0.0609	0.0622	-0.0092	0.078
	5.0	0.0072	-0.0323	0.0518	-0.0044	0.0596
	7.0	0.0051	-0.0141	0.0385	-0.0019	0.0438
	10.0	0.0034	0.0264	0.0264	0.0034	0.0336
	15.0	0.0053	0.041	0.041	0.0053	0.0475
	20.0	0.0035	0.0225	0.0279	0.0029	0.0301
	30.0	0.0017	0.0095	0.0136	0.0012	0.0154
1	0.25	0.0439	-0.249	0.249	-0.0439	0.2741
	0.5	0.05	-0.2888	0.2888	-0.05	0.3099
	0.75	0.0462	-0.272	0.272	-0.0462	0.2952
	1.0	0.0468	-0.2804	0.2804	-0.0468	0.3003
	2.0	0.0419	-0.2668	0.2668	-0.0419	0.2852
	3.0	0.0375	-0.2491	0.2491	-0.0375	0.2676
	5.0	0.0292	-0.2057	0.2057	-0.0292	0.2463
	7.0	0.0237	-0.173	0.173	-0.0237	0.2217
	10.0	0.0179	-0.1346	0.1346	-0.0179	0.2081
	15.0	0.0149	-0.1149	0.1149	-0.0149	0.1876
	20.0	0.0137	-0.1073	0.1073	-0.0137	0.1763
	30.0	0.0065	-0.0338	0.051	-0.0042	0.087
2	0.25	0.0093	0.0584	0.0584	0.0093	0.0671
	0.5	0.0067	0.0422	0.0422	0.0067	0.0487
	0.75	0.0059	0.0374	0.0374	0.0059	0.0463
	1.0	0.0056	0.0352	0.0352	0.0056	0.0456
	2.0	0.0057	0.0358	0.0358	0.0057	0.0411
	3.0	0.0063	0.0398	0.0398	0.0063	0.0423
	5.0	0.0083	0.0537	0.0537	0.0083	0.0552
	7.0	0.0112	0.0738	0.0738	0.0112	0.075
	10.0	0.0151	0.1023	0.1023	0.0151	0.1044
	15.0	0.0229	0.1611	0.1611	0.0229	0.1703
	20.0	0.0266	0.1934	0.1934	0.0266	0.2106
	30.0	0.0276	0.2084	0.2084	0.0276	0.2365
3	0.25	0.034	0.056	0.3575	0.007	0.4118
	0.5	0.0312	0.0249	0.3298	0.0039	0.3978
	0.75	0.0311	0.0488	0.3283	0.006	0.38
	1.0	0.0312	0.0765	0.3282	0.0086	0.3643
	2.0	0.0378	0.2548	0.3896	0.0255	0.3924
	3.0	0.0425	0.3185	0.4347	0.0319	0.4425
	5.0	0.0471	0.3764	0.4746	0.0381	0.49
	7.0	0.0467	0.3558	0.4641	0.0366	0.4766
	10.0	0.0419	0.2572	0.4115	0.0272	0.4154
	15.0	0.0338	0.1591	0.3281	0.0173	0.3337
	20.0	0.0272	0.0828	0.2627	0.0094	0.2815
	30.0	0.0207	0.0099	0.2	0.0017	0.2481

Table 15: MS-ARIMA forecasting results for the YTM.

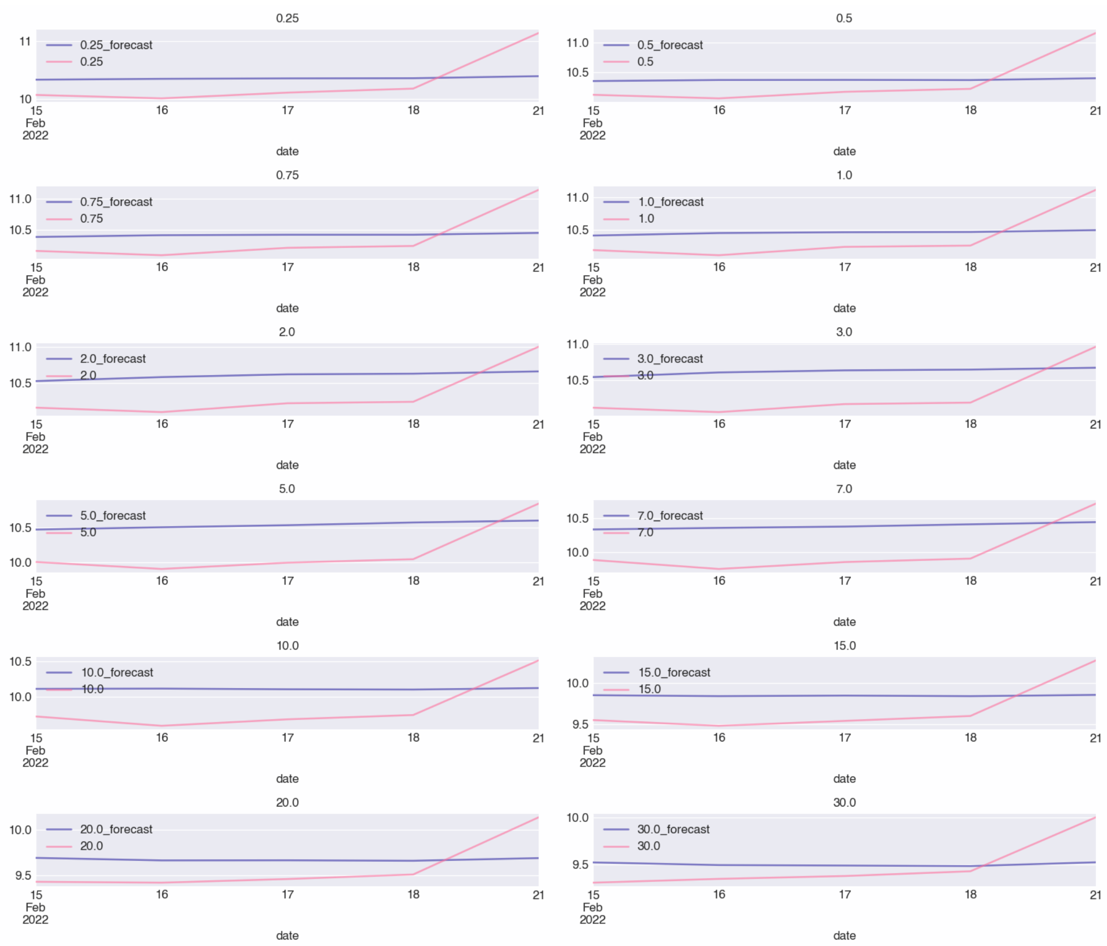

Figure 9: MS-ARIMA forecasting.

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