ДЗ

$$\begin{cases} \dfrac{y_{k+1} - 2y_k + y_{k-1}}{h^2} = -\lambda y_k, & h = \dfrac{1}{N - 1/2} \\ y_0 = y_1, \; y_N = 0 \end{cases}$$

$\triangle$ Есть какое-то фикс. $\lambda \Rightarrow y_{k+1} - 2p\,y_k + y_{k-1} = 0$, $p = 1 - \frac{\lambda h^2}{2} =$

Имеем теор. Виета $\mu_1 \cdot \mu_2 = 1$, $\frac{\mu_1 + \mu_2}{2} = p \Rightarrow \mu_{1,2} = p \pm \sqrt{p^2 - 1}$

$\Rightarrow$ 1). $y_k = C_1 \mu_1^k + C_2 \mu_2^k$

2) $y_k = C_1 \mu_2^k + C_2 k \mu_2^k$

2). $\begin{cases} C_1 \mu^N + C_2 N \mu^N = 0 \\ C_1 = C_1 \mu + C_2 \mu \end{cases} \Rightarrow \begin{cases} C_1 = -NC_2 \\ -NC_2 = -\mu_2 NC_2 + C_2 \mu_2 \end{cases} \Rightarrow$

$-N = -\mu N + \mu$

$\Rightarrow \mu = \frac{N}{N-1} \Rightarrow y_k = C\left(-N \cdot \left(\frac{N}{N-1}\right)^k + k\left(\frac{N}{N-1}\right)^k\right)$

1). $\begin{cases} C_1 + C_2 = C_1 \mu_1 + C_2 \mu_2 \\ C_1 \mu_1^N + C_2 \mu_2^N = 0 \end{cases} \Rightarrow \begin{cases} 1 + \frac{C_2}{C_1} = \mu_1 + \frac{C_2}{C_1}\mu_2 \\ \frac{C_2}{C_1}\mu_2^N + \mu_1^N = 0 \end{cases} \Rightarrow K = \frac{C_2}{C_1}$

$C_1(1 - \mu_1) = C_2(\mu_2 - 1)$ и $\frac{C_1}{C_2} = -\frac{\mu_2^N}{\mu_1^N} \Rightarrow \frac{\mu_2 - 1}{1 - \mu_1} = -\frac{\mu_2^N}{\mu_1^N}$

$\Rightarrow \frac{\mu_2(1 - \mu_1)}{1 - \mu_1} = \mu_2 = -\frac{\mu_2^N}{\mu_1^N} = -\mu_2^{2N} \Rightarrow \mu_2^{2N-1} = -1$

$\Rightarrow \mu_{2,m} = e^{\frac{(\pi + 2\pi m)}{2N-1} i}$, $m = 0, 1, 2, \dots, N-1$ и т.к. $\frac{C_2}{C_1}\mu_2^{2N} = -1$

$\Rightarrow C_1 = -C_2$

$\Rightarrow y_k^{(m)} = C_1 \sin \frac{\pi(2m-1)(N-k)}{2N-1}$

Найдем с.ч.

$$\lambda : 1 - \frac{\lambda h^2}{2} = \frac{\mu_1 + \mu_2}{2} = \cos \frac{\pi \cancel{m} (2m-1)}{N - \frac{1}{2}} \iff \lambda^{(m)} = \frac{4}{h^2} \sin^2 \frac{\pi (2m-1)}{2(2N-1)}.$$

$m = 1, \dots, N-1.$