Matrix Tools

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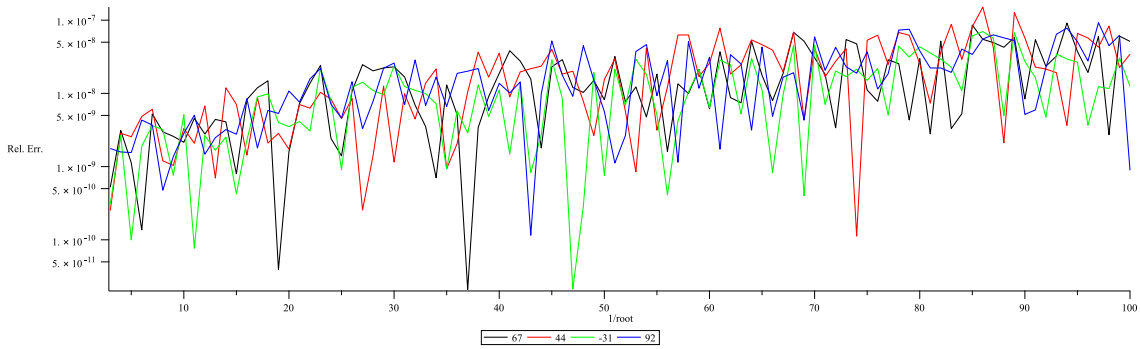
1 MatrixRoot

`MatrixRoot` does what you expect: it returns the nth root of a given matrix.

It is used as follows: Given a matrix $A \in \mathcal{M}_{n\times n}(\mathbb{C})$, with eigenvalues in $\lambda \in \mathbb{R}^+$, and a root, $r \in (0,1)$, `MatrixRoot(r,A)` returns the matrix A^r. For $r \neq \frac{1}{2}$, the result is numerically evaluated. To evaluate the errors, we consider the sample matrix,

$$M := \begin{pmatrix} 44 & -31 \\ 92 & 67 \end{pmatrix}. \tag{1}$$

The error for each term (that is, comparing each element of M with each element of `MatrixRoot(r,M)^r`) from $r = 1/3$ to $r = 1/100$ is shown in Fig. 1. The largest relative error within this range was $\sim 10^{-7}$.



The complexity to root this matrix from $r = 1/3$ to $r = 1/100$ was $\sim \mathcal{O}(\frac{1}{r}^{1.44})$, however, the trend began to diverge quickly. Odd roots tended to be longer calculations: having said that, the longest calculation was for $r = 1/78$, which took $\sim 6\,\text{s}$. On average, it took $\sim 1.4\,\text{s}$.

2 Partial Trace

`PartialTrace` does what you expect, it calculates the partial trace of a given matrix. It is used as follows: Given a matrix $A \in \mathcal{M}_{n\times n}(\mathbb{C})$ where n is even and $n \geq 2$, and a blocksize within said matrix, `PartialTrace(A,size)` returns the partially traced matrix.

For example, consider the matrix

$$M := \begin{pmatrix} -62 & 99 & 24 & 31 & 50 & -38 & -93 & 8 \\ -33 & 60 & 65 & -50 & 10 & -18 & -76 & 69 \\ -68 & -95 & 86 & -80 & -16 & 87 & -72 & 99 \\ -67 & -20 & 20 & 43 & -9 & 33 & -2 & 29 \\ 22 & -25 & -61 & 25 & -50 & -98 & -32 & 44 \\ 14 & 51 & -48 & 94 & -22 & -77 & -74 & 92 \\ 16 & 76 & 77 & 12 & 45 & 57 & -4 & -31 \\ 9 & -44 & 9 & -2 & -81 & 27 & 27 & 67 \end{pmatrix}. \tag{2}$$

The command `PartialTrace(M,2)` creates the matrix

$$\begin{pmatrix} -2 & -26 & 32 & -24 \\ -88 & 129 & 17 & -43 \\ 73 & 33 & -127 & 60 \\ -28 & 75 & 72 & 63 \end{pmatrix}. \tag{3}$$

The command `PartialTrace(M,4)` creates the matrix

$$\begin{pmatrix} 127 & -11 \\ 148 & -64 \end{pmatrix}. \tag{4}$$

Hopefully, these examples suffice.