

MATH 230A: DIFFERENTIAL GEOMETRY

Fall 2023

Instructor: Fan Ye
Email: fanye@math.harvard.edu

Time: 3:30-4:15 pm, M-W
Place: Science Center 113

Course Page: <https://scholar.harvard.edu/fanye/classes/math230a-differential-geometry-2023>

Course Assistant: Keeley Hoek, khoek@math.harvard.edu

Office Hours: Fan Ye, SC 505H, 4:20-5:20 pm, M-W.
Keeley Hoek, SC 333G, 1:30-2:30 pm Tuesday.

Main Reference: Differential Geometry: Bundles, Connections, Metrics and Curvature, Chapters 1-16, by Clifford H. Taubes.

Other suggested references:

1. An Introduction to Differentiable Manifolds and Riemannian Geometry, William M. Boothby.
2. Foundations of differentiable manifolds and Lie groups, Frank W. Warner.
3. Differential forms in algebraic geometry, Raoul Bott and Loring W. Tu.
4. Vector bundles & K-theory, Allen Hatcher.

Prerequisites: An undergraduate-level understanding of manifolds. An undergraduate may look at the first chapter of Cliff's book for preparation.

Tentative Course Outline: Notes will be posted on the course page after the classes.

- Weeks 1-2 (Sept. 6-13): smooth manifolds and Lie groups [Taubes, Chapters 1-2]
- Weeks 3-4 (Sept. 18-27): vector bundles [Taubes, Chapters 3-6]
- Weeks 5-6 (Oct. 2-11, with holiday on Oct. 9): metrics and geodesics [Taubes, Chapters 7-9]
- Weeks 7-8 (Oct. 16-25): de Rham cohomology and covariant derivative [Taubes, Chapters 12]
- Weeks 9-10 (Oct. 30-Nov. 8): Levi-Civita connections and Riemann curvature tensors [Taubes, Chapters 15, 14.1]
- Weeks 11-13 (Nov. 13-20, with holidays on Nov. 22-26): characteristic classes and principal bundles [Taubes, Chapters 14, 10-11]
- Week 14 (Nov. 27-Dec. 4): Yang-Mills equation and applications.

Grading Policy: Homework (60% = 5 × 12%) and Final exam (40%).

Homework: There will be 6 assignments, posted on the course page every two weeks, which are related to the materials in the next two weeks. The first assignment will be posted on Sept. 6. The homework should be submitted via Canvas (see Important Dates for the exact dates). Late homework is accepted only by my permission. LaTeX, scanned copies, photos of writings, and any other types of writings are accepted if they are clear and understandable. The lowest homework score will be dropped and it is allowed to only finish and submit homework for 5 times.

Final Exam: There will be a take-home final exam, posted on the course page on Dec. 6. It should be submitted via Canvas before Dec. 10.

Collaboration: Discussion and collaboration are encouraged for homework but students should write up their own solutions. It is good to write the names of collaborators on the first page of the homework. Collaboration on the final exam is prohibited.

Important Dates:

Sept. 6, Wed	Assignment 1 will be posted
Sept. 20, Wed	Assignment 2 will be posted
Oct. 4, Wed, 11:59 pm	Deadline of Assignment 1
Oct. 4, Wed	Assignment 3 will be posted
Oct. 18, Wed, 11:59 pm	Deadline of Assignment 2
Oct. 18, Wed	Assignments 4 will be posted
Nov. 1, Wed, 11:59 pm	Deadline of Assignment 3
Nov. 1, Wed	Assignment 5 will be posted
Nov. 15, Wed, 11:59 pm	Deadline of Assignments 4
Nov. 15, Wed	Assignment 6 will be posted
Nov. 22-26, Wed-Sun	Thanksgiving break
Nov. 29, Wed, 11:59 pm	Deadline of Assignment 5
Dec. 6, Wed, 11:59 pm	Deadline of Assignment 6
Dec. 6., Wed	Final exam will be posted
Dec. 10, Sun, 11:59 pm	Deadline of final exam