MATH 285Z ASSIGNMENT 5

This assignment is due at 11:59 pm on Wednesday, Apr. 17, 2024.

Problem 1: Let $M = S^1 \times D^2$ and $\gamma = S^1 \times \{4\text{pt}\} \subset \partial M$ with opposite orientations on adjacent components. Compute the sutured Floer homology SFH, SHM, SHI for (M, γ). (Hint: consider the disk $D = \text{pt} \times \partial D^2$. It intersects γ at 4 points, so the adjunction formula implies there are only two nontrivial grading summands for D. It suffices to compute the sutured Floer homology of the two sutured manifolds obtained from (M, γ) by decomposing along D and $-D$.)

Problem 2: Suppose P is a principal $SO(3)$ bundle over a closed, oriented, connected 3-manifold Y. Suppose $\omega \subset Y$ is a 1-submanifold representing the Poincaré dual $PD(w_2(P))$. Moreover, suppose there exists an embedded oriented surface $\Sigma \subset Y$ with $\omega \cdot \Sigma$ odd. Prove that any flat connection on P is irreducible, i.e. the corresponding homomorphism $\rho : \pi_1(Y) \to SO(3)$ has nonabelian image.