

## MATH 231A ASSIGNMENT 4

This assignment is due at 11:59 pm on Wednesday, Nov. 13, 2024.

**Problem 1:** Read Lectures 15 and 17 in Miller's notes. You do not need to write anything for this problem.

**Problem 2 (Exercise 15.7):** Complex projective space  $\mathbb{CP}^n$  admits a CW structure in which  $\text{Sk}_{2k} = \text{Sk}_{2k+1} = \mathbb{CP}^k$  for  $0 \leq k \leq n$ . Verify this by describing characteristic maps  $D^{2k} \rightarrow \mathbb{CP}^k$ .

**Problem 3 (Exercise 16.8):** Let  $p, q \in \mathbb{Z}$ , and let  $X$  be the 2-dimensional CW complex obtained by attaching two 2-cells to  $S^1$  using maps of degree  $p$  and  $q$ . Compute  $\pi_1(X)$  and  $H_*(X)$ .

**Problem 4 (Exercise 18.7):** Show that the Euler characteristic is a “cut-and-paste” invariant, in the following sense. Let  $X$  and  $Y$  be subcomplexes of the finite CW complex  $X \cup Y$ . Show that

$$\chi(X \cup Y) = \chi(X) + \chi(Y) - \chi(X \cap Y).$$

**Problem 5 (Exercise 19.2):** Let  $X$  be a finite CW complex. Show that for any field  $\mathbb{F}$ ,

$$\chi(X) = \sum (-1)^k \dim_{\mathbb{F}} H_k(X; \mathbb{F}).$$