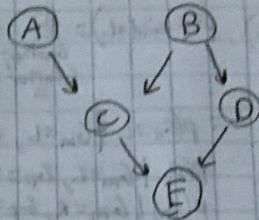


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CS 470

Hw 3 - Bayesian Networks

- 1. a) ~~False~~ false
- b) ~~True~~ true
- c) ~~False~~ False
- d) ~~False~~ false
- e) ~~False~~ False

- F) True
- g) ~~False~~ False
- h) ~~False~~ False
- I) ~~False~~ false
- J) ~~False~~ False



$$\begin{aligned}
 2. a) P(m) &= \sum_{B, a, e} P(B) P(e) P(a|B, e) P(m|a) = \sum_B P(B) \sum_e P(e) \sum_a P(a|B, e) P(m|a) \\
 &= P(B) P(e) P(a|B, e) P(m|a) + P(\neg B) P(e) P(a|\neg B, e) P(m|a) + P(B) P(\neg e) P(a|B, \neg e) P(m|a) + \\
 &\quad P(\neg B) P(\neg e) P(a|\neg B, \neg e) P(m|a) + P(B) P(e) P(a|B, e) P(m|\neg a) + P(\neg B) P(e) P(a|\neg B, e) P(m|\neg a) \\
 &\quad + P(\neg B) P(\neg e) P(a|\neg B, \neg e) P(m|\neg a) + P(B) P(\neg e) P(a|B, \neg e) P(m|\neg a)
 \end{aligned}$$

$$\begin{aligned}
 b) J = \text{True} \quad P(m|J) &= \frac{P(m, J)}{P(J)} \quad \frac{P(m, J)}{P(J)} = \frac{P(m, J, a)}{P(J)} + \frac{P(m, J, \neg a)}{P(J)} \\
 \frac{P(m|J, a) P(a|J) P(J)}{P(J)} + \frac{P(m|J, \neg a) P(\neg a|J) P(J)}{P(J)} &= P(m|J, a) P(a|J) + P(m|J, \neg a) P(\neg a|J) \\
 P(m|J, a) \rightarrow P(m|a) &= \cancel{\frac{P(m|J, a) P(J|a) P(a)}{P(J)} + \frac{P(m|J, \neg a) P(J|\neg a) P(\neg a)}{P(J)}}
 \end{aligned}$$

3. a) ~~(C)~~

b) ~~(A)~~ ~~(B)~~, a is most correct But ~~this~~ can follow some paths as a,

~~(A)~~, because while b can be the same, there are additional edges that don't follow the hypothesis.

d) $G_{\text{child}} = \frac{P(\text{child}) P(\text{parent})}{R}$

L	L	1-m	m
L	R	0.5	0.5
R	L	0.5	0.5
R	R	m	m 1-m

$$\begin{aligned}
 e) p(G_{child} = L) &= \sum_{\substack{\text{Gmother} \\ \text{Gfather}}} p(G_{child} = L | G_{mother}, G_{father}) p(G_{mother}, G_{father}) \\
 &= p(G_c = L | G_m = L, G_f = L) p(G_{mother}) p(G_{father}) + \\
 &\quad p(G_c | G_m = L, G_f = R) p(G_{mother}) p(G_{father}) + \\
 &\quad p(G_c | G_m = R, G_f = L) p(G_{mother}) p(G_{father}) + \\
 &\quad p(G_c | G_m = R, G_f = R) p(G_{mother}) p(G_{father})
 \end{aligned}$$

$$(1-m)q^2 + 0.5q(1-q) + 0.5q(1-q) + m(1-q)^2 = P(G_{child} = L)$$

$$q^2 - mq^2 + 0.5q - 0.5q^2 + 0.5q - 0.5q^2 + m(1-q)(1-q)$$

$$q + m(1-q)(1-q) - mq^2$$

$$q + (1 - q - q + q^2)m = q - mq^2 + m - 2mq + mq^2 = \boxed{q + m - 2mq}$$

$$f) p(G_{child}) p(G_{mother} = L) p(G_{father} = L)$$

$$P(G_{child}) = P(G_{mother} = L) = P(G_{father} = L) = q + m - 2mq = q$$

$$\frac{m - 2mq}{-m} = 0$$

The hypothesis must be false because if each generation has 50% chance to be lefthanded, there should be $\frac{-2mq}{-m} = \frac{1}{2}$ more lefthanded people. 50% of the population would be lefthanded, which is not true.

$$\begin{array}{c}
 a \quad (m+1) \quad = \quad b \\
 m \quad m+1 \quad \downarrow \quad \downarrow \\
 0.5 \quad 0.5 \quad R \quad L \\
 0.5 \quad 0.5 \quad L \quad R \\
 m+1 \quad m \quad R \quad R
 \end{array}$$