CS 470 Homework
Filtering
85 Points

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1. **Bayesian Networks and Chocolate [55 points]**

   You work for *The Bayesian Chocolate Company*, where the slogan is: *Randomly good chocolate!*. Suppose you're watching chocolate being produced on a conveyor belt in the factory. There are two levers controlling production, one that controls whether the chocolate is plain or almond, and another that controls whether coconut is added or not. On each time step the levers stay in the position they're in with probability 0.7 and switch (independently of each other) to the other setting with probability 0.3. (People buy this chocolate just for this randomness!) At the start of the day (time step 1) you initialize the randomness by instructing the levers to be arranged in their positions with equal probability. That is, each lever has an equal probability of being in either of its two positions. As the chocolates are produced, you can observe the conveyor belt, which only tells you the color of the chocolate. You have the following table of probabilities for the color given what's inside.

Inside	P(Color=Light | Inside)	P(Color=Dark | Inside)
P: Plain (no coconut)	0.1	0.9
A: Almond (no coconut)	0.3	0.7
C: Coconut (plain chocolate)	0.8	0.2
B: Almond + Coconut	0.9	0.1

   Let X_i represent the state of the two levers at time step i, and let Y_i represent the observation at time step i. So, the domain of X_i is {**P**(Plain), **A**(Almond), **C**(Coconut), **B**(Almond + Chocolate)} and the domain of Y_i is {**L**(Light), **D**(Dark)}.

   (a) **[8 points]** Draw the complete Bayesian Network corresponding to the first two time steps of this dynamic stochastic process. Be sure to include all nodes (X_i, Y_i at each time step i), and all edges between nodes. Do not include the conditional probability tables in your answer.

   (b) **[4 points]** What is the $P(X_1)$, the (prior) distribution over states at time step 1 (beginning of the day)? Please give your answer as a vector of length 4 (1 for each possible state), where each entry gives the probability that the state takes on one possible value.

   (c) **[3 points]** What is the sensor model for this problem? For each possible value the state could be in, give a distribution over possible observations (**L** and **D**).

   (d) **[10 points]** Now calculate $P(X_1 \mid Y_1 = D)$. Show your work. Give your answer as a vector of length 4, one for each possible value the state could take on at time step 1. This should use your answer to the last two parts (sensor model and $P(X_1)$).

(e) **[5 points]** What is the transition model ($P(X_{t+1} \mid X_t)$) for this problem? For each possible state X_t at time t (**A,P,C** and **B**), give the probability distribution over possible states X_{t+1}

(f) **[15 points]** Using your transition model and $P(X_1 \mid Y_1 = D)$, what is $P(X_2 \mid Y_1 = D)$, the probability distribution over states at time step 2 given the observation of $Y_1 = D$ at time step 1?

(g) **[10 points]** Now, use the previous result and the sensor model again to calculate $P(X_2 \mid Y_1 = D, Y_2 = L)$. Show your work. Again report your answer as a vector representing the probability of each possible value X_2 could take on.

2. **Particle Filters [15 points]**

Consider the use of a particle filter to localize a robot. At a certain stage of the process we have the following particles. Each shows the probability returned from the sensor model for that particle and the current sensor reading.

Particle	$P(z \mid x)$
1	0.93
2	0.08
3	0.18
4	0.46
5	0.49
6	0.38
7	0.17
8	0.86
9	0.60
10	0.02

(a) **[10 points]** For each particle, given the probability that it will be sampled for inclusion in the set of particles at the next time step.

(b) **[5 points]** Assume that 5 particles are resampled from that previous set of particles, with the probabilities you determined in part (a). Which set of particles is more likely to occur? (Each particle is referenced by its number, which is the number used in part (a)).

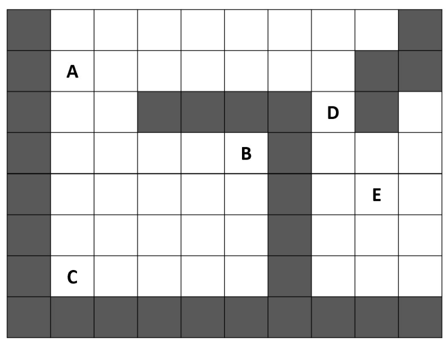
i. Set 1: { 1, 8, 8, 4, 5 }
ii. Set 2: { 2, 1, 1, 8, 8 }
iii. Set 3: { 5, 4, 9, 1, 1 }

3. **Sensor Models [15 points]** Consider a sonar sensor model for a mobile robot operating on a grid. (*Note:* This is the sensor model that is used in this module's programming assignment!)

   The sensor model is characterized by a single parameter: p_s. The robot is equipped with 4 sonar sensors, one pointed in each direction (up, down, right, left). The sonar sensor is designed to detect walls. Each sonar will return the correct reading (wall/no wall) with probability p_s, while it will return the incorrect reading with probability (1-p_s).

   A sonar reading consists of a string of 4 (`0`/`1`) characters, each specifying whether or not there was a wall detected in each direction. The order of the directions is (Up, Down, Right, Left). For example, the sensor reading `1001` means that the sonar detected a wall in the up and left directions, but not in the down and right directions.

   On the following map there are 5 locations marked. Each question will ask you to give the probability of a sensor reading, given that the robot is at one of the locations, and that the sensor model parameter p_s has some specific value. In other words, in each case you will be asked to provide $P(z \mid x)$ for that z and x. Please make sure you write out the answer as a function of p_s, in addition to giving the final numerical answer.



   (a) **[3 points]** Location: A, Sensor string: `0001`, $p_s = 0.9$
   (b) **[3 points]** Location: B, Sensor string: `1011`, $p_s = 0.75$
   (c) **[3 points]** Location: C, Sensor string: `0111`, $p_s = 0.7$
   (d) **[3 points]** Location: D, Sensor string: `0110`, $p_s = 0.6$
   (e) **[3 points]** Location: E, Sensor string: `1011`, $p_s = 0.8$