CS 470 Homework
Bayesian Networks
100 Points

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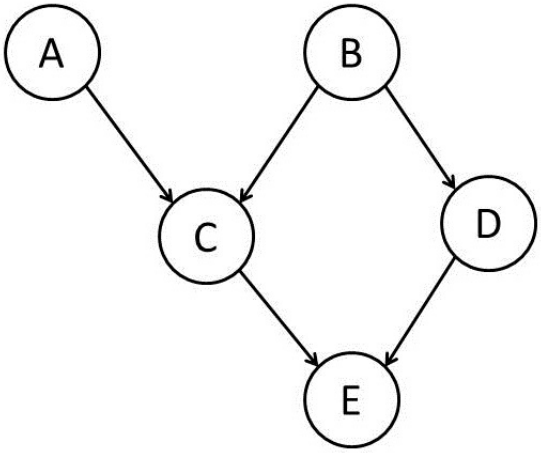
1. **Independence in Bayesian Networks [20 points]**

   Consider the following Bayesian Network with the random variables A, B, C, D, E, and F.



   For each of the following statements, determine whether it is true or false, given the Bayesian Network above. The notation $X \perp\!\!\!\perp Y$ states that X and Y are independent, while $X \perp Y \mid Z$ means that X and Y are conditionally independent, given Z.

   (a) **[2 points]** $A \perp\!\!\!\perp E$

   (b) **[2 points]** $A \perp\!\!\!\perp B$

   (c) **[2 points]** $C \perp\!\!\!\perp D$

   (d) **[2 points]** $A \perp\!\!\!\perp B \mid C$

   (e) **[2 points]** $A \perp\!\!\!\perp B \mid E$

   (f) **[2 points]** $C \perp\!\!\!\perp D \mid B$

   (g) **[2 points]** $C \perp\!\!\!\perp D \mid B, E$

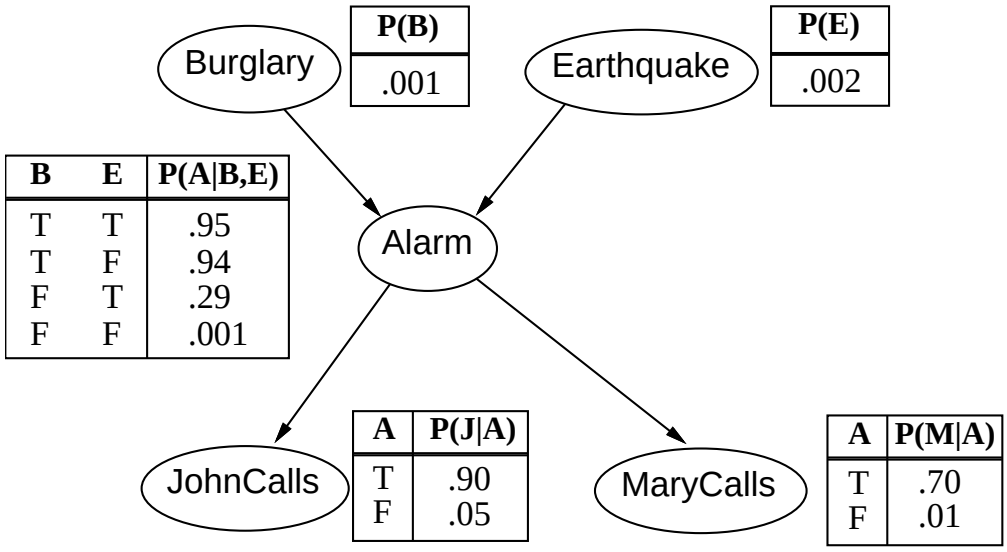
   (h) **[2 points]** $A \perp\!\!\!\perp D \mid C$

   (i) **[2 points]** $A \perp\!\!\!\perp D \mid E$

   (j) **[2 points]** $B \perp\!\!\!\perp E \mid D$

2. **Inference in Bayesian Networks [30 points]**

   You are given the following Bayesian network

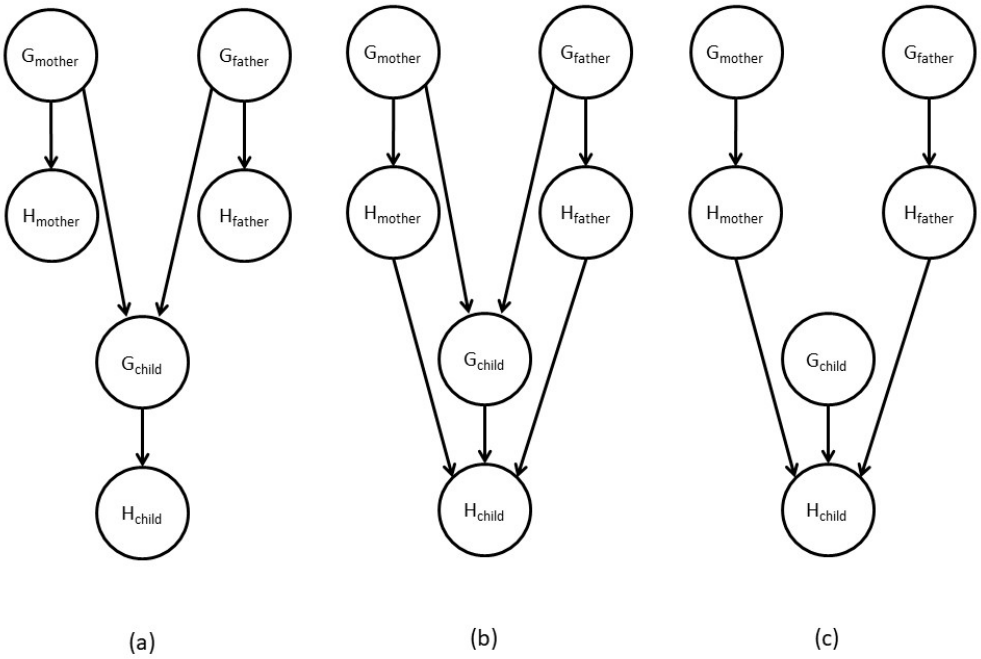


   (a) **[15 points]** Prior to making any observations, show how you would compute the (prior) probability that $MaryCalls$ is true? [We don't ask you to do the numerical computation!] We ask you to express the probability that $MaryCalls$ is true, $P(m)$, using only quantities that are given in the Bayes' Net. To simplify notations, replace $MaryCalls = true$ by m, $Earthquake = true$ by e, etc and let $P(m)$, $P(\neg m)$, $P(e)$, ... denote the probability of m, not m, e, etc ...]

   (b) **[15 points]** Now, assume that you observe that $JohnCalls$ is True. How would you compute the new (posterior) probability that $MaryCalls$ is true? [Again, we don't ask you for numerical values. Express the quantity $P(m \mid j)$ in terms of the quantities given in the Bayes' Net's CPT tables. In this explanation, you may need to use the prior probability of $JohnCalls$, $P(j)$. There is no need to reduce this term further or explain how to compute it, since its computation is very similar to that of $P(m)$ in Question 2a.]

3. **Bayes Nets Construction (14.6 in book) [50 points]**

Let H_x be a random variable denoting the handedness of an individual x, with possible values l or r. A common hypothesis is that left- or right-handedness is inherited by a simple mechanism; that is, perhaps there is a gene G_x, also with values l or r, and perhaps actual handedness turns out mostly the same (with some probability s) as the gene an individual possesses. Furthermore, perhaps the gene itself is equally likely to be inherited from either of an individual's parents, with a small nonzero probability m of random mutation flipping the handedness.



(a) **[4 points]** Which of the three networks shown above claim that

$$P(G_{father}, G_{mother}, G_{child}) = P(G_{father})P(G_{mother})P(G_{child})?$$

(b) **[8 points]** Which of the three networks make independence claims that are consistent with the hypothesis about the inheritence of handedness?

(c) **[8 points]** Which of the three networks is the best description of the hypothesis?

(d) **[10 points]** Write down the CPT for the G_{child} node in network (a), in terms of s and m.

(e) **[10 points]** Suppose that $P(G_{father} = l) = P(G_{mother} = l) = q$. In network (a), derive an expression for $P(G_{child} = l)$ in terms of m and q only, by conditioning on its parent nodes.

(f) **[10 points]** Under condition of genetic equilibrium, we expect the distribution of genes to be the same across generations. Use this to calculate the value of q, and, given what you know about handedness in humans, explain why the hypothesis described at the beginning of this question must be wrong.