CS 470 Homework
Probability
65 Points

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1. **Probabilistic Reasoning I [10 points]** After your recent annual visit to the doctor, you are given some bad news and some good news. The bad news is that your lab results came back with a positive test for a serious disease, and that the test is 99% accurate (i.e. the probability of testing positive when you do have the disease is 0.99, as is the probability of testing negative when you don't have the disease). The good news is that this is a rare disease, striking only 1 in 10,000 people of your age. What is the probability that you actually have the disease, given your positive test result? Please show your work.

2. **Probabilistic Inference with a Joint Distribution [30 points]**

	$L = t$		$L = f$	
	$C = t$	$C = f$	$C = t$	$C = f$
$X = t$	0.336	0.144	0.084	0.036
$X = f$	0.016	0.024	0.144	0.216

Table 1: Full joint distribution for the random variables X, L, and C

In Table 1 you are given the joint distribution over the random variables X (whether or not a robot is in the goal location), L (whether or not the laser sensor says that the robot is in the goal location), and C (whether or not the camera sensors says that the robot is in the goal location). Each of these random variables can take on one of two values t (true), or f (false).

(a) **[18 points]** Using the joint distribution shown, determine each of the following probabilities. Please show your work.

i. **[1 points]** $P(X = t)$
ii. **[1 points]** $P(L = t)$
iii. **[1 points]** $P(C = f)$
iv. **[1 points]** $P(X = t, L = t)$
v. **[1 points]** $P(X = f, L = t)$
vi. **[1 points]** $P(C = t, L = f)$
vii. **[1 points]** $P(X = f, L = f, C = t)$
viii. **[2 points]** $P(L = t \mid C = t)$
ix. **[2 points]** $P(L = f \mid C = t)$
x. **[2 points]** $P(C = t \mid L = t)$
xi. **[2 points]** $P(L = f \mid X = t)$
xii. **[3 points]** $P(X = t \mid C = t, L = t)$

(b) **[12 points]** Recall that for two random variables A and B, A and B are *independent* (written $A \perp\!\!\!\perp B$), if $P(A, B) = P(A)P(B)$ for all possible values A and B can take on.

For example, if A and B can be either true (t) or false (f), then $A \perp\!\!\!\perp B$ means that all the following are true

- $P(A = t, B = t) = P(A = t)P(B = t)$
- $P(A = t, B = f) = P(A = t)P(B = f)$
- $P(A = f, B = t) = P(A = f)P(B = t)$
- $P(A = f, B = f) = P(A = f)P(B = f)$

An equivalent definition of this is that A and B are independent if $P(A \mid B) = P(A)$, again for all possible values that A and B can take on. In other words, having information about the value of B tells us nothing about the distribution over A's values. A and B are *conditionally independent given* C if $P(A, B \mid C) = P(A \mid C)P(B \mid C)$. This is written as $A \perp\!\!\!\perp B \mid C$. Again, an equivalent definition of this is that $A \perp\!\!\!\perp B \mid C$ if $P(A \mid B, C) = P(A \mid C)$. Intuitively, given information about the value of C, information about B's value doesn't tell us anything more about the distribution over A's values.

For each of the following independence statements, use the joint distribution represented in Table 1 to determine if it is True or False. Please show your work.

i. **[6 points]** $L \perp\!\!\!\perp C$

ii. **[6 points]** $L \perp\!\!\!\perp C \mid X$

3. **Probabilistic Reasoning II [25 points]**

Suppose you are given a bag containing n unbiased coins. You are told that $n - 1$ of these coins are normal, with heads on one side and tails on the other, whereas one coin is a fake, with heads on both sides.

(a) **[5 points]** Suppose you reach into the bag, pick out a coin at random, flip it, and get a head. What is the (conditional) probability that the coin you chose is the fake coin? Please show your work.

(b) **[10 points]** Suppose you continue flipping the coin for a total of k times after picking it and see k heads. Now what is the conditional probability that you picked the fake coin? Please show your work.

(c) **[10 points]** Suppose you wanted to decide whether the chosen coin was fake by flipping it k times. The decision procedure returns $fake$ if all k flips come up heads; otherwise it returns *normal*. What is the (unconditional) probability that this procedure makes an error? Please show your work.