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CS 470

Hw 2 - Probability

I. $T = \text{Test is positive}$ $T^- = \text{Test neg}$
 $D = \text{Disease}$ $D^- = \text{not disease}$

$$\begin{aligned} P(T|D) &= .99 \\ P(D) &= .0001 \\ P(D^-) &= .9999 \end{aligned}$$

$$\begin{aligned} P(T|D^-) &= .99 \\ P(T) &= P(T|D) + P(T|D^-) \end{aligned}$$

$$P(D|T) = \frac{P(T|D)P(D)}{P(T)}$$

$$P(T) = P(D)P(T|D) + P(D^-)P(T|D^-)$$

$$P(T) = (.0001)(.99) + (.9999)(.01)$$

$$.000099 + .009999 = .010098$$

$$\frac{.99 \cdot .0001}{.010098} =$$

$$\boxed{.009803}$$

2. a) i. $P(X=+) = .336 + .144 + .084 + .036 = \boxed{.6}$

ii. $P(L=+) = .336 + .144 + .016 + .024 = \boxed{.52}$

iii. $P(C=f) = .144 + .024 + .036 + .216 = \boxed{.42}$

iv. $P(X=+, L=+) = .336 + .144 = \boxed{.48}$

v. $P(X=f, L=+) = .016 + .024 = \boxed{.04}$

vi. $P(C=+, L=f) = .084 + .144 = \boxed{.228}$

vii. $P(X=f, L=f, C=+) = \boxed{.144}$

viii. $P(L=+ | C=+) = \frac{P(L=+, C=+)}{P(C=+)} = \frac{P(L=+, C=+)}{P(C=+)} = \frac{.352}{.58} = \boxed{.60684}$

ix. $P(L=f | C=+) = \frac{P(L=f, C=+)}{P(C=+)} = \frac{P(L=f, C=+)}{P(C=+)} = \frac{.228}{.58} = \boxed{.3931}$

x. $P(C=+ | L=+) = \frac{P(C=+, L=+)}{P(L=+)} = \frac{.352}{.52} = \boxed{.6769}$

xi. $P(L=f | X=+) = \frac{P(L=f, X=+)}{P(X=+)} = \frac{.12}{.6} = \boxed{.2}$

xii. $P(X=+ | C=+, L=+) = \frac{P(X=+, C=+, L=+)}{P(C=+, L=+)} = \frac{.336}{.336 + .144 + .074} = \boxed{.7088}$

b) i. $L \perp\!\!\!\perp C \Rightarrow P(L, C) = P(L)P(C) \rightarrow L=+ C=+ \rightarrow .336 + .016 = .52 \cdot .58 \Rightarrow \boxed{.352 = .3016}$

$$L=+ C=- \rightarrow .144 + .024 = .168 = .52 \cdot .42 = \boxed{.2184}$$

$$L=- C=+ \rightarrow .228 = .48 \cdot .58 = \boxed{.2784}$$

$$L=- C=- \rightarrow$$

$$3.a) P(C=fake | \text{heads}) = \frac{P(\text{fake}, \text{heads})}{P(\text{heads})}$$

$$\frac{P(\text{fake}, \text{heads})}{P(\text{real})P(\text{heads}|\text{real}) + P(\text{fake})P(\text{heads}|\text{fake})}$$

$$P(\text{heads}) = P(\text{heads}|\text{real}) + P(\text{heads}|\text{fake})$$

$$P(\text{heads}) = P(\text{real})P(\text{heads}|\text{real}) + P(\text{fake})P(\text{heads}|\text{fake})$$

$$\frac{1/n}{\left(\frac{n-1}{n}\right)\frac{1}{2} + \frac{1}{n} \cdot 1} = \frac{2}{n+1}$$

$$b) P(\text{fake} | k \text{ heads}) = \frac{P(\text{fake}, k \text{ heads})}{P(k \text{ heads})}$$

$$\frac{P(\text{fake}, k \text{ heads})}{P(\text{real})P(k \text{ heads}|\text{real}) + P(\text{fake})P(k \text{ heads}|\text{fake})} = \frac{\frac{1}{n}}{\frac{1}{2^n} \left(\frac{n-1}{n}\right) + 1 \cdot \frac{1}{n}} \rightarrow \frac{2^k}{(n-1) + 2^n}$$

$$c) \cancel{P(k \text{ heads} | \text{real})} = P(k \text{ heads} | \overset{\text{real}}{\cancel{\text{real}}}) = \cancel{P(k \text{ heads} | \cancel{\text{real}})} / \cancel{P(\text{real})}$$

$$P(k \text{ heads} | \text{real}) P(\text{real})$$

$$\cancel{\text{real}} \frac{1}{2^n} \left(\frac{n-1}{n}\right) \rightarrow \frac{n-1}{2^n}$$

$$2.b.ii \quad L \perp\!\!\!\perp C | X \quad P(L | C, X) = P(L | X) \quad \checkmark L \perp\!\!\!\perp C \Rightarrow P(L, C | X) = P(L | X) \cdot P(C | X) = \frac{P(L | X)}{P(C | X)} = \frac{P(L | X)}{P(X)} = \frac{.336}{.336 + .094} = .9 = \frac{.48}{.52} = .9$$

True

for each L, C, X

$$\frac{P(L, C, X)}{P(C, X)} = \frac{P(L, X)}{P(X)}$$

$$\checkmark L \perp\!\!\!\perp C \Rightarrow P(L, C | X) = P(L | X) \cdot P(C | X) = \frac{P(L | X)}{P(C | X)} = \frac{.016}{.016 + .4} = \frac{.04}{.4} = .1$$

$$\checkmark L \perp\!\!\!\perp C \Rightarrow P(L, C | X) = P(L | X) \cdot P(C | X) = \frac{P(L | X)}{P(C | X)} = \frac{.016}{.016 + .4} = \frac{.04}{.4} = .1$$

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