CS 470 Homework
Dynamic Bayesian Networks
100 Points

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1. **Dynamic Bayes Nets [25 points]**

   Two robots, Romeo and Juliet, are moving around the Talmage Building. They are on different floors and never meet. Each robot moves using a random walk (i.e. at each time step moving from one position to a random neighbor in the grid.) In general, the robots do not know their location, and have to estimate it based on their sensors. Each has a standard noisy sonar sensor of the environment, which gives one reading per time step. In addition, they have access to an inter-robot proximity sensor (a single one for both of them), that provides a noisy measurement of their distance to each other. (e.g., if Romeo is directly below Juliet, they will probably "feel" closer that if they are on opposite sides of the building. The entire model is known to both robots; the map of the building, the sensor models of all sensors, and the random walk model used by both robots.

   (a) **[10 points]** Draw a Bayesian network that represents the first three time steps of this process. Define random variables as needed. Be sure to include all relevant edges and nodes. You do not need to fill in any Conditional Probability Distributions (CPDs) or Conditional Probability Tables (CPTs).

   (b) **[5 points]** Juliet wants to localize herself - getting the most informed probability distribution about her location. Does it help Juliet to accomplish this if Romeo transmits his sonar observations to her (i.e. is Juliet's location independent of Romeo's sonar observations)? Justify this using the Bayes net structure.

   (c) **[5 points]** Now assume that at time step 2, the robot's proximity sensor breaks. Would it be useful for Romeo to transmit his sonar readings for time steps 3+ (i.e. is Juliet's location independent of Romeo's sonar observations now)? Justify this using the Bayes net structure.

   (d) **[5 points]** Now assume that at time step 2, when the sensor breaks, Juliet happens to know exactly where she is, due to a unique landmark on her floor. Would it now be useful for Romeo to transmit his sonar readings for time steps 3+ (i.e. is Juliet's location independent of Romeo's sonar observations now)? Justify this using the Bayes net structure.

2. **Dynamic Bayesian Sleep Deprivation [75 points]**

   You are a professor with an early morning class. You like to know if students in your class are getting enough sleep (X_t). On each day t, you observe whether the students have red eyes, R_t, and whether they sleep in class, S_t.

   You know the following probabilities:

   - The prior probability of a student getting enough sleep, with no observations, $P(X_1 = t)$, is 0.7
   - The probability of a student getting enough sleep on night t is 0.8 given that the student got enough sleep the previous night, and 0.3 if not.
   - The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.
   - The probability of sleeping in class is 0.1 if the student got enough sleep, and 0.3 if not.

   (a) **[10 points]** Draw the first 2 time steps of the Dynamic Bayesian Network that best captures this situation.

   (b) **[20 points]** Provide the following for this problem. Be sure to provide the complete CPTs in each case.

      i. **[3 points]** The prior probability distribution $P(X_1)$

      ii. **[7 points]** The transition model $P(X_t \mid X_{t-1})$

      iii. **[5 points]** The sensor model for red eyes $P(R_t \mid X_t)$

      iv. **[5 points]** The sensor model for sleeping in class $P(S_t \mid X_t)$

   (c) **[45 points]** You will now run the recursive filtering process to compute $P(X_t \mid e_{1:t})$ for each of $t = 1, 2, 3$. The evidence for each day is:

      - e_1 = not red eyes, not sleeping in class
      - e_2 = red eyes, not sleeping in class
      - e_3 = red eyes, sleeping in class

      For each of the following questions, please show all of your work and steps to allow for partial credit when grading.

      i. **[15 points]** What is $P(X_1 \mid e_1)$?

      ii. **[15 points]** What is $P(X_2 \mid e_{1:2})$?

      iii. **[15 points]** What is $P(X_3 \mid e_{1:3})$?