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CS 312

- 0.1 a) $f = \Theta(g)$
b) $f = O(g)$
c) $f = \Omega(g)$
d) $f = \Theta(g)$
e) $f = \Theta(g)$
f) $f = \Theta(g)$
m) $f = O(g)$

0.2 a) if $c < 1$

as the exponent of c increases to n , the value of a number c^n approaches 0, so the limit of $g(n)$ only approaches $\frac{1}{1-c}$, which is constant, and so $g(n) = \Theta(1)$ if $c < 1^{1-c}$

b) if $c = 1$, then each c^n will be 1, so the complexity is constant, and depends on n . If $n=6$ then we get 7, which is $n+1$, so $g(n) = \Theta(n)$ if $c=1$

c) if $c > 1$

Then each c^n is going to get exponentially bigger, but the highest complexity would be c^n , which is significantly bigger than the rest of the c^n variables, so $g(n) = \Theta(c^n)$ if $c > 1$

```

def fab(n):
    if n <= 2:
        return 1
    else:
        return fab(n-1) + fab(n-2) * fab(n-3)

```

For this exponential function, we see that for each n in $\text{fab}(n)$, we get a total big O complexity of around 3^n . Each call to $\text{fab}()$ increases the total calls by 3, thus tripling the work done by each function call at each level, making the big O 3^n . We can compare the $\text{fab}()$ function to the Fibonacci function analysis on page 14 to show this concept as well.

```

def linFab(n):
    fn = f1 = f2 = f3 = 1
    if n <= 2:
        return 1
    else:
        for x in range(2, n):
            fn = f1 + f2 * f3
            f3 = f2
            f2 = f1
            f1 = fn
    return fn

```

For this linear function of the fibonacci function, we see that for each $n > 2$, we get a single addition and a single multiplication for $n - 2$ iterations. For example, if $n = 3$, we will only ever get $n - 2$ addition and multiplications, so we only will ever add and multiply once. If $n = 10$, the same analysis holds because we will have 8 additions and 8 multiplications. If $n \leq 2$, then not a single multiplication or addition will be used.