

Sam Hopkins  
Hw 3  
1.18, 1.20, 1.27

1.18) gcd(210, 588)

$$\begin{array}{l} 210 \\ \wedge \\ (2) \ 105 \\ \wedge \\ (3) \ 35 \\ \wedge \\ (7) \ 5 \end{array}$$

$$\begin{array}{l} 588 = 2 \cdot 3 \cdot 7 = \boxed{42} \\ \wedge \\ 2 \cdot 294 \\ \wedge \\ (2) \ 147 \\ \wedge \\ (3) \ 49 \\ \wedge \\ (7) \ 7 \end{array}$$

b)

X	Y	mod
588	% 210	= 168
210	% 168	= $\boxed{42}$
168	% 42	= 0

1.20) inverse of: 20 mod 79

~~79 = 20 \cdot 3 + 19~~  

$$\begin{aligned} 79 &= 20 \cdot 3 + 19 \\ 20 &= 19 \cdot 1 + 1 \\ 1 &= 20 - 19 \\ 1 &= 20 - (79 - (20 \cdot 3)) \\ 1 &= 20 - 79 + 20(3) \\ 1 &= 20(4) - 79 \\ \frac{1}{20} &= 4 \pmod{79} \end{aligned}$$

b) 3 mod 62

$$\begin{aligned} 62 &= 3 \times 20 + 2 \\ 3 &= 2 \times 1 + 1 \\ 1 &= 3 - 2 \times 1 \\ 2 &= 62 - (3 \times 20) \\ 1 &= 3 - (62 - (3 \times 20)) \\ 1 &= 3 - 62 + 3 \times 20 \\ 1 &= 3(21) - 62 \\ \frac{1}{3} &= 21 \pmod{62} \end{aligned}$$

c) 21 mod 91

$$\begin{aligned} 91 &= 21 \times 4 + 7 \\ 21 &= (7) \times 3 + 0 \end{aligned}$$

There is no inverse because gcd = 7

d) 5 mod 23

$$\begin{aligned} 23 &= 5 \times 4 + 3 \\ 5 &= 3 \times 1 + 2 \\ 3 &= 2 \times 1 + 1 \\ 1 &= 3 - 2 \times 1 \\ 2 &= 5 - 3 \times 1 \\ 1 &= 3 - (5 - 3) \\ 3 &= 23 - 5 \times 4 \\ 1 &= 3 - (5 - (23 - (5 \times 4))) \\ 1 &= 3 - (5 - 23 + (5 \times 4)) \\ 1 &= 3 - 5 + 23 - 5 \times 4 \\ 1 &= (23 - 5 \times 4) + 23 - 5 \times 5 \\ 1 &= (23 \times 2) + (-5 \times 9) \\ 1 &= 5(9) - 23 \\ \frac{1}{5} &= 9 \pmod{23} \end{aligned}$$

1.27)  $(p-1)(q-1) = 352$   $e=3$

$$\begin{aligned} 352 &= 3 \times 117 + 1 \\ 1 &= 352 - 3 \times 117 \\ 1 &= -3 \times 117 \quad -117 \rightarrow 235 \\ \frac{1}{3} &= 235 \pmod{352} \\ \text{So } d &= 235 \pmod{352} \end{aligned}$$

$$E(m) = 41^e \pmod{N} = 41^3 \pmod{391} = 105 \pmod{391}$$