

2. Dynamic Programming

- a) Basically, the solution for $T(i) = \sum T(i-1)$. The recursive relationship would rely on the current state/solution of the iteration before, like we did with Gene alignment. We can show the structure in part B but basically in each recursive step we rely upon the current score for the solution before us. We make values for matches, indels, and substitutions and use those to find the best possible solution before recursing down again.

- B) This structure would be a 2d array of integers values. Its size would be $L \times R$ with L and R being the number of books in each stack. It might look like this for the example:

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| R | Y | G | G | B | G | Y | B | G |
| B | | | | | | | | |
| Y | | | | | | | | |
| B | | | | | | | | |
| G | | | | | | | | |
| Y | | | | | | | | |
| B | | | | | | | | |
| Y | | | | | | | | |
| G | | | | | | | | |
| Y | | | | | | | | |

filled with
optimization
numbers
for priority

Each place would contain a value to show optimization, where we get an Indel, we can show where a book should be removed

c) Create 2d array with ∞ current value
 for i in $\text{len}(L)$: # shortest on top
 for j in $\text{len}(R)$: # longest on side
 at $\text{arr}[i][j]$, look at top, left, and top-left-diagonal values
 check that $L[i]$ and $R[j]$ are equal colors
 if so, $\text{arr}[i][j] = \max(\text{diag}, \text{top}, \text{left}) + 1$
 # Only increment max height if book colors are =
 else if $\max(\text{diag}, \text{top}, \text{left}) = \text{diag}$ # remove book from both stacks
 else if $\max(\text{diag}, \text{top}, \text{left}) = \text{left}$ # remove book from L
 else if $\max(\text{diag}, \text{top}, \text{left}) = \text{top}$ # remove book from R
 return $\text{array}[L][R]$ if bottom right corner for max height

d) Both time and space are $O(mn)$ with m being length of L and n being length of R . Space complexity comes with the two $2d$ arrays, initialised to be $O(mn)$ space, changing values within does not alter the space as changes are $O(1)$. Time is also $O(mn)$ because we iterate through each element in arr , each iteration being $O(1)$, so double for loops need give us $O(mn)$ time.

e) You could create two boolean arrays of size $n(L)$ and $m(R)$. As we run the program, we maintain pointers to which positions in arr we have come from, which produces an optimal path when we start from the lower right corner (max height). We then back track, keeping track of the max height at each position i, j we visit. If the height doesn't change, we mark the position at $\text{Larray}[i] = \text{false}$ and $\text{Rarray}[j] = \text{false}$. If its height doesn't change and we came from below, $\text{Rarray}[j]$ is marked false. If we came from right index, $\text{Larray}[i] = \text{false}$. This leaves us with the arrays where false means removing that book at that position from the pile.
 and we come from a diagonal