Precept 5

October 8th, 2025

Reminders

- Project 1 should be decently far along at this point
    - **No office hours during fall break. Project due right after fall break.**
- Midterm exam after fall break
    - Precept after fall break will be a midterm review
- Problem set 2 has been posted (fundamentals of orbital mechanics)
    - We will try to grade the problem set before the exam, but definitely look at the solutions yourselves
- Be aware that you have problem set, project, and midterm all within a very short period of time

Reminders

- Please post questions on **Ed** instead of email (unless the question is sensitive in nature)
  - So everyone can read the answer / we don’t answer the same questions repeatedly

- New Canvas resource: Fundamentals of Astrodynamics.pdf

- Common Project Q’s: “This method vs. this method?”
  - Answer: Experiment, try things, compare your results
  - We provide methods but you don’t have to do it our way(s)

Agenda

- Content review
  - Conic sections
  - Orbital elements
  - Orbital elements again but faster
- Practice problems !!!
- Some parting words of wisdom

Main goals: really understand orbital elements, apply in some practice problems

Content Review: Conic Sections

Resources: Lecture notes, these NASA slides, Fundamentals of Astrodynamics textbook

Conic Sections

Eccentricity	Orbit Shape
$e = 0$	Circle
$0 < e < 1$	Ellipse
$e = 1$	Parabola
$e > 1$	Hyperbola

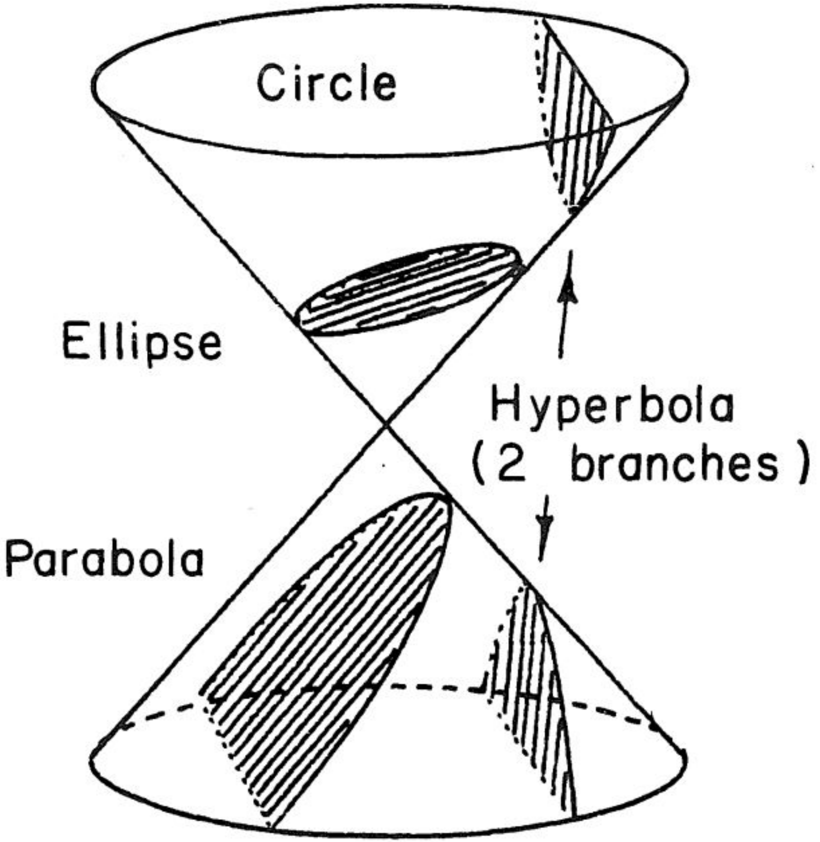
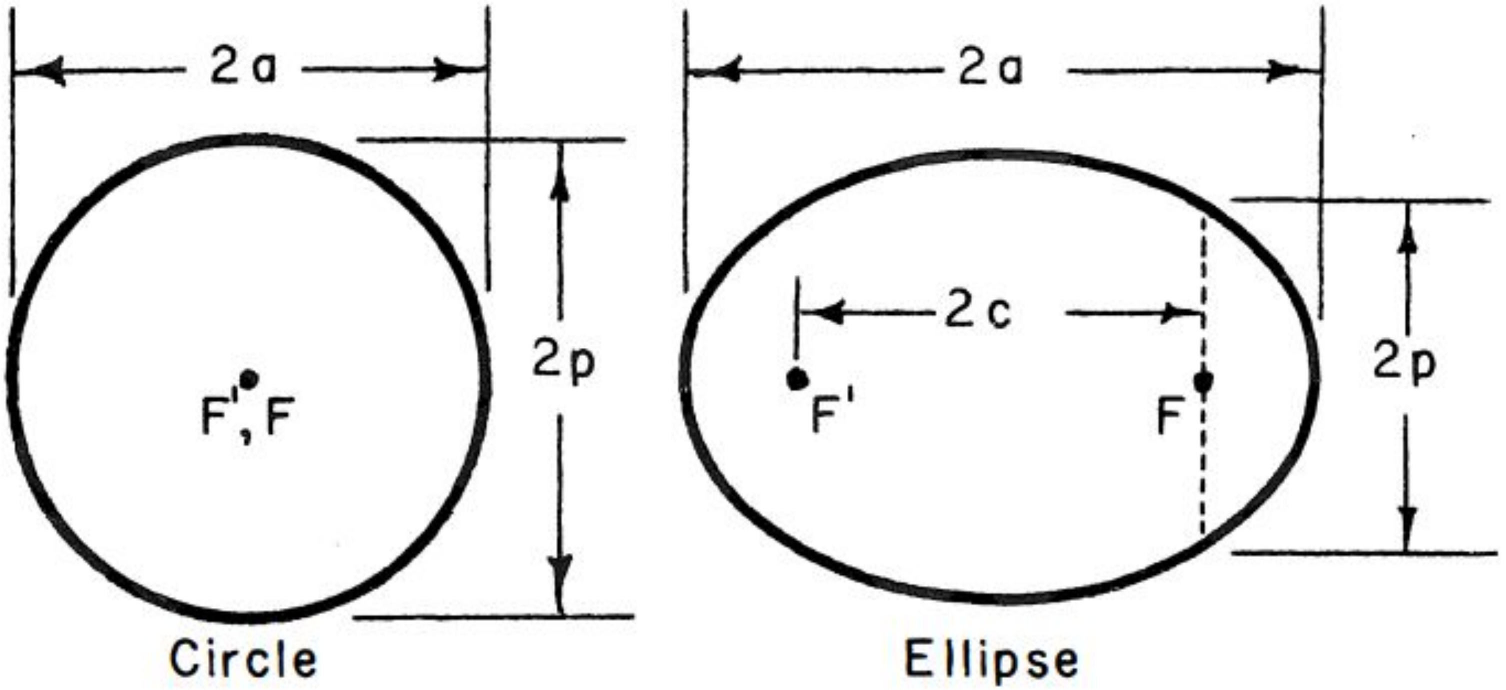
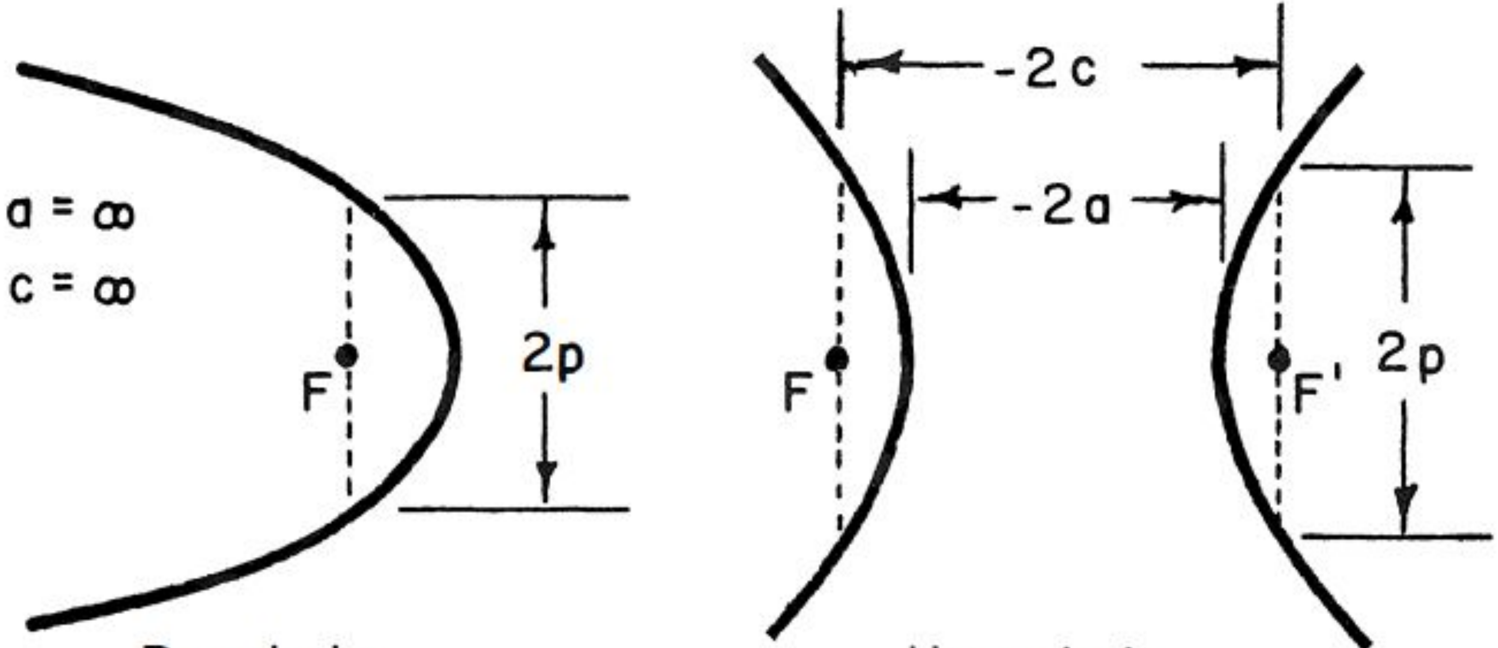


Figure 1.5-2 The conic sections

Geometrical Dimensions Common to All Conic Sections



Geometrical Dimensions Common to All Conic Sections



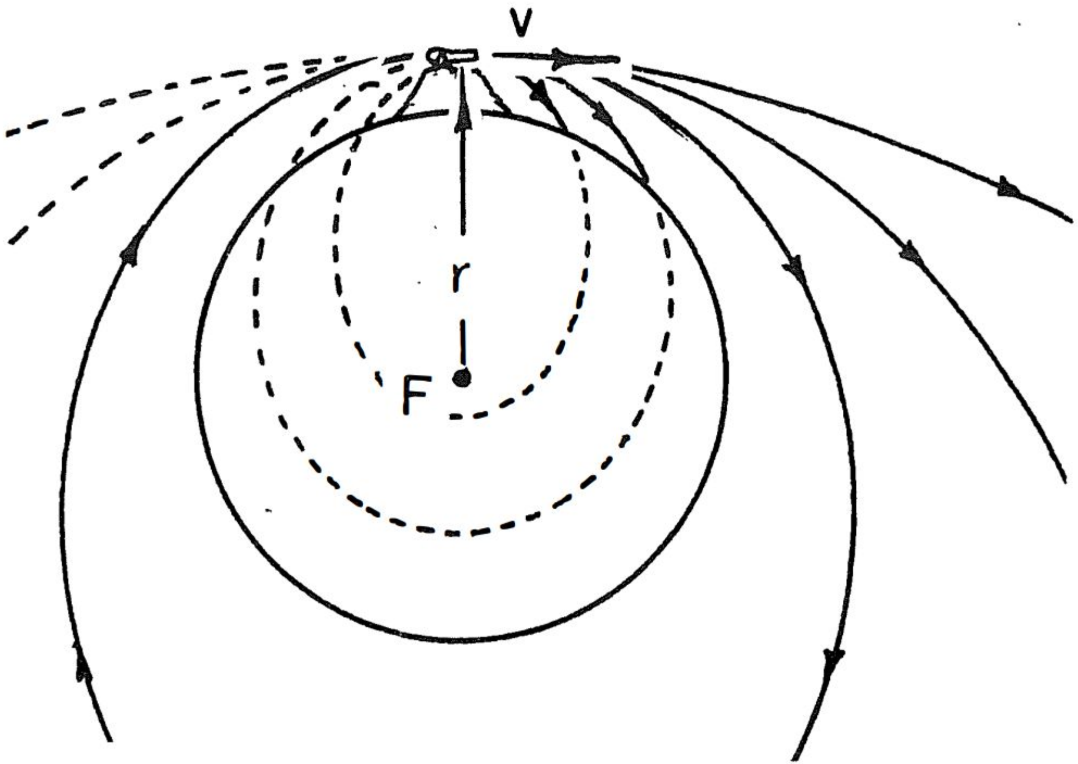
Parabola

Hyperbola

Cannonball Satellite

Purpose: See intuitively why an increase in h should result in a larger value for p

See textbook pg. 26



Content Review: Classical Orbital Elements

Resources: Lecture notes, these NASA slides, Fundamentals of Astrodynamics textbook, Delft University of Technology Section 11.5 Orbital Elements

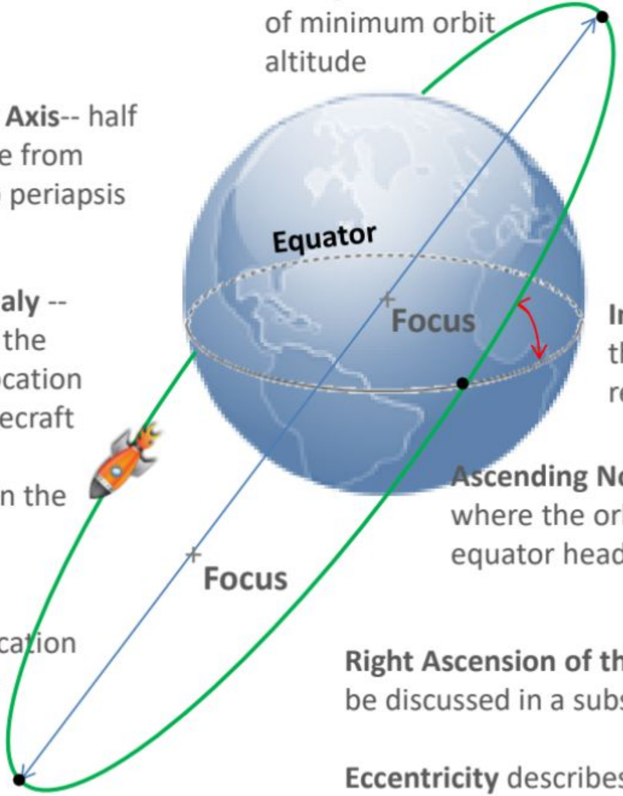
Five independent quantities called "orbital elements" are sufficient to completely describe the size, shape and orientation of an orbit. A sixth element is required to pinpoint the position of the satellite along the orbit at a particular time

Aside: Anatomy of an Orbit

Periapsis -- the location of minimum orbit altitude

a **Semimajor Axis**-- half the distance from apoapsis to periapsis

ν **True Anomaly** -- angle from the periapsis location to the spacecraft location, measured in the orbit plane



Apoapsis -- the location of maximum orbit altitude

Argument of Periapsis -- the angle, measured in the orbit plane, from the ascending node to the periapsis ω

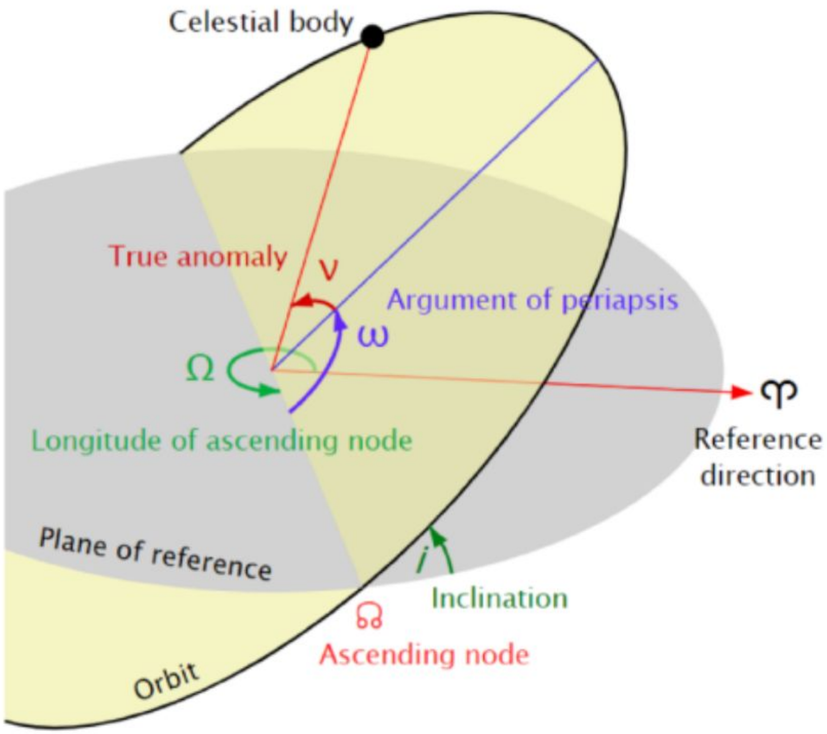
Inclination -- the tilt of the orbit plane with respect to the equator i

Ascending Node -- the location where the orbit crosses the equator headed south to north

Right Ascension of the Ascending Node will be discussed in a subsequent section Ω

Eccentricity describes the shape of the orbit and will be discussed in a subsequent section e

Draw a schematic illustrating the 6 orbital elements

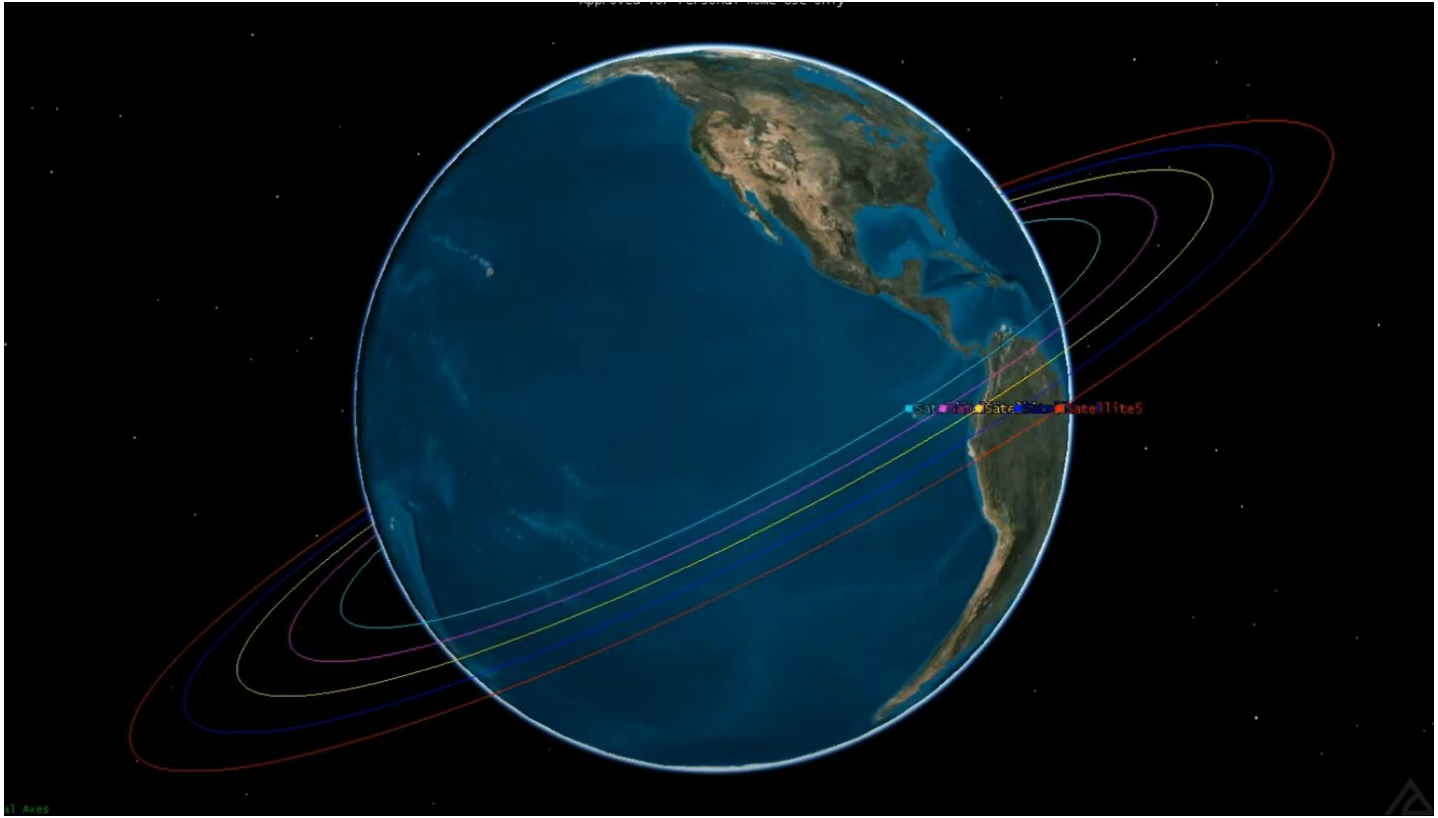


a - semi-major axis

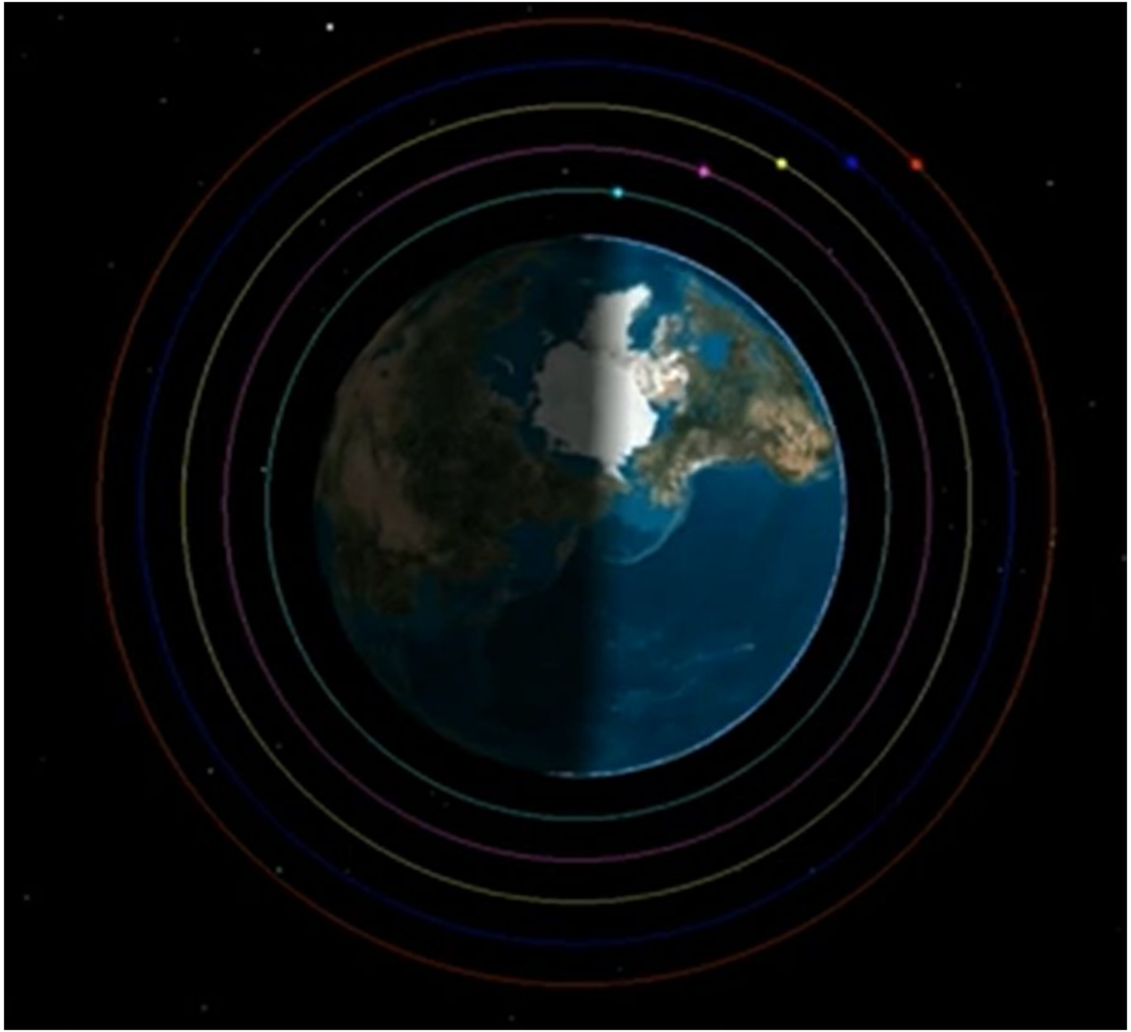
A constant defining the **size** of the conic orbit

m, km, astronomical units (AU)

a - semi-major axis



a - semi-major axis



a - semi-major axis

e - eccentricity

A constant defining the **shape** of the conic orbit

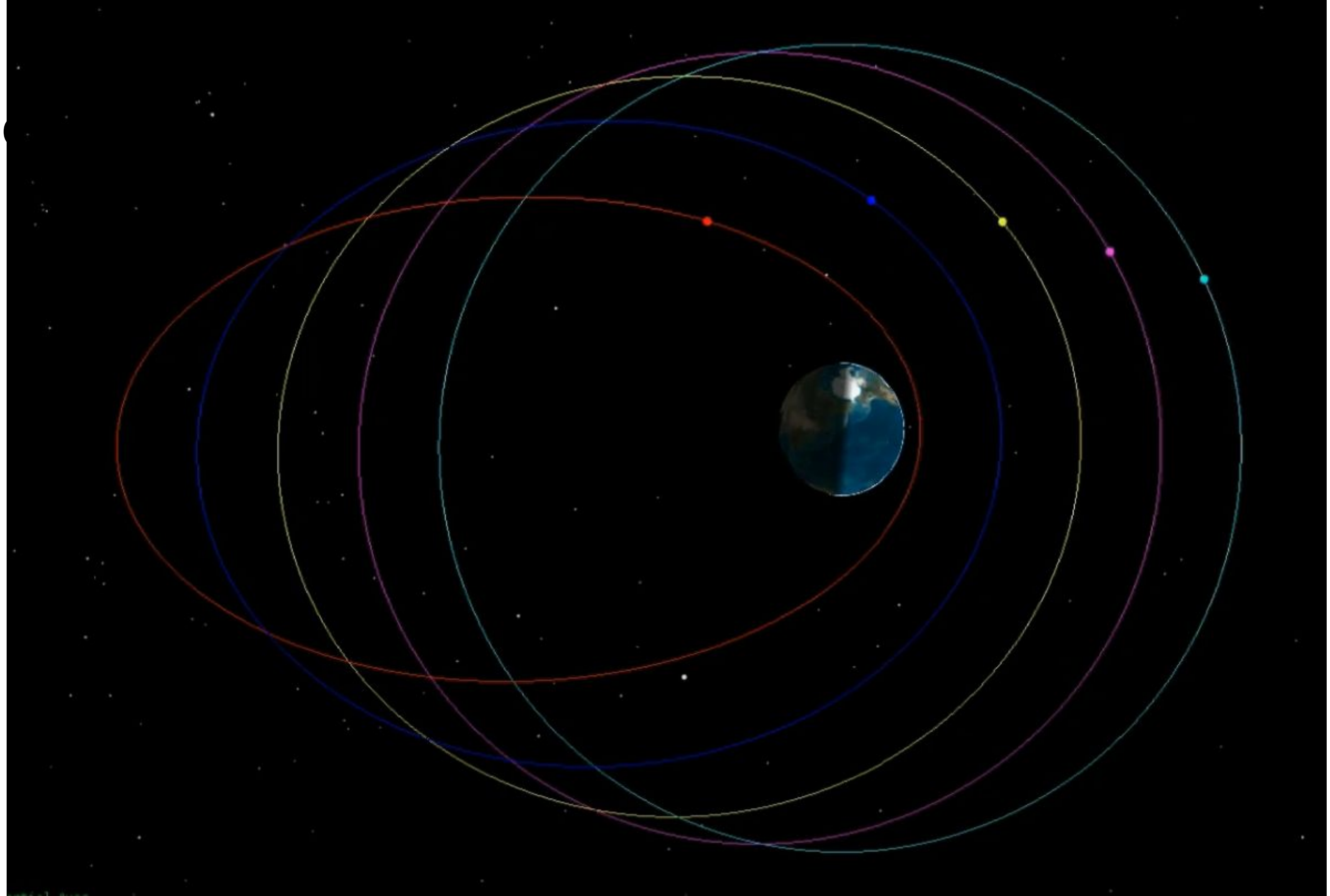
What are the units for eccentricity? Why?

$e = c / a$

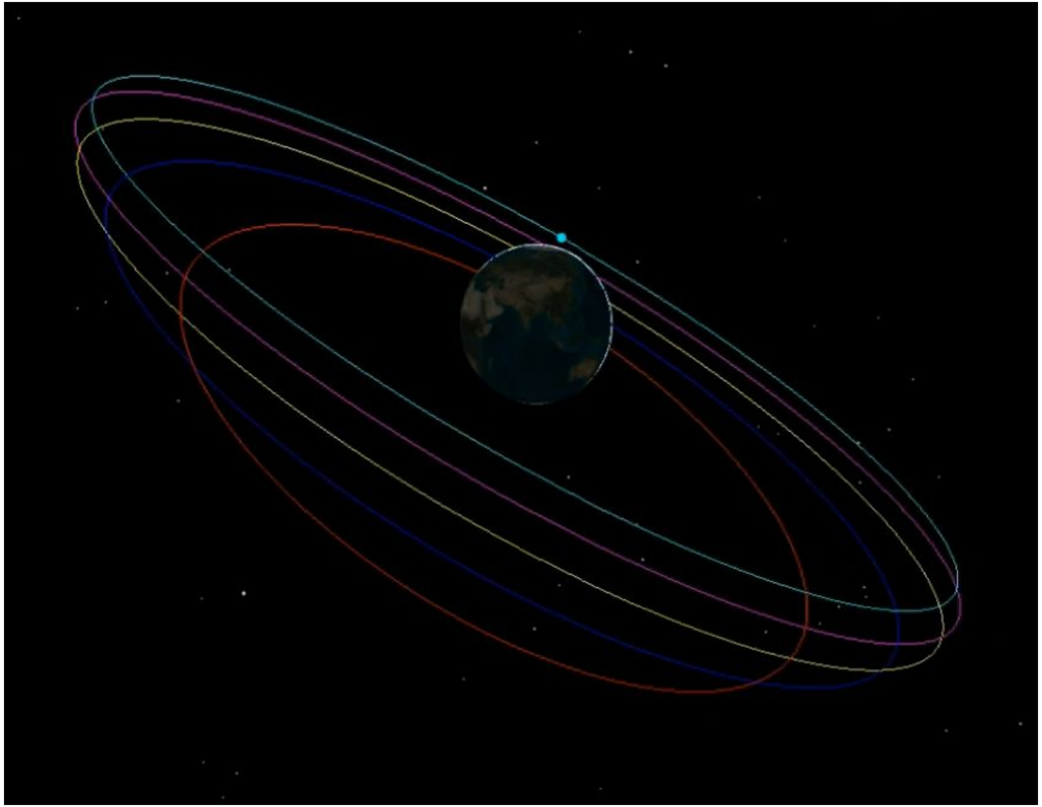
Ratio of distance between two foci and length of major axis

Unitless

e -



e - eccentricity

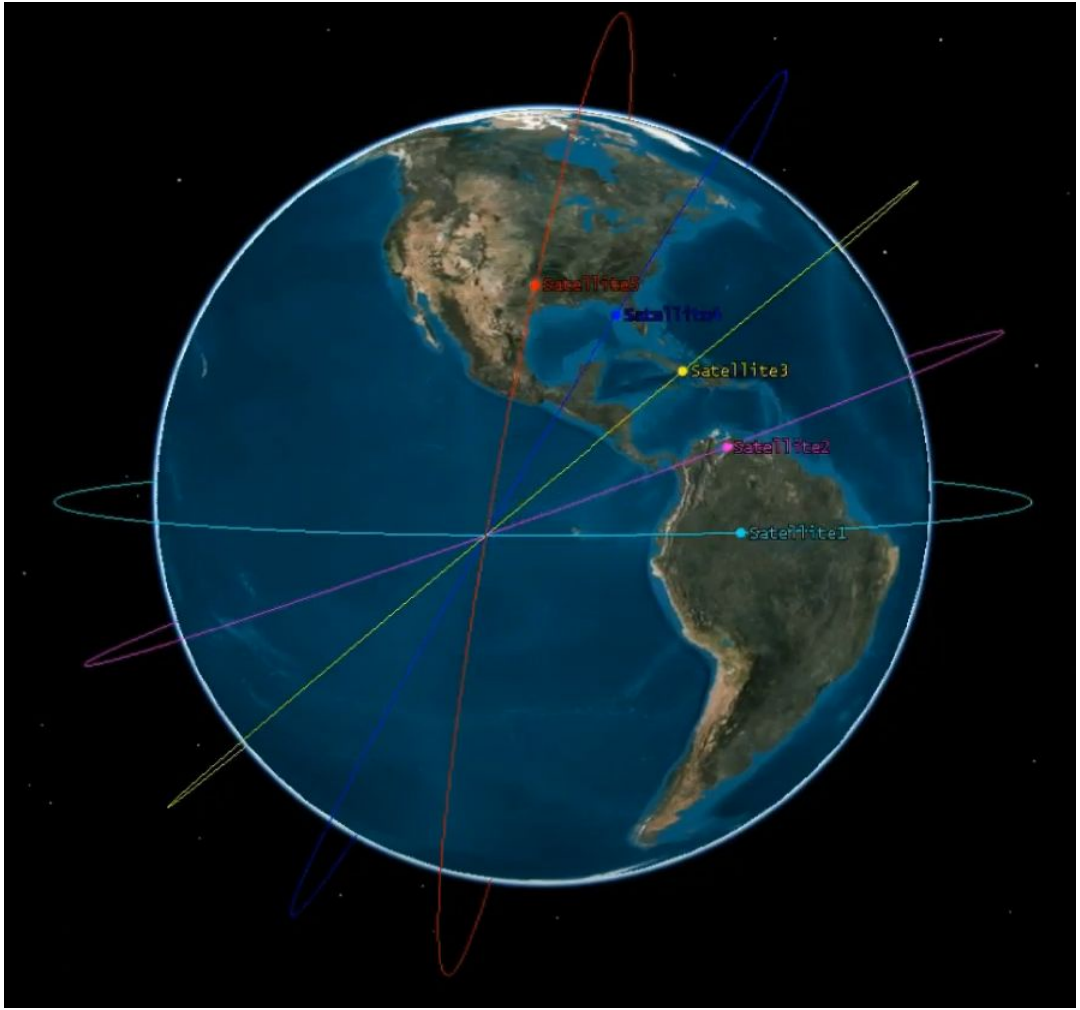


i - inclination

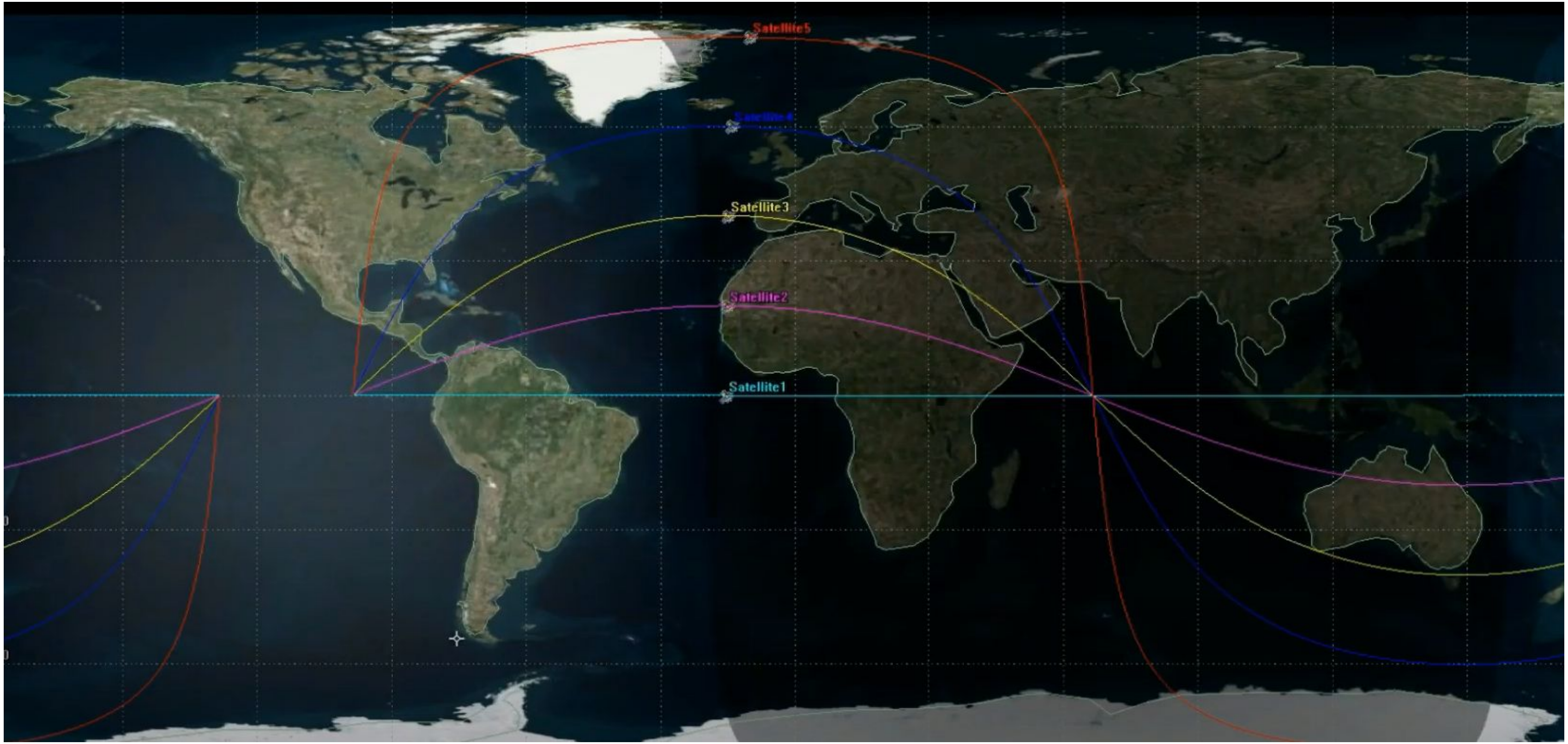
The **angle** between the **K** unit vector and the angular momentum vector **h**

Degrees, radians

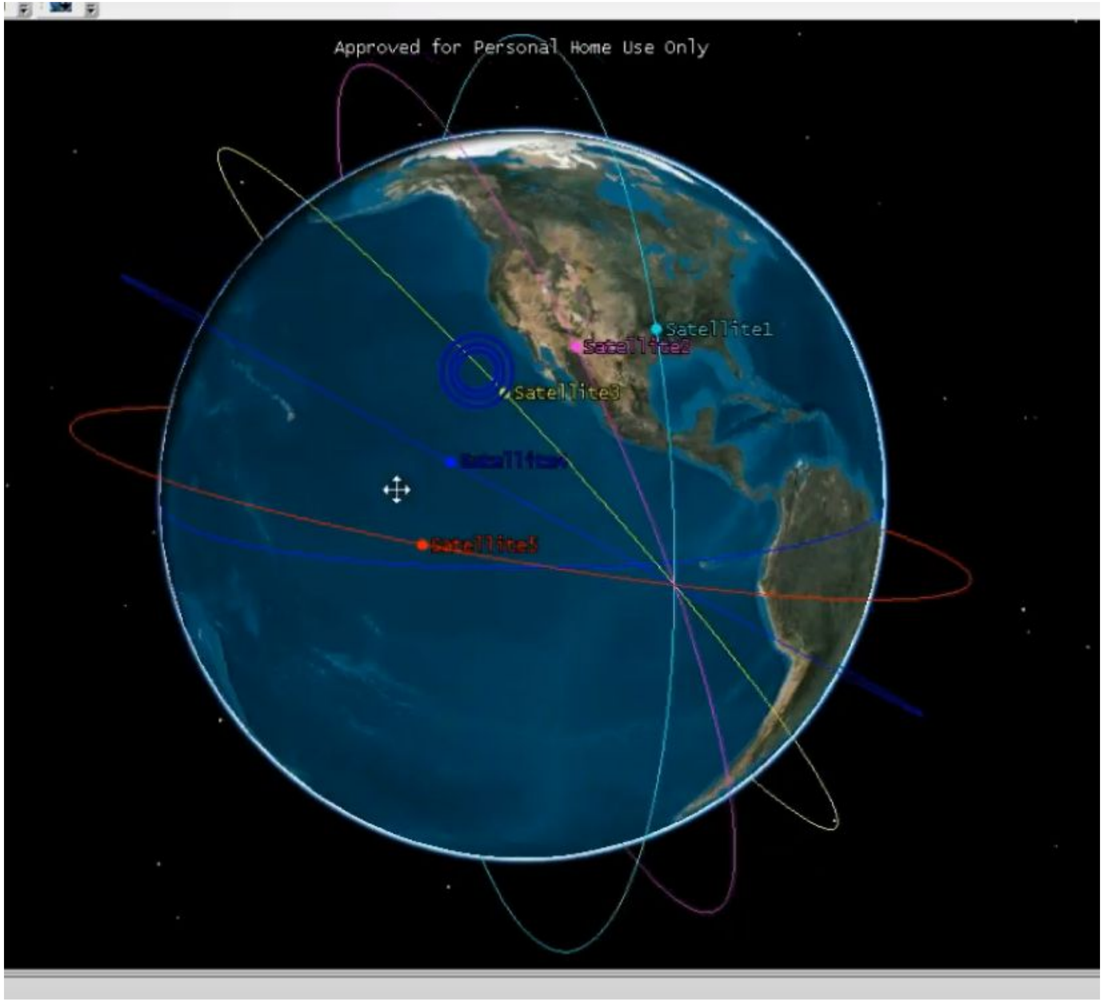
i - inclination



i - inclination



i - inclination:
Retrograde



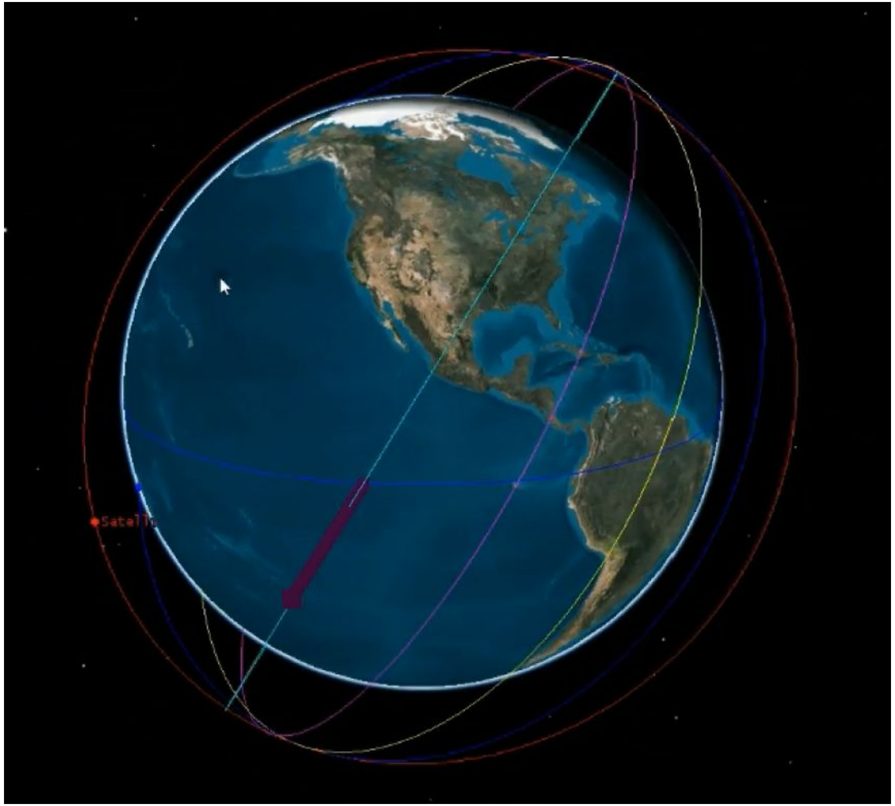
Ω - longitude of the ascending node

The angle, in the fundamental plane, between the **I** unit vector and the point where the satellite crosses through the fundamental plane in a northerly direction (ascending node) measured counterclockwise when viewed from the north side of the fundamental plane

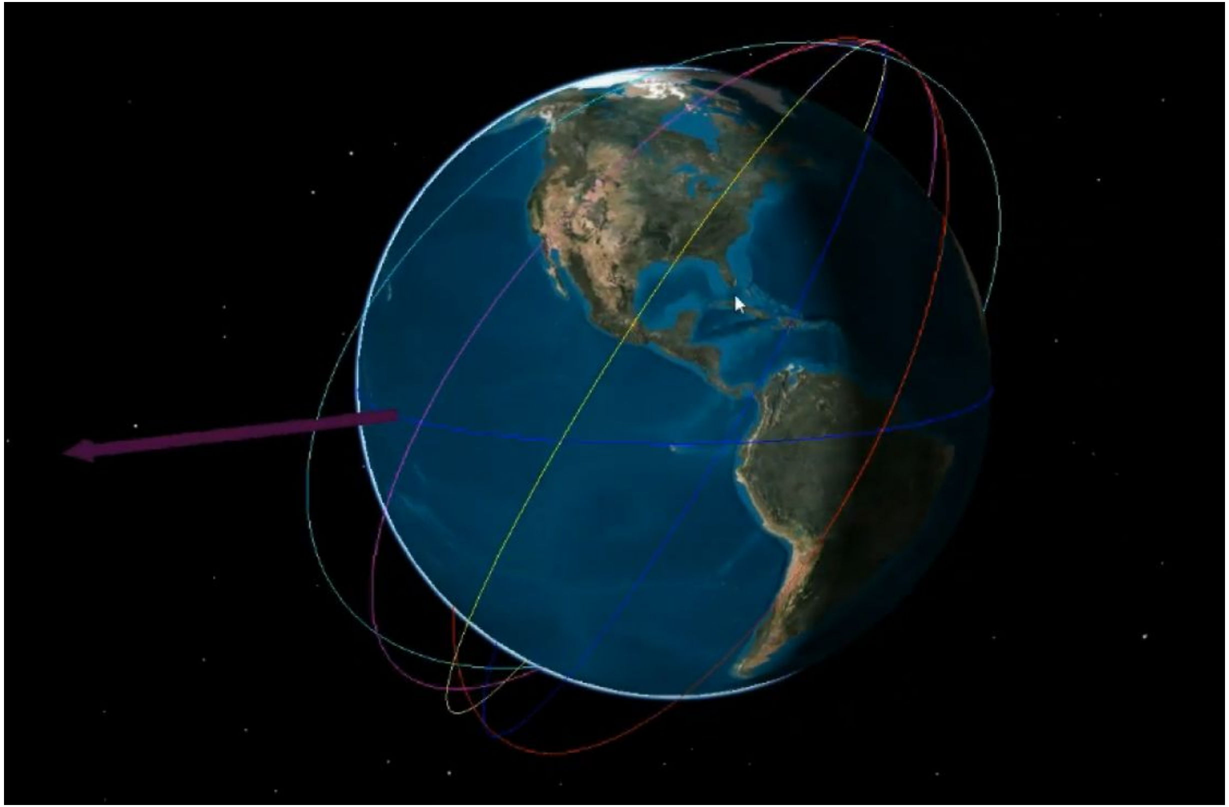
Sometimes called “right ascension of the ascending node” (RAAN) – same thing

Degrees or radians

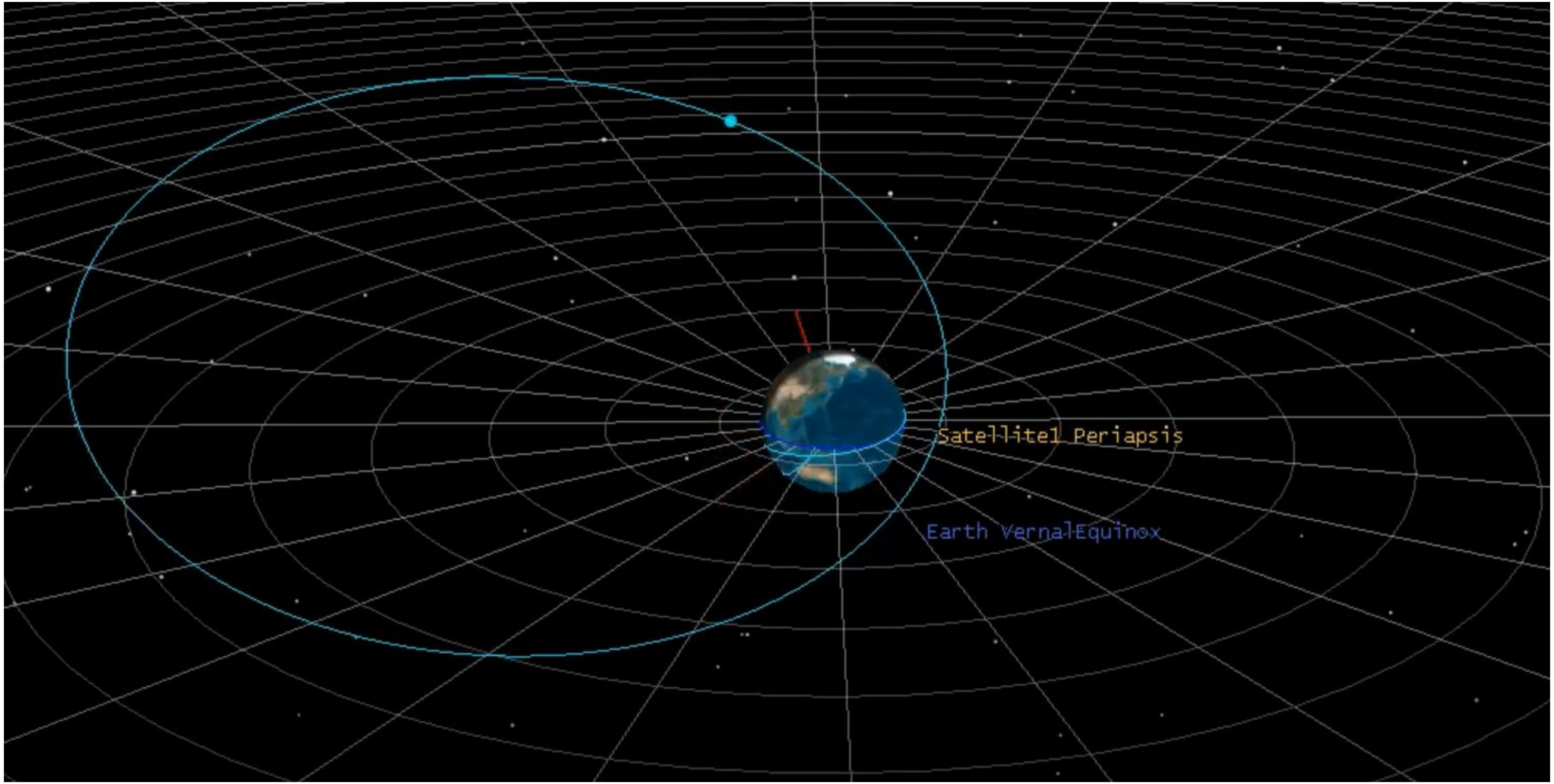
Ω - longitude of the ascending node



Ω - longitude of the ascending node



Ω - longitude of the ascending node

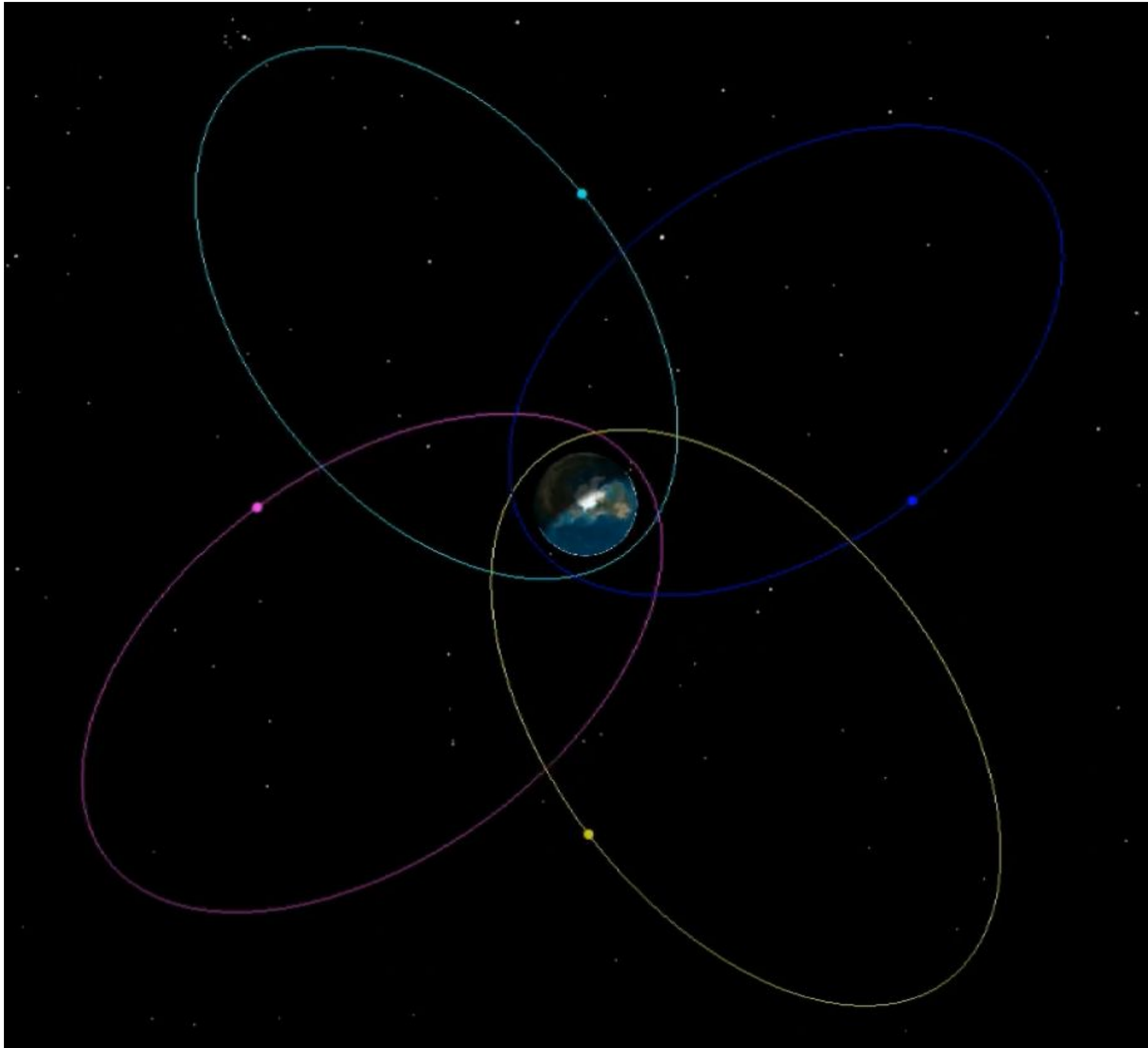


ω - argument of periapsis

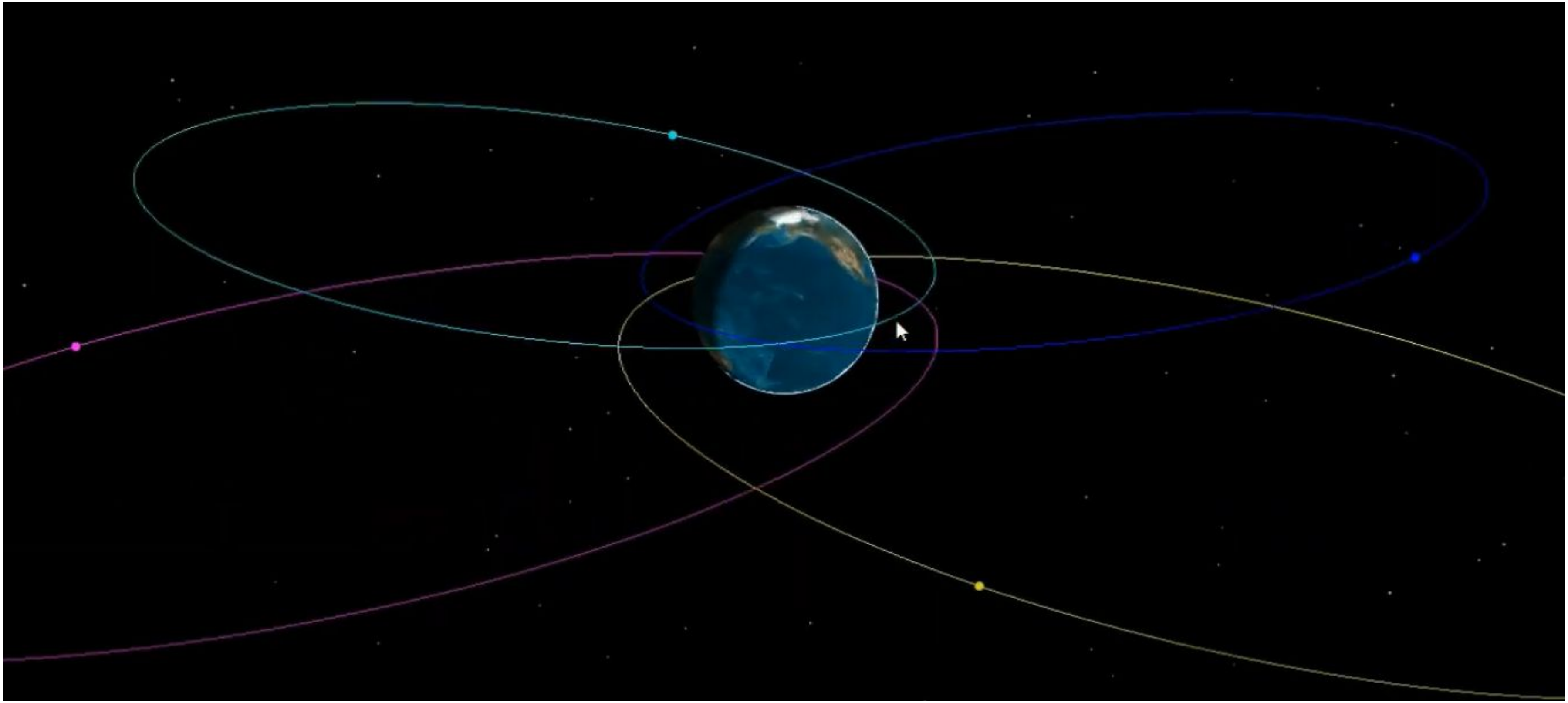
The angle, in the plane of the satellite’s orbit, between the ascending node and the periapsis point, measured in the direction of the satellite’s motion

Degrees or radians

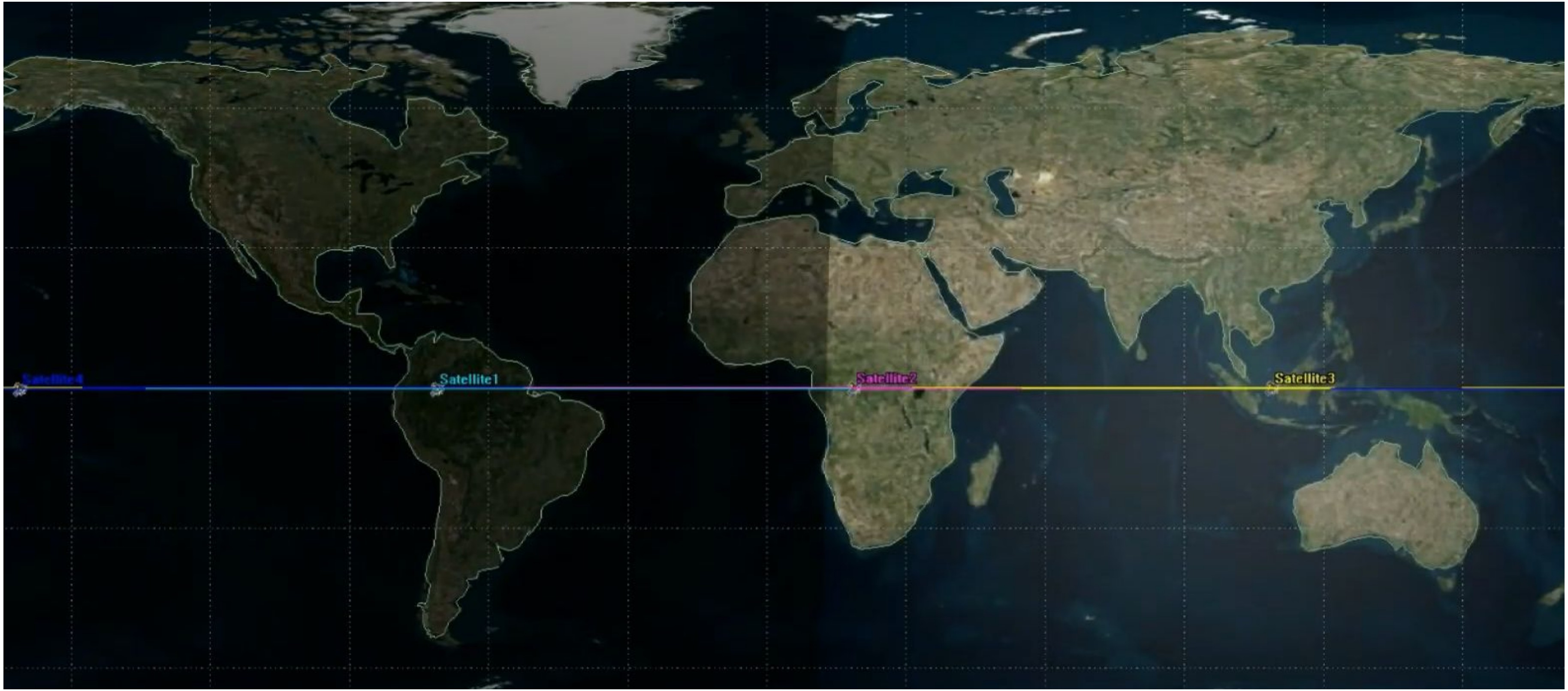
ω - argument of periapsis



ω - argument of periapsis



ω - argument of periapsis



T - time of periapsis passage

(Not considered one of our “six” for this class, but some sources consider it ⅙)

The time when the satellite was at periapsis

Note: T (time of periapsis passage), v (true anomaly at epoch), and u (argument of latitude at epoch) are all somewhat substitutable and are sufficient to locate the satellite at a particular time

v - true anomaly at epoch

The angle, in the plane of the satellite's orbit, between periapsis and the position of the satellite at a particular time t_o called the epoch

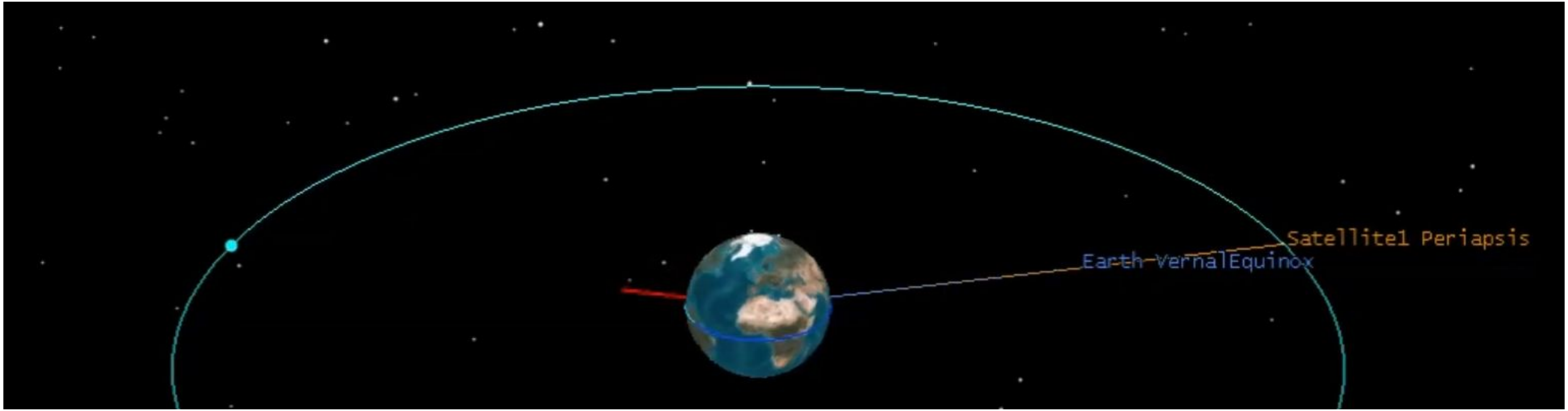
(actual geometric angle b/t direction of periapsis and the s/c's current position – tells you where s/c is in the orbit at that moment aka the epoch)

Degrees or radians

Note: T (time of periapsis passage), v (true anomaly at epoch), and u (argument of latitude at epoch) are all somewhat substitutable and are sufficient to locate the satellite at a particular time

v - true anomaly at epoch

True anomaly = what the red line is pointing at

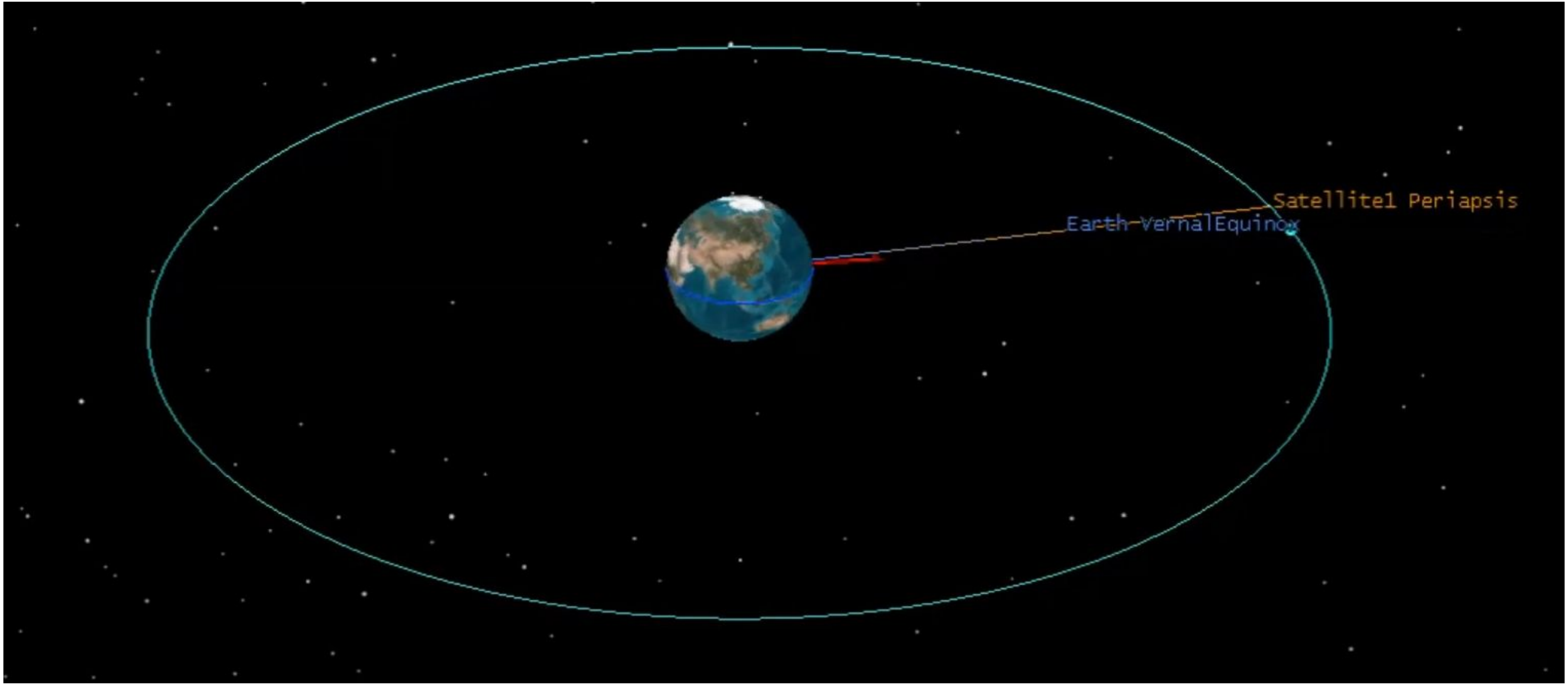


Bonus quantities

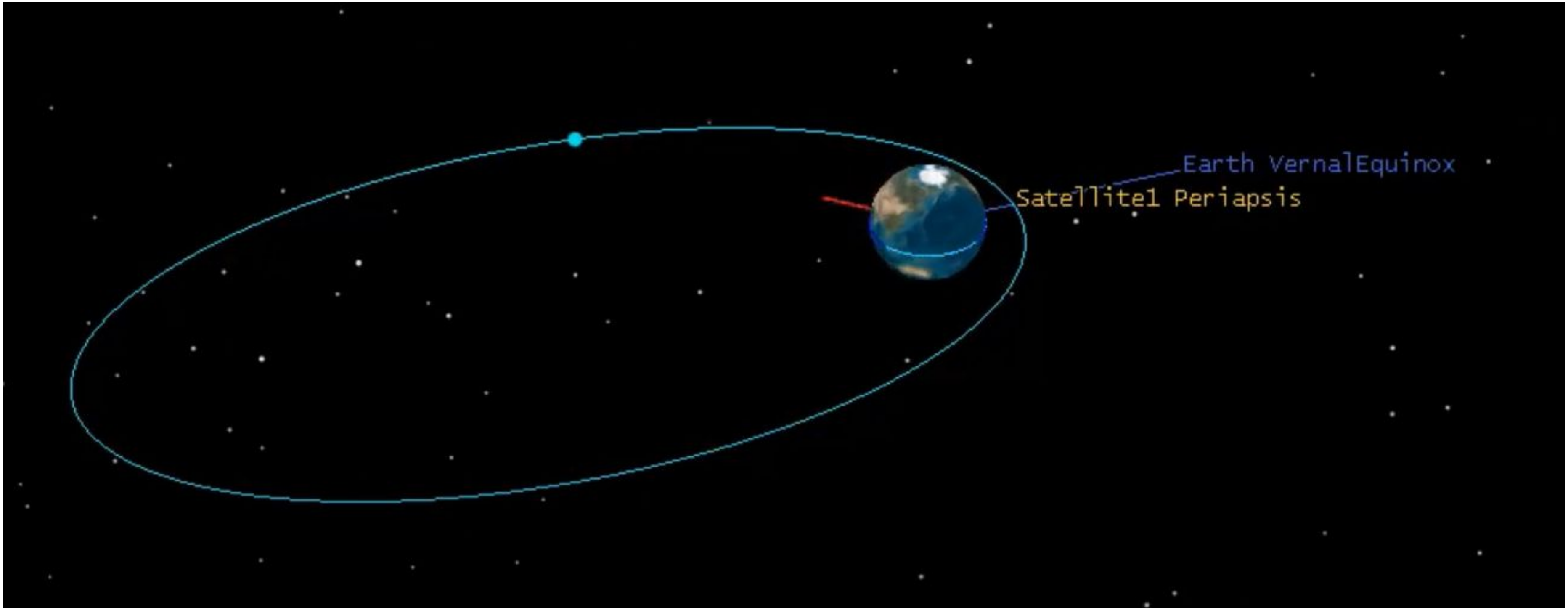
- Orbital period – time for one full revolution around primary body – [s, min, hr, days]
- Orbital radius – r – [km]

AGAIN!

a - semi-major axis

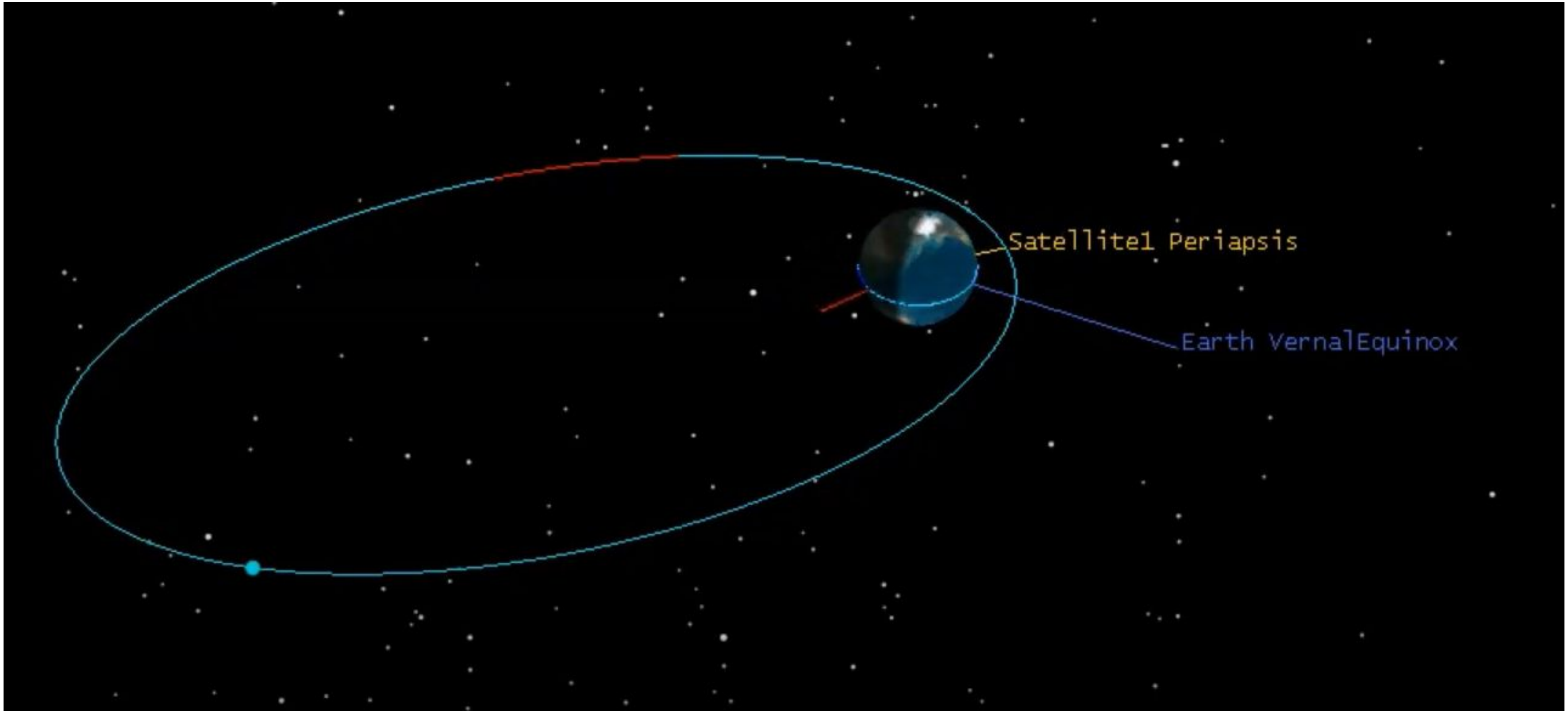


e - eccentricity

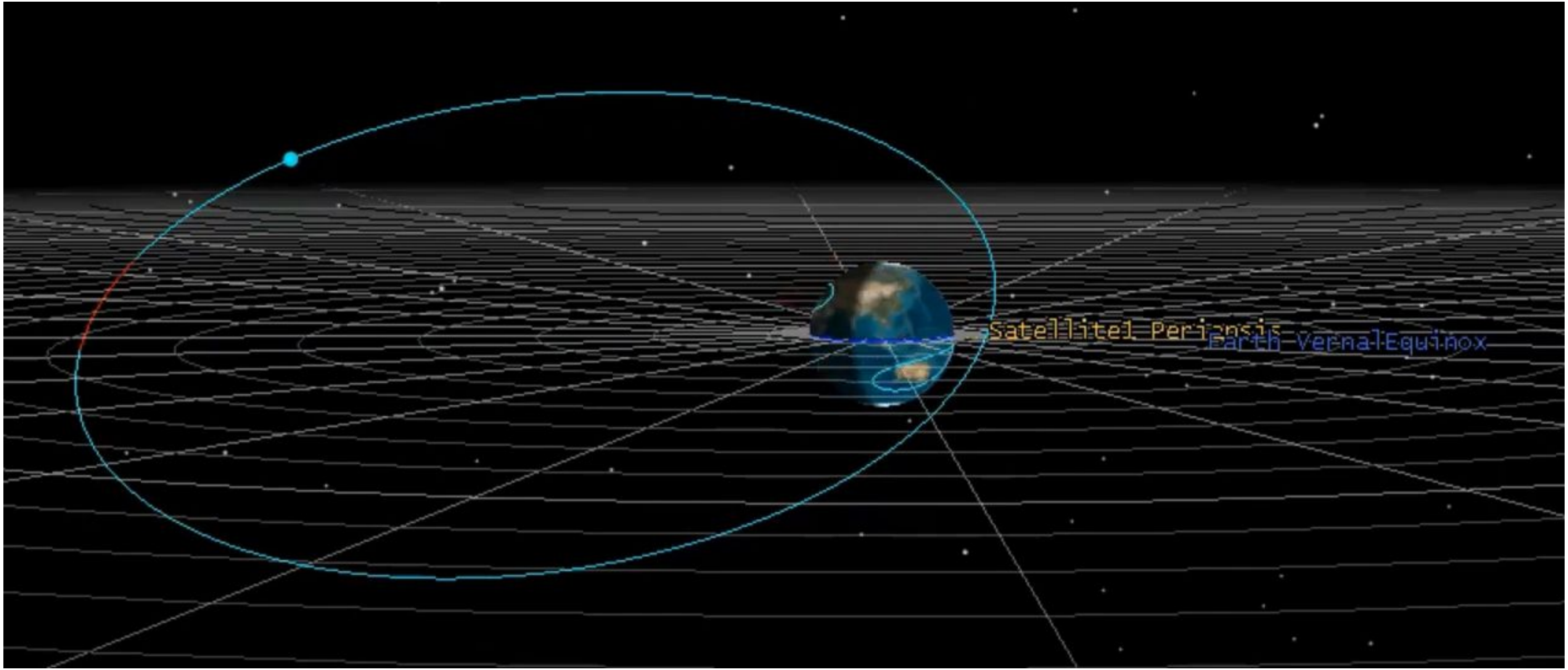


ω - argument of periapsis

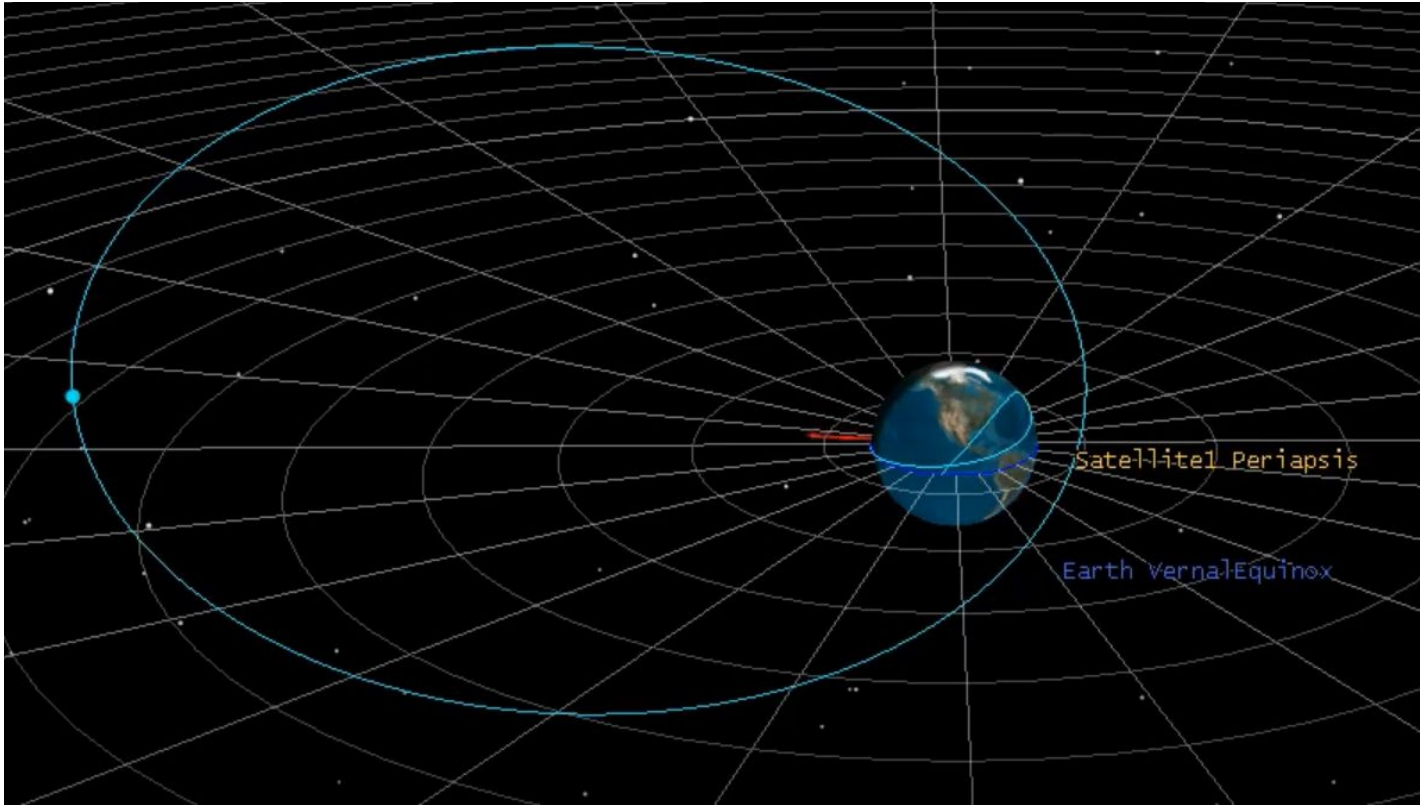
Note change in angle between satellite periapsis and earth vernal equinox



i - inclination

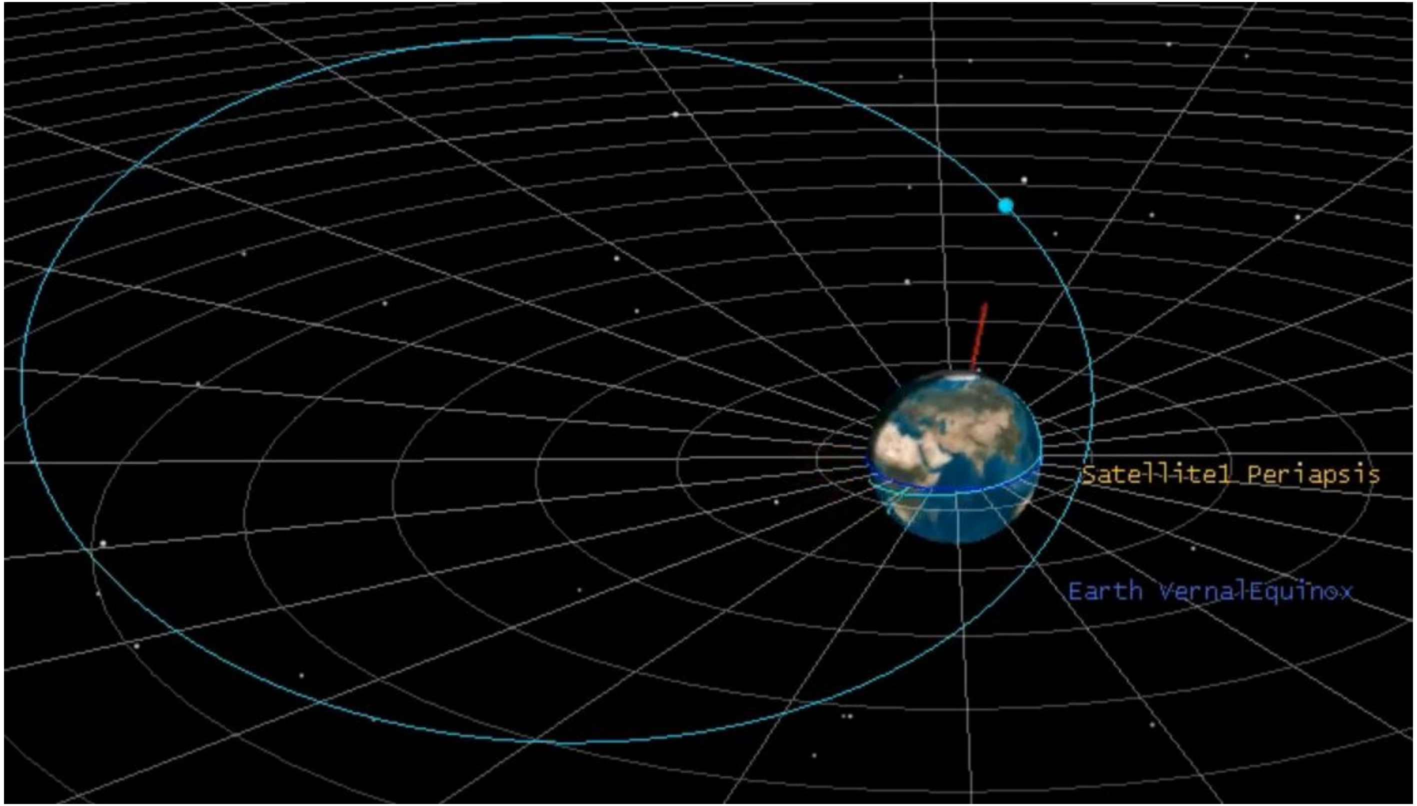


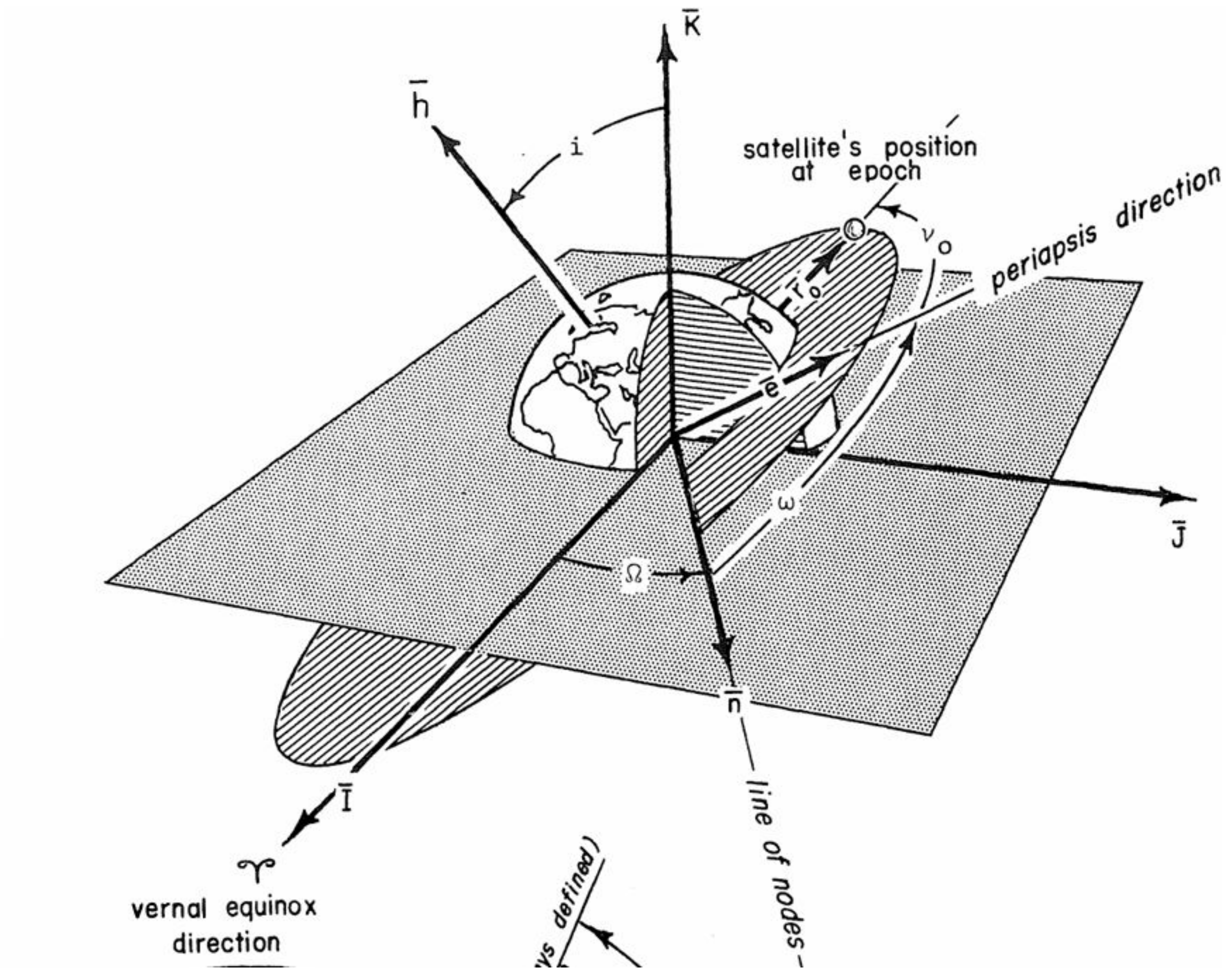
Ω - longitude of the ascending node



v - true anomaly at epoch

True anomaly = what the red line is pointing at





Textbook pg 59

Practice Problems

What are the last names of the three pioneers considered by historians of astronautics to be the “Fathers of Rocketry”?

Tsiolkovsky, Goddard, and Oberth

The unit of energy is Joule. Can we talk of a satellite having a total energy of −1KJ (yes or no)? If yes, what does it mean that the energy is negative? If no, why?

Yes

It means that the satellite is in a bound (closed) orbit (i.e. circular or elliptic) (because its kinetic energy is not enough to surmount the gravitational well k/r.)

Practice Problem

For a certain satellite the observed velocity and radius at v =90 degrees is observed to be 45,000 ft/sec and 4,000 n mi, respectively. Find the eccentricity of the orbit.

Practice Problem

Answer: $e = 1.581$

See textbook page 44

Practice Problem

An earth satellite is observed to have a height of perigee of 100 n mi and a height of apogee of 600 n mi. Find the period of the orbit.

Example Problem: Parabolic Orbit

EXAMPLE PROBLEM. A space probe is to be launched on an escape trajectory from a circular parking orbit which is at an altitude of 100 n mi (185 km) above the earth. Calculate the minimum escape speed required to escape from the parking orbit altitude. Make a sketch of the escape trajectory and the circular parking orbit.

Solution

Escape Velocity from a Circular Orbit

We begin with a **circular orbit**, which has eccentricity

$$e = 0.$$

The **orbital radius** is given by the sum of Earth's radius and the orbital altitude:

$$r = R_{\oplus} + 185 \text{ km}.$$

From the **vis-viva equation**:

$$v = \sqrt{\mu \left(\frac{2}{r} - \frac{1}{a} \right)},$$

where

- v is the orbital velocity,
- $\mu = GM$ is Earth's standard gravitational parameter,
- r is the distance from Earth's center, and
- a is the semi-major axis.

Solution

For a **parabolic (escape) trajectory**, the semi-major axis tends to infinity:

$$a \to \infty.$$

Thus, the second term vanishes, and the equation reduces to

$$v = \sqrt{\frac{2\mu}{r}}.$$

This expression gives the **escape velocity** from a circular orbit of radius r.

Solution

See textbook page 36

a. Escape Speed:

Earth gravitational parameter is

$$\mu = 1.407654 \times 10^{16}\ \text{ft}^3/\text{sec}^2$$

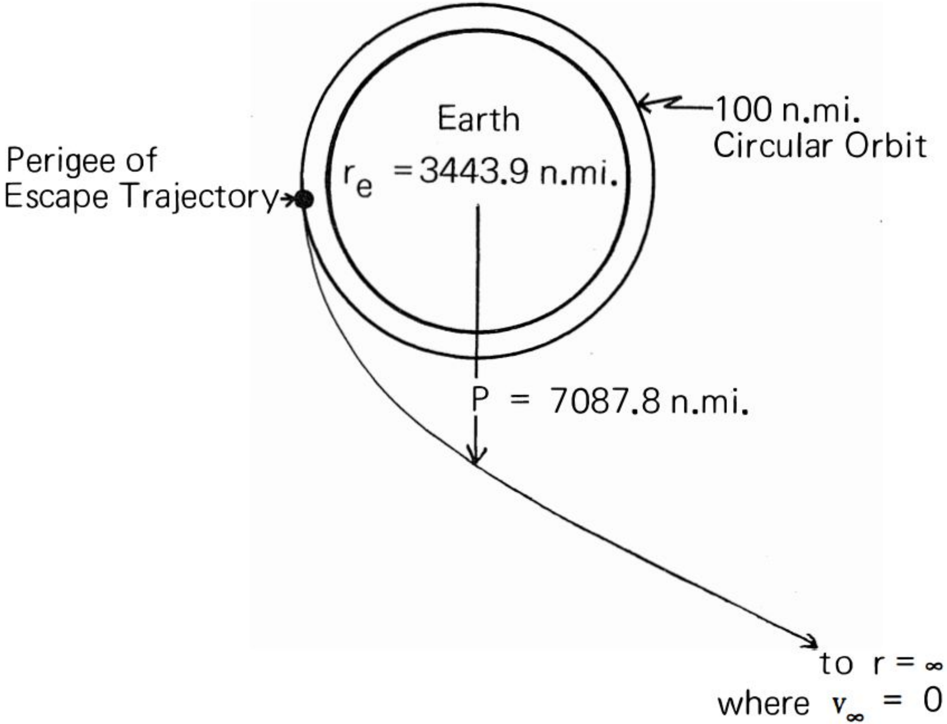
Radius of circular orbit is

$$r = r_{earth} + \text{Altitude Circular Orbit}$$

$$= 21.53374 \times 10^6\ \text{ft}$$

From equation (1.9-2)

$$v_{esc} = \sqrt{\frac{2\mu}{r}} = 36{,}157.9\ \text{ft/sec}$$

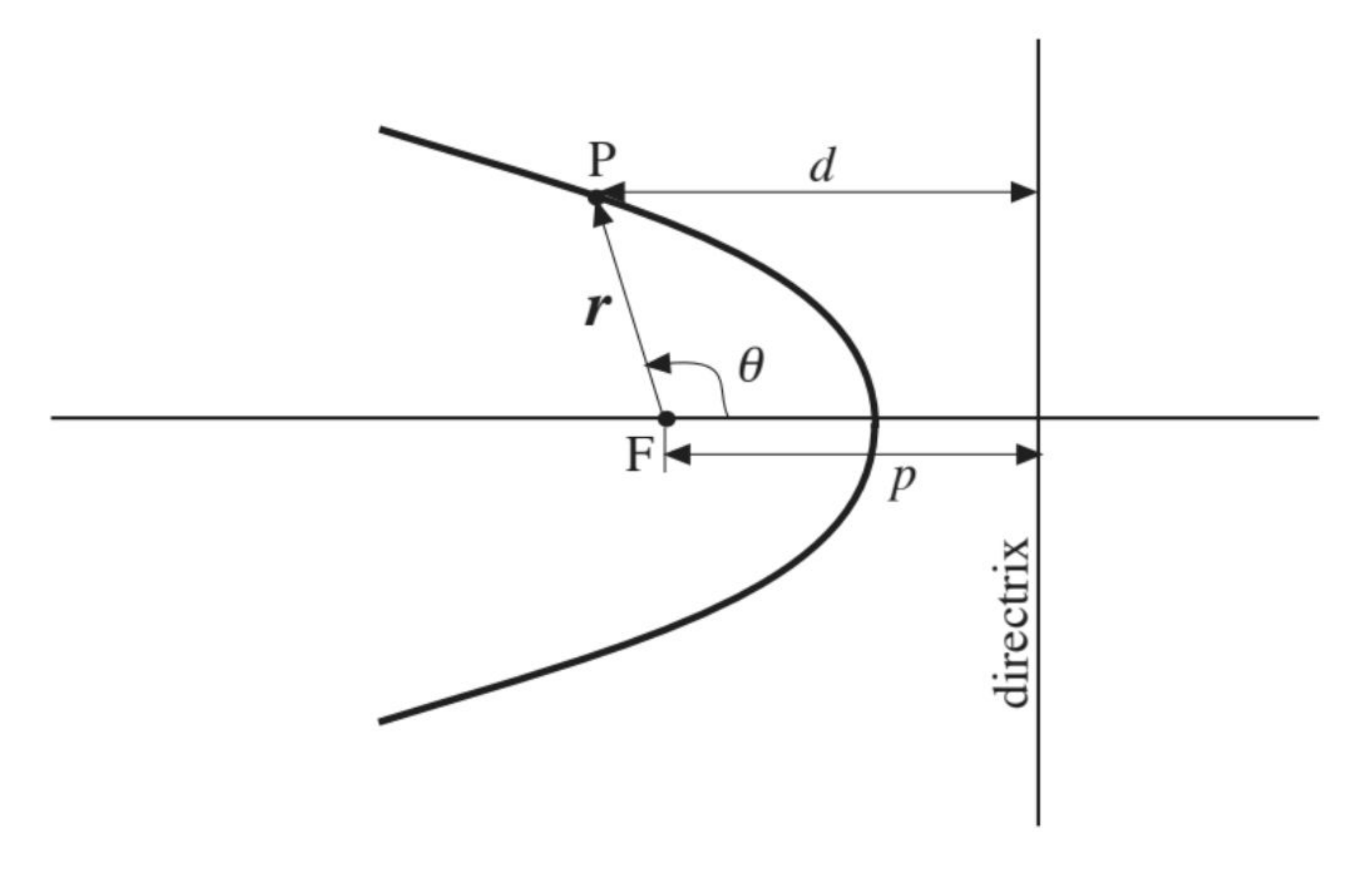


Drawing

Draw a schematic showing a generic conic section and define on the drawing the following 4 parameters used in the lecture notes and discussed in class: r, θ, p and d. Using only 2 of these 4 parameters, write down a simple expression that defines the eccentricity e.

Solution

e = r/d



Conceptual Practice Problem

State Kepler’s third law (the one about the dependence of a planet’s orbital period on its mean distance from the Sun). Prove that law starting with one of the equations you were asked to memorize. What assumption must you make to get to Kepler’s exact statement from your derivation?

Solution

(a) A planet's orbital period is proportional to its mean distance from the sun raised to the power 3/2.

$$\tau_e = 2\pi\sqrt{\frac{ma_e^3}{k}}.$$

(b) The third law can be seen by taking the ratio of the periods of two planets of masses m_1 and m_2 orbiting around the sun of mass $m_\odot$,

$$\left(\frac{\tau_1}{\tau_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3 \left(\frac{m_2 + m_\odot}{m_1 + m_\odot}\right),$$

(c) Kepler, implicitly made the approximation that both m_1, m_2 are much smaller than $m_\odot$. Under this good approximation we have,

What we (TAs) recommend you do before next precept

- Review main topics from history & space environment
- Do both problem sets & review solutions
- Be able to draw orbital elements diagram
  - Know all elements
- Intuitively understand different types of orbits
- Be able to manipulate vis-viva (derive), go between orbital elements and (r, v) frame

External materials to optionally review (other than lecture notes, problem sets, practice exam)

- YouTube video showing Animated Orbital Elements with STK
    - Don’t watch whole thing, just see what each element is like
- NASA fundamentals of orbital mechanics (nice overview)
- Textbook (on Canvas) – first few chapters
-