MAE 341 Space Flight Precept 3

Wed Sept 24, 2025

Reminders

- Look at the project
- Homework 1 (attitude dynamics) due Fri, September 26th
  - We will post solutions after the deadline
  - Strong recommendation: Complete the pset before the midterm
- Project now due after fall break
  - Be aware this means you need to ask course staff your questions **before** fall break as much as possible – no office hours then
- Midterm exam coming up
  - ~30-40% attitude dynamics
  - Rest is history, space environment, orbital mechanics
  - Practice midterm exam will be posted
- Let us know if you need a copy of the dynamics textbook

Agenda

1. Finish attitude dynamics
2. What is an orbit (build intuition)
3. What are the orbital elements
4. Project math

Finishing up attitude dynamics lectures

Kinematic Differential Equations

Gives you an expression for how a reference is changing with time given an angular velocity

$$\dot{C}_{B/N}(t) = -{}^{B}\tilde{\omega}_{B/N}(t)C_{B/N}(t)$$

How Can Different KDE's be expressed using different rotation methods?

$$\omega = \dot{\psi}\hat{n}_3 + \dot{\theta}\hat{b}_2' + \dot{\phi}\hat{b}_1.$$

$$\dot{\beta} = \frac{1}{2}\begin{bmatrix} \beta_0 & -\beta_1 & -\beta_2 & -\beta_3 \\ \beta_1 & \beta_0 & -\beta_3 & \beta_2 \\ \beta_2 & \beta_3 & \beta_0 & -\beta_1 \\ \beta_3 & -\beta_2 & \beta_1 & \beta_0 \end{bmatrix}\begin{pmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix} = \frac{1}{2}\overline{B}\begin{pmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}.$$

$$\beta_0 = \pm\frac{1}{2}\sqrt{C_{11} + C_{22} + C_{33} + 1}$$

$$\beta_1 = \frac{C_{23} - C_{32}}{4\beta_0}$$

$$\beta_2 = \frac{C_{31} - C_{13}}{4\beta_0}$$

$$\beta_3 = \frac{C_{12} - C_{21}}{4\beta_0}$$

What if you don’t have constant angular velocity?

We need to solve for the change in internal angular velocity based on the moment of inertia

$$I_T\dot{\omega}_1 = -(I_{33} - I_T)\omega_2\omega_3$$

$$I_T\dot{\omega}_2 = -(I_T - I_{33})\omega_3\omega_1$$

$$I_{33}\dot{\omega}_3 = 0.$$

Concept Questions

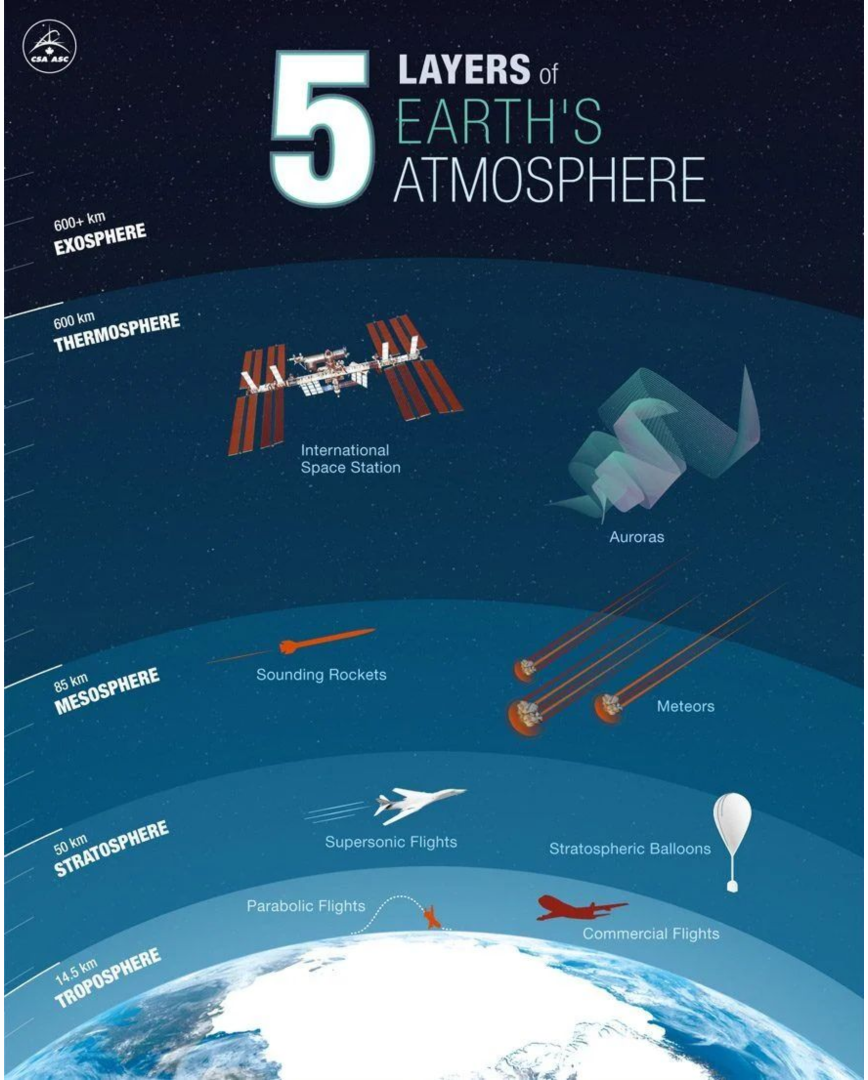
Explain the coupling of attitude kinematic equations and Euler’s rotational equations.

What is the dual spin concept + why does the dual spin concept work?

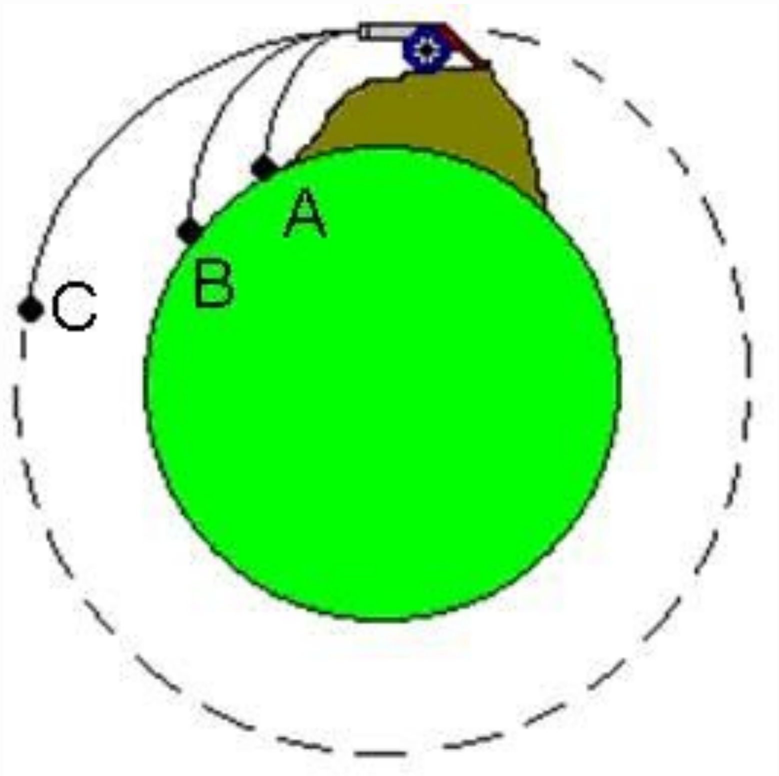
Precept Part 2 - Orbital Mech

How do orbits work?
Intuitive explanation

Space Environment

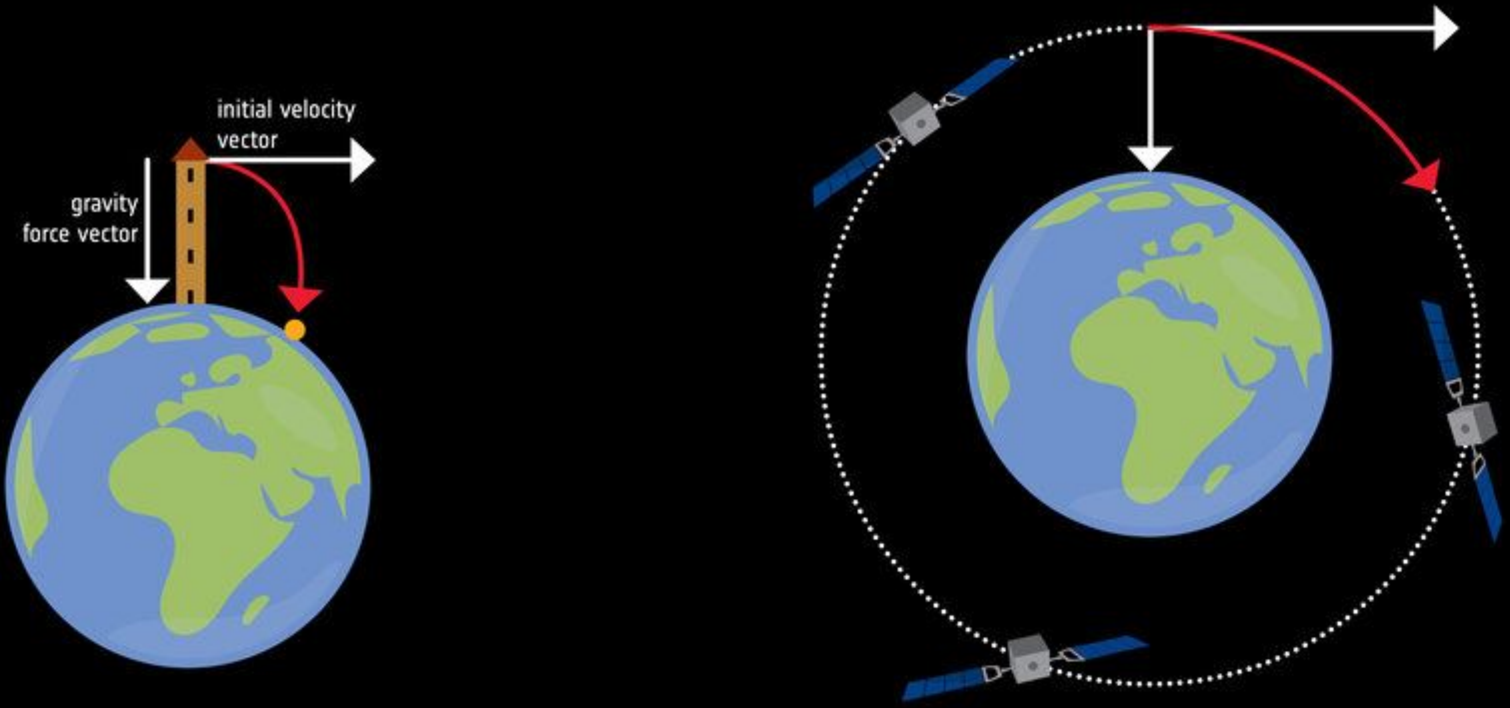


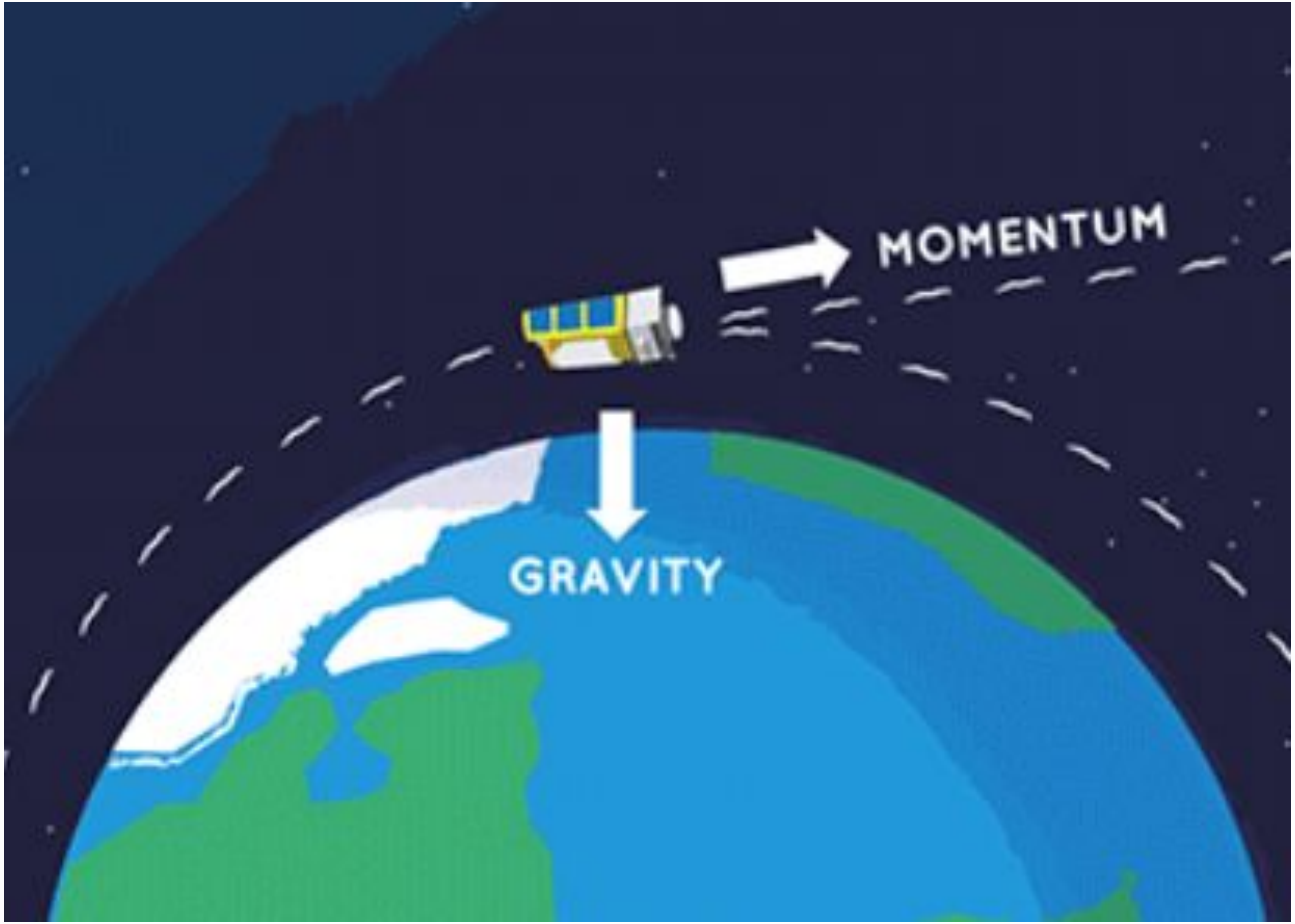
Newton’s Cannonball Thought Experiment



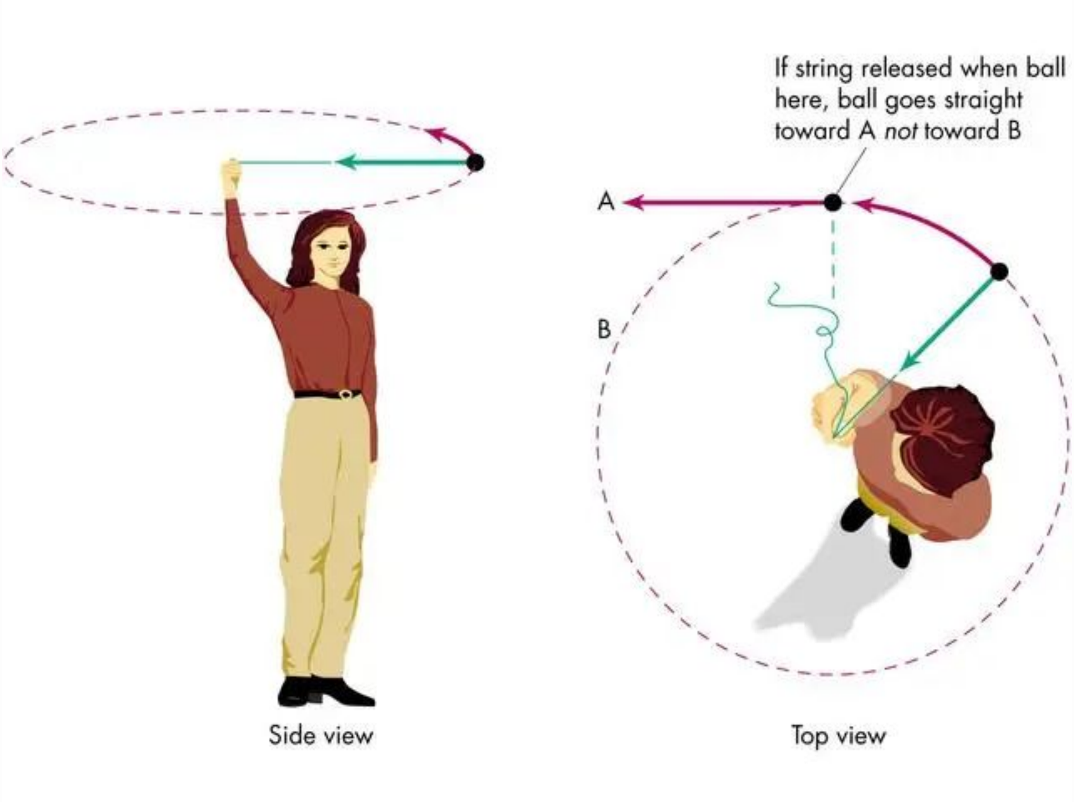
- An orbit is just **continuous free fall** around Earth.
- Instead of hitting the ground, the spacecraft’s horizontal velocity makes it miss the surface as the Earth curves away.

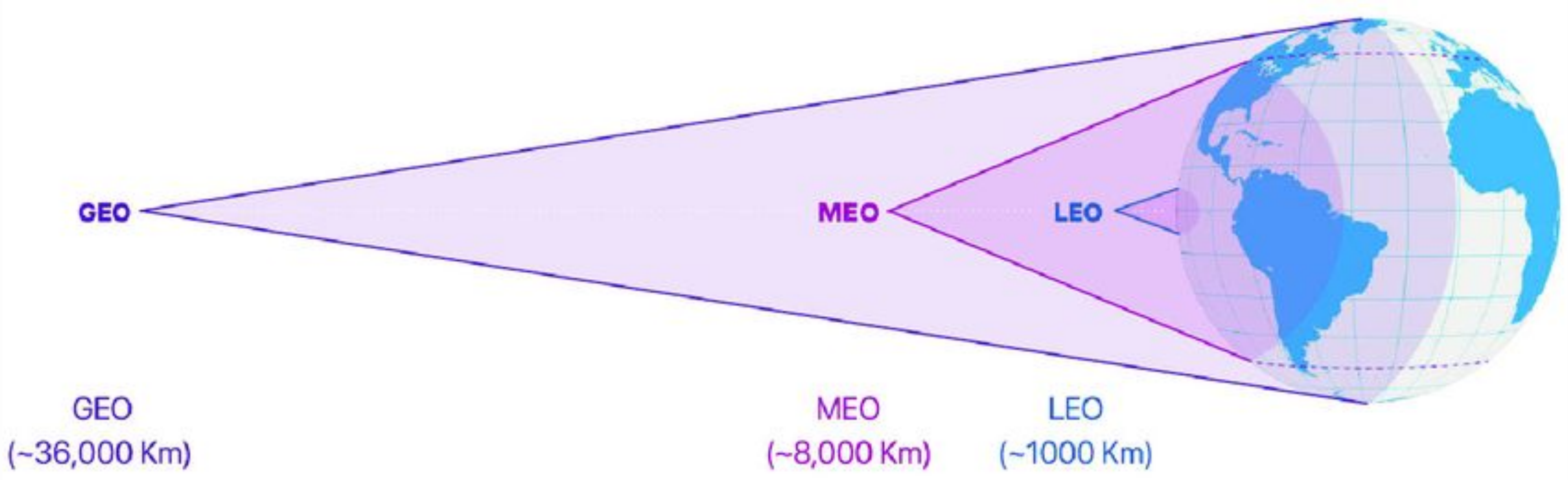
Reaching orbit




MOMENTUM
GRAVITY

Everyday Analogies



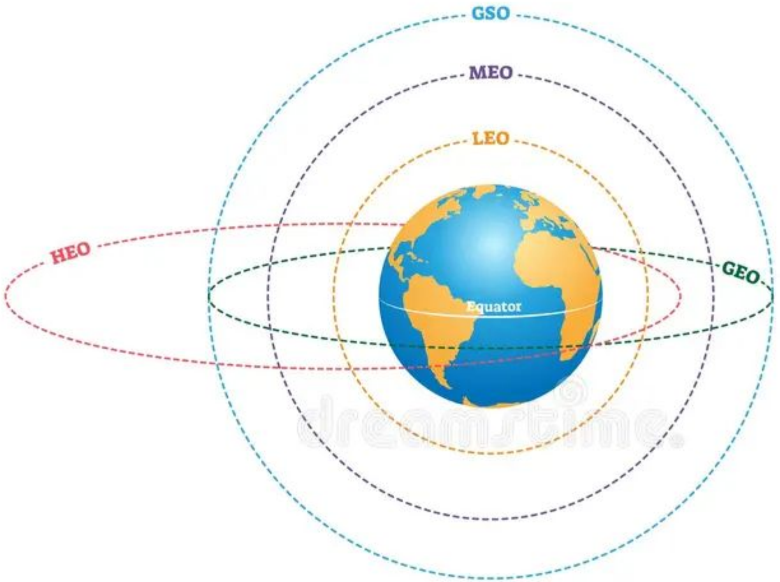

GEO
MEO
LEO
GEO
(~36,000 Km)
MEO
(~8,000 Km)
LEO
(~1000 Km)

Concept Check

- What goes faster, satellites in LEO or in MEO? Approximate speeds?

Orbit Altitude (km)	Orbital Velocity (km/sec)	Orbit Period (min)	Orbit Period (hour)	Comments
200	7.78	88.5	1.47	
370	7.69	91.9	1.53	Low end of ISS orbit
400	7.67	92.6	1.54	
460	7.63	93.8	1.56	High end of ISS orbit
600	7.56	96.7	1.61	
1,000	7.35	105.1	1.75	
2,000	6.90	127.2	2.12	
5,000	5.92	201.3	3.36	
10,000	4.93	347.7	5.79	
15,000	4.32	518.5	8.64	
20,000	3.89	710.6	11.84	
20,230	3.87	719.9	12.00	GPS satellites
29,429	3.34	1,123.9	18.73	
35,786	3.07	1,436.1	23.9345	GSO satellites
385,000*	1.02	39,340.8	655.68	Moon orbits in 27.32 days

ORBIT TYPES



Over the Equator

LEO
Low Earth Orbit

- Altitude: 160-2.000 km
- Speed: ~ 8 km/sec
- Orbital period: ~ 90 min

Example:
Globalstar - 48 satellites
Voice and Data Services

MEO
Medium Earth Orbit

- Altitude: 2.000-35.786 km
- Speed: ~ 3-8 km/sec
- Orbital period: ~ 2-24 hours

Example:
GPS - 24 satellites
Global Positioning System

GEO
Geostationary Orbit

- Altitude: 35.786 km
- Speed: ~ 3 km/sec
- Orbital period: 24 hours

Example:
Communications satellites,
Broadcast satellites

GSO
Geosynchronous Orbit

- Altitude: 35.786 km
- Speed: ~ 3 km/sec
- Orbital period: 24 hours

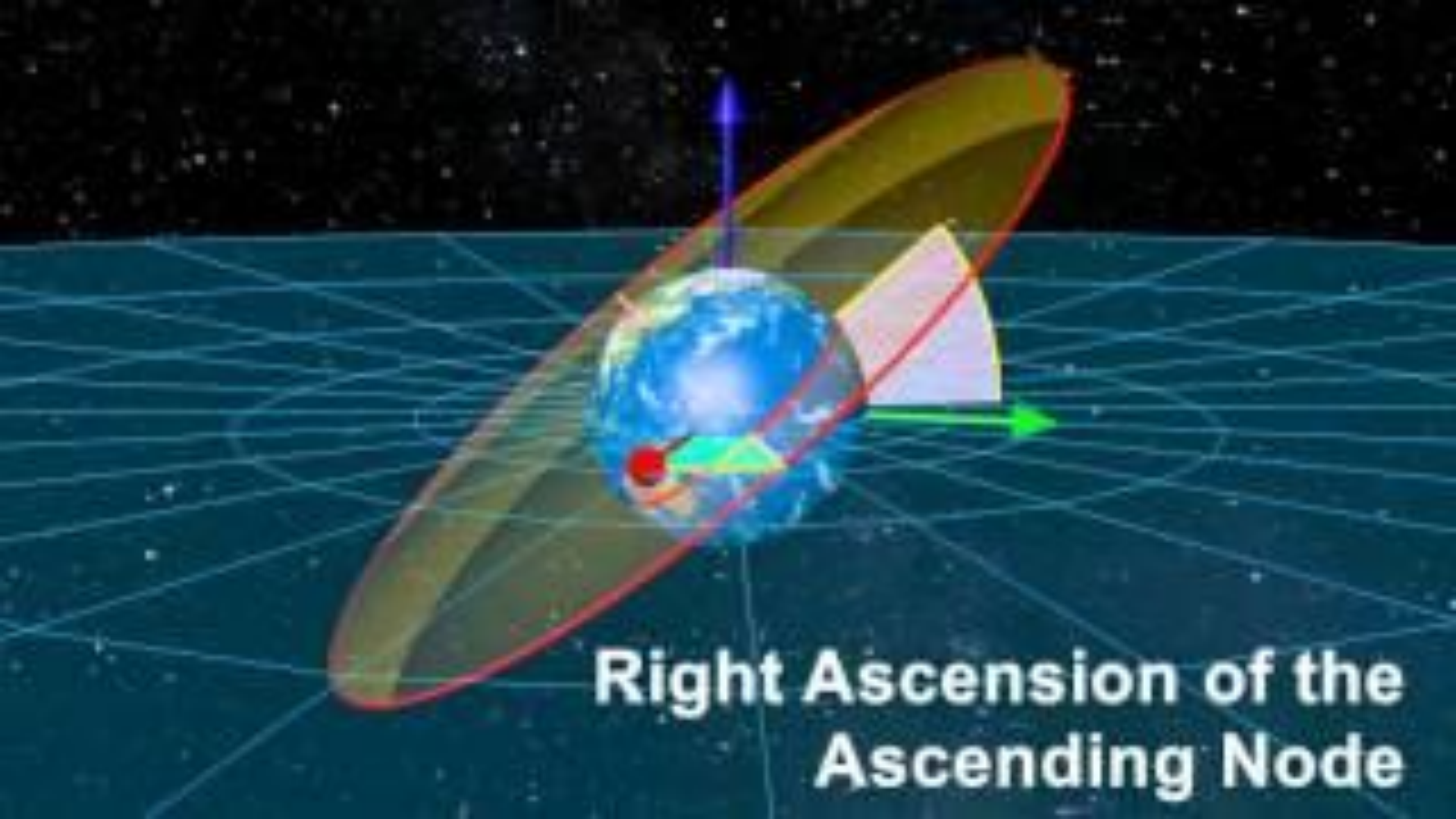
Example:
SBAS
Weather satellites

HEO
Highly Elliptical Orbit

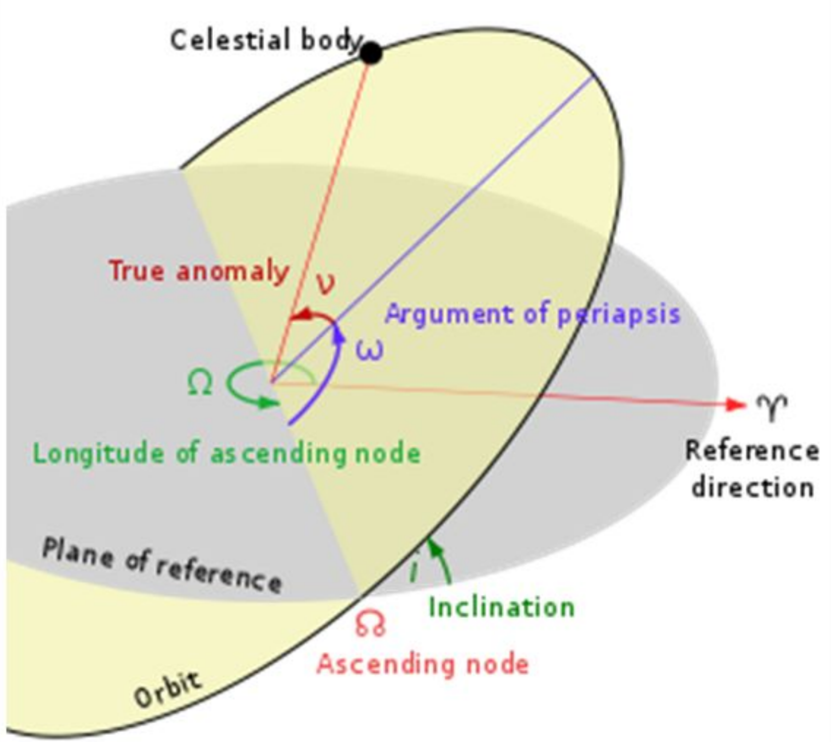
- Apogee altitude: 40.000 km
  Perigee altitude: 1.000 km
- Speed: ~ 1.5-10.0 km/sec
- Orbital period: ~ 12 hours

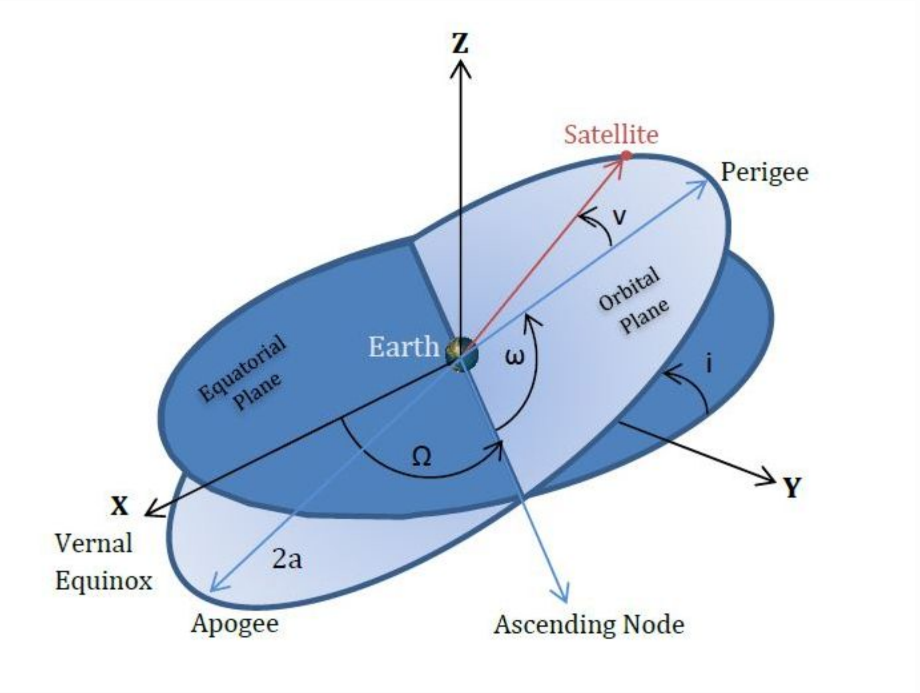
Example:
Communications,
Remote sensing

The Orbital Elements


Right Ascension of the
Ascending Node

Orbital Elements





How to Do Project 1

Satellite Selection

Choose a satellite that has an eccentricity ~<= 0.01

Our method of calculation assumes circular orbit

Less circular = more error; use judgement

Ex: Ajisai satellite (ex from project intro precept) e = 0.001
Earth's orbit e is 0.0167

Satellite Selection

Choose a satellite from the options in your satellite tracker app (not every satellite will have publicly available data in these apps)

Potential Apps for Tracking

- Heavens Above (website/app)
- Satellite Tracker (mobile app by Vito Technology)
  - $0.49/month with 1 week free trial
- In The Sky (website)
  - > spacecraft > world map of sat positions > select Princeton on map

- Don’t just scalp TLE data from internet – do some “data collection”

Measuring Period & Angular Velocity Method 1 (Recommended)

Record the time that a satellite takes to reach the same point in the sky in two different passes

The time difference between them is your period, and since the orbit is circular you can calculate the distance it travels using your radius

Measuring Period & Angular Velocity Method 2

Record a few times, elevation, and azimuth angles at least 10 minutes apart.

With these measurements, you can convert to true anomaly using a code we can give you. By finding the difference in true anomaly with time, you can get angular velocity.