Space Flight Precept 4

Oct 1, 2025

Reminders

- Solutions to problem set 1 are posted
- You should have started your Project 1 data collection
  - No office hours during fall break. Project due right after fall break.
- Midterm exam after fall break
  - ~30-40% attitude dynamics
  - Rest is history, space environment, orbital mechanics
  - Practice midterm exam will be posted
- Problem set 2 will be posted soon (today-ish) & due one week after posting
  - This problem set will be extremely similar to midterm questions

Agenda

- Project 1 methods discussion
- Lecture Review
  - Kepler's Laws
  - Vis-viva equation
- Practice Problems

Project 1

Company Name

Date

Project 1: Measuring Period

See document on Canvas outlining many different methods

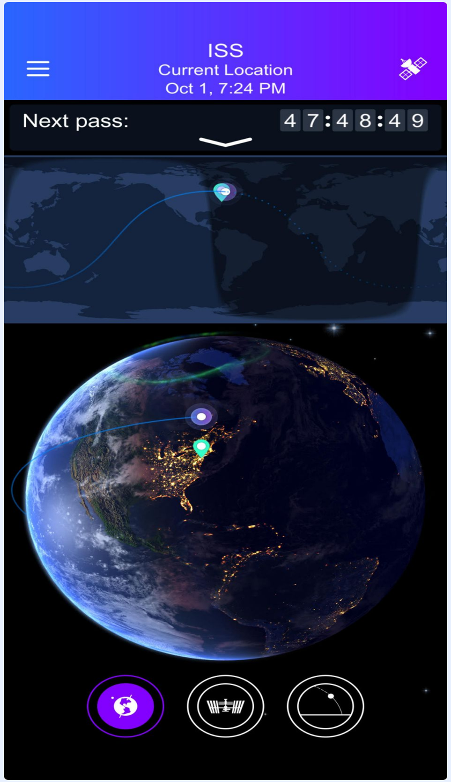
Generally, this project has you find approximate solutions for your different orbital parameters. You can use many different methods. Your answer will be a little bit wrong. Just explain why & compare against other methods/online TLE data.

We will give bonus points if you develop and execute an *exceptional* method.

Project 1: Elevation Angle to Measure Period

- Have BOTH Star Tracker app and the In The Sky website
  - https://in-the-sky.org/
- Use your app to know approximately when your satellite is over Princeton. Use this to know when to look at In The Sky website
- Use In The Sky to measure time at the SAME elevation angle two passes in a row
  - Do this many times – need ~10 measurements for period
  - > spacecraft > world map of sat positions > select Princeton on map

Project 1: Elevation Angle to Measure Period



Project 1: Sample Projects

Sample projects with redacted data have been posted. These projects are good, but are not necessarily perfect.

These are old examples & may not include some specific instructions for this year. (Ex: Need more discussion of error, more comparison of methods and results)

Generally, these are a good resource if you're confused about how to do the project

Acceptable uses of AI on the Project

AI can help you write code to plot your results

You can ask AI basic conceptual questions such as “What does inclination mean?” for your own understanding

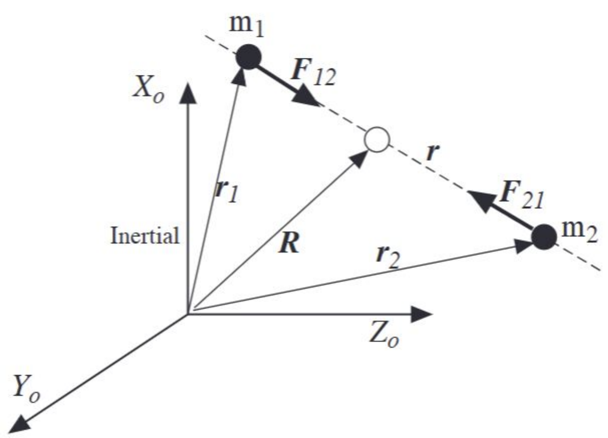
Please do <u>not</u> use AI to do the actual calculations or write question responses

Lecture Review

Company Name

Date

Understanding Derivations in Lecture



$$m\ddot{r} - mr\dot{\theta}^2 = F = -\frac{\partial V}{\partial r}, \tag{17}$$

$$mr^2\dot{\theta} = H, \tag{18}$$

where H is the magnitude of the angular momentum defined earlier.

A single second order differential equation called the *differential equation of the orbit* can be found by rewriting the last equation as the operator

$$\frac{d}{dt} = \frac{H}{mr^2}\frac{d}{d\theta}, \tag{19}$$

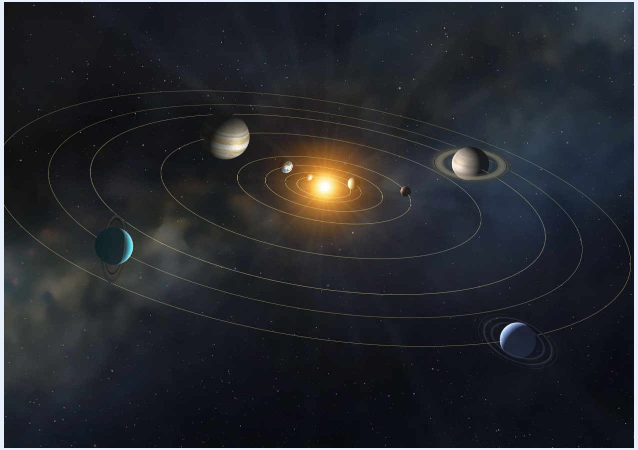
and using it to eliminate time in Eq. (17), yielding

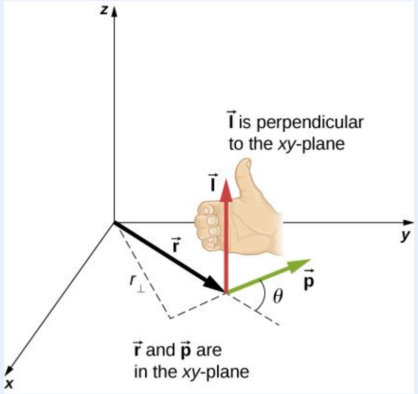
$$\frac{d^2u}{d\theta^2} + u = -\frac{mF}{H^2u^2} = -\frac{m}{H^2}\frac{\partial V}{\partial u}, \tag{20}$$

Angular & Linear Momentum Conservation

What does it mean for angular and linear momentum to be conserved?

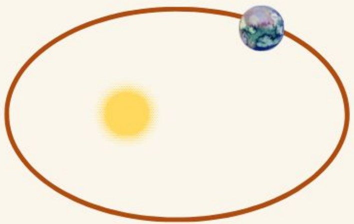
Does this work for more than 2 bodies?





Kepler's Laws

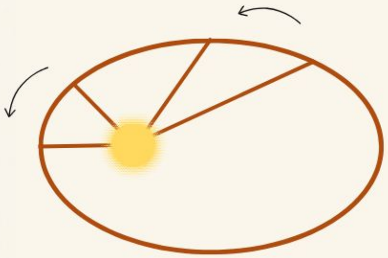
First Law



ellipse

Planets orbit in ellipses with the Sun at one focus.

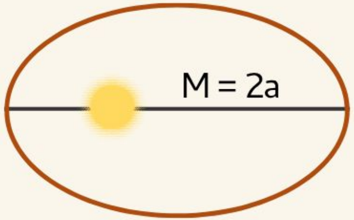
Second Law



Planets sweep out equal areas in equal times.

Third Law

P: period (time for one cycle)
2a: length of major axis



$P^2 \propto a^3$

The square of the orbital period is proportional to the cube of the semi-major axis

Vis-Viva Equation

What is it? What does it represent? Why do we care?

Orbital velocity

$$v = \sqrt{\frac{k}{m}\left(\frac{2}{r} - \frac{1}{a}\right)}$$

Orbits - Concept Questions

Basic shapes? What makes them different? Eccentricity? Impact on vis-viva?

Circular, elliptical, parabolic, hyperbolic....

(see Fundamentals of Orbital Mechanics.pdf – section on each)

Practice Problems

Company Name

Date

Deriving Vis-Viva

My First Satellite Calculations

A geocentric satellite is in an elliptic orbit having a perigee altitude of 610 km and an apogee altitude of 2430 km.

Calculate the orbit’s eccentricity, energy, angular momentum and period (i.e. time to make a single orbit)

Solutions

First we need the actual perigee and apogee radii:

$$
\begin{aligned}
r_{earth} &= 6378\ [km] \\
r_p &= 610 + 6378\ [km] = 6988\ [km] \\
r_a &= 2430 + 6378\ [km] = 8808\ [km]
\end{aligned}
$$

Now we get the semi-major axis and focus:

$$
a = \frac{r_p + r_a}{2} = 7898\ [km]
$$

$$
c = a - r_p = 910\ [km]
$$

Solution cont.

From this we can get the eccentricity:

$$e = \frac{c}{a} = 0.115 \;\checkmark$$

We can also get the period via:

$$\tau = 2\pi\sqrt{\frac{a^3}{\mu}} = 6985 \;[s]\checkmark$$

Angular momentum from Eqn 33 in the notes:

$$H = \sqrt{mka(1-e^2)} = \sqrt{mm_sGM_Ea(1-e^2)}$$

but

$$m = \frac{m_sM_E}{m_s + M_E}$$

which gives us:

$$km = Gm_sM_e\frac{m_sM_E}{m_s + M_E} = G\frac{m_s^2M_E^2}{m_s + M_E}$$

Solution cont.

Assuming $m_s << M_E$:

$$km \approx G\frac{m_s^2 M_E^2}{M_E} = GM_E m_s^2 = \mu_E m_s^2$$

Subbing this into H we get:

$$H = m_s\sqrt{\mu_E a(1-e^2)}$$

We will solve for the specific angular momentum:

$$h = H/m_s = \sqrt{\mu_E a(1-e^2)} = 5.5735 \times 10^{10}\ [m^2/s] \checkmark$$

Energy:

$$E = -\frac{k}{2a} = -\frac{Gm_s M_E}{2a}$$

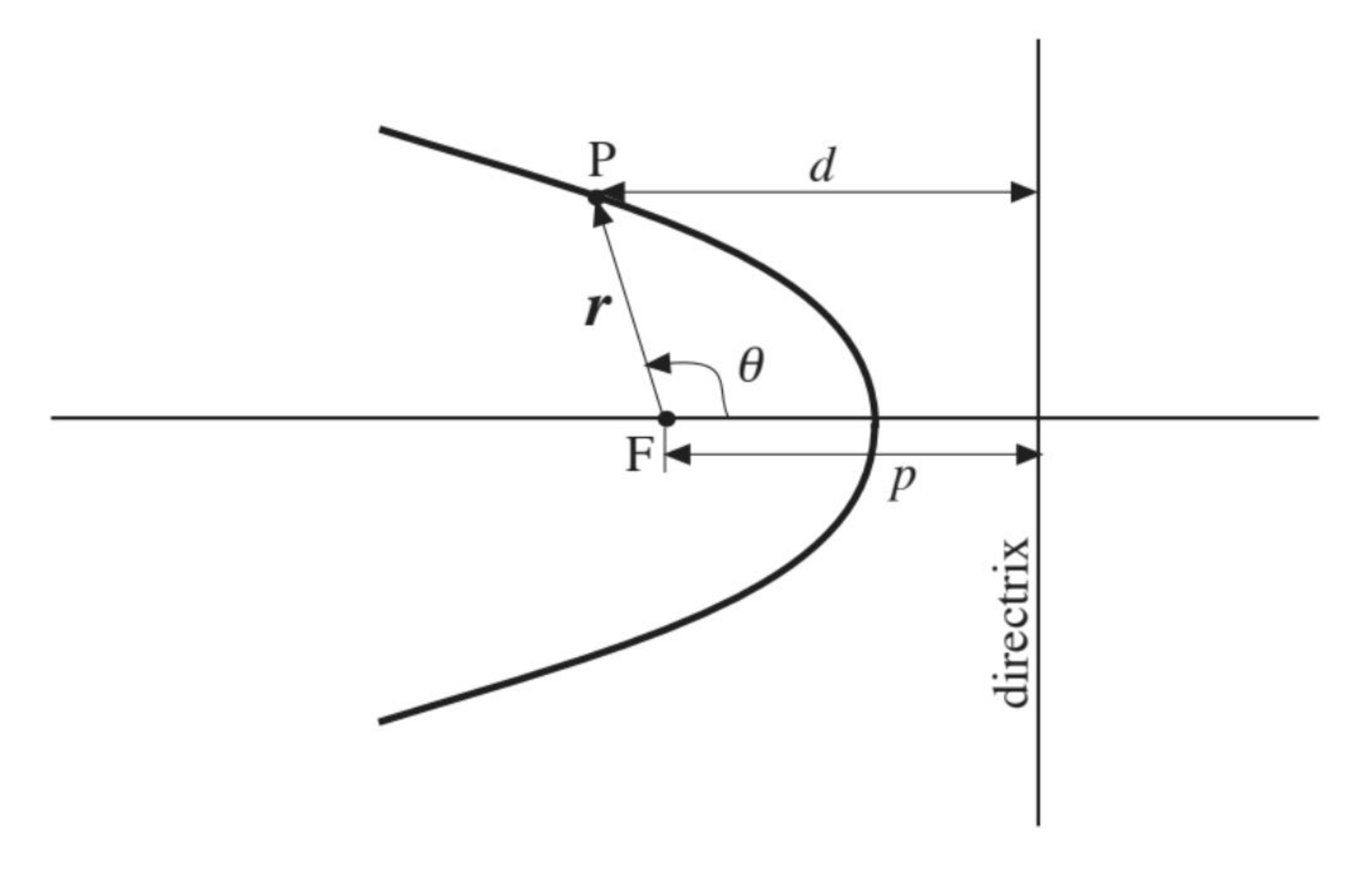
$$\frac{E}{m_s} = -\frac{\mu_E}{2a} = -2.5234 \times 10^7\ [J/kg] \text{ or } [m^2/s^2] \checkmark$$

Drawing

Draw a schematic showing a generic conic section and define on the drawing the following 4 parameters used in the lecture notes and discussed in class: r, θ, p and d. Using only 2 of these 4 parameters, write down a simple expression that defines the eccentricity e.

Solution

e = r/d



Conceptual Practice Problem

State Kepler’s third law (the one about the dependence of a planet’s orbital period on its mean distance from the Sun). Prove that law starting with one of the equations you were asked to memorize. What assumption must you make to get to Kepler’s exact statement from your derivation?

Solution

(a) A planet's orbital period is proportional to its mean distance from the sun raised to the power 3/2.

$$\tau_e = 2\pi\sqrt{\frac{ma_e^3}{k}}.$$

(b) The third law can be seen by taking the ratio of the periods of two planets of masses m_1 and m_2 orbiting around the sun of mass $m_\odot$,

$$\left(\frac{\tau_1}{\tau_2}\right)^2 = \left(\frac{a_1}{a_2}\right)^3 \left(\frac{m_2 + m_\odot}{m_1 + m_\odot}\right),$$

(c) Kepler, implicitly made the approximation that both m_1, m_2 are much smaller than $m_\odot$. Under this good approximation we have,