Empirical Observation of the ISS

[redacted]

MAE 341

October 6, 2023

1 Introduction

The goal of this project was to observe the motion of a satellite orbiting Earth and use observational data to compile and calculate information on the characteristics of the satellite's orbital path. The International Space Station (ISS) was selected as the satellite to be analyzed because of its favorable trajectory during the observational period and its rich history as part of humanity's effort to push beyond the limits of Earth.

Figure 1: The ISS with Earth in the Background

[redacted]

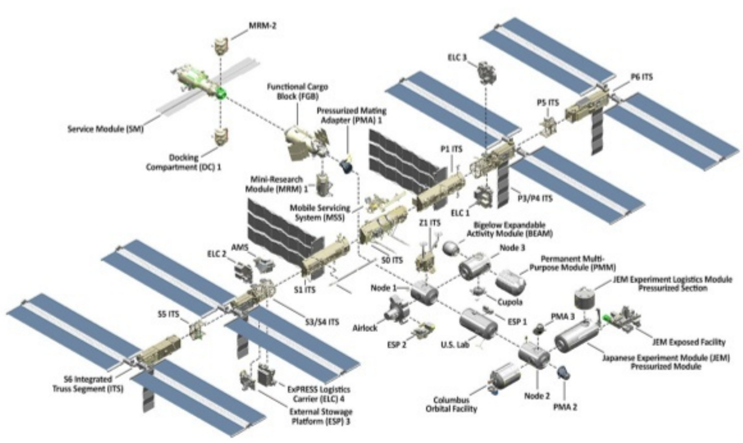

Figure 2: Configuration of the ISS

[1]https://issuu.com/faircountmedia/docs/nasa-international-space-station-20/s/23954
[2]https://www.nasa.gov/reference/international-space-station/#hds-sidebar-nav-6

[redacted]

2 Data Collection & Methods

The ISS was observed over the course of three days. The iOS app “Satellite Tracker” was the primary source of data. When the station began its pass overhead in Princeton, NJ, a screen recording was taken of the app’s data, which included elevation angle, azimuth, and distance from the observer to the station. Time stamps were taken along with the screen recording in order to accurately know when the data was collected. While attempts were made to visually observe the space station as it went overhead, weather conditions and location did not allow for direct viewing. Additional data was collected via websites including Heavens Above and In The Sky. Any data that is derived from these websites will be noted in the results section of the report. The raw data is seen below.

Date	Time	Elevation Angle	Azimuth	Distance (mi)	Distance (km)
9/28/23	8:21:37	[redacted]			
9/28/23	8:22:06	[redacted]			
9/28/23	8:22:51	[redacted]			
9/28/23	8:23:25	[redacted]			
9/28/23	8:23:55	[redacted]			
9/28/23	8:24:28	[redacted]			
9/28/23	8:25:10	[redacted]			
9/28/23	8:25:37	[redacted]			
9/28/23	8:26:08	[redacted]			
9/28/23	8:26:27	[redacted]			

Figure 3: Raw Data from 9/28/23

Date	Time	Elevation Angle	Azimuth	Distance (mi)	Distance (km)
9/29/23	7:30:16	[redacted]			
9/29/23	7:30:36	[redacted]			
9/29/23	7:31:28	[redacted]			
9/29/23	7:32:03	[redacted]			
9/29/23	7:32:37	[redacted]			
9/29/23	7:32:58	[redacted]			
9/29/23	7:33:37	[redacted]			
9/29/23	7:34:11	[redacted]			
9/29/23	7:34:35	[redacted]			
9/29/23	7:34:54	[redacted]			
9/29/23	7:35:45	[redacted]			
9/29/23	7:36:22	[redacted]			
9/29/23	7:36:54	[redacted]			
9/29/23	7:37:21	[redacted]			
9/29/23	7:37:54	[redacted]			

Figure 4: Raw Data from 9/29/23

[3] https://www.nasa.gov/international-space-station/space-station-research-and-technology/

Date	Time	Elevation Angle	Azimuth	Distance (mi)	Distance (km)
9/30/23	8:18:51	[redacted]	[redacted]	[redacted]	[redacted]
9/30/23	8:19:12	[redacted]	[redacted]	[redacted]	[redacted]
9/30/23	8:19:36	[redacted]	[redacted]	[redacted]	[redacted]
9/30/23	8:20:02	[redacted]	[redacted]	[redacted]	[redacted]
9/30/23	8:20:35	[redacted]	[redacted]	[redacted]	[redacted]
9/30/23	8:21:00	[redacted]	[redacted]	[redacted]	[redacted]
9/30/23	8:21:25	[redacted]	[redacted]	[redacted]	[redacted]
9/30/23	8:21:44	[redacted]	[redacted]	[redacted]	[redacted]
9/30/23	8:22:07	[redacted]	[redacted]	[redacted]	[redacted]
9/30/23	8:22:32	[redacted]	[redacted]	[redacted]	[redacted]
9/30/23	8:22:51	[redacted]	[redacted]	[redacted]	[redacted]
9/30/23	8:23:33	[redacted]	[redacted]	[redacted]	[redacted]

Figure 5: Raw Data from 9/30/23

3 Results

All preceding calculations were done with the assistance of Python (through Google Colab Notebooks), MATLAB, and/or Google Sheets. All formulas and one set of calculations will be laid out here, while the code and majority of the data will be included in the appendix. The selected data point we will be considering for all calculations will be that taken on 9/28/23 at 8:24:28pm, with the data over time to be from the previous point at 8:23:55pm.

3.1 Data Conversion

The angles taken as data using "Satellite Tracker" are measured from the point of observation. However, in order to perform calculations, the angles and distance needed to be converted into measurements from the center of the Earth. The provided Python code was used to convert the data from Elevation Angle, Azimuth, and Distance into Right Ascension (RA), Declination (Dec), and radius.

3.2 Period (τ)

The Period of the ISS's orbit was calculated in two different ways. The first method involved assuming the orbit is a circle, which we will later see is a fair assumption to make. The equation for period of a circular orbit is:

$$\frac{\psi}{2\pi} = \frac{\Delta T}{\tau} \tag{1}$$

where ψ is the angle through which the satellite travels in time ΔT. The angle was calculated using the following approximation for two perpendicular angles:

$$cos(\psi) = sin(\delta_1)sin(\delta_1) + cos(\delta_1)cos(\delta_2)cos(\alpha_1 - \alpha_2) \tag{2}$$

where δ is the declination and α is the Right Ascension, and points 1 and 2 are the two points separated by ΔT. For the data point we are analyzing, we get:

ψ [redacted] (3)

and

$\tau = \frac{2\pi\Delta T}{\psi}$ [redacted] (4)

The average τ calculated using this method wa[redacted] The other method of calculating τ is with Kepler's Third Law[redacted]

$$\tau = 2\pi * \sqrt{\frac{ma^3}{k}} \tag{5}$$

where m is the mass of the satellite, a is the semi-major axis, and k is the constant $G * m_1 * m_2$ or $m * \mu$. The calculations for a will come in section 3.4. For our data point, we get:

τ [redacted] (6)

3.3 Angular Velocity (ω)

The Angular Velocity was calculated using the change in angle over time.

$$\omega = \frac{\psi}{\Delta T} \text{[redacted]} \tag{7}$$

The ω values for all data points were remarkably consistent, with an average [redacted] rad/s and most values being eith[redacted]

3.4 Semi-Major Axis (a)

The semi-major axis was calculated using two methods. The first is with the assumption that the orbit is circular so we can use Kepler's Third Law with the period values calculated in the first method of Section 3.2.

$$\tau = 2\pi * \sqrt{\frac{ma^3}{k}} \rightarrow a = (\mu(\frac{\tau}{2\pi})^2)^{1/3} \tag{8}$$

This gives us:

$$a = \text{[redacted]} \tag{9}$$

The other method involves Lambert's Theorem. Lambert's theorem is:

$$TOF = \sqrt{\frac{a^3}{\mu}}((\alpha - sin\alpha) - (\beta - sin\beta)) \tag{10}$$

where alpha and beta are as defined as:

$$cos\alpha = 1 - \frac{r_1 + r_2 + d}{2a} \tag{11}$$

$$sin\alpha = 1 - \frac{r_1 + r_2 - d}{2a} \tag{12}$$

where d is defined as:

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1r_2cos\psi} \tag{13}$$

This equation does not have a discrete solution, so MATLAB was used to numerically iterate the equation until the Time of Flight equaled the ΔT that was measured. The solution for our select point was: a = 6782km. The results from both methods were fairly similar. The a value obtained from Lambert's theorem was used for all succeeding calculations, but it would be an interesting investigation to see the difference in results if the first a value was used.

3.5 Location Prediction

In order to predict the location of the satellite at a future time, Kepler's equation must be used. For this section, two points will be analyzed, the standard point we have been using and the data from 8:22:07pm on 9/30.

$$2\pi\frac{t - t_0}{\tau} = \epsilon - esin\epsilon \tag{14}$$

where epsilon is the eccentric anomaly and t_0 is the time the satellite passed perigee. To predict the satellite's future location, first its eccentric anomaly can be determined using:

$$\epsilon = cos^{-1}(\frac{1 - \frac{r}{a}}{e}) \tag{15}$$

For our first data point we get:

$$\epsilon = \text{[redacted]} \tag{16}$$

And for the second:

$$\epsilon = \text{[redacted]} \tag{17}$$

Once we have values for ϵ, we can find ΔT for our selected points:

$$\Delta T = (\epsilon - esin\epsilon)\frac{\tau}{2\pi} \tag{18}$$

Giving us ΔT values o[redacted]seconds for the first point and[redacted] seconds for the se[redacted]d. With ΔT we [redacted] work [redacted]ackwards to find t_0. We find that perigee occurred at 8:[redacted]pm on 9/28 and 7:3[redacted]m on 9/30. Then, a time to predict the location must be select[redacted]. Because of nu[redacted]rical errors, the times which could be [redacted]cted were somewhat limited. For th[redacted]rst point, the location was to be predicted at 8:[redacted]m and for the second, at 8[redacted]pm on the same day.

[redacted]or the first prediction: ΔT is [redacted]seconds, so from Kepler's Equation, $\epsilon =$ [redacted] The equation is has no closed form so[redacted]tion because of the free ϵ and that within t[redacted]n function, so MATLAB was used to [redacted]merically iterate until the correct value wa[redacted]und. For the second prediction, ΔT is [redacted] seconds, so from Kepler's Equation, $\epsilon =$ [redacted]. From eccentric anomaly, we can fi[redacted] and θ quite easily via the following equati[redacted].

$$r = a(1 - ecos\epsilon) \tag{19}$$

$$\theta = a(\frac{cos\epsilon - e}{r}) \tag{20}$$

Thus, for our two points we get:

r_1 [redacted] (21)

θ_1 = [redacted] (22)

r_2 [redacted] (23)

θ_1 [redacted] (24)

These radius values are both within 1% of the radius values [redacted] from the python calculations described in section 3.1. The True Anomal[redacted] however, are less accurate. The measured true anomaly at the first predicted point wa[redacted]degrees. Gi[redacted]n the other data on angular velocity th[redacted]e have, though, a jump from[redacted]degrees t[redacted]de[redacted]es is unreasonable over th[redacted]econd difference in time. Thus, either the o[redacted]al [redacted]egree value is wrong, or th[redacted]degree value is incorrect, maki[redacted]using one value to [redacted]edict the other unreliable. F[redacted]second point, the prediction of[redacted] degrees is much closer to the calculated value [redacted]5 degrees, though it is still not pe[redacted].

3.6 Inclination (i)

The inclination was obtained through data collection on In The Sky. While the value could be obtained through a circular orbit assumption and the vectors for radius and velocity, at t[redacted]dvice of instructors, I researched the value for inclination. The inclination of the ISS i[redacted] degrees.[4] This value is constant over time, and it allows the space station to fly over 90% of the inhabited portion of the planet.[5]

3.7 Eccentricity (e)

The eccentricity of the ISS orbit was calculated using the following equation:

$$e = \sqrt{1 + \frac{2EH^2}{mk^2}} \tag{25}$$

Using specific momentum and specific energy, as well as the definition that a = -k/2E, this simplifies to:

$e = \sqrt{1 - \frac{r^4\omega^2}{\mu a}} =$ [redacted] (26)

The eccentricities across all data points were relatively consistent in their order of magnitude. The average calculated eccentricity was 0.04, implying that the orbit of the ISS is very nearly a circle. Thus, the circular assumption we have previously made is not an extreme approximation, as e = 0 is a circle.

[4] https://in-the-sky.org/spacecraft_ elements.phpid?=25544
[5] https://www.nasa.gov/reference/international-space-station/#hds-sidebar-nav-6

3.8 True Anomaly (θ)

The True Anomaly is the angle that gives the position of the satellite along its orbit, relative to perigee. Once e and a have been determined, the equation for true anomaly is:

$$\theta = cos^{-1}(\frac{(\frac{a}{r}(1-e^2)-1}{e}) \tag{27}$$

which evaluates to:

$$\theta = \text{[redacted]} \tag{28}$$

The true anomaly should progress over time given that the satellite is orbiting Earth. The calculated values for θ were quite inconsistent, though. Many values returned errors as the inside of the inverse cosine function was greater than 1, while other values simply varied drastically. The only somewhat reliable trend for true anomaly was observed on the third day of observations, 9/30. Below is the trajectory over time.

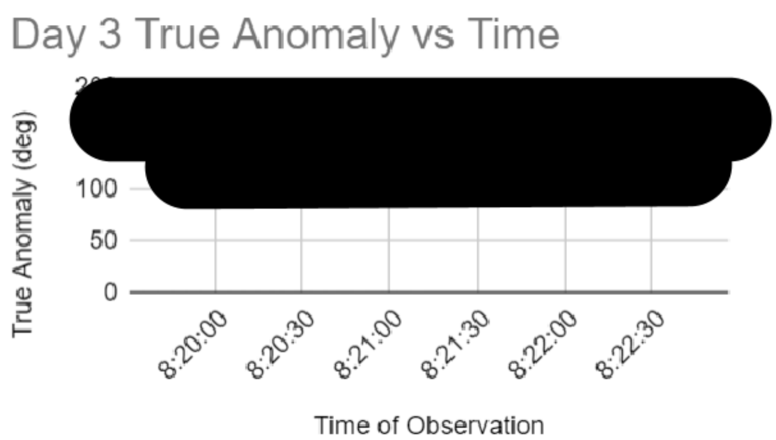

Figure 6: True Anomaly over time on 9/30/23

3.9 Longitude of Ascending Node (Ω)

The Longitude of Ascending Node is where the satellite rises above the reference plane, measured relative to a reference line, which is defined, for Earth, to be the direction of the First Point of Aries. Similar to the inclination, Ω could be calculated using the nodal vector and vector definitions of r and v, but at the advice of instructors, the values were obtained from the internet and are plotted below. Ω precesses along the plane over the course of several days. [6]

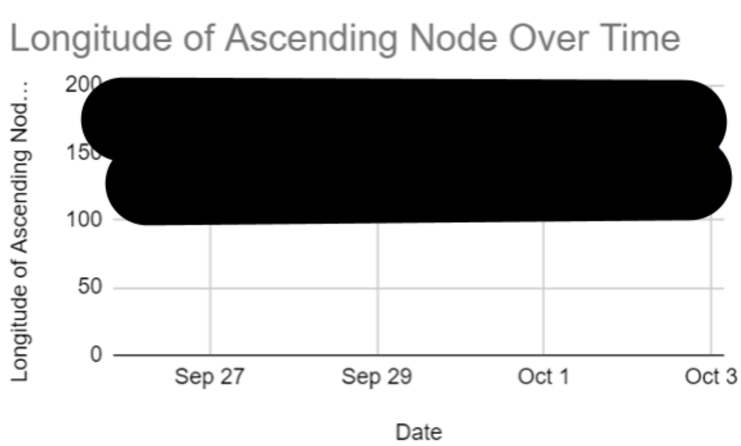

Figure 7: Longitude of Ascending Node over Several Days

3.10 Argument of Periapse (ω)

The Argument of Periapse is the final orbital parameter, and it defines the angle between perigee and the line of nodes. ω was also obtained through research as opposed to calculation. The plot below shows ω over the course of several days. While the plot

[6]https://in-the-sky.org/spacecraft_elements.php?id=25544

oscillates a bit during the period in which the ISS was observed, the general trend over a longer period of time (a month as opposed to a week) sees the value trend upward.[7]

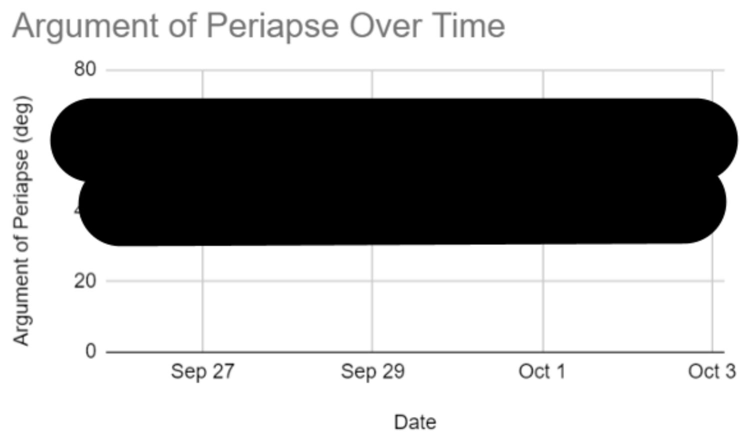

Figure 8: Argument of Periapse over Several Days

4 Discussion

[redacted]

5 Sources

Picture: https://www.issnationallab.org/the-iss-engineering-feat-design/
Picture: https://www.esa.int/kids/en/learn/The_ISS

[7]https://in-the-sky.org/spacecraft_elements.php?id=25544
[8]https://www.nasa.gov/reference/international-space-station/#hds-sidebar-nav-6
[9]https://in-the-sky.org/spacecraft.php?id=25544

“';;.'” ';;' - YouTube, 9 March 2019, https://www.nasa.gov/reference/international-space-station/%5C#hds-sidebar-nav-6. Accessed 9 October 2023.

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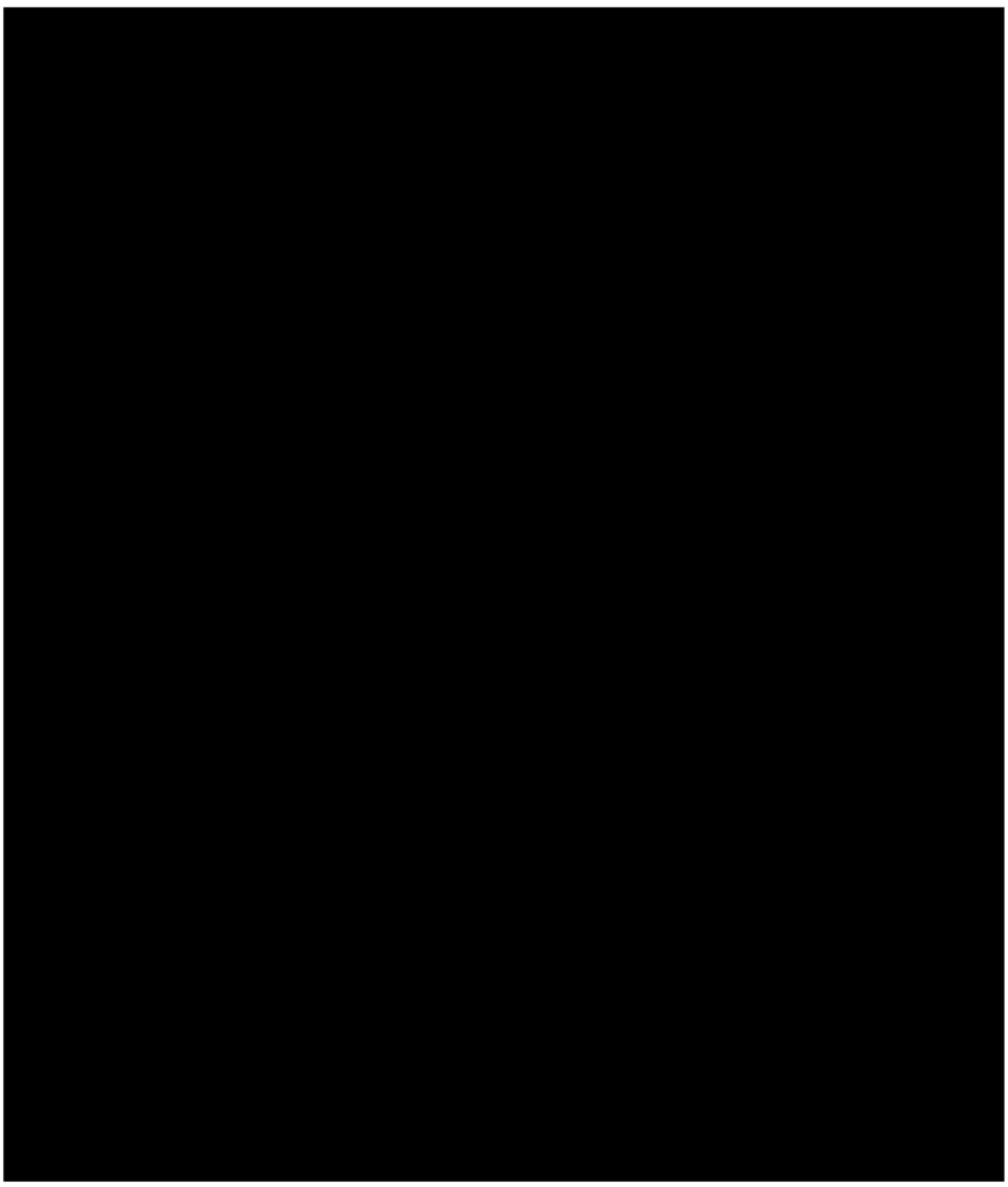
Fundamentals of Orbital Mechanics notes

MAE 341 Lecture + Precept

Satellite Tracker App

6 Appendix



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