

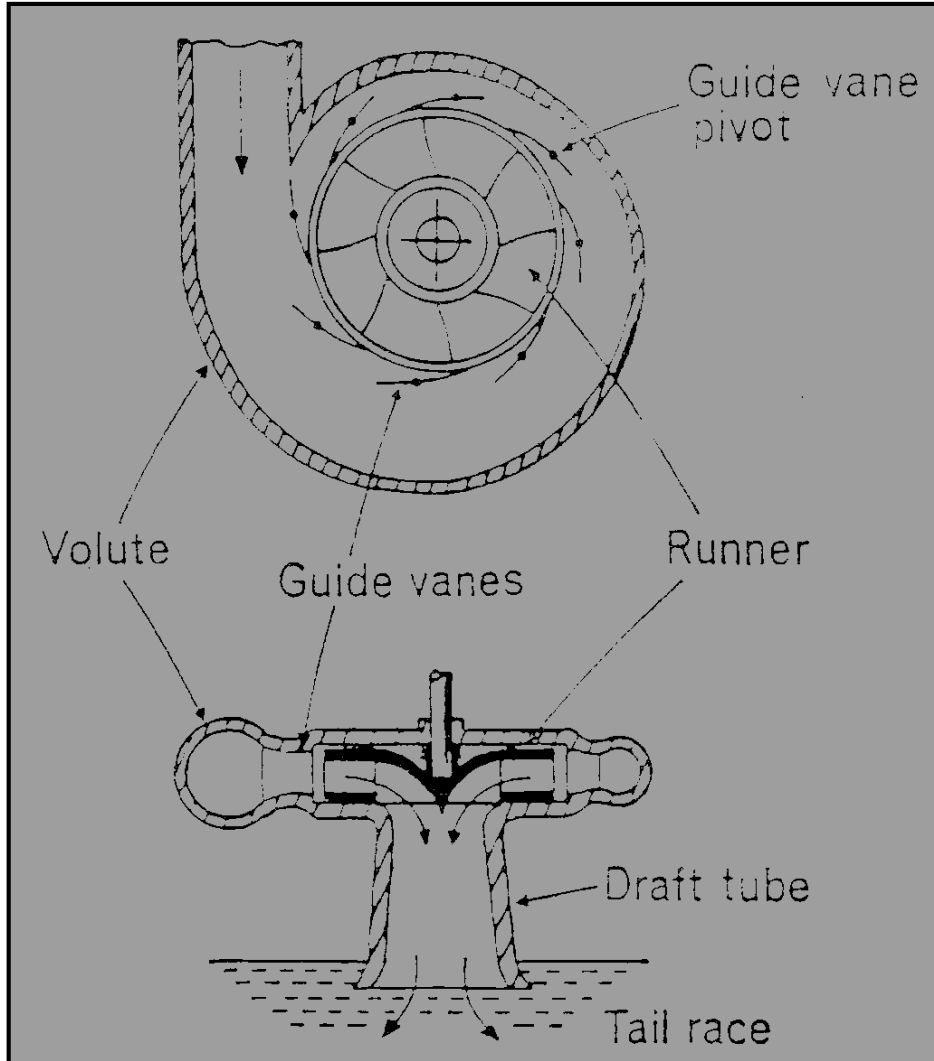
# Mechanical / Civil CAD

## Turbine Fluid Mechanics

### Course Work Module

Prof. Kam Chana  
Trinity Term 2024

# Francis Turbine



Francis turbine is a type of water turbine that was developed by James B. Francis in Massachusetts.

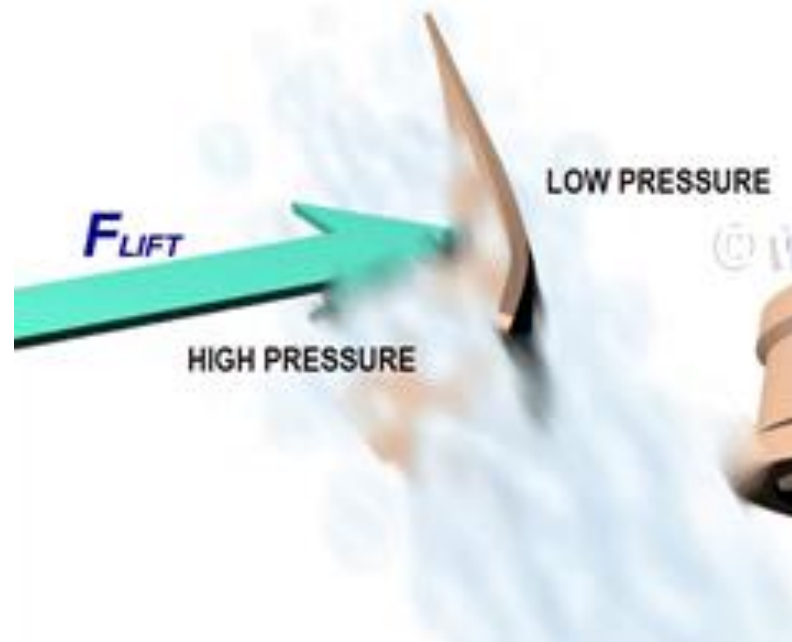
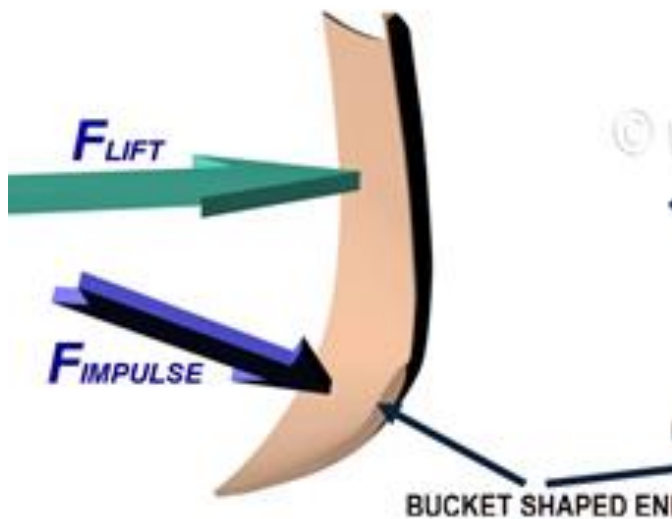
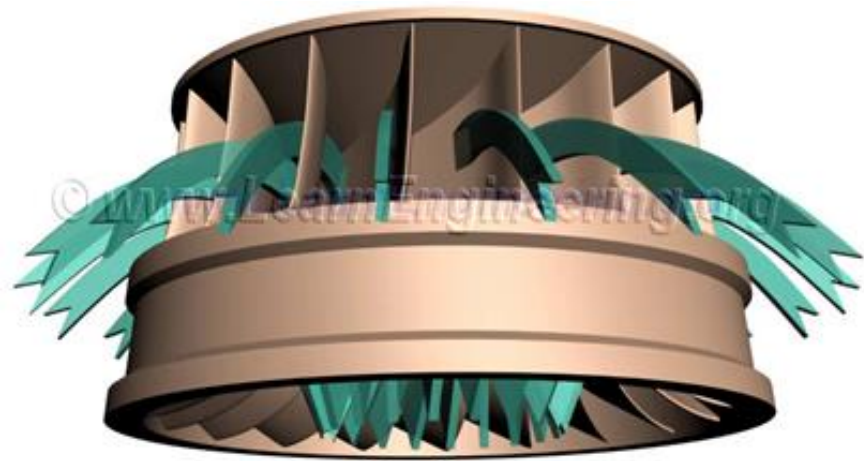
An inward-flow reaction turbine that combines radial and axial flow concepts.

Francis turbines are the most common water turbine in use today. They operate in a water head from 40 to 600 m

Used for electrical power production

# Francis Turbine

## Reaction and Impulse



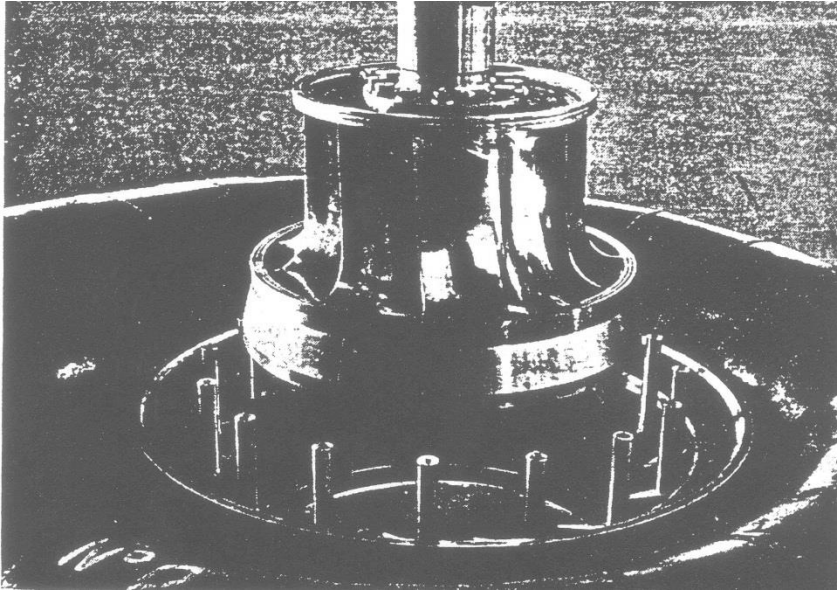
# Main parts (1)

- Spiral casing: around the runner of the turbine is known as the volute, working fluid impounds on the blades of the runner converting the pressure energy of the fluid into momentum energy
- Constant flow rate is maintained despite the fact that numerous openings have been provided for the fluid to gain entry to the blades, the cross-sectional area of this casing decreases uniformly along the circumference.
- Guide or stay vanes: The primary function of the guide or stay vanes is to convert the pressure energy of the fluid into the momentum energy. It also serves to direct the flow at design angles to the runner blades.

## Main parts (2)

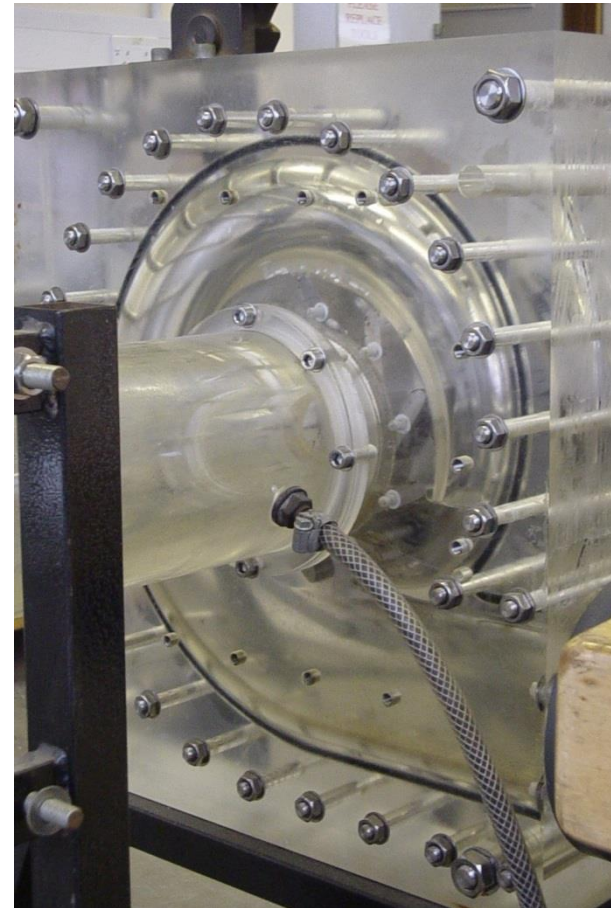
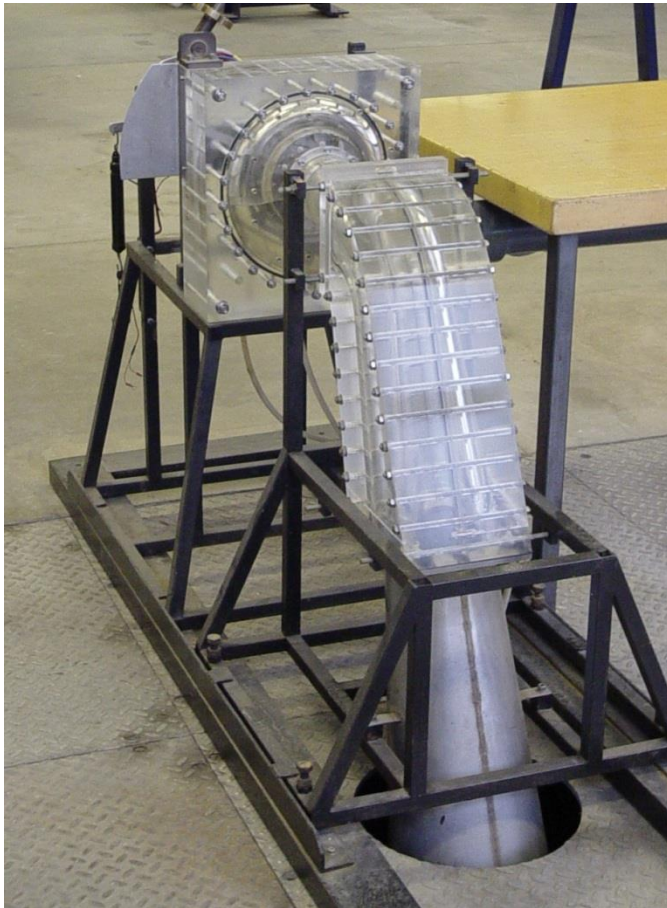
- **Runner blades:** Runner blades where the fluid strikes and the tangential force of the impact causes the shaft of the turbine to rotate. In this part one has to be very careful about the blade angles at inlet and outlet as these are the major parameters affecting the power production.
- **Draft tube:** Draft tube is a conduit which connects the runner exit to the tail race where the water is being finally discharged from the turbine. The primary function of the draft tube is to reduce the velocity of the discharged water to minimize the loss of kinetic energy at the outlet. This permits the turbine to be set above the tail water without any appreciable drop of available head.

# Turbine Runner

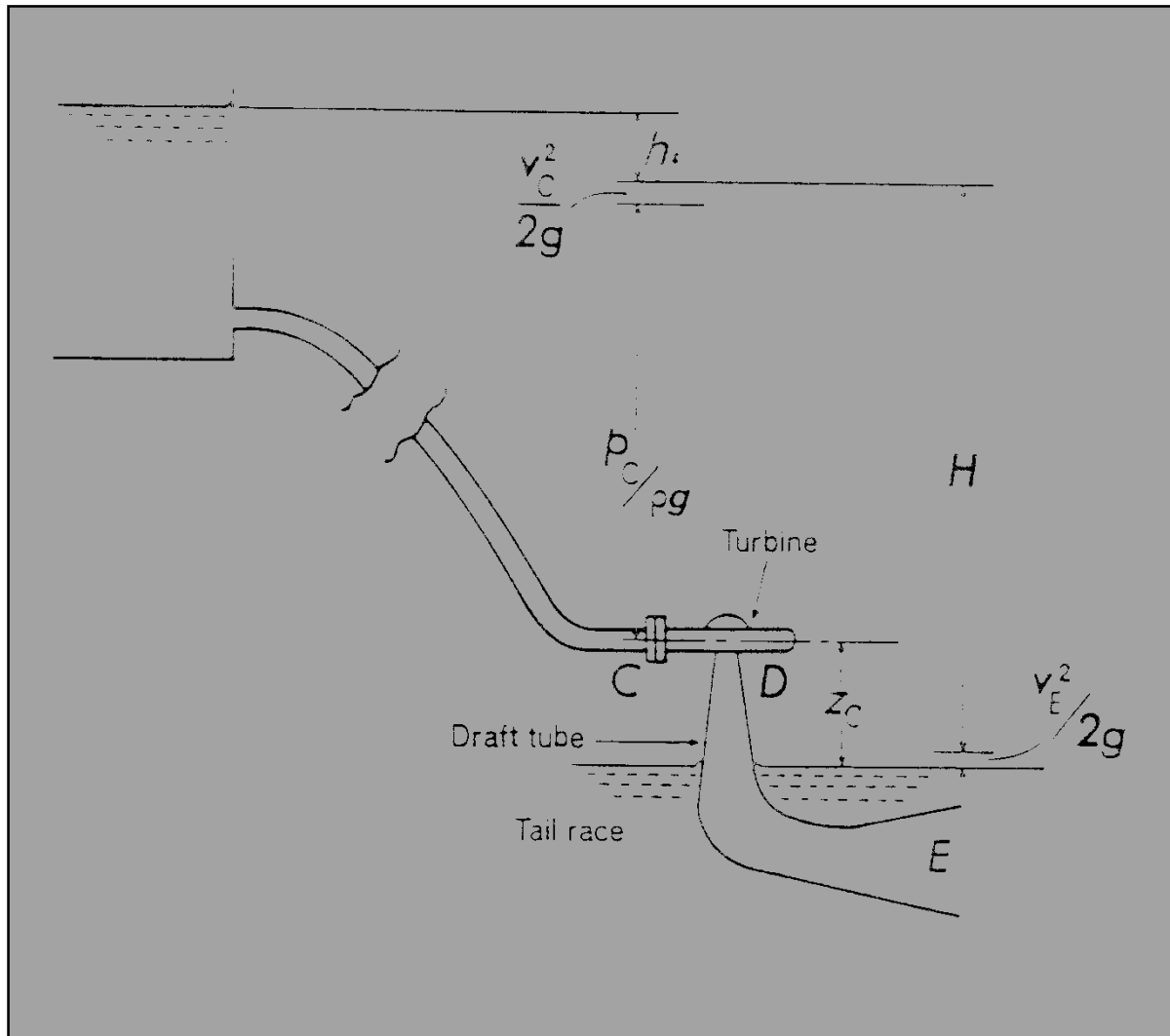




# Francis Turbine



# Description of Energy Heads



Velocity head  $h = v^2 / 2g$   
 Energy of the fluid due  
 to its bulk motion



# Estimating the power produced

- Simple formula for approximating electric power production at a hydroelectric station

$$P = \rho h r g k$$

P power in watts,  $\rho$  density of water ( $\sim 1000 \text{ kg/m}^3$ ),

h height in meters, r flow rate in cubic meters per second,

g acceleration due to gravity  $\text{m/s}^2$ ,

k coefficient of efficiency ranging from 0 to 1. Efficiency is often higher (that is, closer to 1) with larger and more modern turbines.

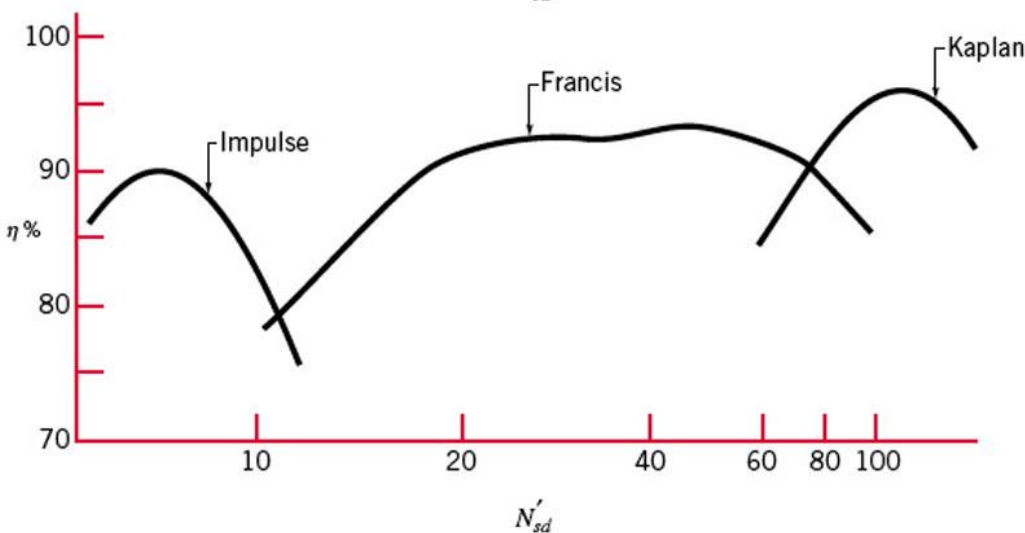
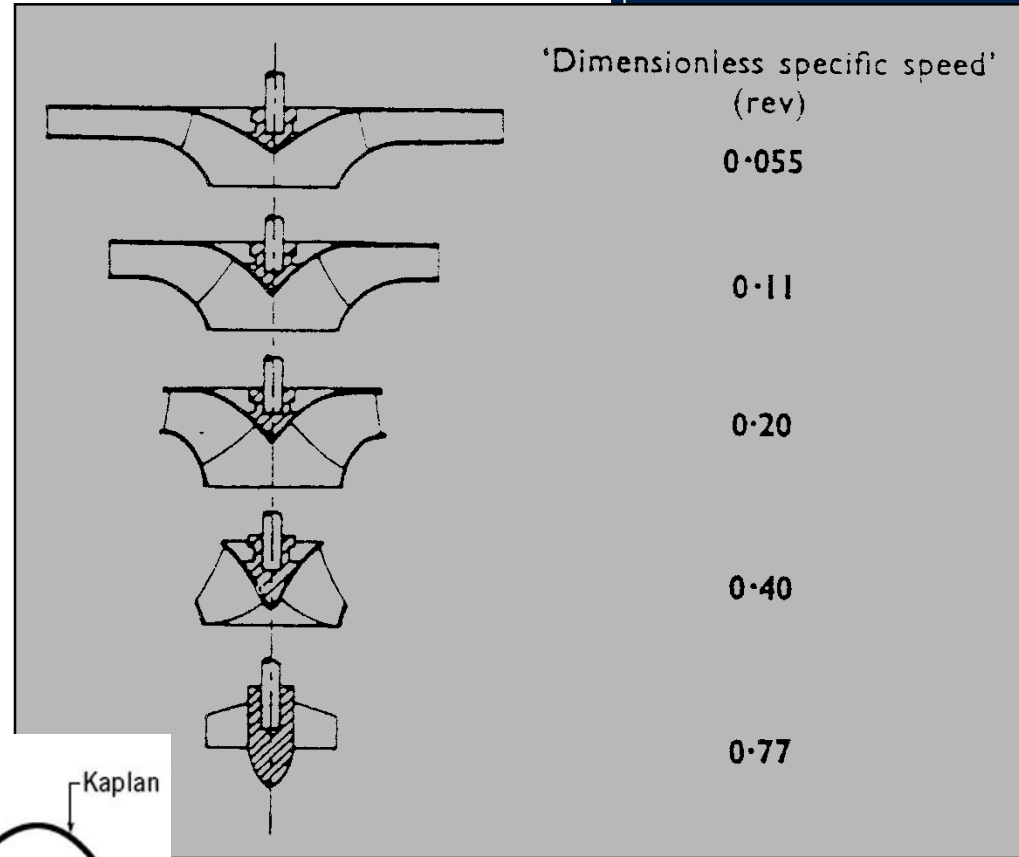
- Annual electric energy production depends on the available water supply.
- In some installations, the water flow rate can vary by a factor of 10:1 over the course of a year.

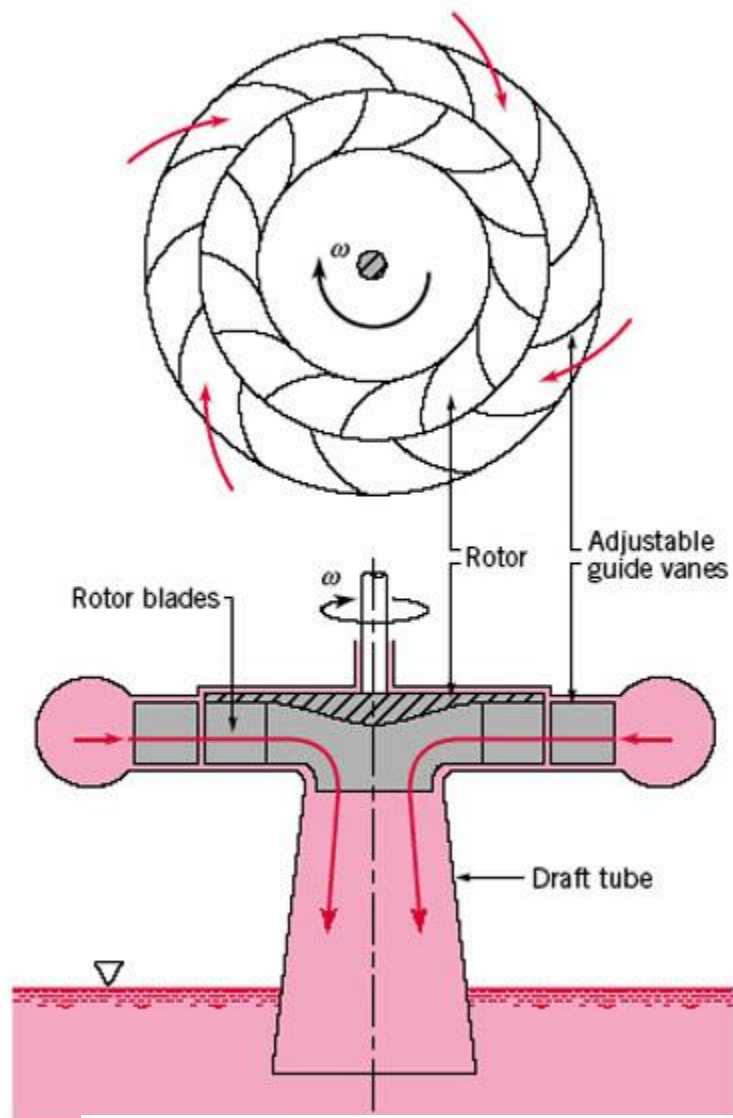
# Effect of Runner Shape - Dimensionless Specific Speed



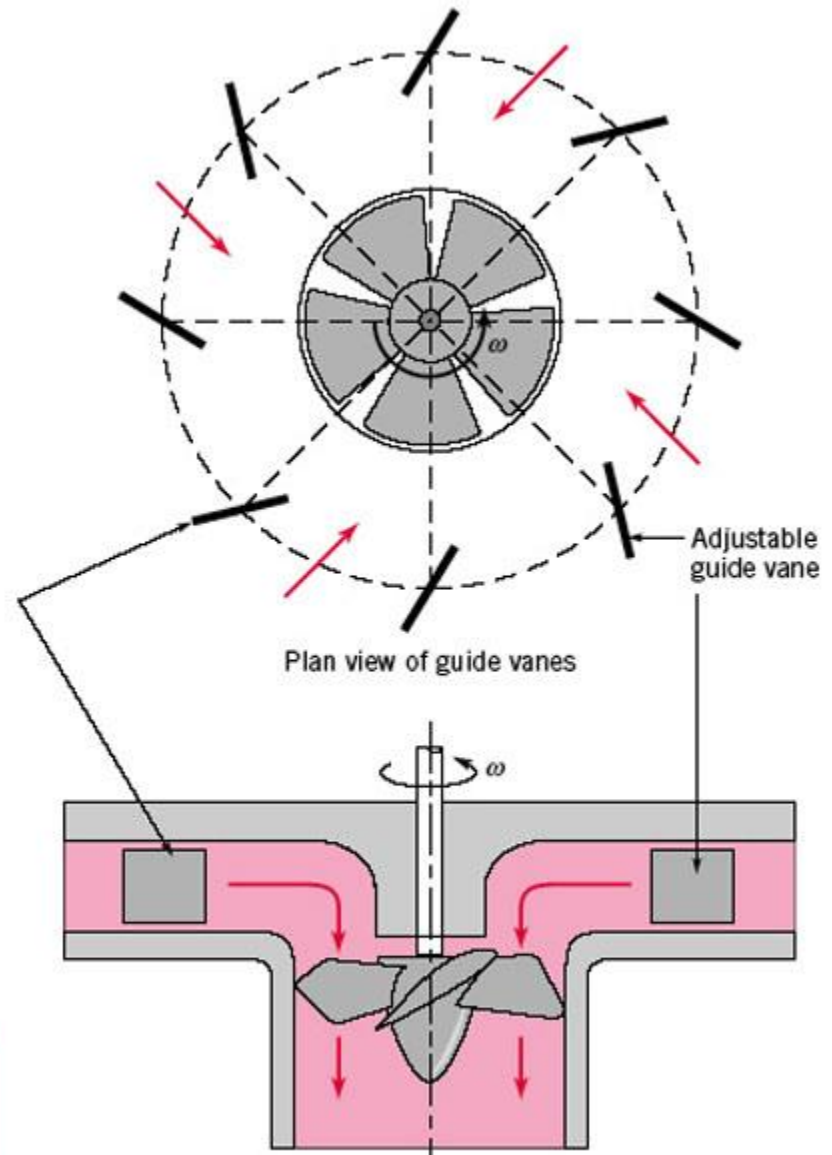
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$$N_s = \frac{n\sqrt{Q}}{(gH)^{3/4}}$$



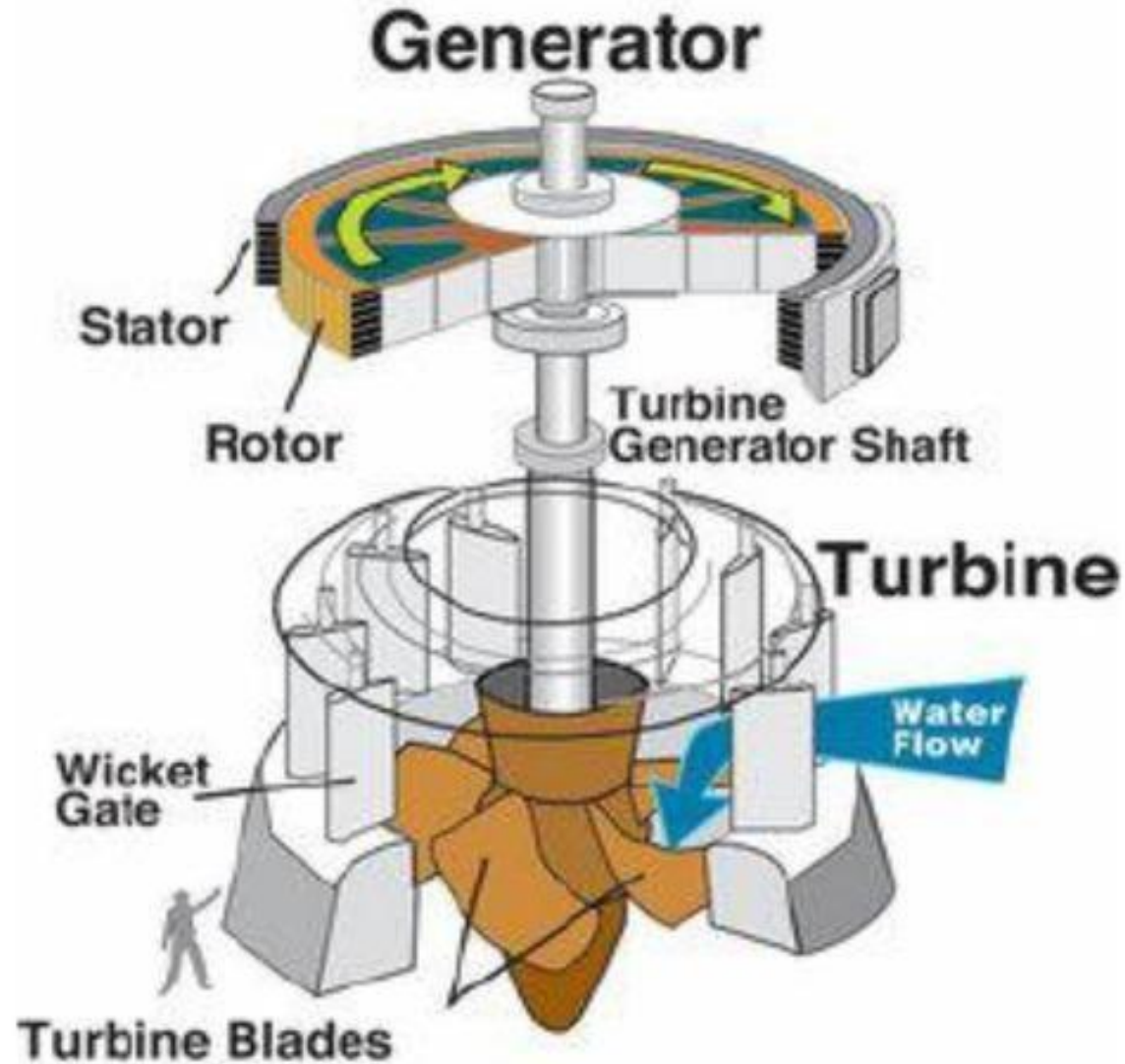


Francis



Kaplan

# Turbine and Generator arrangement

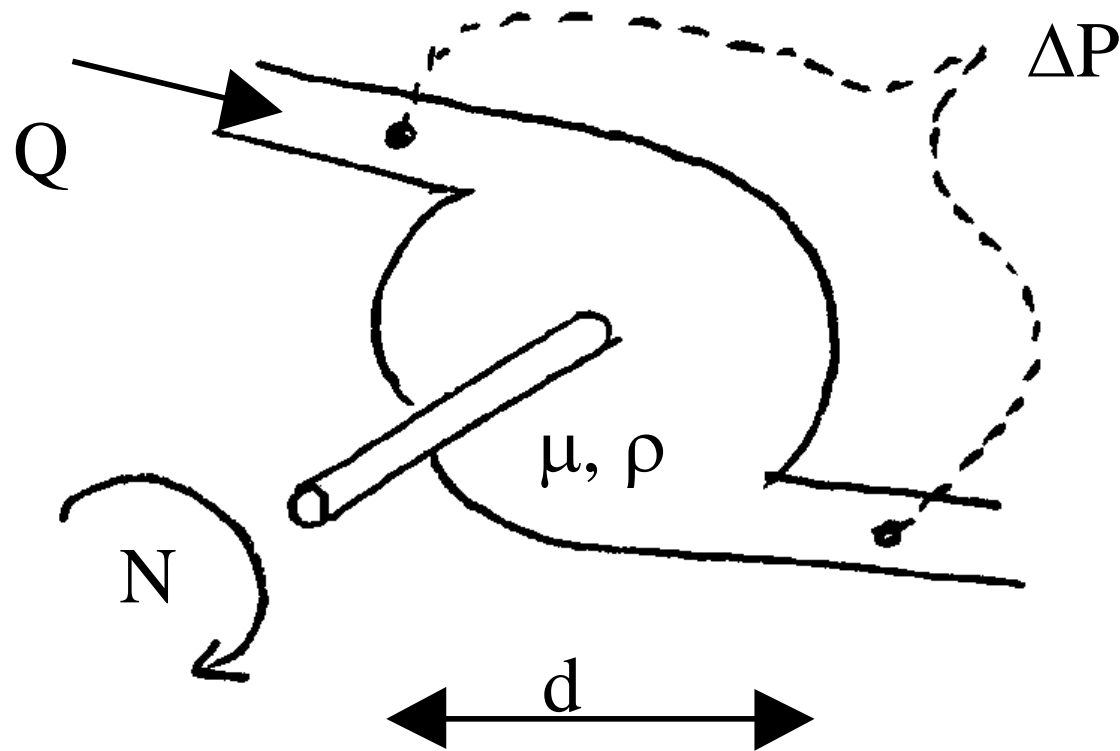


# Application of Dimensional analysis

Considering a pump or fan we are interested in total pressure across the pump,  $\Delta P$ . It is a function of the following

- Size,  $d$ ,
- Speed of rotation,  $N$ ,
- Volume flow rate through the pump,  $Q$ ,
- Fluid properties of density and viscosity,  $\rho$  and  $\mu$ .

# Application of Dimensional analysis to a Pump



Six interrelated variables, three fundamentals, thus three dimensionless groups



# Three Dimensionless Groups

Variables	Symbols	Dimensions
Pressure increase	$\Delta P$	$\text{kg} / \text{ms}^2$
pump size	$d$	$\text{m}$
speed of rotation	$N$	$1 / \text{s}$
Volume flow rate	$Q$	$\text{m}^3 / \text{s}$
Density	$\rho$	$\text{kg} / \text{m}^3$
Viscosity	$\mu$	$\text{kg} / \text{ms}$

- Discharge number

$$(\pi_D): \frac{Q}{Nd^3} \longleftarrow \frac{\text{m}^3 / \text{s}}{(1 / \text{s})(\text{m})^3}$$

- Head number

$$(\pi_h): \frac{\Delta P}{\rho d^2 N^2} \longleftarrow \frac{\text{kg} / (\text{ms}^2)}{(\text{kg} / \text{m}^3)(\text{m})^2 (1 / \text{s})^2}$$

- Reynolds Number

$$(\text{Re}): \frac{\rho Nd^2}{\mu} \longleftarrow \frac{(\text{kg} / \text{m}^3)(1 / \text{s})(\text{m})^2}{\text{kg} / \text{ms}}$$

# Three Dimensionless Groups

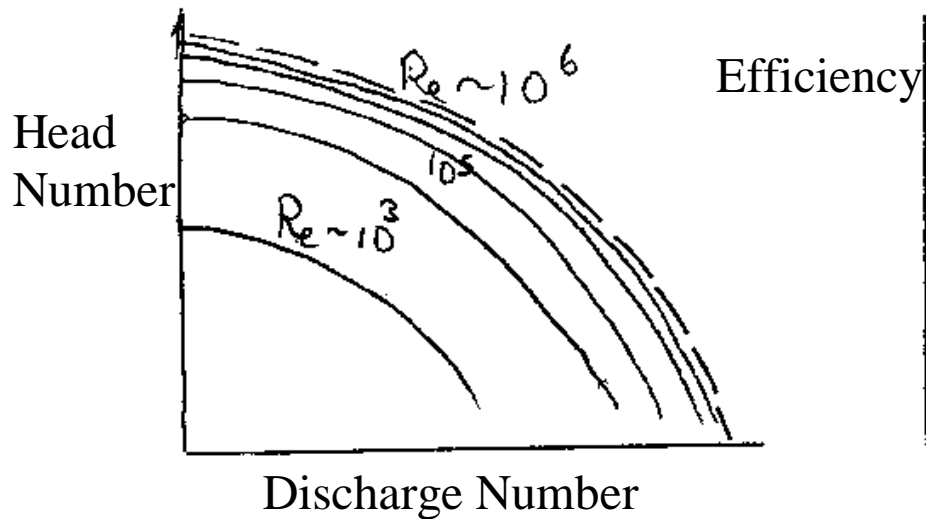
- Discharge number  $(\pi_D)$ :  $\frac{Q}{Nd^3}$

- Power number  $(\pi_p)$ :  $\frac{W}{\rho d^5 N^3}$

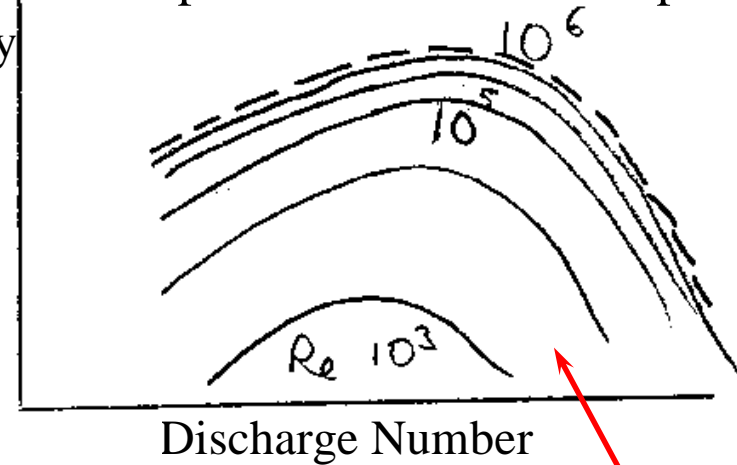
$$\begin{aligned} & \frac{J / s}{(kg / m^3)(m)^5 (1 / s)^3} \\ &= \frac{(kgm / s^2)(m) / s}{(kg / m^3)(m)^5 (1 / s)^3} \end{aligned}$$

- Reynolds Number  $(Re)$ :  $\frac{\rho Nd^2}{\mu}$

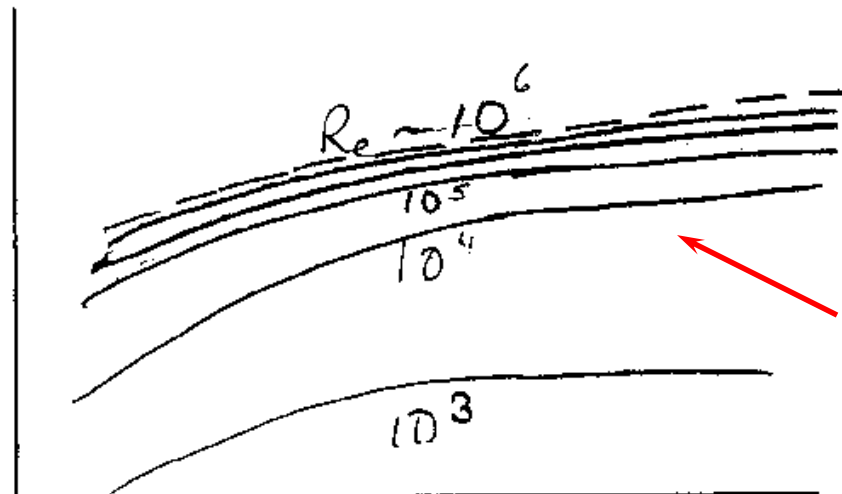
# Non-Dimensional Numbers for a Centrifugal Pump



Pump work over electrical power



Power Number



$$\Delta h_T = \frac{1}{\rho} \Delta P_T$$

$$W = m \times \frac{1}{\rho} \Delta P_T$$

# Addison Shape Number (Specific Speed)

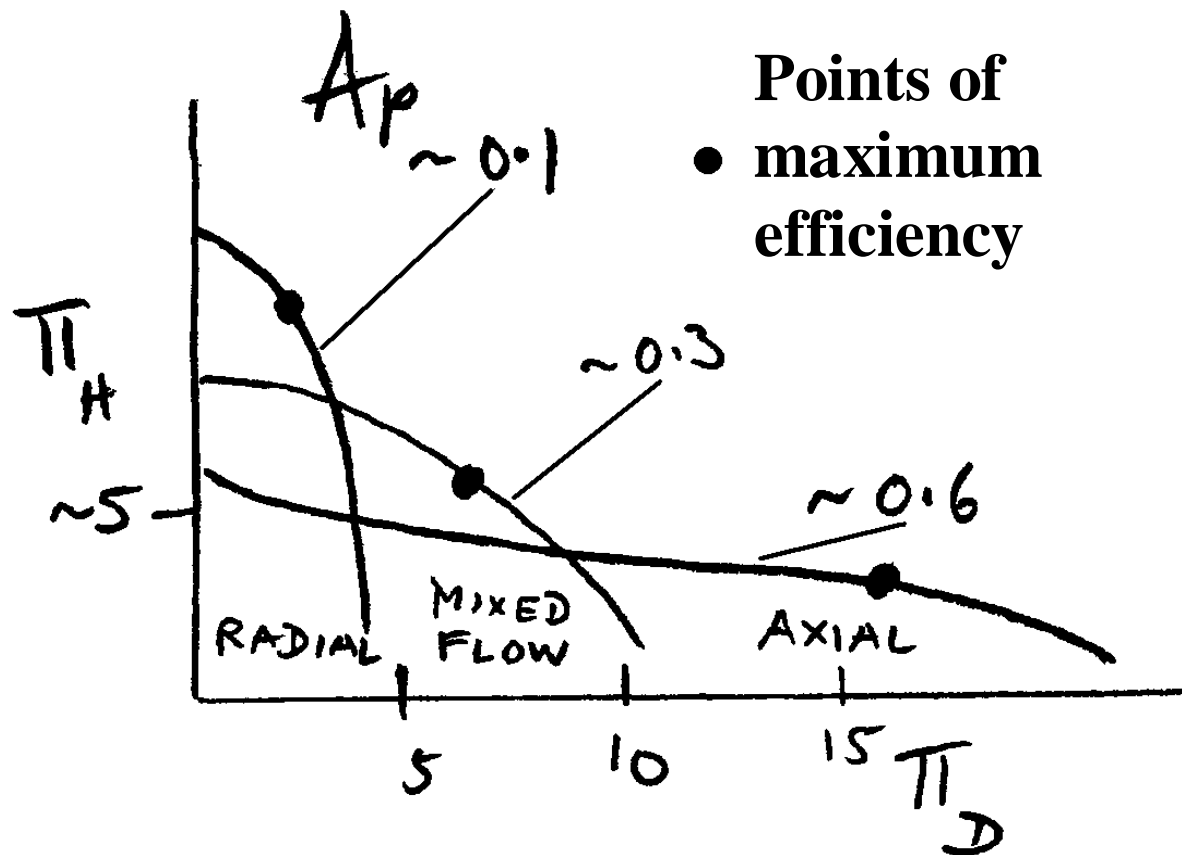
- Eliminating  $d$  between the head number and discharge number

$$\frac{\pi_D^2}{\pi_H^3} = \frac{\frac{Q^2}{N^2 d^6}}{\frac{\Delta P^3}{\rho^3 d^6 N^6}} = \frac{\rho^3 Q^2 N^4}{\Delta P^3}$$

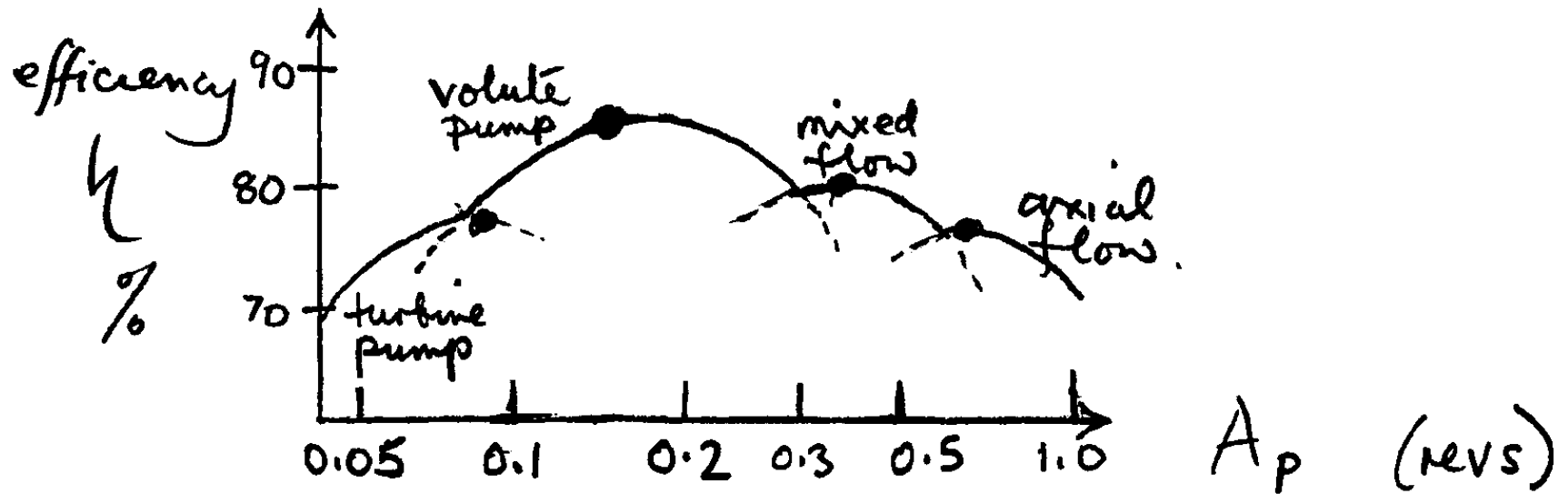
- Taking the fourth root to give  $N$  to first power

$$A_p = \frac{\rho^{3/4} Q^{1/2} N}{\Delta P^{3/4}}$$

# Addison Number as Function of Head Number and Discharge Number for Radial, Mixed and Axial Flow Turbomachines



# Efficiency as the Function of Addison Number for Variety of Turbomachines.





# CORDIER Diagram.

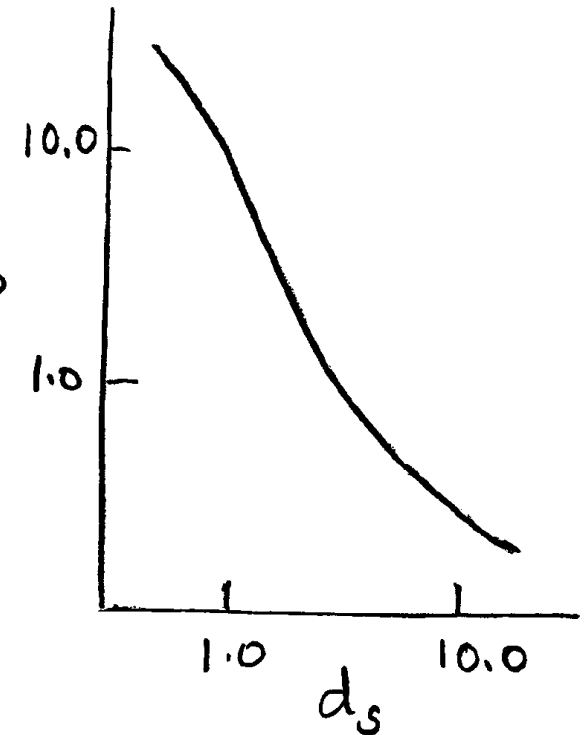
Eliminating  
speed  $N$   
between the  
head number  
and discharge  
number

Specific diameter:  $d_s$

$$\frac{\Delta p^{1/4}}{\rho^{1/4} Q^{1/2}} d$$

$2\pi \times A_p$

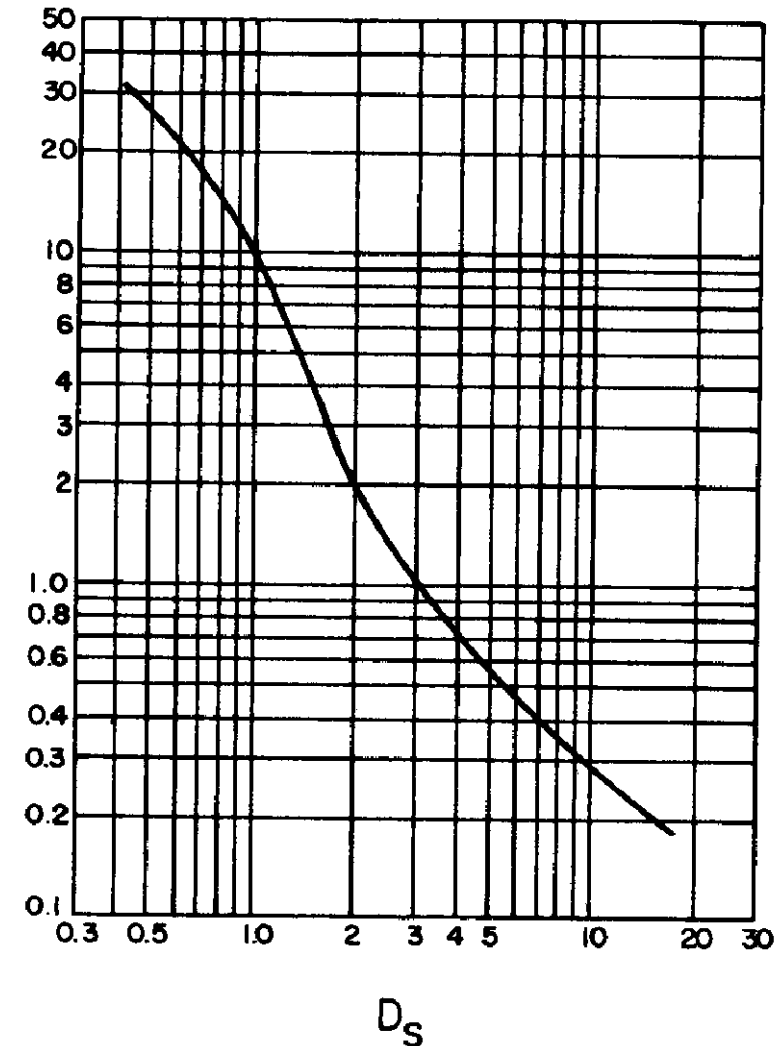
At the point of maximum  
efficiency there will be a given  
 $A_p$  and hence a corresponding  
 $d_s$ .



# The Use of Cordier Diagram

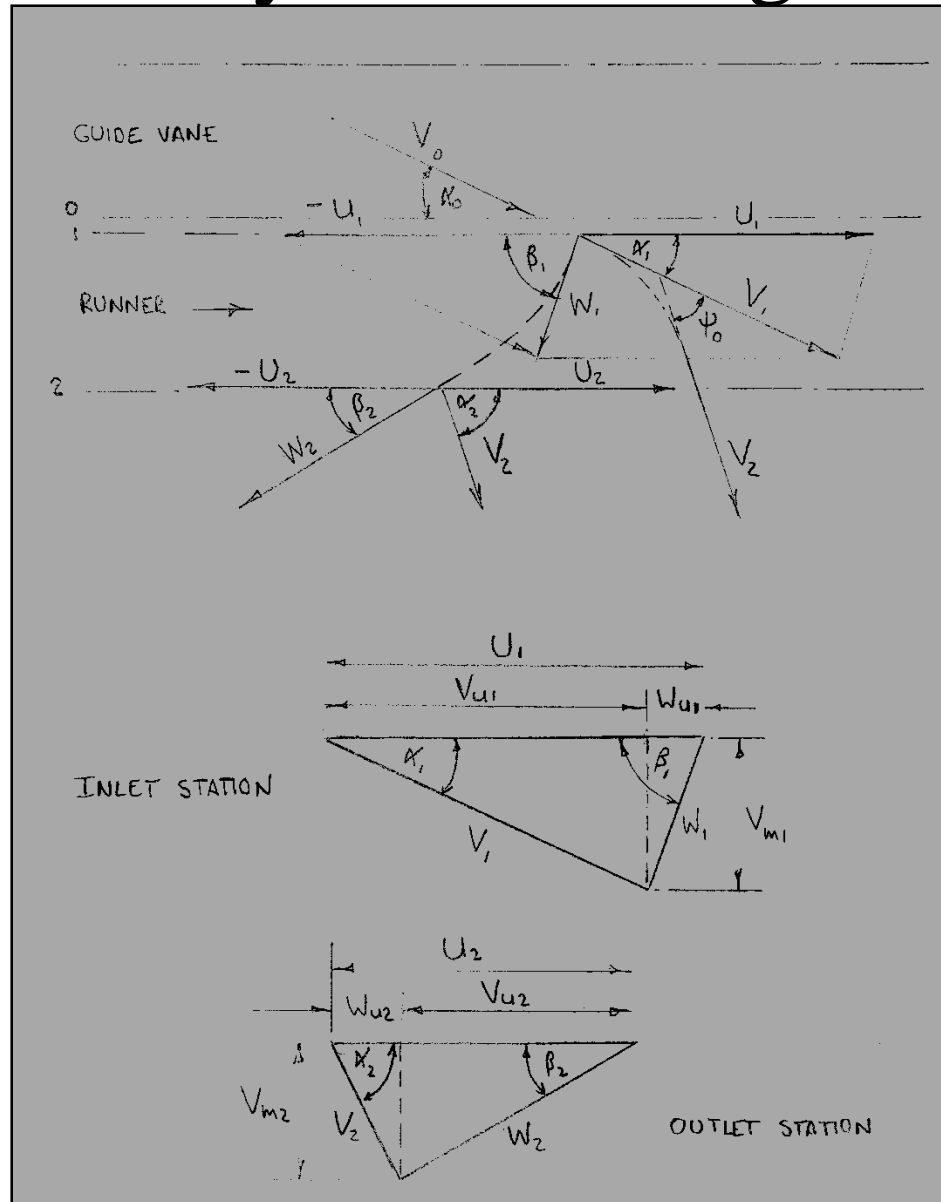
**Table 3.2** Specific Speeds

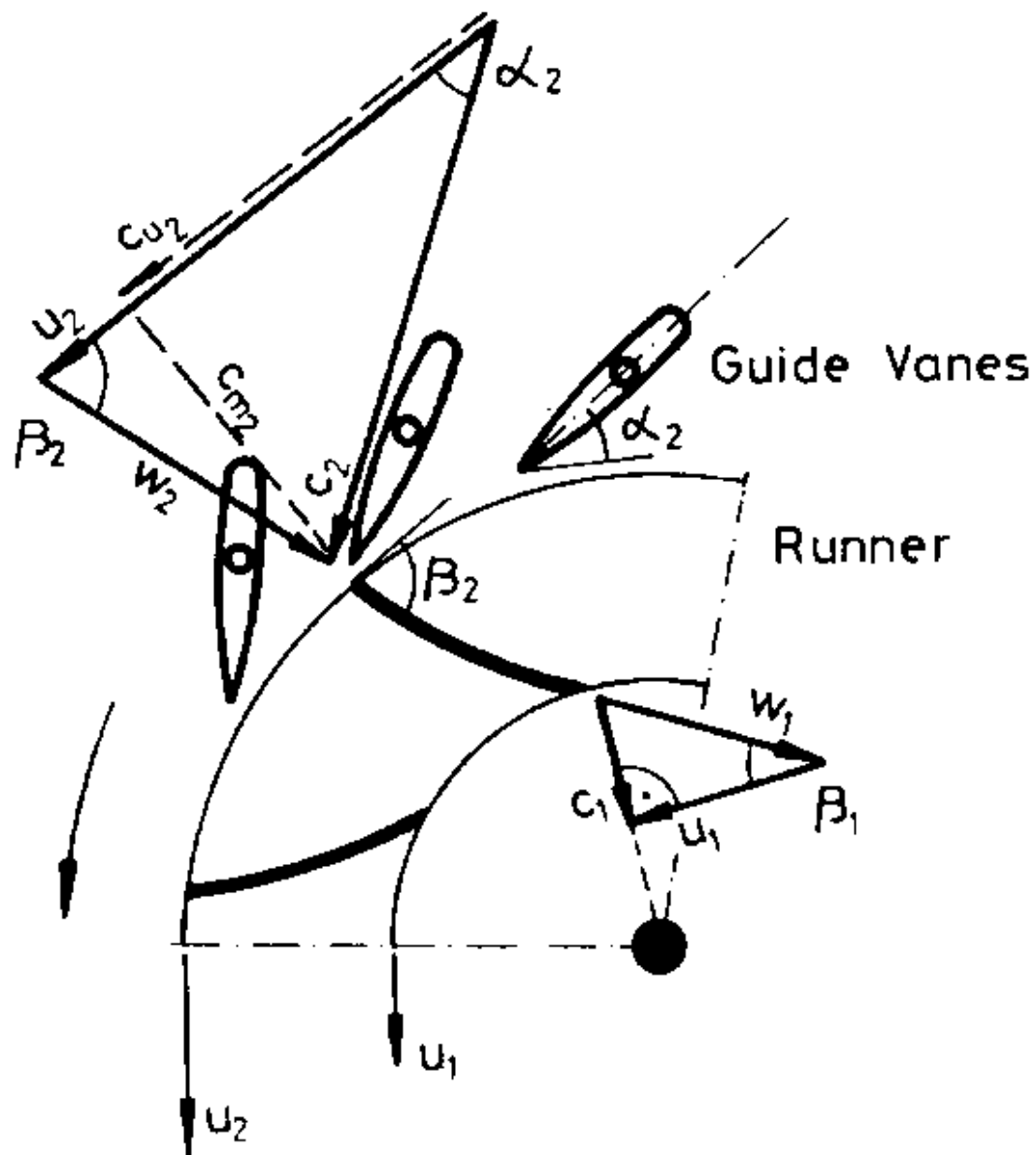
Turbomachine	Specific speed range	$N_s$
Pelton wheel	0.03– 0.3	
Francis turbine	0.3 – 2.0	
Kaplan turbine	2.0 – 5.0	
Centrifugal pumps	0.2 – 2.5	
Axial-flow pumps	2.5 – 5.5	
Centrifugal compressors	0.5 – 2.0	
Axial-flow turbines	0.4 – 2.0	
Axial-flow compressors	1.5 –20.0	



- Find optimized diameter

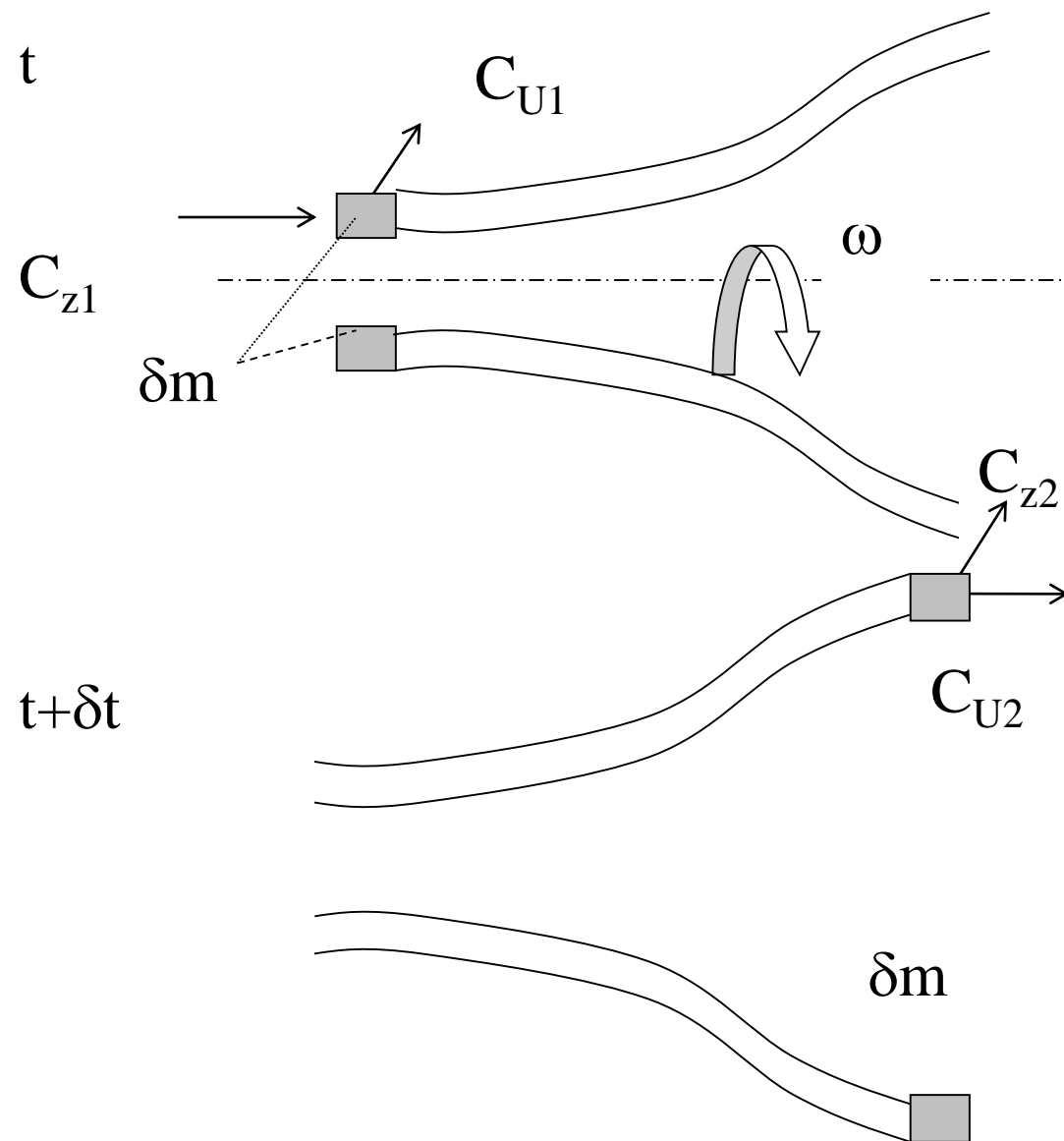
# Velocity Vector Diagrams





**FIGURE B. 7 :**  
**Velocity vector diagrams for a Francis turbine**

# Euler's Equation



**Angular impulse = Change in angular momentum**

At time  $t$ , =  $\delta m r_1 C_{u1}$  + angular momentum within the machine

At time  $t + \delta t$ , =  $\delta m r_2 C_{u2}$  + angular momentum within the machine

$$T \delta t = \delta m (r_2 C_{u2} - r_1 C_{u1})$$

$$T = m (r_2 C_{u2} - r_1 C_{u1})$$

$$\text{Power} = T \omega =$$

$$= m (r_2 \omega C_{u2} - r_1 \omega C_{u1})$$

$$\text{Power} = m (u_2 C_{u2} - u_1 C_{u1})$$

$$w = (u_2 C_{u2} - u_1 C_{u1})$$

# Euler's Equation

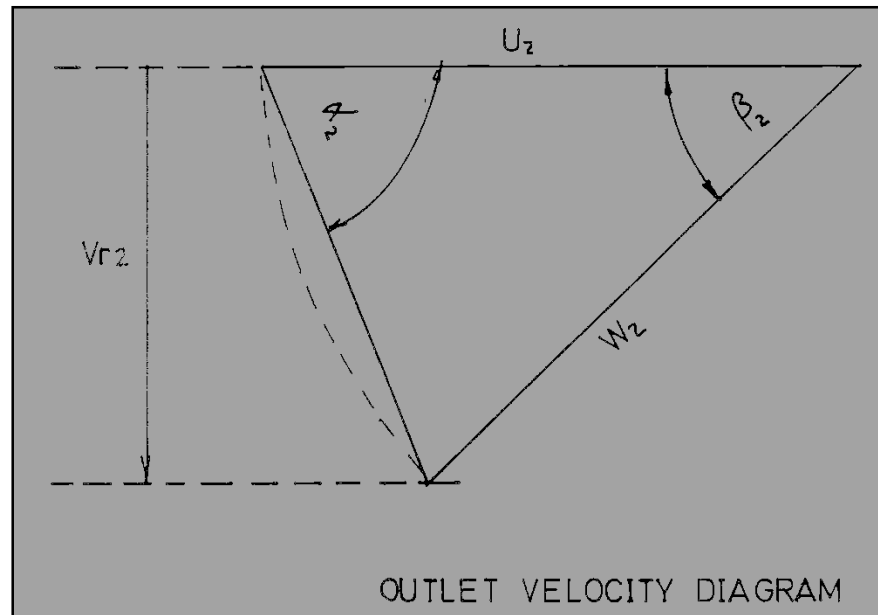
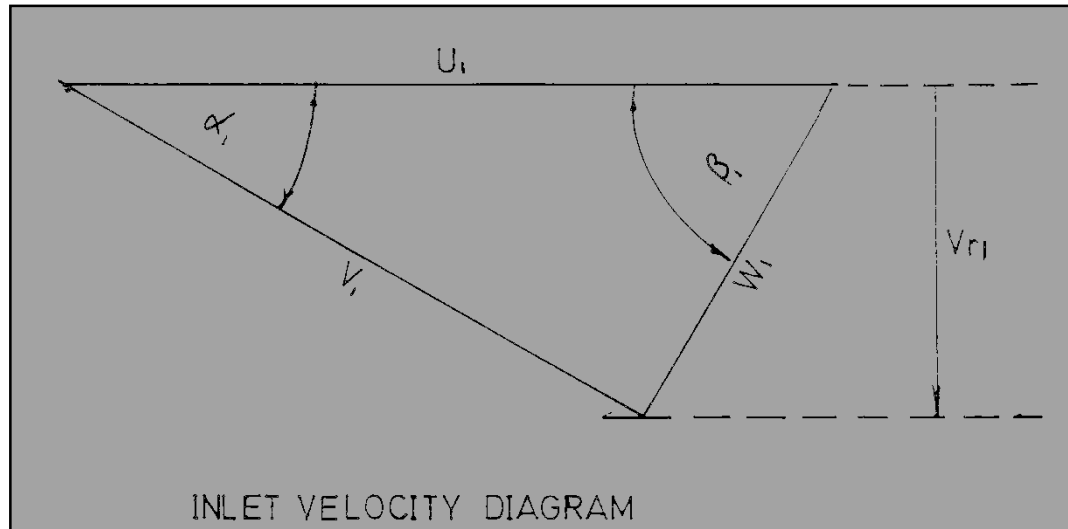
- Conditions are uniform at inlet and outlet
- Power input  $m(U_2 C_{u2} - U_1 C_{u1})$

- From the steady flow energy equation

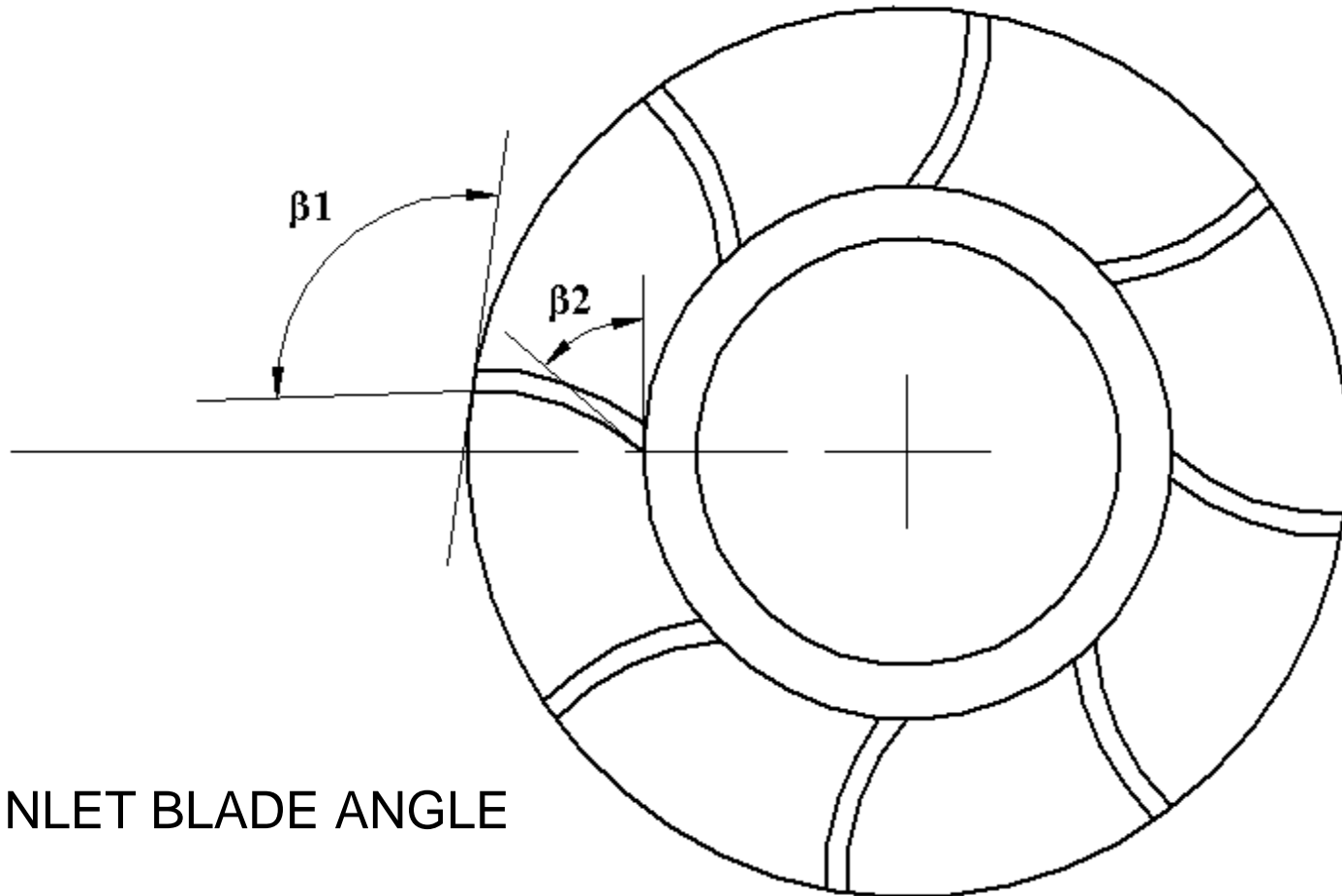
$$h_{T2} - h_{T1} = (U_2 C_{u2} - U_1 C_{u1})$$



# Blade Angle Construction



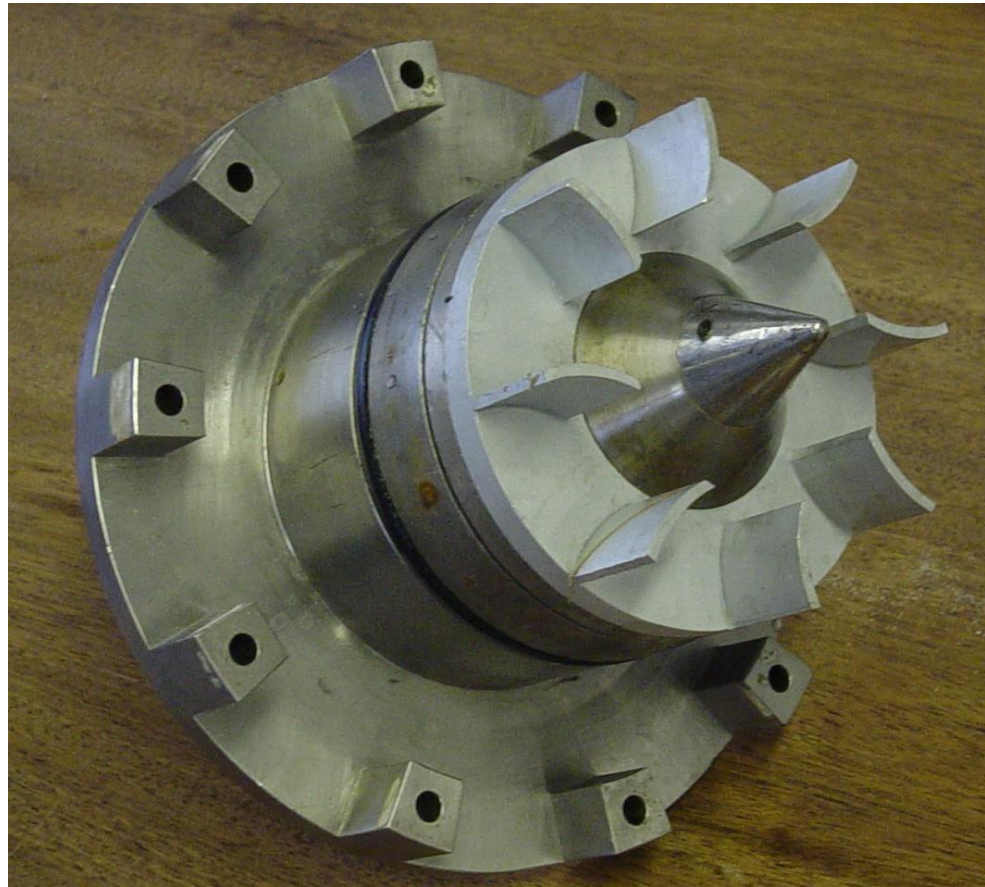
# Blade Angle Construction



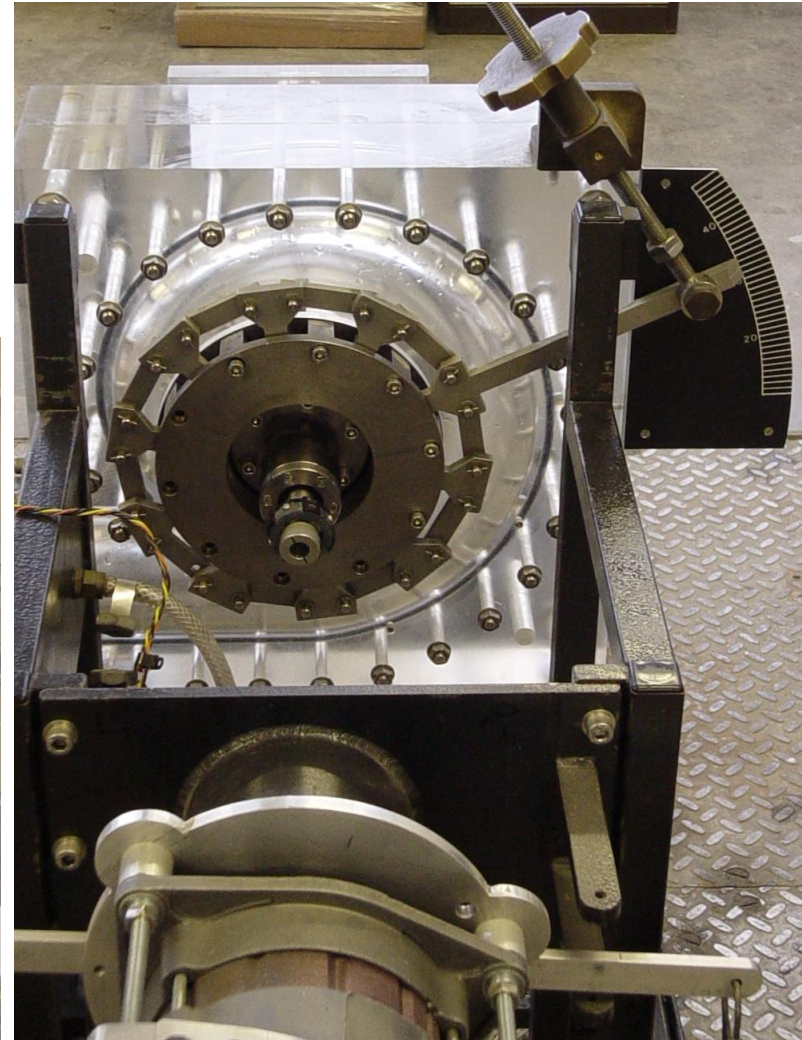
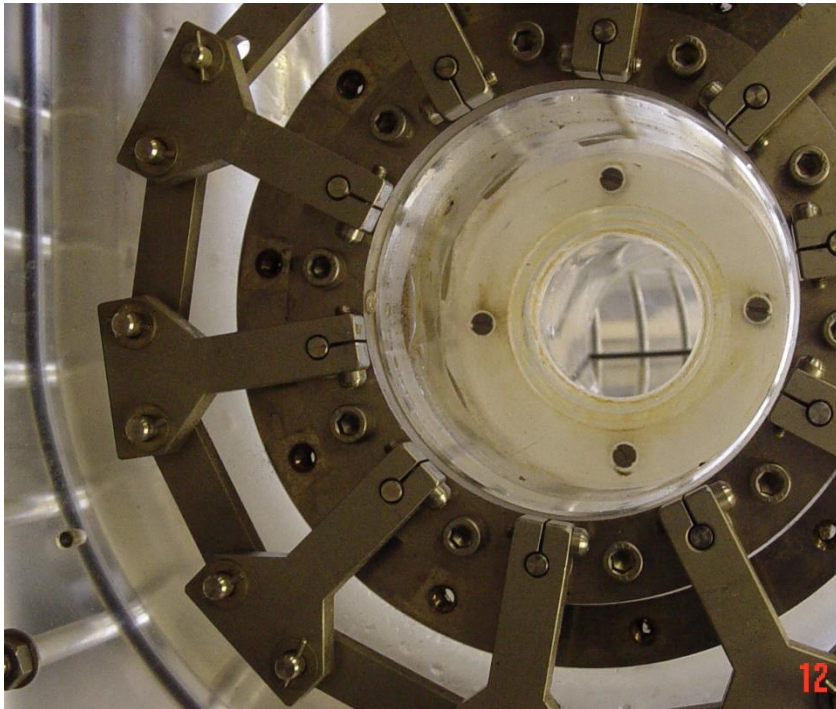
$\beta_1$ - INLET BLADE ANGLE

$\beta_2$ - OUTLET BLADE ANGLE

# Turbine Runner



# Speed Control



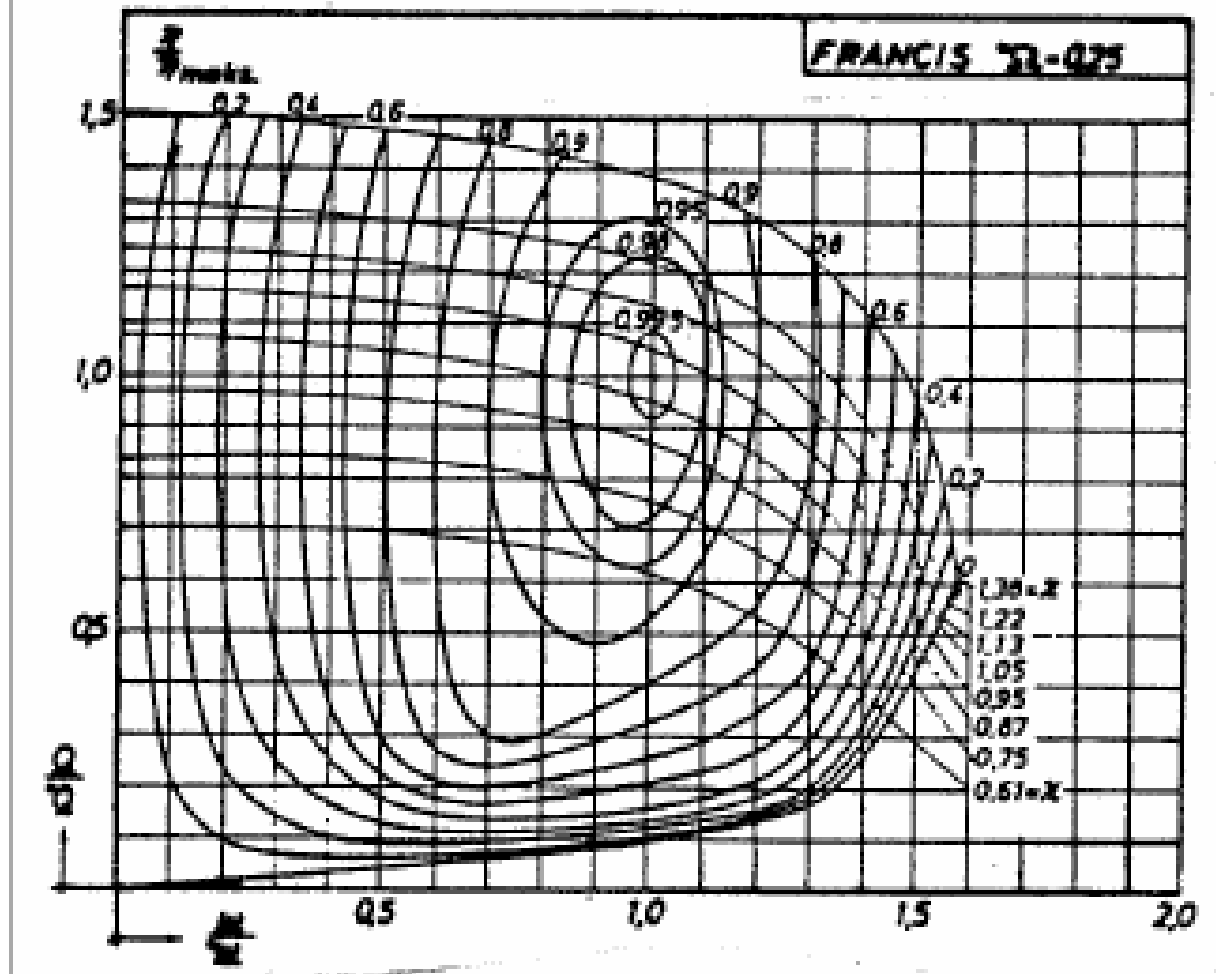
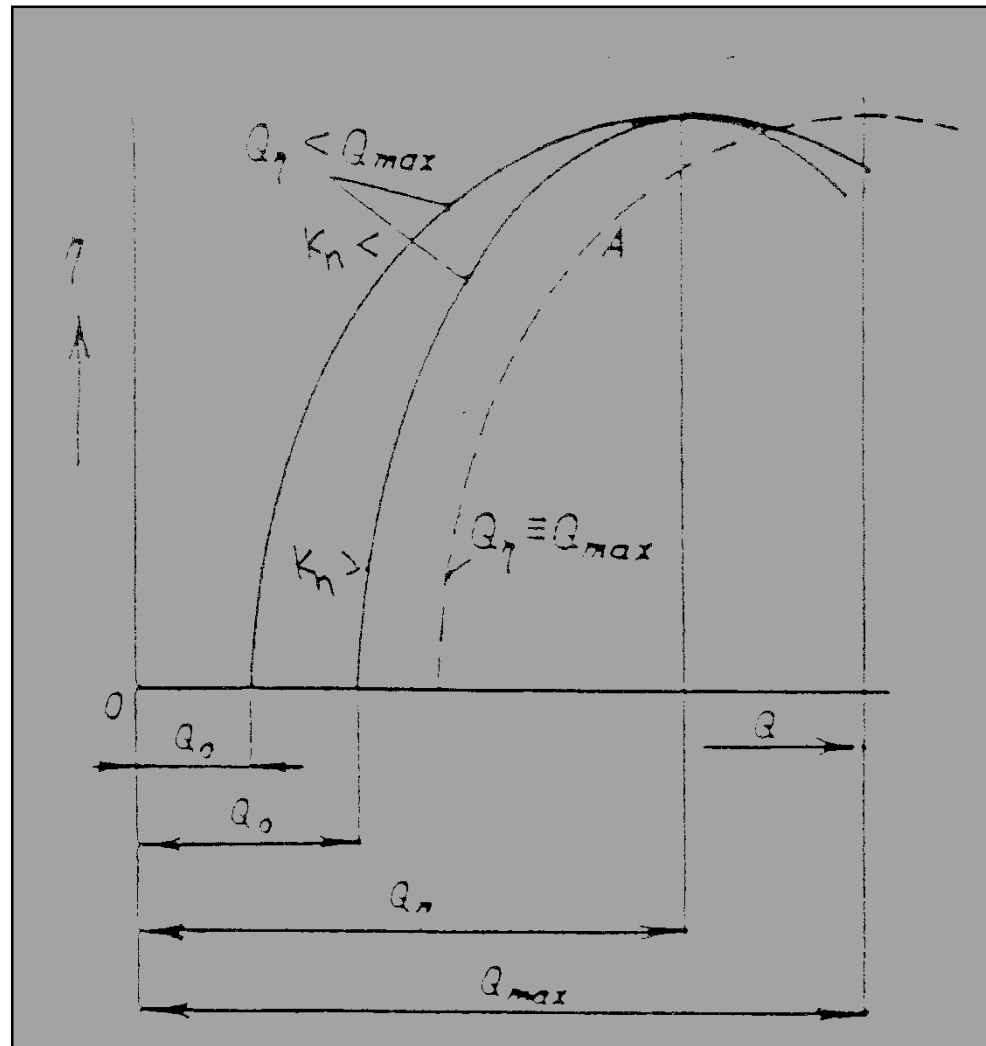


Fig. 3.7 Performance diagram<sup>[5]</sup> of a Francis turbine  
 \*  $\underline{\Omega} = 0.25$

# Efficiency Variation With Flow Rate (Constant Speed)



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# Power Output Variation With Flow Rate

