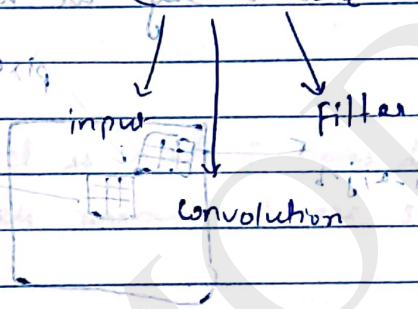
* Convolutional Neural Network

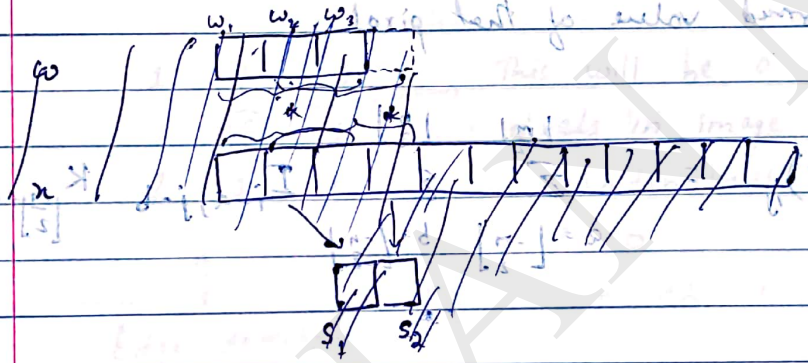
→ 1D - Convolution

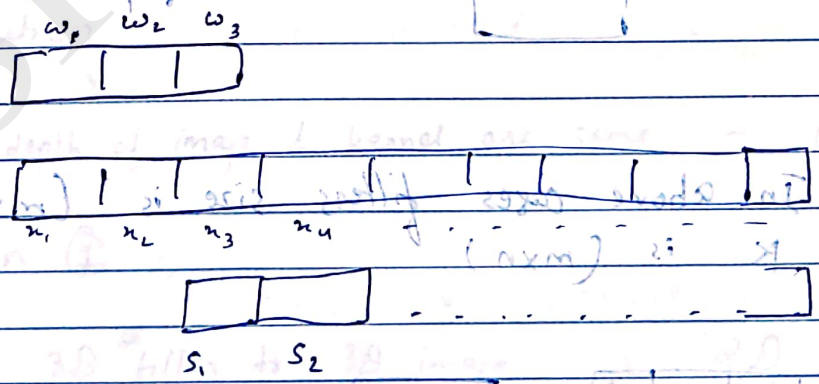
$$s_t = \sum_{a=0}^{n} x_{t-a}\, w_a \qquad (n \rightarrow \text{size of filter})$$

$$s_t = (x * w)_t$$



→ Let's say $n = 3$





$$S_1 = x_1 w_1 + x_2 w_2 + x_3 w_3$$

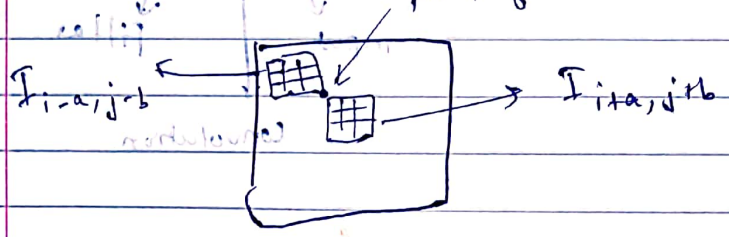
$$S_2 = x_2 w_1 + x_3 w_2 + x_4 w_3$$

:

→ 2D Convolution

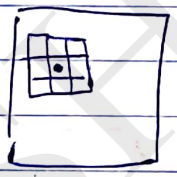
- $$S_{ij} = (I * K)_{ij} = \sum_{a=0}^{m-1} \sum_{b=0}^{n-1} I_{i+a, j+b} K_{a,b}$$

: lets say we have image



we multiply matrix after or before the pixel with filter to get transformed value of that pixel

- $$S_{ij} = (I * K)_{ij} = \sum_{a=\lfloor -\frac{m}{2} \rfloor}^{\lfloor \frac{m}{2} \rfloor} \sum_{b=\lfloor -\frac{n}{2} \rfloor}^{\lfloor \frac{n}{2} \rfloor} I_{i-a, j-b} \; K_{\lfloor \frac{m}{2} \rfloor + a, \lfloor \frac{n}{2} \rfloor + b}$$



get value of pixel by multiplying matrix of pixels around it by filter

Note: In above cases filters size is (m x n) ie size of K is (m x n)

a	b	c	d
e	f	g	h
i	j	k	l

w	x
y	z

$\Rightarrow$

$aw+bx+ey+fz$	$bw+cx+fy+gz$	$cw+dx+gy+hz$
$ew+fx+iy+jz$	$fw+gx+jy+kz$	$gw+hx+ky+lz$

→ Types of Filters

- $\begin{matrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{matrix}$ → replacing this pixel in image with weighted average of all pixel in around it

  3 × 3

  ↑ Blur

- $\begin{matrix}0 & -1 & 0\\ -1 & 5 & -1\\ 0 & -1 & 0\end{matrix}$ → central pixel is made 5 times & all other pixels around it are subtracted

  ↑ Sharpen

- $\begin{matrix}1 & 1 & 1\\ 1 & -8 & 1\\ 1 & 1 & 1\end{matrix}$ → This will be 0 only if all the corresponding pixels in image are same

  $I * f = 1x + 1x + 1x + 1x + -8x + 1x + 1x + 1x + 1x$

  $= 0$

  ↑ Edge detector

→ 3D Convolution

Assume that depth of image & kernel are same — (1)

with assumption (1)

if we apply 3D filter to 3D image a 2D feature map is obtained

refer /notes/ week 11 / 3D conv.png

* Relation (input size, output size, filter size)

→ Let an image have dimensions $W_1 \times H_1$
filter have dimensions $F \times F$

The transformed image will have dimensions $W_2 \times H_2$

$$W_2 = W_1 - F + 1 = W_1 - (F-1)$$
$$H_2 = H_1 - F + 1 = H_1 - (F-1)$$

→ In almost all cases we might have to add padding (Transform all pixels by taking them at centre)
$P = 1$ is added to all four size of input matrix
The formula then becomes as follows

$$W_2 = W_1 + 2P - F + 1$$
$$H_2 = H_1 + 2P - F + 1$$

→ We may keep a stride as per output dimensions needed

$$W_2 = \frac{W_1 + 2P - F}{S} + 1$$
$$H_2 = \frac{H_1 + 2P - F}{S} + 1$$

Main formula

(Q)

$227 \times 227 \times 3$ image * $(11 \times 11 \times 3)$ 96 filter $\Rightarrow$ 9 output

$W_2 \times H_2 \times D$

Ans Since filter depth & img depth are same
Thus for each filter 2D feature map is obtained

Since there are 96 filters

$D = 96$

→ given → $s = 4$, $p = 0$

$$W_2 = \frac{227 - 11}{4} + 1 = 55$$

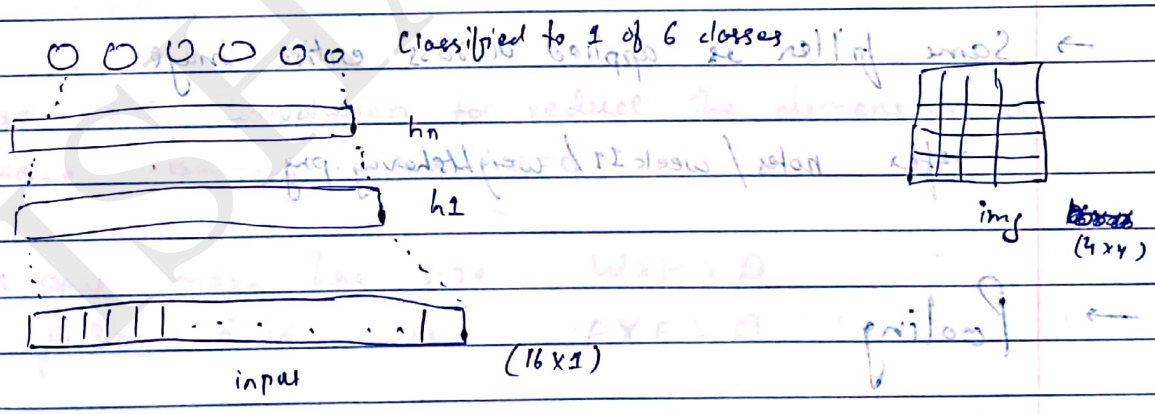
$$H_2 = \frac{227 - 11}{4} + 1 = 55$$

* CNNs

ML approach → Till now we focused on applying predefined static kernels/filters on image. Then we can learn weights to classify

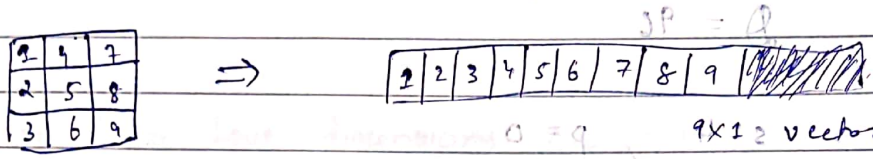
DL approach → Deep learning focuses on learning these filters. Then we learn weights to classify

→ For Feed Forward NN



It is a dense connection and there are a lot of parameters since each neuron in previous layer contributes to formation of neuron in next layer.

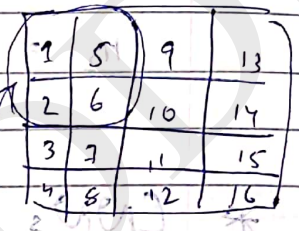
→ Convolution NN

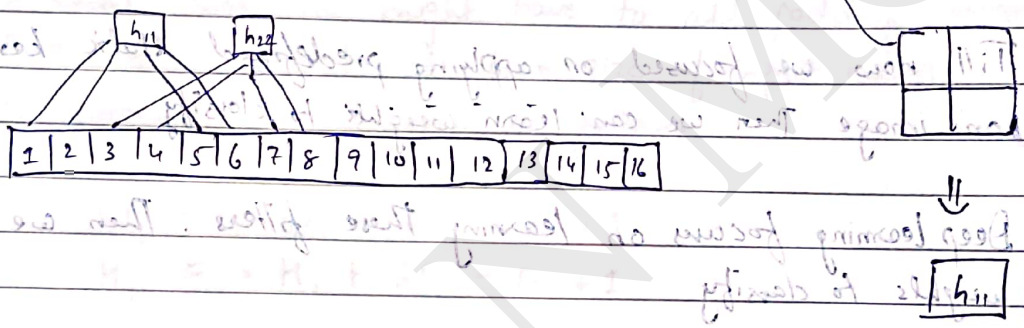


3×3 img. 9×1 vector

(1)

So this is comparatively sparse connection





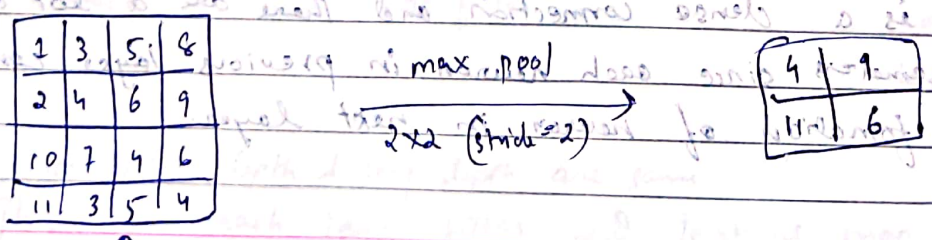
(2) Weight Sharing

→ Same filter is applied across entire image

refer notes / week II / weightsharing.png

→ Pooling

Used to reduce dimension of image



feature map obtained after applying filter on img

Sample Convolutional NN (LeNet-5 Optical Character Recognition Handwritten

Refer - notes / week11 / LeNet-5.png

DELUXE PAGE NO.: DATE:

Similarly avg-pool 2x2 (stride=2)

(1+2+3+5)/4	(5+6+8+9)/4
(10+7+8+11)/4	(4+4+6+5)/4

* Image Net

→ AlexNet

refer notes / week 11 / AlexNet.png

→ ZFNet

refer notes / week 11 / ZFNet.png

→ VGGNet

refer notes / week 11 / VGGNet.png

→ GoogleNet

- Use 1x1 convolution to reduce the dimension of image assuming s=1, p=0

(i) let's say image has size W x H x D
& filter size F x F x D

Total Computations required = F x F x D
by each element of o/p

(ii) if we use 1x1xD convolutions & D' such convolutions then

(W x H x D) * (1 x 1 x D) ⇒ W x H x D'

If $D' < D$: computations are reduced
now applying filter. size $F \times F \times D'$

Total computation for each element of o/p = $F \times F \times D'$ significantly reduced

- GoogleNet has various inception modules having basic layout as follows

refer notes / week 11 / inceptionmodule.png

other inception modules structures

refer ~~x~~ referenceppt / 11 / slides 423 - 442

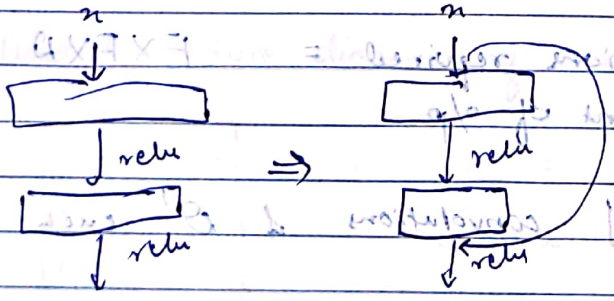
- In the last convolution layer

$7 \times 7 \times 1024 \xrightarrow{\text{avg pool}} 1 \times 1 \times 1024$

↓

equivalent to a vector
can be connected with dense layer with less parameters than earlier

→ ResNet refer / notes / week 11 / resnet.png



Skip connection
(deep network can perform as good as a better shallow network)

In residual network, in forward pass current layer that has finer features is aggregated with previous layers [(2 - 3) layers before the current layer is aggregated]

Generally during backpropagation if network is huge / contains many layers the gradient vanishes during backprop.
But if we have residual connection it doesn't allow the gradient to vanish and thus relevant update is made to the weight & the network learns.

WEEK 12

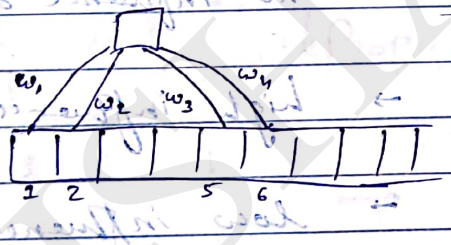
* Visualizing Patches

→ given a neuron in any layer one can always find a patch in the image which is responsible for that neuron to fire

* Visualizing Filters of a CNN

→ refer autoencoders deep learning (2) how to visualize input using weights for a neuron to fire

→ Similar to that we have learned weights to determine the input & thus determine which patch of input is responsible for a neuron to fire



$$\begin{bmatrix} w_1 & w_3 \\ w_2 & w_4 \end{bmatrix} \Rightarrow \boxed{w_1 | w_2 | w_3 | w_4} = w'$$

$$\boxed{x_1 | x_2 | x_5 | x_6} = \frac{w'}{\|w'\|}$$

Thus for these values of input the given neuron will fire.

* Occlusion Experiments

→ Take a patch in image & replace it by black box
~~none~~ now try to classify

If you get correct classification that patch doesn't affect o/p has a very less impact on o/p

Similarly do it for all patches to understand important patches for classification

* Finding influence of Input Pixels using Backprop

→ Gradients with respect to input give the influence

Lets say h_j is a neuron of hidden layer j

Now, if

$\frac{\partial h_j}{\partial x_i} = 0$ → no influence of x_i on h_j

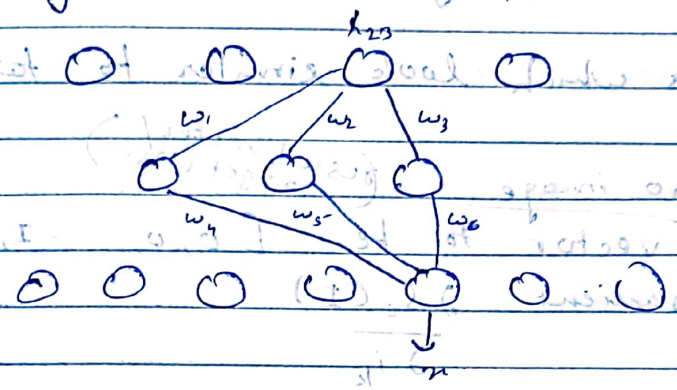
$\frac{\partial h_j}{\partial x_i}$ = large → high influence

$\frac{\partial h_j}{\partial x_i}$ = small → low influence.

So we represent images as gradients

$\frac{\partial h_j}{\partial x_{00}}$	$\frac{\partial h_j}{\partial x_{01}}$	...	$\frac{\partial h_j}{\partial x_{mn}}$
⋮	⋮	⋮	

- Finding gradient wrt input.



So we want to find

$$\frac{\partial h_{23}}{\partial x} = \frac{\partial h_{23}}{\partial h_{11}}\frac{\partial h_{11}}{\partial x} + \frac{\partial h_{23}}{\partial h_{12}}\frac{\partial h_{12}}{\partial x} + \frac{\partial h_{23}}{\partial h_{13}}\frac{\partial h_{13}}{\partial x}$$

$$= w_1 w_4 + w_2 w_5 + w_3 w_6$$

Now, ~~plot~~ ~~image~~ calculate similarly for all input & plot img.

* Guided Backpropagation (Technique used visualizing the impact of image pixels on result)

1	-1	5
2	-5	-7
-3	2	4

→

1	0	5
2	0	0
0	2	4

(-1, -5, -7 become zero because of ReLU) } Forward pass

-2	0	-1
6	0	0
0	-1	3

←

-2	3	-1
6	+3	1
2	-1	3

} Normal Backprop

0	0	0
6	0	0
0	0	3

←

-2	3	-1
6	-3	1
2	-1	3

All negative gradients in back pass are made zero } Guided Back Prop

* Optimization Over Images

→ Generating images which look similar to target class

(1) Start with zero image → (result vector / target class)
(2) Set the (score) vector to be $[0, 0, \ldots -1, 0, 0]$
(3) Compute the gradient $\frac{\partial S_c(I)}{\partial i_k}$
(4) Update pixel $i_k = i_k - \eta \frac{\partial S_c(I)}{\partial i_k}$
(5) Now again do forward Pass
(6) Go to step 2

→ We can also create image such that a particular neuron of any hidden layer fires.

(1) Feeding to network
(2) Make activation such that selected neuron is one others are zero
(3) Backprop to image
(4) $i_k = i_k - \eta \cdot \frac{\partial A(I)}{\partial i_k}$, $A(I)$ is the activation of i^{th} neuron in some layer

* Creating Embeddings for Image

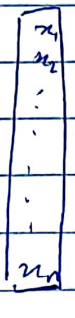
(used to construct an image that would give same embedding at wanted layer)

Any layer in the CNN can be considered as the embedding

Let i_{img} be the original image

We pass this image through the Network

At the fully connected layer some set of values are obtained



These values are called embedding values and entire set of values is one embedding

Now we take a random image $\hat{i}$ and try to reconstruct it such that when it is passed through network the corresponding Fully connected (FC) layer embedding is same.

loss

$$L(i) = \| \underbrace{\phi(x)}_{\text{obtained embedding}} - \underbrace{\phi_0}_{\text{required embedding}} \|^2 + \lambda \| \phi(x) \|_6^6$$

update rule

$$i_k = i_k - \eta \frac{\partial L(i)}{\partial i_k}$$

This abstractions of image can be constructed using image & image embeddings.

Similar is possible for convolution layers as well.

* Deep Dream

→ Here we make our loss function & update rule such that certain neurons can fire more.

So if h_{ij} is a neuron in convolution layer,

we want to maximize $L(I)$

i.e. $\max_I L(I)$

where

$$L(I) = h_{ij}^2$$

During backprop. update pixel of image

$$\frac{\partial L(I)}{\partial i_{mn}} = \frac{\partial L(I)}{\partial h_{ij}} * \frac{\partial h_{ij}}{\partial i_{mn}}$$

update rule

$$i_{mn} = i_{mn} + \frac{\partial L(I)}{\partial i_{mn}}$$

→ Result of this will be as follows:

During training it might have developed relation that if one neuron fires for sky other neurons might fire for bird, castle, etc.

Thus when we give image of sky and increase the probability of neuron denoting sky to fire. Then it automatically takes knowledge from training data & generates birds, castle as foreground.

* Deep Art

→ Making an original image a cartoon type ie styleing the image so it looks like cartoon

(1) Content Target

$$L_{content}(\vec{p}, \vec{x}) = \sum_{ijk} (\vec{p}_{ijk} - \vec{x}_{ijk})^2$$

($\vec{p}$: pixels of embeddings of actual image; $\vec{x}$: pixels of embeddings of new image.)

New image is generated such that all image representations / ~~of original or same~~ embeddings generated using new image are similar to embeddings generated by actual image

(2) Style (cartoon)

According to CV (computer vision) study $V \in R^{64 \times (258 \times 256)}$ (→ dimension of embedding)

$V^T V \in R^{64 \times 64}$ (gives style of image)

$$\therefore L_{style}(\vec{a}, \vec{x}) = \sum_{l=0}^{L} w_l E_l$$

} we want the style to match for all embeddings / layers

Here $E_l = \sum_{ij} (G_{ij}^l - A_{ij}^l)^2$

G_{ij}^l: Style gram matrix of original image [Style matrix generated from embedding obtained from original image]

A_{ij}^l: Style gram matrix of generated image [style matrix generated from embedding obtained from new image]

→ Total loss function

$$L_{total}(\vec{p}, \vec{a}, \vec{x}) = \alpha \cdot L_{content}(\vec{p}, \vec{x}) + \beta \, L_{style}(\vec{a}, \vec{x})$$

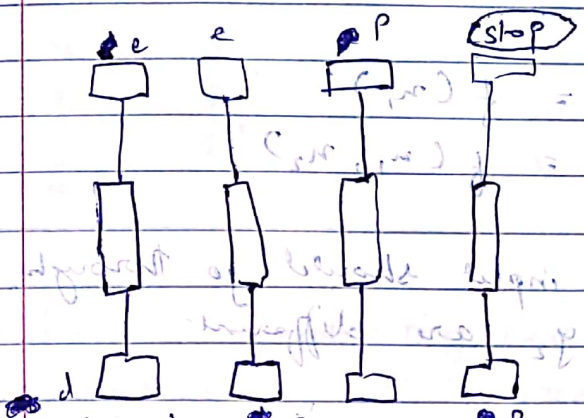
refer notes/ week 12/ deepart.png

* Fooling CNNs

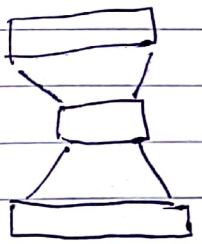
→ Change the target class value & backpropagate and make changes to original image such that you get minimum loss.

WEEK 13

* Sequence Learning Problems

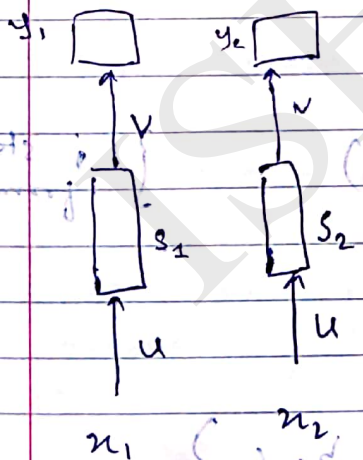


Predicting next character in a word depends on previous characters thus, it is a sequence problem.



Each one of them is a feed forward NN.

* Recurrent Neural Networks (RNN)



$$s_i = \sigma(Ux_i + b)$$

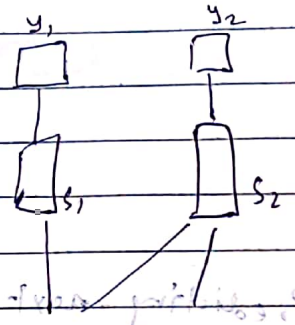
$$y_i = O(Vs_i + c)$$

activation

i = timestamp

But this doesn't account for previous inputs

refer notes/week13/wishlist.png



If we connect it like this

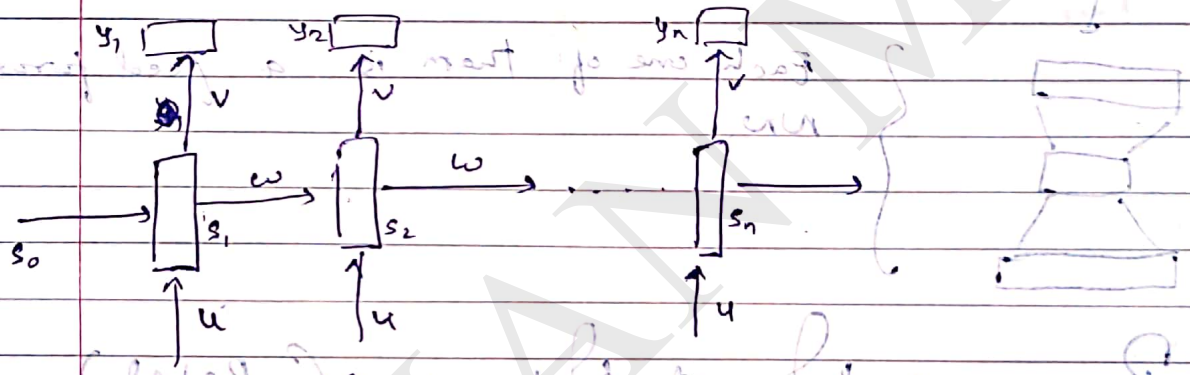
then

$y_1 = f(x_1)$

$y_2 = f(x_1, x_2)$

As per wish list every input should go through same function but y_1 & y_2 are different.

→ Thus we make recurrent connections



Now here

$$s_i = \sigma(U x_i + W s_{i-1} + b)$$; starting from 1

$$y_i = O(V s_i + c)$$

OR

$$y_i = f(x_i, s_i, W, U, V, b, c)$$

Note

s_i is state of network at timestamp i

Weights W, U, V and biases are shared across all timestamps

* Back Propagation Through Time

Lets say,

$x_i \in R^n$ $\quad U \in R^{n \times d}$

$s_i \in R^d$ $\quad W \in R^{d \times d}$

$y_i \in R^k$ $\quad V \in R^{d \times k}$

→ Example is predicting next character in word using RNN.
refer notes / week13 / backprop RNN.py.

- Activation at output layer = Softmax
- Loss

$$L_t(\theta) = -\log(y_{tc})$$ } cross entropy at each time stamps

$$L(\theta) = \sum_{t=1}^{T} L_t(\theta)$$ } total loss

y_{tc} = predicted probability of true character at time-stamp

T = number of timesteps

→ Here during backprop we need to find gradients for,

V, W, U

(1) Gradients for V

$$\frac{\partial L(\theta)}{\partial V} = \sum_{t=1}^{T} \frac{\partial L_t(\theta)}{\partial V}$$

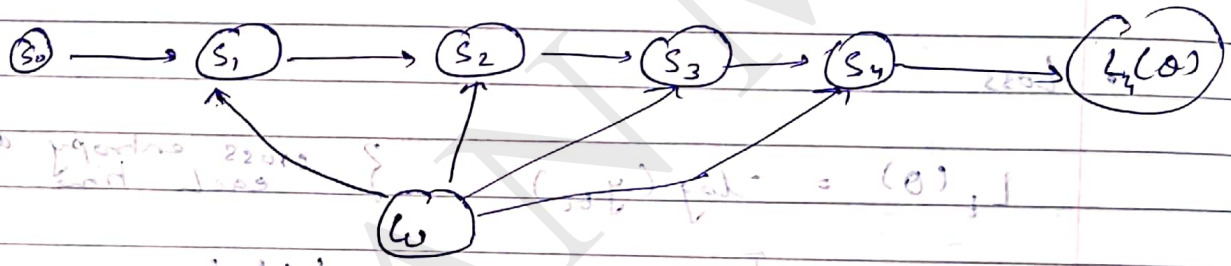
$\frac{\partial L_t(\theta)}{\partial V}$ ↓ matrix

} add gradient matrix obtained at every time stamp

(2) Gradient wrt W

$$\frac{\partial L(\theta)}{\partial W} = \sum_{t=1}^{T} \frac{\partial L_t(\theta)}{\partial W}$$

For the network



Now,

$$\frac{\partial L_4(\theta)}{\partial W} = \frac{\partial L_4(\theta)}{\partial S_4} \frac{\partial S_4}{\partial W}$$

refer Backprop { derivative wrt first hidden layer

Now, $S_4 = \sigma(WS_3 + b)$ W, W, V

Here $S_4, S_3, \ldots, S_1$ are functions of W thus we need as follows

Here we have ignored σ for simplicity

$$\frac{\partial s_4}{\partial w} = \frac{\partial^+ s_4}{\partial w} + \frac{\partial s_4}{\partial s_3}\frac{\partial s_3}{\partial w}$$

Explicit (has direct path to w) — implicit (has indirect path to w)

$$= \frac{\partial^+ s_4}{\partial w} + \frac{\partial s_4}{\partial s_3}\left[\frac{\partial^+ s_3}{\partial w} + \frac{\partial s_2}{\partial s_2}\frac{\partial s_2}{\partial w}\right]$$

Explicit — implicit

$$= \frac{\partial^+ s_4}{\partial w} + \frac{\partial s_4}{\partial s_3}\frac{\partial^+ s_3}{\partial w} + \frac{\partial s_4}{\partial s_3}\frac{\partial s_3}{\partial s_2}\frac{\partial^+ s_2}{\partial w} + \frac{\partial s_4}{\partial s_3}\frac{\partial s_3}{\partial s_2}\frac{\partial s_2}{\partial s_1}\frac{\partial^+ s_1}{\partial w}$$

$$= \frac{\partial s_4}{\partial s_4}\frac{\partial^+ s_4}{\partial w} + \frac{\partial s_4}{\partial s_3}\frac{\partial^+ s_3}{\partial w} + \frac{\partial s_4}{\partial s_2}\frac{\partial^+ s_2}{\partial w} + \frac{\partial s_4}{\partial s_1}\frac{\partial^+ s_1}{\partial w}$$

$$\frac{\partial s_4}{\partial w} = \sum_{k=1}^{4}\frac{\partial s_4}{\partial s_k}\frac{\partial^+ s_k}{\partial w}$$

$$\therefore \frac{\partial L_4(\theta)}{\partial w} = \frac{\partial L_4(\theta)}{\partial s_4}\left[\sum_{k=1}^{t}\frac{\partial s_4}{\partial s_k}\frac{\partial^+ s_k}{\partial w}\right]$$

General

$$\frac{\partial L_t(\theta)}{\partial w} = \frac{\partial L_t(\theta)}{\partial s_t}\left[\sum_{k=1}^{t}\frac{\partial s_t}{\partial s_k}\frac{\partial^+ s_k}{\partial w}\right]$$

(3) Gradient wrt v

use chain rule through v & w and find it

* Exploding & Vanishing Gradients

→ Now, we find

$$\frac{\partial S_t}{\partial S_k}$$

$$\frac{\partial S_t}{\partial S_k} = \frac{\partial S_t}{\partial S_{t-1}} \frac{\partial S_{t-1}}{\partial S_{t-2}} \cdots \cdots \frac{\partial S_{k+1}}{\partial S_k} = \prod_{j=k}^{t-1} \frac{\partial S_{j+1}}{\partial S_j}$$

→ Thus

$\frac{\partial S_j}{\partial S_{j-1}}$ is to be found

let,

$a_j = W S_{j-1} + b + U x_j$

$S_j = \sigma(a_j)$

$a_j = [a_{j1}, a_{j2}, a_{j3} \ldots a_{jd}]$ — ①

$S_j = [\sigma(a_{j1}), \sigma(a_{j2}) \ldots \sigma(a_{jd})]$ — ②

→ Now

$$\frac{\partial S_j}{\partial S_{j-1}} = \frac{\partial S_j}{\partial a_j} \frac{\partial a_j}{\partial S_{j-1}}$$

↓ will be diagonal matrix

↘ W

from (1) & (2)

$$\frac{\partial s_j}{\partial a_j} = \begin{bmatrix} \frac{\partial s_{j1}}{\partial a_{j1}} & \frac{\partial s_{j1}}{\partial a_{j2}} & \cdots \\ 0 & & \\ 0 & & \\ \vdots & & \\ 0 & & \end{bmatrix} \times W$$

(× multiplied)

$$= \begin{bmatrix} \sigma'(a_{j1}) & 0 & \cdots \\ 0 & \sigma'(a_{j2}) & \\ 0 & & \ddots \\ 0 & & \sigma'(a_{jd}) \end{bmatrix} \times W$$

$$= \text{diag}(\sigma'(a_j)) \times W$$

Taking magnitude

$$\left\| \frac{\partial s_j}{\partial s_{j-1}} \right\| = \| \text{diag}(\sigma'(a_j))\, W \|$$

$$\leq \| \text{diag}(\sigma'(a_j)) \| \, \|W\|$$ (by property of norm: $\|ab\| \leq \|a\| \|b\|$)

- $\sigma'(a_j) \leq \frac{1}{4} = \gamma$ (if σ is logistic)

  $\sigma(a_j)(1 - \sigma(a_j)) \leq \frac{1}{4}$

  ↓ max value taken by derivative is $\frac{1}{4}$

- $\sigma'(a_j) \leq 1 = \gamma$ (if σ is tanh)

$\leq \cdot \cdot \gamma \|w\|$

$\leq \gamma \lambda$

Now,

$$\left\| \frac{\partial S_t}{\partial S_k} \right\| = \left\| \prod_{j=k+1}^{t} \frac{\partial S_j}{\partial S_{j-1}} \right\|$$

$$\leq \prod_{j=k+1}^{t} \gamma \lambda$$

$$\leq (\gamma \lambda)^{t-k}$$

→ $\gamma \lambda < 1$ vanishing gradient

→ $\gamma \lambda > 1$ exploding gradient

* Important

$$\frac{\partial L_t(\theta)}{\partial w} = \frac{\partial L_t(\theta)}{\partial S_t} \sum_{k=1}^{t} \frac{\partial S_t}{\partial S_k} \frac{\partial^+ S_k}{\partial w}$$

d x d d x d d x d d x d x d

✓ ✓ ✓

fetched values are known to us

→ lets compute any one element of tensor $dx \times d \times d$

$\frac{\partial^+ s_{kp}}{\partial w_{qr}}$: $(p,q,r)^{th}$ element of tensor

Note: diff wrt w are 0

$a_k = w s_{k-1} + b + U x_k$

$s_k = \sigma(a_k)$

$\therefore a_{ik} = w s_{k-1}$

$$\begin{bmatrix} a_{k1} \\ a_{k2} \\ \vdots \\ a_{kn} \end{bmatrix} = \begin{bmatrix} w_{11} & \cdots \\ \vdots & \\ w_{p1} & w_{p2} \cdots w_{pd} \\ \vdots & \vdots \\ w_{d1} & \cdots w_{dd} \end{bmatrix} \begin{bmatrix} s_{k-1,1} \\ s_{k-1,2} \\ \vdots \\ s_{k-1,p} \\ \vdots \\ s_{k-1,d} \end{bmatrix}$$

$$\therefore a_{kp} = \sum_{i=1}^{d} w_{pi} s_{k-1,i} \quad \text{---} \quad (1)$$

$s_{kp} = \sigma(a_{kp})$

$$\frac{\partial s_{kp}}{\partial w_{qr}} = \frac{\partial s_{kp}}{\partial a_{kp}} \frac{\partial a_{kp}}{\partial w_{qr}}$$

$$= \sigma'(a_{kp}) \frac{\partial a_{kp}}{\partial w_{qr}}$$

$$\frac{\partial a_{kp}}{\partial w_{qr}} = \frac{\partial \sum_{i=1}^{d} w_{pi} \cdot s_{k-1,i}}{\partial w_{qr}}$$

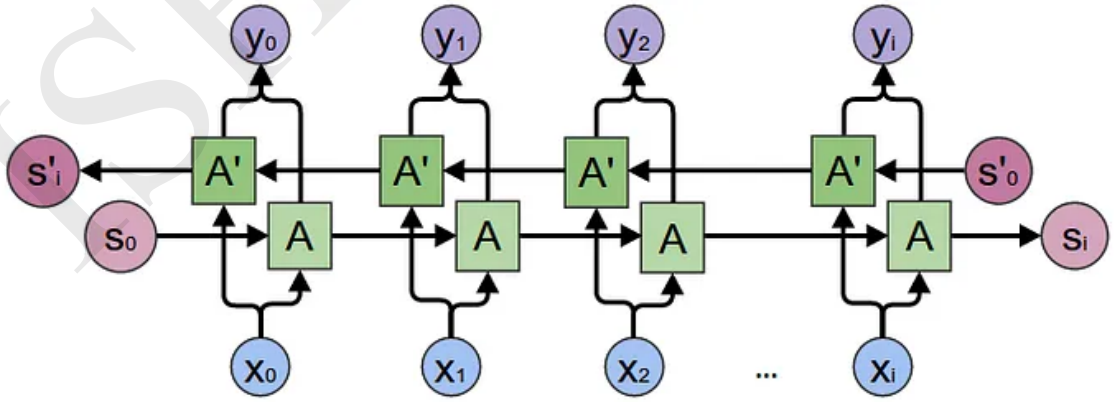
$$= s_{k-1,i} \quad \text{if } p = q \text{ & } i = r$$

$$= 0 \quad \text{otherwise}$$

$$\therefore \frac{\partial s_{kp}}{\partial w_{qr}} = \sigma'(a_{kp})\, s_{k-1,r} \quad \text{if } p = q \text{ & } i = r$$

$$= 0 \quad \text{otherwise}$$

✳ Bi-Directional RNNs



WEEK 14

* Selective Read, Write, Forget

→ In a RNN the problem of vanishing and exploding gradient occurs during backprop.

But, during forward prop also we keep on updating the state vectors. This vector is also of finite dimension so by reaching at end of RNN it will also forget info from 1st state.

During forward pass & Back pass info is tend to be forgotten.

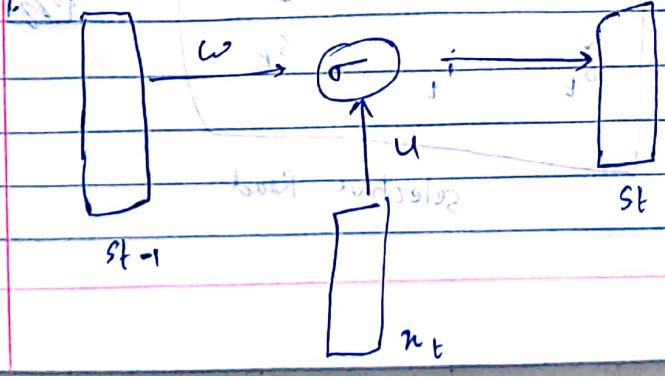
→ Selective read, write, forget using white-board analogy

* LSTM (Long-short term memory)

→ lets consider task of sentiment analysis

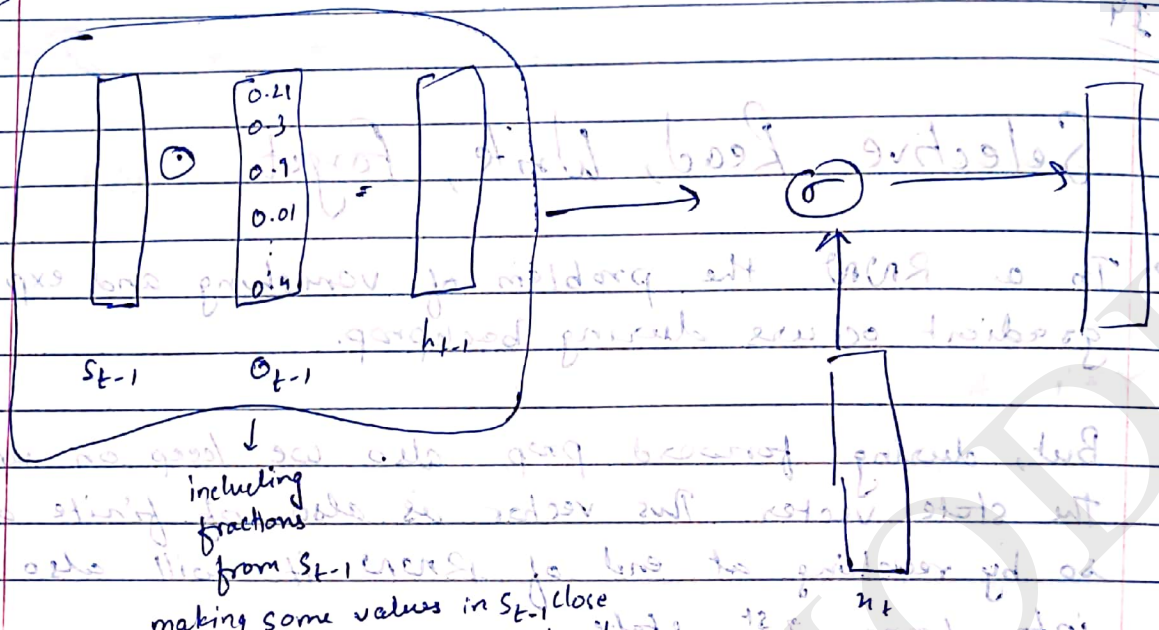
- Forget - the information like stopwords
- Selective Read - the info added by previous sentiment bearing words
- Selective Write - new information from current word to state.

→



Here if we want to selectively write values of S_{t-1}

Selective Write



Selective write gate

including fractions from s_{t-1} making some values in s_{t-1} close to zero & some close to 1 & so on as per importance

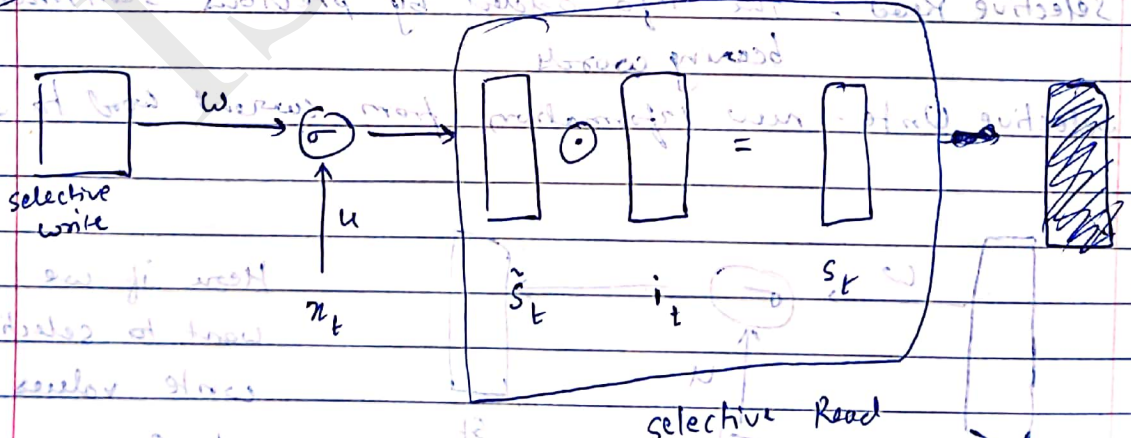
Now, we need to learn O_{t-1} (Output Gate)

$$O_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$$

→ (U, V, W) are parameters of RNN

U_o, b_o, W_o are parameters of RNN (O_{t-1})

Selective Read



$$\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$$

$$s_t = \tilde{s}_t \odot i_t$$

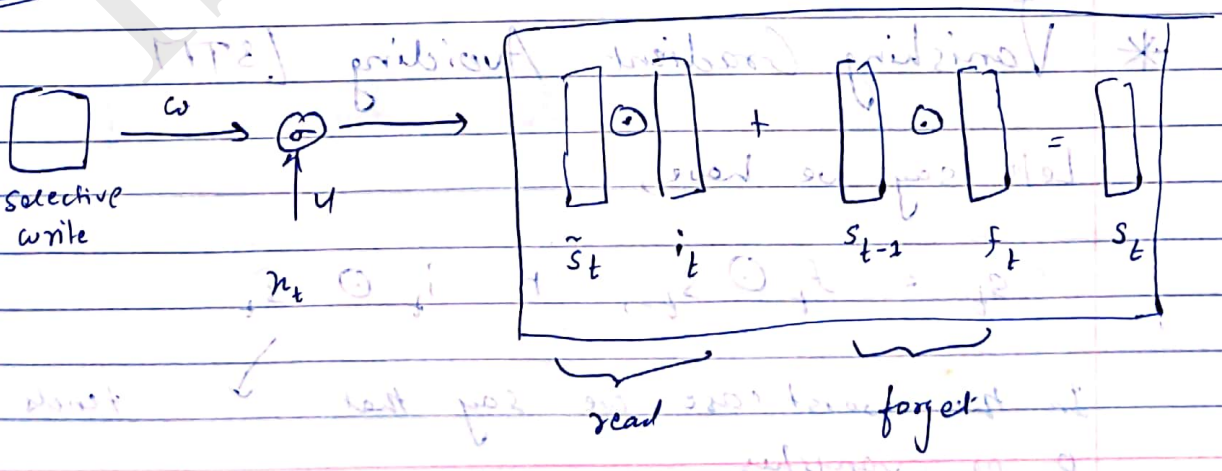
$$i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$$

Note → W, U, b & W_i, U_i, b_i are different parameters

Till now

- Previous state → s_{t-1}
- Output gate → $o_{t-1} = \sigma(W_o h_{t-2} + U_o x_{t-1} + b_o)$
- Selective Write → $h_{t-1} = o_{t-1} \odot \sigma(s_{t-1})$
- Current (temp) state → $\tilde{s}_t = \sigma(W h_{t-1} + U x_t + b)$
- Input gate → $i_t = \sigma(W_i h_{t-1} + U_i x_t + b_i)$
- Selective Read → $i_t \odot \tilde{s}_t$

Selective Forget



$$f_t = \sigma(W_f h_{t-1} + U_f x_t + b_f)$$

$$s_t = s_{t-1} \odot f_t + \tilde{s}_t \odot i_t$$

* GRU (Gated Recurrent Unit)

→ Gates

- $o_t = \sigma(W_o s_{t-1} + U_o x_t + b_o)$
- $i_t = \sigma(W_i s_{t-1} + U_i x_t + b_i)$

→ States

- $\tilde{s}_t = \sigma(W(o_t \odot s_{t-1}) + U x_t + b)$
- $s_t = (1 - i_t) \odot s_{t-1} + i_t \odot \tilde{s}_t$

Note: refer Notes/week14/LSTM.png & Notes/week14/lstm.png
notes/week14/GRU.png & Notes/week14/gru.png

* Vanishing Gradient Avoiding LSTM

Let's say we have,

$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$

In the worst case we say that $\tilde{s}_t$ tends to 0 or vanishes

Thus we have

$$S_t = f_t \odot S_{t-1}$$

$$\frac{\partial S_t}{\partial S_{t-1}} = f_t$$

as we go back pass $f_t \times f_{t-1} \times f_{t-2} \times \ldots \ldots f_1$

$$= (f_t)^T$$

Here also during back pass f_t vanishes

But during forward pass

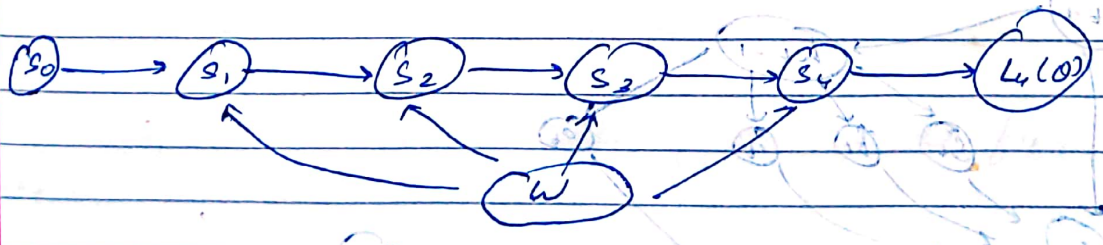
f_t information goes from 1 state to next

$\therefore (f_t)^T$ for forward.

Conclusion

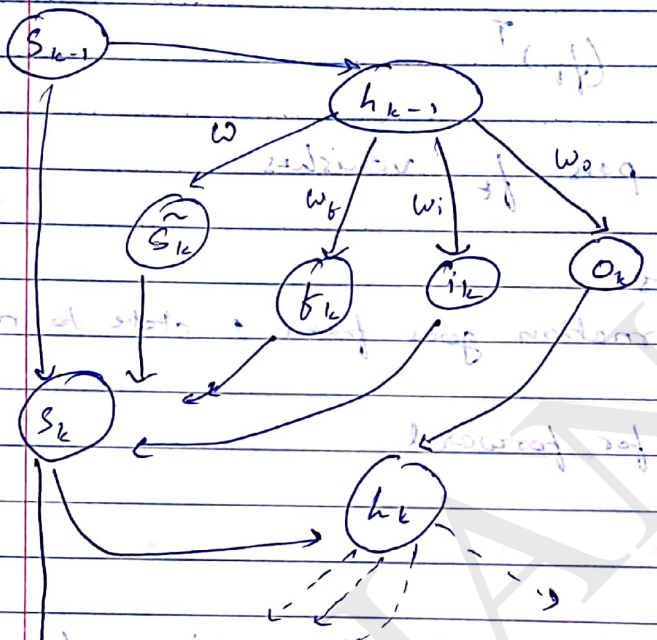
→ If the state at time $t-1$ did not contribute to state at time t (ie, if $\|f_t\| \to 0$ and $\|o_{t-1}\| \to 0$) then during backprop the gradients flowing into s_{t-1} will vanish.

* Proof (Vanishing doesn't occur)



→ In general, the gradient of $L_t(O)$ wrt θ_i vanishes when the gradients flowing through each and every path from $L_t(O)$ to θ_i vanishes.

→ The gradient of $L_t(O)$ wrt θ_i explodes when the gradient flowing through at least one path explodes.



$$o_t = \sigma(w_o h_{t-1} + v_o x_t + b_o)$$

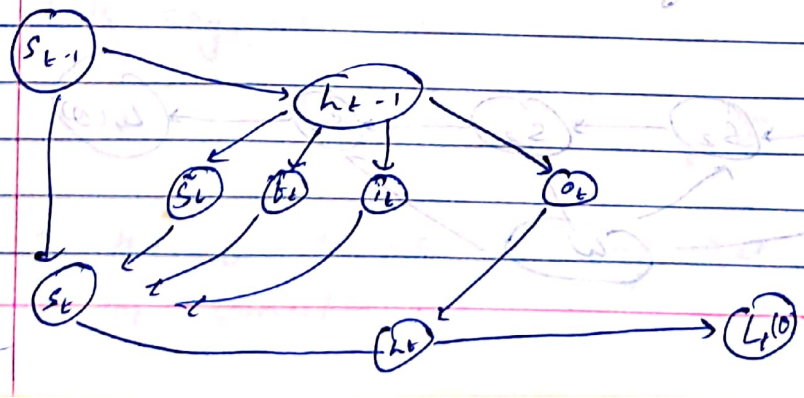
$$i_t = \sigma(w_i h_{t-1} + v_i x_t + b_i)$$

$$f_t = \sigma(w_f h_{t-1} + v_f x_t + b_f)$$

$$\tilde{s}_t = \sigma(w h_{t-1} + v x_t + b)$$

$$s_t = f_t \odot s_{t-1} + i_t \odot \tilde{s}_t$$

$$h_t = o_t \odot \sigma(s_t)$$



Path $L_t(\theta) \to h_t \to s_t \to s_{t-1} \dots s_{k+1} \to s_k$

→ Now lets say w_f causes error in output & we want to backprop gradient to w_f

Then we need to prove that there exist at least one path from $L_t(\theta)$ to s_k that allows gradient to flow so that it doesn't vanish

$$\frac{\partial L_t(\theta)}{\partial s_k} = \frac{\partial L_t(\theta)}{\partial h_t} \frac{\partial h_t}{\partial s_t} \left(\frac{\partial s_t}{\partial s_{t-1}} \cdots \frac{\partial s_{k+1}}{\partial s_k} \right)$$

$\frac{\partial L_t(\theta)}{\partial h_t}$ → can be found

$\frac{\partial h_t}{\partial s_t}$ → ①

$\left(\frac{\partial s_t}{\partial s_{t-1}} \cdots \frac{\partial s_{k+1}}{\partial s_k} \right)$ → ②

~~$\frac{\partial}{\partial s_t} f_t \odot s_t + (i_t \odot \tilde{s}_t)$~~ ignoring in worst case

① $\frac{\partial h_t}{\partial s_t} = \frac{\partial}{\partial s_t} o_t \odot \sigma(s_t) = \text{Diagonal matrix} \left(o_t \odot \sigma'(s_t) \right)$

② $\frac{\partial s_t}{\partial s_{t-1}} = \frac{\partial}{\partial s_{t-1}} f_t \odot s_{t-1} + (i_t \odot s_t)$ → ignoring worst case

$= D(f_t)$

↓ diagonal matrix

$\left(\frac{\partial s_t}{\partial s_{t-1}} \cdots \frac{\partial s_{k+1}}{\partial s_k} \right) = D(f_t \odot \dots \odot f_{k+1})$

$$= L_t'(h_t) \cdot D(o_t \odot \sigma'(s_t)) \cdot D\left(\bigodot_{i=k+1}^{t} f_i\right)$$

Thus gradient vanish in backprop only if it vanished in forward prop. It vanish only when it needs to this proved

* Proof (Exploding can be stopped)

path $= (L_t, L_t(o) \to h_t \to o_t \to h_{t-1} \dots h_k \to o_k \to h_{k-1})$

$$= \frac{\partial L_t(o)}{\partial h_t} \left(\frac{\partial h_t}{\partial o_t} \frac{\partial o_t}{\partial h_{t-1}}\right) \cdots \left(\frac{\partial h_k}{\partial o_k} \frac{\partial o_k}{\partial h_{k-1}}\right)$$

$$= L_t'(h_t) \cdot (D(\sigma(s_t) \odot o_t') \cdot W_o) \cdots (D(\sigma(s_k) \odot o_k') \cdot W_o)$$

$$\|t_1\| \le \|L_t'(h_t)\| (\|k\| \|W_o\|)^{t-k+1}$$

Depending on norm of $\|W_o\|$ gradient may explode

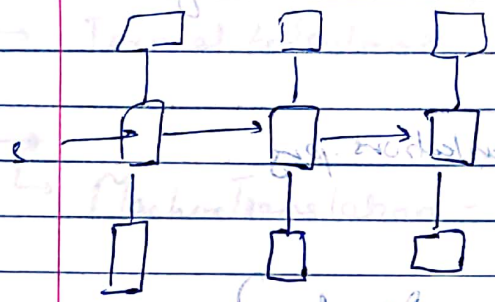
Thus use (Gradient Clipping) . If norm exceeds certain value bring it down to a threshold.

WEEK 15

* Introduction To Encoder Decoder

→ Predicting next word in sequence

$$y^* = \text{argmax}\ P(y_t = j \mid y_1, y_2, \ldots, y_{t-1})$$



at each stage in RNN, it will predict a probability distribution & word with max prob will be chosen.

Using RNN we can compute

$$P(y_t = j \mid \underbrace{y_1^{t-1}}_{(y_1, y_2 \ldots y_{t-1})}) = \underbrace{\text{softmax}(Vs_t + c)_j}_{\text{j}^{th} \text{ element of vector}}$$

$j \in V$, V is vocabulary

$\Downarrow$ i vector of size equal to vocab.

$$P(y_t = j \mid \underbrace{y_1^{t-1}}_{(y_1, y_2, \ldots y_{t-1})}) = P(y_t = j \mid \underbrace{s_t}_{\text{All the information of } (y_1, y_2 \ldots y_{t-1}) \text{ is contained in } s_t})$$

Thus model is

refer notes/week15/lec1/model.png

During training we would have all the input along with sequence

But, during test we will be given only a word and we will have to build entire sequence using it

refer notes/week15/lec1/testcase.png.

→ We will now use following representations for RNN, GRU, LSTM

refer notes/week15/lec1/representations.png.

→ Image Captioning (Encoder - Decoder)

$$P(y_t \mid y_1^{t-1}) = P(y_t \mid s_t)$$

Here we have image as well

$$\therefore P(y_t \mid s_t, I)$$ → cannot take entire image we need representation of image

$$P(y_t \mid s_t, fc_7(I))$$ ← fully connected layer in VGG.

This becomes s_0,

$\therefore s_0 = fc_7(I)$ now we feed it to RNN for caption generation

We can also feed in $f_{CT}(I)$ to all states.

refer notes/week15/lec1/encoderdecoder.png.

* Application of Encoder-Decoder

→ Image Captioning - refer/notes/week15/lec2/imagecaptioning.png

→ Textual Entailment - refer/notes/week15/lec2/textentailment1.png
/ textentailment2.png

→ Machine Translation - refer/notes/week15/lec2/machinetranslation1.png
/ machinetranslation2.png

→ Transliteration - refer/notes/week15/lec2/transliteration1.png
/ transliteration2.png

→ Image QnA - refer/notes/week15/lec2/imageQnA.png

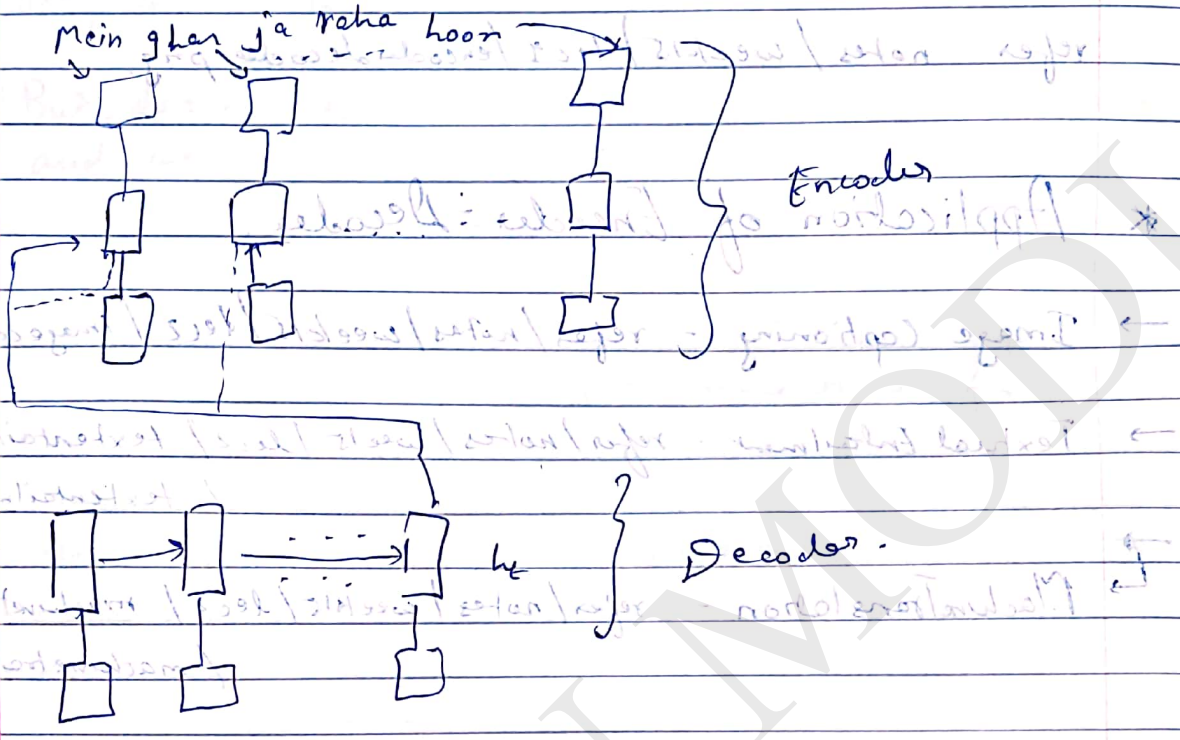
→ Document summarization
refer/notes/week15/lec2/docsummary.png

→ Video Captioning - refer/notes/week15/lec2/videocaption.png

→ Video Classification - refer/notes/week15/lec2/videoclassification.png

→ Dialog - refer/notes/week15/lec2/dialog.png

* Attention Mechanism

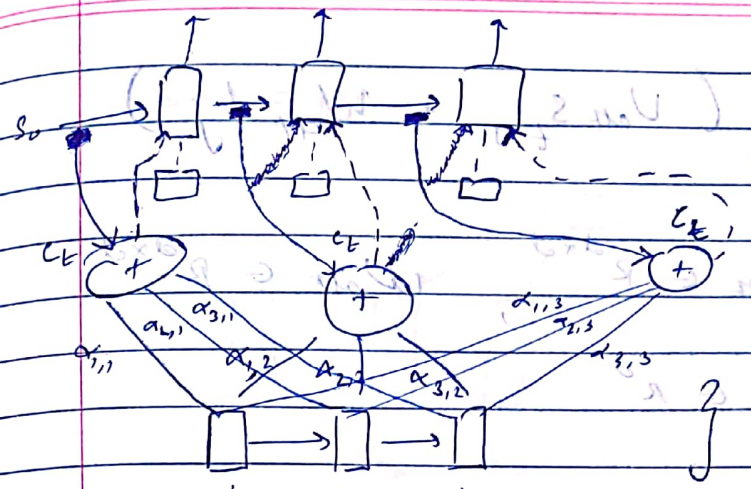


So here we are passing h_t as S_0 for another network, this h_t contains all the information ie (I am going home) but we can observe that:

I → Mein
home → ghar
am → hoon
going → ja raha

These are words that are hugely unrelated

So the intuition is why not pass this relationship instead of h_t



Here $\alpha_{1,1}$ $\alpha_{1,2}$ $\alpha_{1,3}$ | $\alpha_{2,1}$ $\alpha_{2,2}$ $\alpha_{2,3}$ | $\alpha_{3,1}$ $\alpha_{3,2}$ $\alpha_{3,3}$

are all parameters. which are to be learnt

→ Now,

- $e_{jt} = f_{ATT}(s_{t-1}, h_j)$

  (s_{t-1}: decoder state; h_j: encoder state.)

  e_{jt}: j^{th} word in encoder, t^{th} time stamp in decoder

- $\alpha_{jt} = \dfrac{\exp(e_{jt})}{\sum_{j=1}^{M} \exp(e_{jt})}$

This α_{jt} is probability measure of how important is j^{th} word with respect to t^{th} time stamp.

$$e_{jt} = V_{att}^T \tanh\left(U_{att} s_{t-1} + W_{att} h_j\right)$$

scalar

$V_{att}^T \in R^d$, $U_{att} \in R^{d \times d}$, $W_{att} \in R^{d \times d}$

$s_{t-1} \in R^d$, $h_j \in R^d$

This thing works because it is better modeling choice

Also,

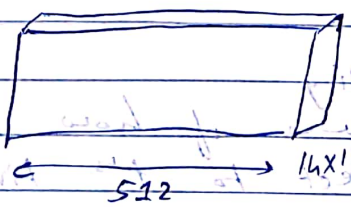
$$c_t = \sum_{j=1}^{T} \alpha_{jt} h_j$$ } for encoder.

refer notes / week 15 / lec 3 / attentionmechanism.png

* Attention Over Images

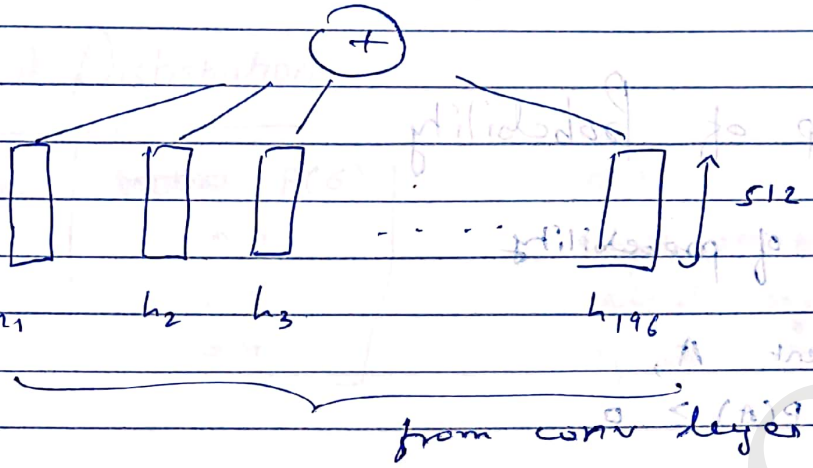
Unlike RNN, where each state gives word CNN for image gives FC_7 which doesn't give us which entry is for which pixel in image

Thus we use the convolution layers before FC_7



Each of pixel from 14X14 represents some portion in image.

14 x 14 = 196



$$\alpha_{ij,t} = f_{ATT}(s_{t-1}, h_j)$$

$$f_{ATT} = v^T \tanh(U s_{t-1} + W h_j + b)$$

* Hierarchical Attention

→ Understanding hierarchical RNN structures

- refer notes/week15/lec5/DialogHierarchyRNN.png
  /DocumentHierarchy.png

→ Now we use attention mechanism in these hierarchical models.

- refer notes/week15/lec5/attmechhierarchyencoder.png
  attmechhierarchydecoder.png.