武漢大學

eKalibr

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目录

插图

1 Dynamics

$$\begin{cases} \mathbf{a}(\tau) = \left(\mathbf{R}_b^{b_0}(\tau)\right)^\top \cdot \left(\mathbf{a}_b^{b_0}(\tau) - \mathbf{g}^{b_0}\right) \\ \boldsymbol{\omega}(\tau) = \left(\mathbf{R}_b^{b_0}(\tau)\right)^\top \cdot \boldsymbol{\omega}_b^{b_0} \end{cases} \tag{1}$$

set the body frame as $\underrightarrow{\mathcal{F}}_{b^i}$, the reference frame as $\underrightarrow{\mathcal{F}}_{b_0^r}$:

$$\begin{cases} \mathbf{a}^i(\tau) = \left(\mathbf{R}_{b^i}^{b_0^r}(\tau)\right)^\top \cdot \left(\mathbf{a}_{b^i}^{b_0^r}(\tau) - \mathbf{g}^{b_0^r}\right) \\ \boldsymbol{\omega}^i(\tau) = \left(\mathbf{R}_{b^i}^{b_0^r}(\tau)\right)^\top \cdot \boldsymbol{\omega}_{b^i}^{b_0^r}(\tau) \end{cases} \tag{2}$$

where $\mathbf{a}_i(\tau)$ and $\boldsymbol{\omega}_i(\tau)$ are the linear acceleration and angular velocity output from the i-th IMU at time τ, and

$$\begin{aligned} \mathbf{R}_{b^i}^{b_0^r}(\tau) &= \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{R}_{b^i}^{b^r} \\ \mathbf{p}_{b^i}^{b_0^r}(\tau) &= \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{p}_{b^i}^{b^r} + \mathbf{p}_{b^r}^{b_0^r}(\tau) \end{aligned} \tag{3}$$

thus

$$\begin{aligned} \boldsymbol{\omega}_{b^i}^{b_0^r}(\tau) &= \boldsymbol{\omega}_{b^r}^{b_0^r}(\tau) \\ \mathbf{v}_{b^i}^{b_0^r}(\tau) &= -\left[\mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{p}_{b^i}^{b^r}\right]_\times \cdot \boldsymbol{\omega}_{b^r}^{b_0^r}(\tau) + \mathbf{v}_{b^r}^{b_0^r}(\tau) \\ \mathbf{a}_{b^i}^{b_0^r}(\tau) &= -\left[\mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{p}_{b^i}^{b^r}\right]_\times \cdot \boldsymbol{\alpha}_{b^r}^{b_0^r}(\tau) - \left[\boldsymbol{\omega}_{b^r}^{b_0^r}(\tau)\right]_\times \cdot \left[\mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{p}_{b^i}^{b^r}\right]_\times \cdot \boldsymbol{\omega}_{b^r}^{b_0^r}(\tau) + \mathbf{a}_{b^r}^{b_0^r}(\tau) \end{aligned} \tag{4}$$

Based on:

$$\boldsymbol{\omega}^i(\tau) = \left(\mathbf{R}_{b^i}^{b_0^r}(\tau)\right)^\top \cdot \boldsymbol{\omega}_{b^i}^{b_0^r}(\tau) = \left(\mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{R}_{b^i}^{b^r}\right)^\top \cdot \boldsymbol{\omega}_{b^r}^{b_0^r}(\tau) \tag{5}$$

the rotation spline of $\underrightarrow{\mathcal{F}}_{b^r}$ and the rotation extrinsics $\mathbf{R}_{b^i}^{b^r}$ could be recovered.

2 Inertial Alignment

we have

$$\mathbf{a}^r(\tau) = \left(\mathbf{R}_{b^r}^{b_0^r}(\tau)\right)^\top \cdot \left(\mathbf{a}_{b^r}^{b_0^r}(\tau) - \mathbf{g}^{b_0^r}\right) \qquad \mathbf{a}^i(\tau) = \left(\mathbf{R}_{b^i}^{b_0^r}(\tau)\right)^\top \cdot \left(\mathbf{a}_{b^i}^{b_0^r}(\tau) - \mathbf{g}^{b_0^r}\right) \tag{6}$$

thus

$$\mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{a}^r(\tau) = \mathbf{a}_{b^r}^{b_0^r}(\tau) - \mathbf{g}^{b_0^r} \qquad \mathbf{R}_{b^i}^{b_0^r}(\tau) \cdot \mathbf{a}^i(\tau) = \mathbf{a}_{b^i}^{b_0^r}(\tau) - \mathbf{g}^{b_0^r} \tag{7}$$

and

$$\int_{\tau_k}^{\tau_{k+1}} \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{a}^r(\tau) \cdot d\tau = \mathbf{v}_{b^r}^{b_0^r}(\tau_{k+1}) - \mathbf{v}_{b^r}^{b_0^r}(\tau_k) - \mathbf{g}^{b_0^r} \cdot (\tau_{k+1} - \tau_k) \tag{8}$$

for the left part, we have

$$\begin{aligned} \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{a}^r(\tau) &= \mathbf{a}_{b^r}^{b_0^r}(\tau) - \left(\mathbf{a}_{b^i}^{b_0^r}(\tau) - \mathbf{R}_{b^i}^{b_0^r}(\tau) \cdot \mathbf{a}^i(\tau)\right) = \mathbf{a}_{b^r}^{b_0^r}(\tau) - \mathbf{a}_{b^i}^{b_0^r}(\tau) + \mathbf{R}_{b^i}^{b_0^r}(\tau) \cdot \mathbf{a}^i(\tau) \\ \mathbf{a}_{b^r}^{b_0^r}(\tau) - \mathbf{a}_{b^i}^{b_0^r}(\tau) &= \left[\mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{p}_{b^i}^{b^r}\right]_\times \cdot \boldsymbol{\alpha}_{b^r}^{b_0^r}(\tau) + \left[\boldsymbol{\omega}_{b^r}^{b_0^r}(\tau)\right]_\times \cdot \left[\mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{p}_{b^i}^{b^r}\right]_\times \cdot \boldsymbol{\omega}_{b^r}^{b_0^r}(\tau) \\ \mathbf{a}_{b^r}^{b_0^r}(\tau) - \mathbf{a}_{b^i}^{b_0^r}(\tau) &= -\left[\boldsymbol{\alpha}_{b^r}^{b_0^r}(\tau)\right]_\times \cdot \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{p}_{b^i}^{b^r} - \left[\boldsymbol{\omega}_{b^r}^{b_0^r}(\tau)\right]_\times \cdot \left[\boldsymbol{\omega}_{b^r}^{b_0^r}(\tau)\right]_\times \cdot \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{p}_{b^i}^{b^r} \\ \mathbf{a}_{b^r}^{b_0^r}(\tau) - \mathbf{a}_{b^i}^{b_0^r}(\tau) &= -\left(\left[\boldsymbol{\alpha}_{b^r}^{b_0^r}(\tau)\right]_\times + \left[\boldsymbol{\omega}_{b^r}^{b_0^r}(\tau)\right]_\times^2\right) \cdot \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{p}_{b^i}^{b^r} \end{aligned} \tag{9}$$

thus

$$\mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{a}^r(\tau) = \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{R}_{b^i}^{b^r} \cdot \mathbf{a}^i(\tau) - \left(\left[\boldsymbol{\alpha}_{b^r}^{b_0^r}(\tau) \right]_\times + \left[\boldsymbol{\omega}_{b^r}^{b_0^r}(\tau) \right]_\times^2 \right) \cdot \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{p}_{b^i}^{b^r} \tag{10}$$

with the left part of (8), we have:

$$\int_{\tau_k}^{\tau_{k+1}} \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{R}_{b^i}^{b^r} \cdot \mathbf{a}^i(\tau) \cdot d\tau - \int_{\tau_k}^{\tau_{k+1}} \left(\left[\boldsymbol{\alpha}_{b^r}^{b_0^r}(\tau) \right]_\times + \left[\boldsymbol{\omega}_{b^r}^{b_0^r}(\tau) \right]_\times^2 \right) \cdot \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot d\tau \cdot \mathbf{p}_{b^i}^{b^r} = \mathbf{b}_{k,k+1}^i - \mathbf{A}_{k,k+1} \cdot \mathbf{p}_{b^i}^{b^r} \tag{11}$$

we rewrite (8):

$$\mathbf{b}_{k,k+1}^i - \mathbf{A}_{k,k+1} \cdot \mathbf{p}_{b^i}^{b^r} = \mathbf{v}_{b^r}^{b_0^r}(\tau_{k+1}) - \mathbf{v}_{b^r}^{b_0^r}(\tau_k) - \mathbf{g}^{b_0^r} \cdot (\tau_{k+1} - \tau_k) \tag{12}$$

$$\begin{bmatrix} -\mathbf{I}_3 & \mathbf{I}_3 & \mathbf{A}_{k,k+1} & -\mathbf{I}_3 \cdot (\tau_{k+1} - \tau_k) \end{bmatrix} \begin{bmatrix} \mathbf{v}_{b^r}^{b_0^r}(\tau_k) \\ \mathbf{v}_{b^r}^{b_0^r}(\tau_{k+1}) \\ \mathbf{p}_{b^i}^{b^r} \\ \mathbf{g}^{b_0^r} \end{bmatrix} = \mathbf{b}_{k,k+1}^i \tag{13}$$

$$\mathbf{H}_{k,k+1} \cdot \mathbf{x}_{k,k+1}^i = \mathbf{b}_{k,k+1}^i \tag{14}$$

$$\left(\mathbf{H}_{k,k+1}^\top \cdot \mathbf{H}_{k,k+1} \right) \cdot \mathbf{x}_{k,k+1}^i = \mathbf{H}_{k,k+1}^\top \cdot \mathbf{b}_{k,k+1}^i \tag{15}$$

marginalize $\mathbf{v}_{b^r}^{b_0^r}(\tau_k)$ and $\mathbf{v}_{b^r}^{b_0^r}(\tau_{k+1})$ when solving.

Given a data sequence from $\mathcal{N}$ IMUs, split it into $\mathcal{M}$ blocks, thus we need to estimate parameters with size of:

$$(\mathcal{N} - 1) \times 3 + 2 + (\mathcal{M} + 1) \times 3 = (\mathcal{N} + \mathcal{M}) \times 3 + 2 \tag{16}$$

we have constraints with size of:

$$\mathcal{M} \times 3 \times \mathcal{N} \tag{17}$$

when:

$$\mathcal{M} \times 3 \times \mathcal{N} \geq (\mathcal{N} + \mathcal{M}) \times 3 + 2 \tag{18}$$

we could recover the extrinsic translations and the gravity vector. Assume that we have two IMUs, i.e., $\mathcal{N} = 2$, then:

$$\mathcal{M} \times 3 \times 2 \geq (2 + \mathcal{M}) \times 3 + 2 \qquad \Rightarrow \qquad \mathcal{M} \geq 3 \geq \frac{8}{3} \tag{19}$$

3 Visual-inertial Alignment

Given the specific force measurements, we have:

$$\int_{\tau_m}^{\tau_{m+1}} \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{a}^r(\tau) \cdot d\tau = \mathbf{v}_{b^r}^{b_0^r}(\tau_{m+1}) - \mathbf{v}_{b^r}^{b_0^r}(\tau_m) - \mathbf{g}^{b_0^r} \cdot (\tau_{m+1} - \tau_m) \tag{20}$$

$$\iint_{\tau_m}^{\tau_{m+1}} \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{a}^r(\tau) \cdot d\tau = \mathbf{p}_{b^r}^{b_0^r}(\tau_{m+1}) - \mathbf{p}_{b^r}^{b_0^r}(\tau_m) - \mathbf{v}_{b^r}^{b_0^r}(\tau_m) \cdot (\tau_{m+1} - \tau_m) - \frac{1}{2} \cdot \mathbf{g}^{b_0^r} \cdot (\tau_{m+1} - \tau_m)^2 \tag{21}$$

where

$$\mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{a}^r(\tau) = \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{R}_{b^i}^{b^r} \cdot \mathbf{a}^i(\tau) - \left(\left[\boldsymbol{\alpha}_{b^r}^{b_0^r}(\tau) \right]_\times + \left[\boldsymbol{\omega}_{b^r}^{b_0^r}(\tau) \right]_\times^2 \right) \cdot \mathbf{R}_{b^r}^{b_0^r}(\tau) \cdot \mathbf{p}_{b^i}^{b^r} \tag{22}$$

thus

$$\int_{\tau_m}^{\tau_{m+1}} \mathbf{R}_{b^r}^{b_0^r}(\tau)\cdot\mathbf{R}_{b^i}^{b^r}\cdot\mathbf{a}^i(\tau)\cdot d\tau - \int_{\tau_m}^{\tau_{m+1}} \left(\left[\boldsymbol{\alpha}_{b^r}^{b_0^r}(\tau)\right]_\times + \left[\boldsymbol{\omega}_{b^r}^{b_0^r}(\tau)\right]_\times^2\right)\cdot\mathbf{R}_{b^r}^{b_0^r}(\tau)\cdot d\tau\cdot\mathbf{p}_{b^i}^{b^r} = \mathbf{b}_{m,m+1}^i - \mathbf{A}_{m,m+1}\cdot\mathbf{p}_{b^i}^{b^r} \tag{23}$$

where

$$\mathbf{b}_{m,m+1}^i - \mathbf{A}_{m,m+1}\cdot\mathbf{p}_{b^i}^{b^r} = \mathbf{v}_{b^r}^{b_0^r}(\tau_{m+1}) - \mathbf{v}_{b^r}^{b_0^r}(\tau_m) - \mathbf{g}^{b_0^r}\cdot(\tau_{m+1}-\tau_m) \tag{24}$$

induce the extrinsics of multiple cameras:

$$\mathbf{p}_{c^m}^{b_0^r}(\tau) = \mathbf{R}_{b^r}^{b_0^r}(\tau)\cdot\mathbf{p}_{c^m}^{b^r} + \mathbf{p}_{b^r}^{b_0^r}(\tau) \quad\to\quad \mathbf{p}_{b^r}^{b_0^r}(\tau) = \mathbf{p}_{c^m}^{b_0^r}(\tau) - \mathbf{R}_{b^r}^{b_0^r}(\tau)\cdot\mathbf{p}_{c^m}^{b^r} \tag{25}$$

对时间求导:

$$\mathbf{v}_{b^r}^{b_0^r}(\tau) = \mathbf{v}_{c^m}^{b_0^r}(\tau) + \left[\mathbf{R}_{b^r}^{b_0^r}(\tau)\cdot\mathbf{p}_{c^m}^{b^r}\right]_\times\cdot\boldsymbol{\omega}_{b^r}^{b_0^r}(\tau) = \mathbf{v}_{c^m}^{b_0^r}(\tau) - \left[\boldsymbol{\omega}_{b^r}^{b_0^r}(\tau)\right]_\times\cdot\mathbf{R}_{b^r}^{b_0^r}(\tau)\cdot\mathbf{p}_{c^m}^{b^r} \tag{26}$$

为方便，记:

$$\mathbf{C}_m = \left[\boldsymbol{\omega}_{b^r}^{b_0^r}(\tau_m)\right]_\times\cdot\mathbf{R}_{b^r}^{b_0^r}(\tau_m) \tag{27}$$

最后有:

$$\mathbf{b}_{m,m+1}^i - \mathbf{A}_{m,m+1}\cdot\mathbf{p}_{b^i}^{b^r} = \mathbf{v}_{c^m}^{b_0^r}(\tau_{m+1}) - \mathbf{C}_{m+1}\cdot\mathbf{p}_{c^m}^{b^r} - \left(\mathbf{v}_{c^m}^{b_0^r}(\tau_m) - \mathbf{C}_m\cdot\mathbf{p}_{c^m}^{b^r}\right) - \mathbf{g}^{b_0^r}\cdot(\tau_{m+1}-\tau_m) \tag{28}$$

$$\mathbf{b}_{m,m+1}^i - \mathbf{A}_{m,m+1}\cdot\mathbf{p}_{b^i}^{b^r} = \mathbf{v}_{c^m}^{b_0^r}(\tau_{m+1}) - \mathbf{v}_{c^m}^{b_0^r}(\tau_m) - (\mathbf{C}_{m+1}-\mathbf{C}_m)\cdot\mathbf{p}_{c^m}^{b^r} - \mathbf{g}^{b_0^r}\cdot(\tau_{m+1}-\tau_m) \tag{29}$$

再次积分得到位置:

$$\iint_{\tau_m}^{\tau_{m+1}} \mathbf{R}_{b^r}^{b_0^r}(\tau)\cdot\mathbf{R}_{b^i}^{b^r}\cdot\mathbf{a}^i(\tau)\cdot d\tau - \iint_{\tau_m}^{\tau_{m+1}} \left(\left[\boldsymbol{\alpha}_{b^r}^{b_0^r}(\tau)\right]_\times + \left[\boldsymbol{\omega}_{b^r}^{b_0^r}(\tau)\right]_\times^2\right)\cdot\mathbf{R}_{b^r}^{b_0^r}(\tau)\cdot d\tau\cdot\mathbf{p}_{b^i}^{b^r} = \mathbf{c}_{m,m+1}^i - \mathbf{B}_{m,m+1}\cdot\mathbf{p}_{b^i}^{b^r} \tag{30}$$

其中:

$$\mathbf{c}_{m,m+1}^i - \mathbf{B}_{m,m+1}\cdot\mathbf{p}_{b^i}^{b^r} = \mathbf{p}_{b^r}^{b_0^r}(\tau_{m+1}) - \mathbf{p}_{b^r}^{b_0^r}(\tau_m) - \mathbf{v}_{b^r}^{b_0^r}(\tau_m)\cdot(\tau_{m+1}-\tau_m) - \frac{1}{2}\cdot\mathbf{g}^{b_0^r}\cdot(\tau_{m+1}-\tau_m)^2 \tag{31}$$

而已有:

$$\mathbf{p}_{b^r}^{b_0^r}(\tau) = \mathbf{p}_{c^m}^{b_0^r}(\tau) - \mathbf{R}_{b^r}^{b_0^r}(\tau)\cdot\mathbf{p}_{c^m}^{b^r} \tag{32}$$

所以:

$$\mathbf{p}_{b^r}^{b_0^r}(\tau_{m+1}) - \mathbf{p}_{b^r}^{b_0^r}(\tau_m) = \mathbf{p}_{c^m}^{b_0^r}(\tau_{m+1}) - \mathbf{R}_{b^r}^{b_0^r}(\tau_{m+1})\cdot\mathbf{p}_{c^m}^{b^r} - \left(\mathbf{p}_{c^m}^{b_0^r}(\tau_m) - \mathbf{R}_{b^r}^{b_0^r}(\tau_m)\cdot\mathbf{p}_{c^m}^{b^r}\right) \tag{33}$$

$$\mathbf{p}_{b^r}^{b_0^r}(\tau_{m+1}) - \mathbf{p}_{b^r}^{b_0^r}(\tau_m) = \mathbf{p}_{c^m}^{b_0^r}(\tau_{m+1}) - \mathbf{p}_{c^m}^{b_0^r}(\tau_m) - \left(\mathbf{R}_{b^r}^{b_0^r}(\tau_{m+1}) - \mathbf{R}_{b^r}^{b_0^r}(\tau_m)\right)\cdot\mathbf{p}_{c^m}^{b^r} \tag{34}$$

为方便，记:

$$\mathbf{D}_{m,m+1} = \mathbf{p}_{c^m}^{b_0^r}(\tau_{m+1}) - \mathbf{p}_{c^m}^{b_0^r}(\tau_m) \qquad \mathbf{E}_{m,m+1} = \mathbf{R}_{b^r}^{b_0^r}(\tau_{m+1}) - \mathbf{R}_{b^r}^{b_0^r}(\tau_m) \tag{35}$$

最后有:

$$\mathbf{c}_{m,m+1}^i - \mathbf{B}_{m,m+1}\cdot\mathbf{p}_{b^i}^{b^r} = \mathbf{D}_{m,m+1} - \mathbf{E}_{m,m+1}\cdot\mathbf{p}_{c^m}^{b^r} - \left(\mathbf{v}_{c^m}^{b_0^r}(\tau_m) - \mathbf{C}_m\cdot\mathbf{p}_{c^m}^{b^r}\right)\cdot(\tau_{m+1}-\tau_m) - \frac{1}{2}\cdot\mathbf{g}^{b_0^r}\cdot(\tau_{m+1}-\tau_m)^2 \tag{36}$$

注意，上文表述中的所有 $\{b_0^r\}$ 可以任意更换，比如更换为 $\{w\}$。

参考文献

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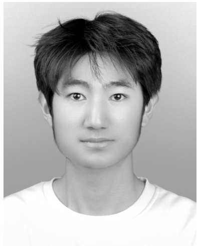
Author Information



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