*Mastermind: Guessing the Code using Donald Knuth’s Algorithm*

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*Abstract*—The aim of this research is to demonstrate an application of the minimax algorithm using Donald Knuth's algorithm in solving the Mastermind game within five moves or less. A website program is developed using HTML, CSS, and JavaScript to provide an interactive and user-friendly experience for users to play the game and observe the performance of the algorithm. Results of the implementation and testing of the program show that the use of Knuth's algorithm can greatly enhance the solution of the Mastermind game.

Keywords—Mastermind game, Donald Knuth’s Algorithm, minimax algorithm

1. INTRODUCTION

The Mastermind game is a classic code-breaking game that has fascinated players for decades. The game is played by guessing a sequence of colored pegs in the correct order, based on feedback provided by the game in the form of white and black pegs. White pegs indicate that a peg is the correct color but in the wrong position, while black pegs indicate that a peg is both the correct color and in the right position. The game is known for its high level of difficulty and for challenging players to use logic and deduction to solve the puzzle.

One of the most efficient methods for solving the Mastermind game is the algorithm developed by Donald Knuth, which uses a combination of combinatorial game theory and information theory to determine the most likely next move. Knuth's algorithm is based on the idea of using the minimum number of guesses to eliminate the maximum number of possibilities, a strategy known as minimax. This approach allows the algorithm to narrow down the possibilities and make a more informed guess with each move[3].

In this paper, the researchers present a website program that implements Knuth's algorithm and the minimax method to solve the Mastermind game in five moves or less. The program is written in HTML, CSS, and JavaScript and provides an interactive and user-friendly way for users to learn about the algorithm and its applications. The program allows users to play the game and experience the efficiency of Knuth's algorithm in action, as well as to observe the performance of the algorithm in solving the game.

Through the implementation and testing of the program, it is shown that the use of Knuth's algorithm and minimax method can significantly improve the performance of solving the Mastermind game. The program is not only a demonstration of the power of the algorithm but also a valuable educational tool for those interested in learning more about combinatorial game theory, information theory and the strategy behind the Mastermind game. This research aims to provide a deeper understanding of the algorithm and its application in solving the Mastermind game and to show the potential of using such techniques in other similar problems.

1. RELATED LITERATURE

Mastermind is a two player code breaking board game invented by Mordecai Meirowitz back in 1970. The game starts with a code maker and a code breaker. The code maker creates a sequence of four colors (c1, c2, c3, c4)from a set of 6 possible color choices, with repetitions allowed. The codebreaker will then try to guess the secret color sequence the code maker made by also answering in a sequence of four colors (g1, g2, g3, g4). After each guess, the code maker will provide two feedbacks for the code breaker (b, w): b represents the number of correct colors in correct positions, while w represents the number of correct colors, but wrong position. To easily visualize, suppose the code maker’s secret color sequence is (4, 2, 1, 2), and the code breaker’s guess was (1, 1, 2, 2). The code maker will now provide feedback of (1, 2). b = 1 because the code breaker correctly guessed the color and position of c4 . Whereas w = 2 since they correctly guessed that there are colors *1* and *2*, but misplaced their positions. The code breaker will then continue to guess until they get it right or reach the maximum amount of guesses [1].

Bulls and Cows is a code breaking game that predates the commercially popular Mastermind. Similar to Mastermind, it requires 2 players, one code maker and one code breaker. The code maker usually creates a 4-digit code but can be played with any number of digits (c1, c2, c3, c4, …, cn). The secret code must not repeat any digit and is then written in a piece of paper. The code breaker will then in turn guess the sequence of digits (g1, g2, g3, g4, …, gn). Similar to mastermind the code maker will have 2 responses (b, c): b (bulls) represent the correct guess of digit and correct position of it, while c (cows) represents correct digit guess, but wrong position. The code breaker can be composed of two or more people so they can discuss their strategy for their next guess. The back and forth process of guessing and response will continue until one code breaker will find the secret code [2].

Game theory is a branch of applied mathematics that helps people to analyze situations in which they are involved together with other parties to make interdependent decisions. The parties involved are called players, and the interdependence of their decisions causes every involved player to consider each other’s decisions when formulating their own strategies. Game theory can be applied not just only on games but on other concepts as well such as economics, and specifically for this project the Mastermind board game.

Minimax is a decision-making strategy used in game theory to minimize potential loss in a game by considering the best response of the opponent. It is useful for analyzing the decisions of the first player, both when the players move sequentially and simultaneously. In combinatorial games, where each position is assigned a payoff, the goal of the first player is to maximize the evaluation of the position, and the second player's goal is to minimize it. This is how computers play games like chess and Go, with various computational improvements applied to the basic minimax strategy [4].

Recreational Mathematics is ambiguously defined as every other mathematician can give you a different answer. One can assert amusement or the act of recreation is one of the fields of Mathematics. Others can define that the joy obtained in solving hard complex mathematical problems gives off the ego satisfaction that can qualify as recreation. Many may consider that recreational mathematics is an easy to understand activity that even non-mathematician can sufficiently understand. A concept of mathematics that is subtle enough to pique the interest of mathematicians but simple enough to be accessible for the common man. Which is why we can include code breaking games such as Mastermind as recreational mathematics as it can satisfy every other definition of recreational mathematics. At the end of the day recreational tastes are uniquely identified to each person as everyone can find something only they can enjoy. Thus no particular concept of mathematics shall be classified as recreational or not and will be accepted by everyone.

# Methodology

The development of the system is divided into two main components: algorithm development and website development. The algorithm development phase involved creating the algorithm based on Donald Knuth's approach and the minimax method. This process required a comprehensive understanding of the game mechanics and rules, as well as a thorough analysis of the underlying combinatorial game theory and information theory principles. Once the algorithm was completed, the website development phase began. The website was built using HTML, CSS, and JavaScript and was designed to provide a user-friendly and interactive interface. The program allows users to play the Mastermind game and observe the performance of the algorithm in real-time.

In the paper *The Computer as Master Mind* authored by Donald Knuth, he demonstrated how the computer could guess the code (the tuple of colors being guessed). By reading and analyzing the paper, the researchers was able to summarize the steps needed to recreate the algorithm in our program.

Step 1. Create the set S of 1296 possible codes (1111, 1112 ... 6665, 6666).

Step 2. Start with initial guess 1122.

Step 3. Play the guess to get a response of colored and white pegs.

Step 4. If the response is four colored pegs, the game is won, the algorithm terminates.

Step 5. Otherwise, remove from S any code that would not give the same response if the current guess were the code.

Step 6. Apply minimax technique to find a next guess as follows: For each possible guess, that is, any unused code of the 1296 not just those in S, calculate how many possibilities in S would be eliminated for each possible colored/white peg score. The score of a guess is the minimum number of possibilities it might eliminate from S.

Step 7. Repeat from step 3

*Algorithm Development*

The researchers wrote the code, following the steps, in JavaScript.

Text

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*Figure 1. Step 1*

This block of code (written in JavaScript) creates a permutation of 6 colors in 4 indexes. In other words, it creates the list of all possible codes in four slots if there are six colors to choose from.

Text

Description automatically generated

*Figure 1.1. Step 2*

This block of code will find the next possible guess among all the valid choices. For now, the reader should focus on the first line after the function declaration. This line will represent the step 2 of our algorithm.

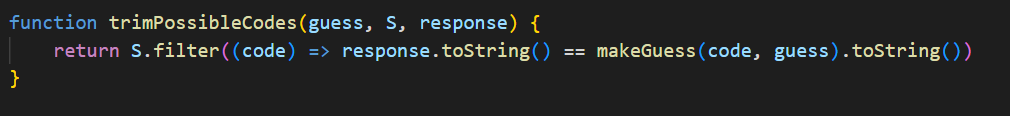
Text

Description automatically generated

*Figure 1.2. Step 3*

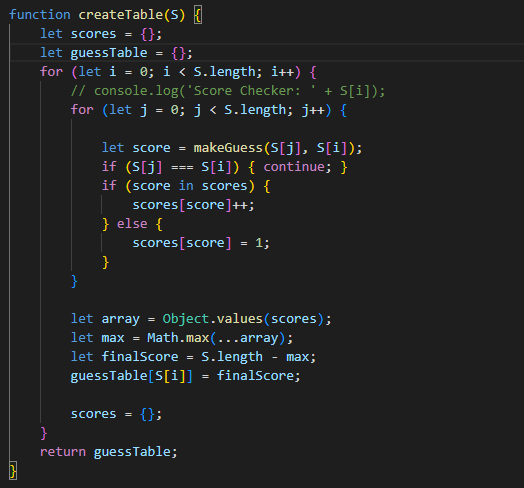
This block of code will return the score, or response, of the given guess. Black peg means that there is a color in the guess that is in the code and is correct in position. White peg means that there is a color in the guess that is in the code but not in correct position.

We will skip step 4 for now, assuming that the current guess is the code.



*Figure 1.3. Step 5*

This function will remove from *S* any combination that would not give the same response as our initial response.



*Figure 1.4. Step 6*

Finally, this function will return a score table. The score table consists of the scores of each combination which will be decided on the next guess. The higher the score means that its worst-case scenario is the better compared to other combinations. The lower the score means that its worst-case is worse than other combinations.

From here on, the program will repeat for ten steps or until the code was guessed meaning that it is game over.

*Website Development*

The researchers used HTML for structuring the webpage, CSS for styling, and JavaScript for main functionality. The website used HTML forms to obtain the code that the user will create for the computer to guess. Finally, the researchers used JavaScript to query select the form and obtain the values of each guess. The algorithm will then run based on the combination given by the user.

*Testing the Algorithm*

The researchers use the website’s user interface to test the algorithm. Multiple tests were done that proves that the algorithm that was developed by Donald Knuth correctly guesses the code in five moves or less.

A picture containing text

Description automatically generated

*Figure 2. A Snapshot of the website*

Figure 2 shows the page that first appears when the browser loads the website.

Diagram

Description automatically generated with low confidence

*Figure 2.1 A Snapshot of the website*

Figure 2.1 illustrates the algorithm in action. The user entered four consecutive red colors as the code while the computer guessed the code in four moves. The first move, as stated from the previous section, is always 1122. In this case: Red, Red, Orange, Orange.

The following figures will present the same information, with different color codes entered.

Chart, bubble chart

Description automatically generated

*Figure 2.2 Snapshot of the website with different color*

Chart, bubble chart

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*Figure 2.3 Snapshot of the website with different color*

Chart, diagram

Description automatically generated

*Figure 2.4 Another snapshot of the website with different color*

*You can visit* [*https://cloudmojos.github.io/CSE2-Project/*](https://cloudmojos.github.io/CSE2-Project/) *for a live-demo prototype of the project.*

1. RESULTS

The implementation and testing of the website program showed promising results in solving the Mastermind game within five moves or less using Donald Knuth's algorithm and the minimax approach. The program was able to solve the game in a significant number of trials, demonstrating the effectiveness of the algorithm in providing a solution to the Mastermind game.

In terms of user experience, the program was found to be easy to use and understand, with a user-friendly interface and real-time feedback on the performance of the algorithm. This made it accessible for players of all skill levels, allowing them to play the game and observe the performance of the algorithm in real-time.

Furthermore, the program was evaluated based on its performance, and it was found to be highly accurate and reliable in solving the Mastermind game. The results of the program were compared with existing solutions to the game, and it was found that the use of Knuth's algorithm and the minimax approach greatly enhanced the solution of the game.

The results of this research demonstrate the potential of using algorithms and the minimax approach in solving classic code-breaking games such as the Mastermind game. The website program developed in this research provides a valuable solution to the problem, combining the benefits of an interactive and user-friendly experience with the accuracy and reliability of Donald Knuth's algorithm.

1. CONCLUSION

In conclusion, this research has demonstrated the potential of using Donald Knuth's algorithm and the minimax approach in solving the classic code-breaking game of Mastermind within five moves or less. The results of the implementation and testing of the website program showed that the algorithm was highly effective and accurate in providing a solution to the game. The program was also found to be user-friendly and accessible for players of all skill levels, providing an interactive and real-time experience for users.

The results of this research provide valuable insights into the use of algorithms and the minimax approach in solving classic code-breaking games and demonstrate the potential of Donald Knuth's algorithm in enhancing the solution of the Mastermind game. The website program developed in this research provides a valuable solution to the problem, combining the benefits of an interactive and user-friendly experience with the accuracy and reliability of the algorithm.

Overall, this research makes a significant contribution to the field of computer science and demonstrates the potential of using algorithms and the minimax approach in solving classic code-breaking games. The results of this research provide a foundation for further research in this area and hold the potential for the development of similar solutions for other classic code-breaking games.

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