Simulation of Queueing Customers at Gentlemen's Barbershop

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Abstract— The service systems and waiting lines are integral to our everyday lives. During business hours, services such as hospitals and clinics, banks, hairdressers, and restaurants see an inflow of customers. Every system wants to avoid losing clients because of a lengthy wait in line. Increasing the number of servers improves wait times, however it is not the optimal approach since it incurs more costs to pay the staff. Conversely, disregarding wait time will result in consumer discontent and, eventually, the loss of valuable clients. This article gives an analysis of the total time a client must spend in a barbershop with the existing number of barbers, as well as the effect of additional servers (barbers) on waiting time. It simulates the barbershops queue system to minimize the waiting time for customers who enter the barbershop. In addition, it gives a foundation for making an informed choice about increasing servers. The system is modeled as a multi-server configuration system with a single queue in steady state.

Keywords—queue system, multi-server configuration system, steady state Keywords:

I. INTRODUCTION

Allocation Everywhere we go, we encounter circumstances in which we must wait in a line on a regular basis. Nobody enjoys standing in a lengthy line. To avoid this, decreasing service time is beneficial. However, depending on the nature of the employment, this is not always possible. Some procedures, such as registration for a medical examination at the entry, hair styling in salons, etc., need almost the same amount of time to service each client, which cannot always be decreased below a certain threshold. Therefore, the second option is to increase the number of servers. However, just raising the number of servers is not the optimal approach, since this would result in higher staff compensation expenditures. Alternatively, if a consumer sees a lengthy line, he may return without joining it, causing the service provider to lose valuable customers. Therefore, service providers strive to minimize waiting time while incurring the fewest costs feasible. This necessitates a comprehensive system study to optimize the waiting time. This may be accomplished by applying the principle of queuing. The mathematical examination of the waiting process in lines is known as queueing theory. When the demand for a service exceeds the capacity of the supplier, queues emerge. A simple queuing system consists of three processes: the arrival process, the queuing process, and the service process. Most queuing issues revolve on establishing the amount of service a business should deliver (Ragsdale, 2008). This study presents a steady-state analysis of a barbershop modeled as a single-queue, multiple-server system with first-come, first-served service to clients. The primary goals of this study are to determine the ideal waiting time for a client in a line and, therefore, to determine the total number of servers to be deployed by the service provider (barbershop) as the best feasible solutions under the current conditions. The issue is that consumers often have to wait a long time for their turn, and occasionally they even leave when they see the length of the line. Two barbers (servers) are installed as an existing solution. The constraint of the existing approach is that the two available servers cannot effectively supply the service to the barber's clients. On average, the consumer must wait in line for 24 minutes before receiving a haircut. In order to handle this problem, a mathematical model based on queuing theory is built, and the optimization of waiting time is analyzed. Barbershops are almost equally dispersed in this area. In a region with a high population density, the daily consumer flow is essentially constant. Therefore, a typical workday was chosen for examination.

II. METHODOLOGY

The information was gathered for eight hours, from 10 am to 6 pm. The total number of clients coming at the barbershop (N), the time they came, and the total time-duration (D) for their service were observed by researchers on a particular day at the barbershop. Using this information, the average client arrival rate (λ) is computed.:

$$\lambda = \frac{N}{8} \text{ per hour (1)}$$

From the frequency distribution, average service duration (Davg) and hence the average service rate (μ) per server are calculated as:

$$D_{avg} = \frac{\sum f.D}{\sum f} \quad (2)$$

$$\mu = \frac{60}{D} \text{ per hour (3)}$$

The arrival of the customers was random and hence the arrival rate is approximated using the Poisson random variable. The probability of arrival of x number of entities in a specified period (per hour in this case) is given by:

$$\mathrm{P(x)} = \frac{\lambda e^{-x}}{x!} \quad (4)$$

where, x=0,1,2,3,…

On the other hand, the inter-arrival time followed the exponential probability distribution with mean 1/λ. The frequency distribution of service time revealed that it followed an exponential distribution and hence it was modeled as an exponential random variable.

- The arrivals followed Poisson distribution and occurred at an average rate of λ per hour.
- Each of the servers available provided service at an average rate of μ per hour, and the actual service times followed an exponential distribution.
- Arrivals waited in a single queue and were served by the first available server.
- Finally, λ < sμ.

The various queuing characteristics pertaining to M/M/s mode are:

$$\mathrm{Rho} = \frac{\lambda}{s\mu} \quad (5)$$

$$\frac{1}{p_0} = \sum_{n=0}^{s-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^s}{s!} x \frac{s\mu}{s\mu - \lambda} \quad (6)$$

$$L_q = P_0 x \frac{(\lambda/\mu)^{s+1}}{(s-1)!} x (s - \lambda/\mu)^{-2} \quad (7)$$

$$L = L_q + \frac{\lambda}{\mu} \quad (8)$$

$$W_q = \frac{L_q}{\lambda} \quad (9)$$

$$W = W_q + \frac{1}{\mu} \quad (10)$$

Using the aforementioned formulae, the average waiting time and the total average-time a customer spends in the system (in line and in service) were calculated. Then, in the next two iterations, the number of servers was increased and again the values of those parameters were calculated and the comparison of three results was made. Thus, the calculations were done for up to 5 number of servers.

CALCULATIONS

Altogether, 50 customers visited the barbershop for the haircut within the considered time frame of 8 hours: 10 am to 6 pm.

Service Duration (D, Mins)	Frequency (f)	f.D
15	9	135
18	6	108
20	7	140
22	11	242
25	6	150
27	3	81
28	3	84
30	2	60
32	1	32
35	1	35
40	1	40
	Σf =50	Σf.D =1107

Table 1. Frequency distribution of service time

A steady-state analysis of a barbershop as a single-queue, multi-server system to optimize the waiting time in a queue.

$$\lambda = \frac{50}{8} = \mathbf{6.25}\ \boldsymbol{per\ hour}$$

$$D_{avg} = \frac{\sum f.D}{\sum f} = \frac{1107}{50} = \mathbf{22.24}\ \boldsymbol{min}$$

$$\mu = \frac{60}{D} = \frac{60}{22.24} = \mathbf{2.71}\ \boldsymbol{Customer\ per\ hour}$$

For 3 servers, various queuing parameters are calculated using equations (5) to (10) as follows:

$$Rho = \frac{\lambda}{s\mu} = \frac{6.25}{3*2.71} = 0.7688 = \mathbf{76.88}\%$$

$$\frac{1}{p_0} = \sum_{n=0}^{s-1} \frac{(6.25/2.71)^n}{n!} + \frac{(6.25/2.71)^3}{3!} x \frac{3x2.71}{3x2.71-6.25} = 14.8063344045$$

$$\therefore P_0 = \frac{1}{14.8063344045} = \mathbf{0.067538661}$$

$$L_q = 0.067538661x \frac{(6.25/2.71)^{3+1}}{(3-1)!} x(3 - 6.25/2.71)^{-2} = \mathbf{1.984915352}$$

$$L = 1.9849 + \frac{6.25}{2.71} = \mathbf{4.291165352}$$

$$W_q = \frac{1.9849}{6.25} = \mathbf{0.317586456}\ \boldsymbol{hr} = \mathbf{19.06}\ \boldsymbol{mins}$$

$$W = 0.6864 + \frac{1}{2.71} = \mathbf{0.686586456}\ \boldsymbol{hr} = \mathbf{41.20}\ \boldsymbol{mins}$$

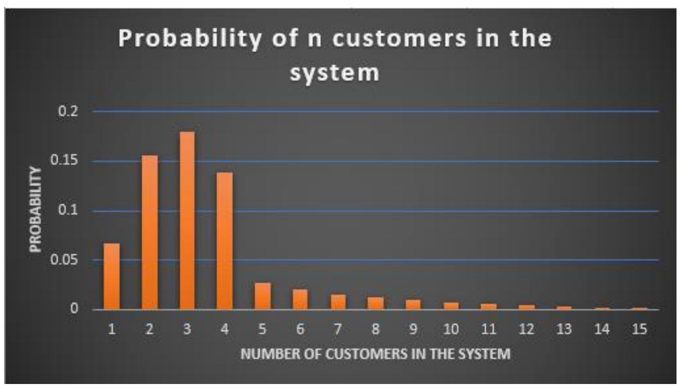
It is seen that, on average, a customer has to stay approximately 19 mins in the queue waiting for his turn, with his total time in system (the barbershop) being nearly 41 minutes

Computations in Excel

Inputs		
Time Unit	hour	
Arrival Rate (λ)	6.25	customers/hour
Service Rate per server (mu)	2.71	customers/hour
Number of servers (s)	3	servers
Intermediate Calculations		
Ave time between arrivals	0.16	hour
Ave service time per server	0.369	hour
Combined service rate (s*mu)	8.130081301	customers/hour
Performance Measures		
Rho (Average server Utilization)	0.76875	
P0 (probability the system is empty)	0.067538661	
L (average number in system)	4.291165352	customers
Lq (average number waiting in the queue)	1.984915352	customers
W (average time in system)	0.686586456	hour
Wq (average time in the queue)	0.317586456	hour

Table 2. Computation of Data on Excel

Probability of a specific no of customers in the system	
no	**Probablility**
0	0.067538661
1	0.155761036
2	0.179611945
3	0.138076683
4	0.026536612
5	0.020400021
6	0.015682516
7	0.012055934
8	0.009267999
9	0.007124775
10	0.00547717
11	0.004210575
12	0.003236879
13	0.002488351
14	0.00191292
15	0.001470557



Let's find out these two parameters again by increasing the number of servers (barber) to 4 i.e. s = 4.

$$Rho = \frac{\lambda}{s\mu} = \frac{6.25}{4*2.71} = 0.5766 = \mathbf{57.66\%}$$

$$\frac{1}{p_0} = \sum_{n=0}^{s-1} \frac{(6.25/2.71)^n}{n!} + \frac{(6.25/2.71)^4}{4!} x \frac{4x2.71}{4x2.71 - 6.25} = \mathbf{10.7937704229}$$

$$\therefore P_0 = \frac{1}{10.7937704229} = \mathbf{0.092646032}$$

$$L_q = 0.092646032x \frac{(6.25/2.71)^{4+1}}{(4-1)!} x(4 - 6.25/2.71)^{-2} = \mathbf{0.351163101}$$

$$L = 0.351163101 + \frac{6.25}{2.71} = \mathbf{2.6574131101}$$

$$W_q = \frac{0.351163101}{6.25} = \mathbf{0.056186096\ hour = 3.37\ mins}$$

$$W = 0.056186096 + \frac{1}{2.71} = \mathbf{0.425186096\ hour} = \mathbf{25.51\ mins}$$

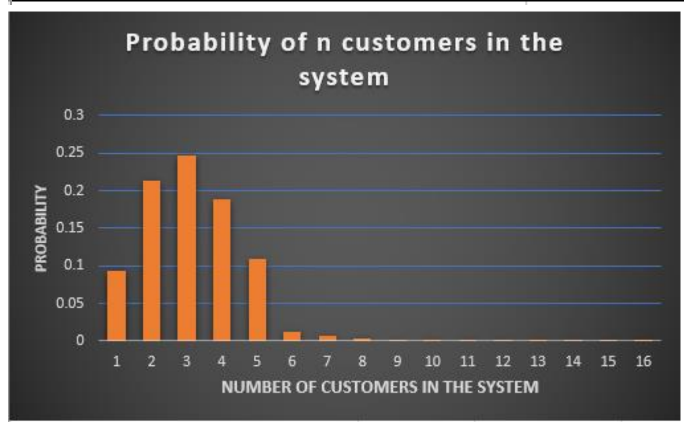
Here, we can see that on increasing the number of servers to 4, the waiting time significantly drops down to 3.37 minutes from 25.51 minutes.

Computations in Excel

Inputs		
Time Unit	hour	
Arrival Rate (λ)	6.25	customers/hour
Service Rate per server (mu)	2.71	customers/hour
Number of servers (s)	4	servers
Intermediate Calculations		
Ave time between arrivals	0.16	hour
Ave service time per server	0.369	hour
Combined service rate (s*mu)	10.8401084	customers/hour
Performance Measures		
Rho (Average server Utilization)	0.5765625	
P0 (probability the system is empty)	0.092646032	
L (average number in system)	2.657413101	customers
Lq (average number waiting in the queue)	0.351163101	customers
W (average time in system)	0.425186096	hour
Wq (average time in the queue)	0.056186096	hour

Table 3. Computation of Data on Excel

Probability of a specific no of customers in the system	
no	**Probablility**
0	0.092646032
1	0.213664912
2	0.246382351
3	0.189406433
4	0.109204646
5	0.012592661
6	0.007260456
7	0.004186107
8	0.002413552
9	0.001391564
10	0.000802323
11	0.00046259
12	0.000266712
13	0.000153776
14	8.86615E-05
15	5.11189E-05



Now, let's repeat the process by increasing the number of servers to 5 to see the effect on the average times.

$$Rho = \frac{\lambda}{s\mu} = \frac{6.25}{5*2.71} = 0.46125 = \mathbf{46.13\%}$$

$$\frac{1}{p_0} = \sum_{n=0}^{s-1} \frac{(6.25/2.71)^n}{n!} + \frac{(6.25/2.71)^5}{5!} x \frac{5x2.71}{5x2.71 - 6.25} = 10.19795173$$

$$\therefore P_0 = \frac{1}{10.19795173} = \mathbf{0.098058907}$$

$$L_q = 0.098058907x \frac{(6.25/2.71)^{5+1}}{(5-1)!} x(5 - 6.25/2.71)^{-2} = \mathbf{0.08472265}$$

$$L = 0.08472265 + \frac{6.25}{2.71} = \mathbf{2.39097265}$$

$$W_q = \frac{0.08472265}{6.25} = \mathbf{0.013555624\ hour = 0.81\ mins}$$

$$W = 0.056186096 + \frac{1}{2.71} = \mathbf{0.38255624\ hour}$$
$$= \mathbf{25.95\ mins}$$

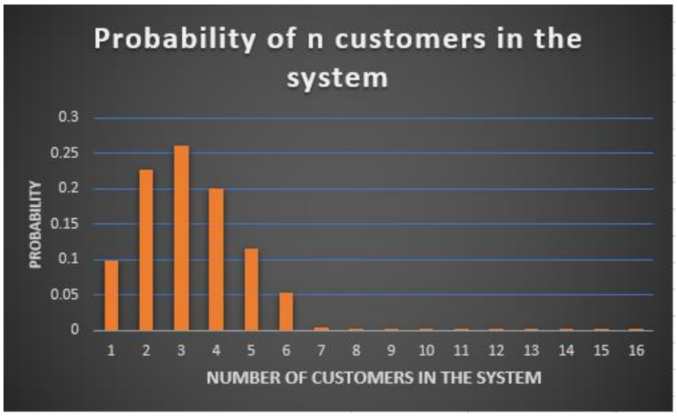
It is seen that, on increasing the number of servers to 4, the average waiting time reduces to 0.81 minutes while the total time to be spent in the system reduces to 25.95 minutes.

Computations in Excel

Inputs		
Time Unit	hour	
Arrival Rate (λ)	6.25	customers/hour
Service Rate per server (mu)	2.71	customers/hour
Number of servers (s)	5	servers
Intermediate Calculations		
Ave time between arrivals	0.16	hour
Ave service time per server	0.369	hour
Combined service rate (s*mu)	13.5501355	customers/hour
Performance Measures		
Rho (Average server Utilization)	0.46125	
P0 (probability the system is empty)	0.098058907	
L (average number in system)	2.39097265	customers
Lq (average number waiting in the queue)	0.08472265	customers
W (average time in system)	0.382555624	hour
Wq (average time in the queue)	0.013555624	hour

Table 4. Computation of Data on Excel

Probability of a specific no of customers in the system	
no	**Probablility**
0	0.098058907
1	0.226148353
2	0.26077732
3	0.200472565
4	0.115584963
5	0.053313564
6	0.00409848
7	0.001890424
8	0.000871958
9	0.000402191
10	0.00018551
11	8.55667E-05
12	3.94676E-05
13	1.82044E-05
14	8.3968E-06
15	3.87302E-06



It can be observed that increasing the number of servers further beyond 3 doesn't reduce the waiting time significantly in relative comparison with the number of servers. Just adding servers may result in increased expenses despite giving added benefit. Thus, it doesn't look attractive to increase the number of servers further.

III. SUMMARY OF FINDINGS

The average amount of time a client spent waiting in line and on the system to be serviced in a barbershop was analyzed. In-person observations were conducted for eight hours, from 10 a.m. to 6 p.m., in order to determine the average arrival rate and average service rate. It was discovered that the arrival followed the Poisson distribution, but the service time followed the exponential distribution. The average rate of arrival was determined to be 6.25 per hour, while the average service duration was 22.24 minutes, resulting in a service rate of 2.71

per hour. The system was represented as an M/M/s system according to the Markovian inter-arrival and service times. Three servers (Barber) were present in the barbershop. For 3 servers, the arrival rate of 6.25 per hour, and the service rate of 2.71 per hour, the equations (5) were used to determine different queuing parameters (10). It was determined that a client waited an average of 19 minutes in line for his turn and 41 minutes in the barbershop to get service. The number of servers was then raised to four, and the average waiting time and average time spent in the system were obtained using the same method. It was found that the average wait time declined dramatically, from 25 minutes to 3 minutes (about) (approx.). When the number of servers was expanded to five, the average wait time dropped to 0.81 minutes, and the total time spent in the system was lowered to 25.95 minutes. However, the reduction in waiting time is not as impressive as when it decreased from around 24 minutes to 3 minutes (approx.). Additionally, it may raise the expense of hiring an additional barber. Taking into account the practically constant flow of clients into the barbershop on a daily basis, it is advised that the owner hire a total of four servers at the barbershop so that customers wait no more than three minutes in line and complete their service within twenty-five minutes.

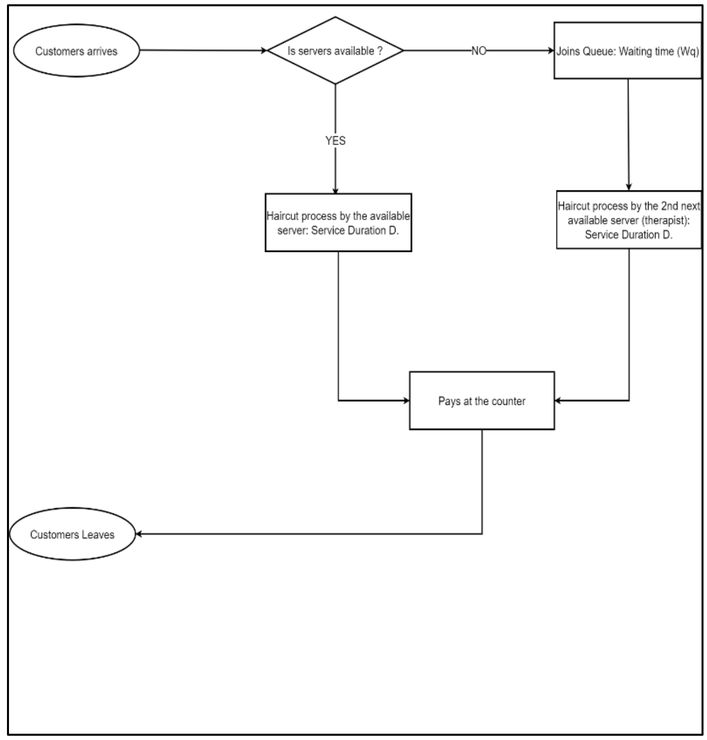


Fig 1. Flowchart Diagram of the System

IV. IMPLEMENTATION

To solve this problem, a mathematical model based on queuing theory is built, and the analysis for waiting time optimization is conducted. This area has a rather equal distribution of barbershops. Consequently, a typical workday was chosen for investigation. Using a flowchart, the fundamental elements of this barbershops queuing procedure may be shown (Fig. 1.). If one of the servers is available upon arrival, the consumer may immediately get service. Otherwise, the consumer must wait until one of the servers is available before receiving service. On average, they wait there for Wq minutes. The consumer departs the barbershop after W minutes, on average, from the time of arrival to service completion. A M/M/s model is used for the system's study. The first 'M' in the Kendall Notation above represents Markovian inter-arrival times (following an exponential distribution), the second 'M' represents Markovian service times (following an exponential distribution of service times), and's' represents the number of available servers. Therefore, the notation M/M/1 indicates that the inter-arrival time of the customers follows an exponential distribution, as does the time required by the server to service each client, and that the total number of servers, i.e. barbers, is three. In this particular study, however, the average arrival rate (because it is a steady-state analysis) and, therefore, the average service length are used.

V. CONCLUSION

A steady-state study of the operation of a barbershop was conducted by modeling the system as an M/M/s model in order to determine the best waiting time for a client and, as a result, to make choices on the total number of servers to be deployed in order to minimize costs. It is determined that the ideal waiting time is around 3 minutes, for which 4 barbers (servers) should be installed. 30 A steady-state study of barbershop as a single-queue, multi-server system to optimize queue wait time. Since, in the Philippines, consumers are not enticed to return after just three minutes of wait time, the proposed solution seems optimal in light of the existing customer flow, personnel costs, and income earned by customer service. Therefore, it is suggested that the proprietor hire four barbers so that a client may have his hair cut in around 25 minutes, while the line wait time is approximately 3 minutes. In places with a comparable pattern of client flow rates (within the city), a total of four servers seems to be the optimal wait time option. It should be noted, however, that in situations where labor expenses are too low relative to revenue generation and customer impatience is high (causing customers to return after arrival), additional analysis may be necessary to make a decision based on a relative comparison of the benefit and the expenses/costs incurred. A basic instance was explored without taking into consideration complicated situations in which clients balk (by not entering a line upon arriving) or back out (leaving the queue before being served). Additionally, the arrival rate is consistent throughout the day.

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