CS 7641 Problem Set 2

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1. You have to communicate a signal in a language that has 3 symbols A, B and C. The probability of observing A is 50% while that of observing B and C is 25% each. Design an appropriate encoding for this language. What is the entropy of this signal in bits?

Answer:

The probability of observing A / The probability of observing B = P(A) / P(B) = 50% / 25% = 2

The probability of observing A / The probability of observing C = P(A) / P(C) = 50% / 25% = 2

Therefore we could encode symbol A with 1 bit, symbol B with 2 bits and symbol C with 2 bits. The entropy of this signal in bits is:

Entropy = 1bit x P(A) + 2bits x P(B) + 2bits x P(C) = 1bit x 0.5 + 2bits x 0.25 + 2bits x 0.25 = 1.5bits

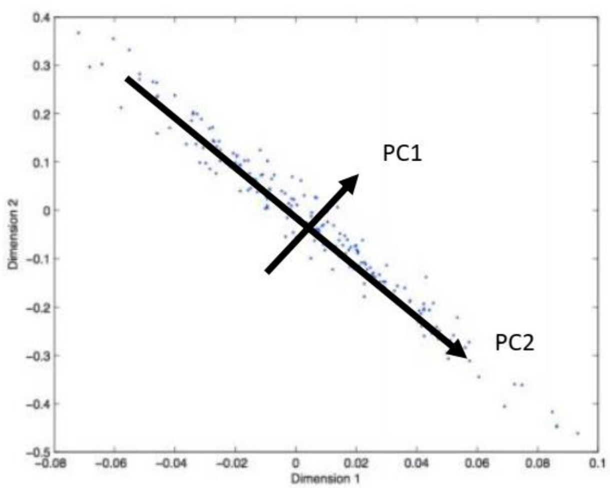
2. Show that the K-means procedure can be viewed as a special case of the EM algorithm applied to an appropriate mixture of Gaussian densities model.

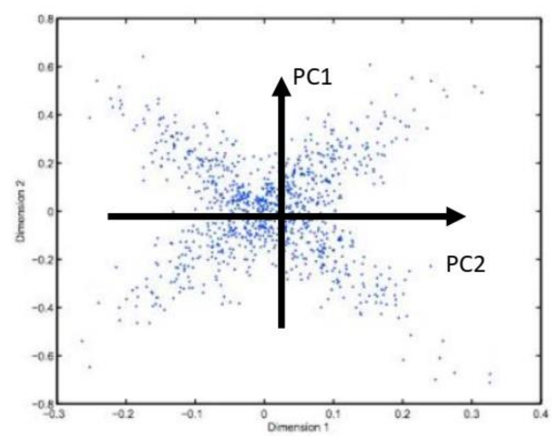
Answer:

For clustering problem, the probability of a data point belonging to a certain cluster is unknown. So in the E step, the EM algorithm builds a model with estimated parameters and computes the ownership probability of each data point. Then in the M step, the estimated parameters (mean, variances and mixing weights for GMM) are updated to maximize likelihood function with the given ownership probabilities.

According to the K-means procedure, the E step is to assign each data point to a cluster, and the M step is to recompute the mean of each cluster. Therefore comparing with GMM, the E step of K-means is a hard assignment without ownership probability information, and the M step of K-means doesn't consider the variances between samples as well as the mixing weights. So the K-means procedure can be viewed as a special case of the EM algorithm applied to an appropriate mixture of Gaussian densities model.

3. Plot the direction of the first and second PCA components in the figures given.





6. Use the Bellman equation to calculate Q(s, a1) and Q(s, a2) for the scenario shown in the figure. Consider two different policies:

● Total exploration: All actions are chosen with equal probability.

● Greedy exploitation: The agent always chooses the best action.

Note that the rewards/next states are stochastic for the actions a1', a2' and a3'. Assume that the probabilities for the outcome of these actions are all equal. Assume that reward gathering / decision making stops at the empty circles at the bottom.

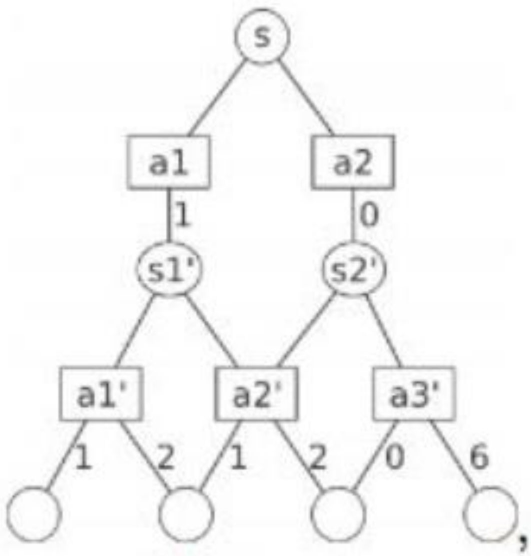


Figure 1.

Answer:

From Figure 1 we could have the following information:

Q(S1’, a1’) = 1 x 0.5 + 2 x 0.5 = 1.5

Q(S1’, a2’) = 1 x 0.5 + 2 x 0.5 = 1.5

Q(S2’, a2’) = 1 x 0.5 + 2 x 0.5 = 1.5

Q(S2’, a3’) = 0 x 0.5 + 6 x 0.5 = 3

For Total exploration:

Q(S, a1) = R(S, a1, S1’) + 0.5 x Q(S1’, a1’) + 0.5 x Q(S1’, a2’) = 1 + 1.5 x 0.5 + 1.5 x 0.5 = 2.5

Q(S, a2) = R(S, a2, S2’) + 0.5 x Q(S2’, a2’) + 0.5 x Q(S2’, a3’) = 0 + 1.5 x 0.5 + 3 x 0.5 = 2.25

For Greedy exploration:

Q(S, a1) = R(S, a1, S1’) + 0 x Q(S1’, a1’) + 1 x Q(S1’, a2’) = 1 + 1.5 x 0 + 1.5 x 1 = 2.5

Q(S, a2) = R(S, a2, S2’) + 0 x Q(S2’, a2’) + 1 x Q(S2’, a3’) = 0 + 1.5 x 0 + 3 x 1 = 3