```
import numpy as np
import pandas as pd
from scipy.spatial.distance import cdist
import matplotlib.pyplot as plt

# Given points
points = np.array([
    [1, 2],
    [4, 5],
    [3, 4],
    [6, 2],
    [7, 3],
    [8, 5],
    [9, 6],
    [10, 2],
    [5, 5],
    [11, 6]
])

# Compute pairwise Euclidean distances
distance_matrix = cdist(points, points, metric='euclidean')

# Convert to DataFrame for better readability
distance_df = pd.DataFrame(distance_matrix, columns=[f"P{i+1}" for i in range(len(points))],
                           index=[f"P{i+1}" for i in range(len(points))])

distance_df
```

	P1	P2	P3	P4	P5	P6	P7	P8	P9	P10
P1	0.000000	4.242641	2.828427	5.000000	6.082763	7.615773	8.944272	9.000000	5.000000	10.770330
P2	4.242641	0.000000	1.414214	3.605551	3.605551	4.000000	5.099020	6.708204	1.000000	7.071068
P3	2.828427	1.414214	0.000000	3.605551	4.123106	5.099020	6.324555	7.280110	2.236068	8.246211
P4	5.000000	3.605551	3.605551	0.000000	1.414214	3.605551	5.000000	4.000000	3.162278	6.403124
P5	6.082763	3.605551	4.123106	1.414214	0.000000	2.236068	3.605551	3.162278	2.828427	5.000000
P6	7.615773	4.000000	5.099020	3.605551	2.236068	0.000000	1.414214	3.605551	3.000000	3.162278
P7	8.944272	5.099020	6.324555	5.000000	3.605551	1.414214	0.000000	4.123106	4.123106	2.000000
P8	9.000000	6.708204	7.280110	4.000000	3.162278	3.605551	4.123106	0.000000	5.830952	4.123106
P9	5.000000	1.000000	2.236068	3.162278	2.828427	3.000000	4.123106	5.830952	0.000000	6.082763
P10	10.770330	7.071068	8.246211	6.403124	5.000000	3.162278	2.000000	4.123106	6.082763	0.000000

Next steps: View recommended plots | New interactive sheet

```
# Define distance threshold
threshold = 4

# Count neighbors within threshold (excluding self)
neighbor_counts = np.sum(distance_matrix <= threshold, axis=1) - 1  # Exclude self

# Compute average neighbor density
avg_density = np.mean(neighbor_counts)

# Compute KLE Score
kle_scores = neighbor_counts / avg_density

# Create neighbor list for each point
neighbor_list = []
for i in range(len(points)):
    neighbors = [f"P{j+1}" for j in range(len(points)) if distance_matrix[i, j] <= threshold and i != j]
    neighbor_list.append(", ".join(neighbors) if neighbors else "None")

# Create DataFrame for KLE scores
kle_df = pd.DataFrame({
    "Point": [f"P{i+1}" for i in range(len(points))],
    "Neighbors": neighbor_list,
    "Num Neighbors": neighbor_counts,
    "KLE Score": kle_scores
})

kle_df
```

	Point	Neighbors	Num Neighbors	KLE Score
0	P1	P3	1	0.238095
1	P2	P3, P4, P5, P6, P9	5	1.190476
2	P3	P1, P2, P4, P9	4	0.952381
3	P4	P2, P3, P5, P6, P8, P9	6	1.428571
4	P5	P2, P4, P6, P7, P8, P9	6	1.428571
5	P6	P2, P4, P5, P7, P8, P9, P10	7	1.666667
6	P7	P5, P6, P10	3	0.714286
7	P8	P4, P5, P6	3	0.714286
8	P9	P2, P3, P4, P5, P6	5	1.190476
9	P10	P6, P7	2	0.476190

Next steps: View recommended plots | New interactive sheet

```
# Define outliers as points with KLE Score < 1
outliers = kle_df[kle_df["KLE Score"] < 1]
normal_points = kle_df[kle_df["KLE Score"] >= 1]

# Extract outlier points
outlier_indices = [int(p[1:]) - 1 for p in outliers["Point"]]
outlier_coords = np.array(points)[outlier_indices]

# Format and print outlier points
outlier_points_str = ", ".join(f"({x}, {y})" for x, y in outlier_coords)
print(f"Outlier points: {outlier_points_str}")
```

```
Outlier points: (1, 2), (3, 4), (9, 6), (10, 2), (11, 6)
```

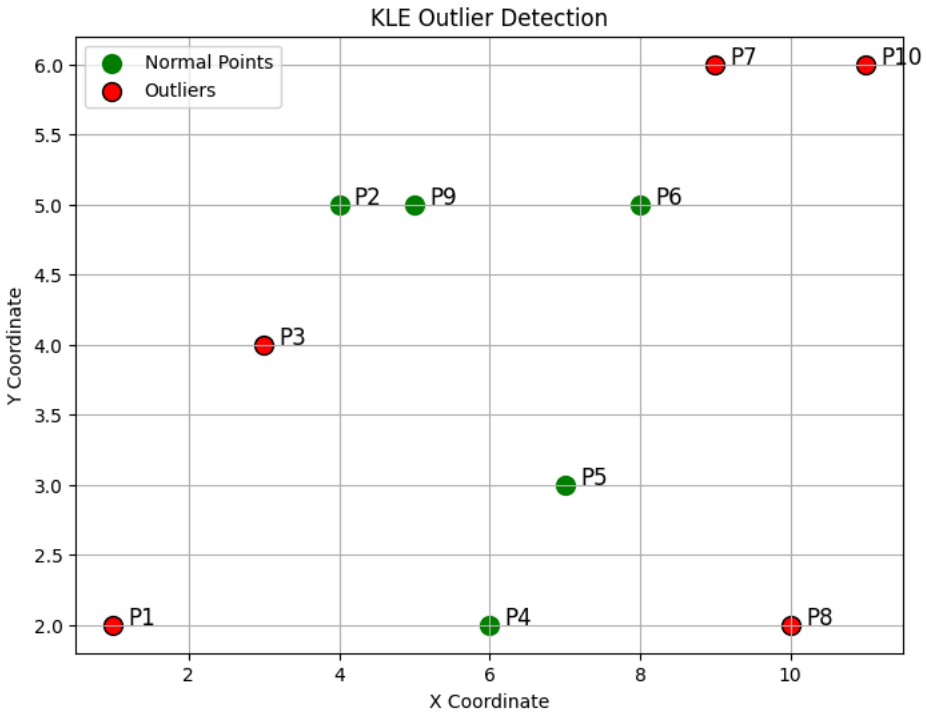
```
# Extract coordinates for plotting
outlier_indices = [int(p[1:]) - 1 for p in outliers["Point"]]  # Convert P1 -> 0, P2 -> 1, ...
normal_indices = [int(p[1:]) - 1 for p in normal_points["Point"]]

outlier_coords = np.array(points)[outlier_indices]
normal_coords = np.array(points)[normal_indices]

# Plot points
plt.figure(figsize=(8, 6))
plt.scatter(normal_coords[:, 0], normal_coords[:, 1], color='green', label='Normal Points', s=100)
plt.scatter(outlier_coords[:, 0], outlier_coords[:, 1], color='red', label='Outliers', s=100, edgecolors='black')

# Annotate points
for i, (x, y) in enumerate(points):
    plt.text(x + 0.2, y, f"P{i+1}", fontsize=12)

plt.xlabel("X Coordinate")
plt.ylabel("Y Coordinate")
plt.title("KLE Outlier Detection")
plt.legend()
plt.grid(True)
plt.show()
```


KLE Outlier Detection
Normal Points
Outliers
P1
P2
P3
P4
P5
P6
P7
P8
P9
P10
Y Coordinate
X Coordinate
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
2
4
6
8
10