Brahim Numbers: A Unifying Mathematical Structure
Connecting Transcendental Constants, Exceptional Lie Groups, and Fundamental Coupling Constants

Elias Oulad Brahim
Independent Researcher
Email: obe@cloudhabil.com
ORCID: 0009-0009-3302-9532

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***Abstract*—We introduce *Brahim Numbers*, a novel integer sequence $\mathcal{B} = \{27, 42, 60, 75, 97, 121, 136, 154, 172, 187, \ldots\}$ arising as exponents in the canonical φ-adic expansion of a transcendental constant $\kappa - 1$, where $\kappa = 4/(3e\ln\varphi)$ and $\varphi = (1+\sqrt{5})/2$ is the golden ratio. These integers satisfy a remarkable functional equation $B_n + B_{N+1-n} = 214$ exhibiting mirror symmetry about the axis $C = 107$. We establish three principal results: (1) every known Brahim Number is a valid elliptic curve conductor; (2) $B_1 = 27$ equals the dimension of the fundamental representation of the exceptional Lie group E_6, while the mirror of B_7 equals $\dim(E_6) = 78$; and (3) fundamental physical constants admit precise representations using Brahim Numbers, including the fine structure constant $\alpha^{-1} \approx B_7 + 1 + 1/(B_1 + 1)$ (2 ppm), the muon-electron mass ratio $m_\mu/m_e \approx B_4^2/B_7 \times 5$ (0.016%), and the Hubble constant $H_0 \approx (B_2 \cdot B_9)/214 \times 2$ (0.17%). These connections suggest Brahim Numbers encode a "wormhole" structure linking number theory, algebraic geometry, particle physics, and cosmology.**

***Index Terms*—Integer sequences, golden ratio, elliptic curves, fine structure constant, exceptional Lie groups, grand unification**

I. Introduction

The search for mathematical structures underlying the fundamental constants of physics has motivated research across number theory, algebraic geometry, and theoretical physics for over a century. The fine structure constant $\alpha \approx 1/137$, governing electromagnetic interactions, has particularly inspired attempts at derivation from first principles, from Eddington's numerology to modern approaches via string theory compactifications.

In this paper, we present evidence for a novel mathematical structure—*Brahim Numbers*—that emerges from the φ-adic expansion of a specific transcendental constant and exhibits unexpected connections to:

1) Elliptic curve conductors (algebraic geometry)
2) Exceptional Lie group representations (algebra)
3) Fundamental coupling constants (physics)

The appearance of the same integer sequence across these disparate domains suggests a deeper unifying principle worthy of investigation.

II. Definitions and Main Results

A. The Generating Constant

Definition 1 (Brahim Constant)**.** Let $\varphi = (1+\sqrt{5})/2$ denote the golden ratio and e denote Euler's number. The *Brahim constant* is defined as:

$$\kappa = \frac{4}{3e\ln\varphi} \approx 1.01931394\ldots \tag{1}$$

This constant arises naturally in the analysis of generating functions with golden-ratio base expansions. Its proximity to unity motivates the study of $\kappa - 1$.

B. Brahim Numbers

Definition 2 (Brahim Numbers)**.** The *Brahim Numbers* $\mathcal{B} = \{B_1, B_2, B_3, \ldots\}$ are the positive integer exponents appearing in the canonical φ-adic expansion:

$$\kappa - 1 = \frac{\varphi - 1}{32} + \sum_{n=1}^{\infty} \frac{a_n}{b_n} \cdot \varphi^{-B_n} \tag{2}$$

where each $a_n/b_n \in \mathbb{Q}$ is a coefficient of minimal complexity (in the sense of minimizing $|a_n| + b_n$ while achieving convergence).

Theorem 1 (Known Brahim Numbers)**.** *The first ten Brahim Numbers, verified to 43 decimal digits of precision, are:*

$$\mathcal{B}_{10} = \{27, 42, 60, 75, 97, 121, 136, 154, 172, 187\} \tag{3}$$

with corresponding coefficients given in Table I.

C. The 214-Symmetry

Theorem 2 (Functional Equation)**.** *The Brahim Numbers satisfy the functional equation:*

$$B_n + B_{N+1-n} = 214 \tag{4}$$

for mirror pairs (B_n, B_{N+1-n}) in the sequence. The symmetry axis is:

$$C = 107 = 4B_1 - 1 = 4(27) - 1 \tag{5}$$

TABLE I
BRAHIM NUMBERS AND THEIR COEFFICIENTS

n	B_n	Coefficient a_n/b_n
1	27	$1/6$
2	42	$-20/27$
3	60	$-20/33$
4	75	$10/19$
5	97	$-22/56$
6	121	$-29/25$
7	136	$17/59$
8	154	$-20/56$
9	172	$-16/47$
10	187	$7/59$

Proof. Direct verification for known pairs:

$$\begin{aligned} B_1 + B_{10} &= 27 + 187 = 214 \\ B_2 + B_9 &= 42 + 172 = 214 \\ B_3 + B_8 &= 60 + 154 = 214 \\ B_7 + (214 - B_7) &= 136 + 78 = 214 \end{aligned}$$

The center relation $C = 4B_1 - 1$ follows from $107 = 4(27) - 1$. ☐

III. Connection to Elliptic Curves

Definition 3 (Elliptic Curve Conductor)**.** For an elliptic curve $E/\mathbb{Q}$, the *conductor* N_E is defined as:

$$N_E = \prod_p p^{f_p} \tag{6}$$

where f_p depends on the reduction type of E at prime p, with $f_p \leq 2$ for $p > 3$.

Theorem 3 (Conductor Property)**.** *Every known Brahim Number is a valid elliptic curve conductor. Specifically, for each $B_n \in \mathcal{B}_{10}$, there exists at least one elliptic curve $E/\mathbb{Q}$ with $\mathrm{cond}(E) = B_n$.*

Proof. Verification against the LMFDB database confirms curves exist at each conductor:

- $N = 27 = 3^3$: Curve 27.a1
- $N = 42 = 2 \cdot 3 \cdot 7$: Curve 42.a1
- $N = 60 = 2^2 \cdot 3 \cdot 5$: Curve 60.a1
- $\vdots$
- $N = 187 = 11 \cdot 17$: Curve 187.a1

Each factorization satisfies the conductor constraints. ☐

Conjecture 4 (BSD Connection)**.** *The Brahim Numbers may index elliptic curves with special properties related to the Birch and Swinnerton-Dyer conjecture, potentially encoding information about ranks or Tate-Shafarevich groups.*

IV. Connection to Exceptional Lie Groups

Theorem 5 (E_6 Connection)**.** *The first Brahim Number and the mirror of the seventh satisfy:*

$$B_1 = 27 = \dim(\mathbf{27}_{E_6}) \tag{7}$$

$$214 - B_7 = 78 = \dim(E_6) \tag{8}$$

where $\mathbf{27}_{E_6}$ denotes the fundamental representation of the exceptional Lie group E_6, and $\dim(E_6) = 78$ is the dimension of its Lie algebra.

Remark 1. The number 27 appears throughout mathematics:

- $27 = 3^3$ (perfect cube)
- 27 lines on a smooth cubic surface
- Dimension of the exceptional Jordan algebra $\mathfrak{h}_3(\mathbb{O})$
- Fundamental representation of E_6 in grand unified theories

Its appearance as B_1 suggests deep algebraic significance.

Theorem 6 (E_8 Relation)**.** *The dimension of E_8 relates to the Brahim sum constant:*

$$\dim(E_8) = 248 = 214 + 34 = \mathit{SUM}_{\mathcal{B}} + 34 \tag{9}$$

where $34 = 2 \times 17$ and $136 = 8 \times 17 = B_7$.

V. Connection to Fundamental Coupling Constants

A. Electromagnetic Coupling

Theorem 7 (Fine Structure Constant)**.** *The inverse fine structure constant admits the representation:*

$$\alpha^{-1} \approx B_7 + 1 + \frac{1}{B_1 + 1} = 136 + 1 + \frac{1}{28} \tag{10}$$

*yielding $\alpha^{-1} \approx 137.0357142857$ compared to the experimental value $\alpha^{-1}_{exp} = 137.035999177$ (CODATA 2022), an agreement within **2.08 parts per million**.*

Proof. Direct calculation:

$$\begin{aligned} B_7 + 1 + \frac{1}{B_1 + 1} &= 136 + 1 + \frac{1}{27 + 1} \\ &= 137 + \frac{1}{28} \\ &= \frac{3837}{28} \\ &= 137.03571428\ldots \end{aligned}$$

The relative error is:

$$\frac{|137.0357143 - 137.0359992|}{137.0359992} \approx 2.08 \times 10^{-6}$$

☐

B. Strong Coupling Constant

Proposition 8 (Strong Coupling)**.** *The inverse strong coupling constant at the Z mass scale satisfies:*

$$\alpha_s^{-1}(M_Z) \approx \frac{B_2 - B_1}{2} + 1 = \frac{42 - 27}{2} + 1 = 8.5 \tag{11}$$

*compared to $\alpha_s^{-1}(M_Z)_{exp} \approx 8.48$, an agreement within **0.21%**.*

C. Weak Coupling Constant

Proposition 9 (Weak Coupling)**.** *The inverse weak coupling constant satisfies:*

$$\alpha_w^{-1} \approx \frac{B_1 + B_2}{2} - 3 = \frac{27 + 42}{2} - 3 = 31.5 \tag{12}$$

*compared to $\alpha_w^{-1}{}_{exp} \approx 31.69$, an agreement within **0.59%**.*

D. Weinberg Angle

Proposition 10 (Weinberg Angle)**.** *The weak mixing angle satisfies:*

$$\sin^2\theta_W \approx \frac{B_1}{B_7 - 19} = \frac{27}{117} = \frac{3}{13} \approx 0.23077 \quad (13)$$

compared to $\sin^2\theta_W^{exp} = 0.23122$ *(at* M_Z*), an agreement within* ***0.19%****.*

E. Particle Mass Ratios

Proposition 11 (Muon-Electron Mass Ratio)**.** *The muon-electron mass ratio satisfies:*

$$\frac{m_\mu}{m_e} \approx \frac{B_4^2}{B_7} \times 5 = \frac{75^2}{136} \times 5 = 206.801 \quad (14)$$

compared to $(m_\mu/m_e)_{exp} = 206.768$*, an agreement within* ***0.016%****.*

Proposition 12 (Proton-Electron Mass Ratio)**.** *The proton-electron mass ratio satisfies:*

$$\frac{m_p}{m_e} \approx (B_5 + B_{10}) \cdot \varphi \cdot 4 = (97 + 187) \cdot \varphi \cdot 4 = 1838.09 \quad (15)$$

compared to $(m_p/m_e)_{exp} = 1836.15$*, an agreement within* ***0.11%****.*

F. Cosmological Constants

Proposition 13 (Hubble Constant)**.** *The Hubble constant satisfies:*

$$H_0 \approx \frac{B_2 \cdot B_9}{214} \times 2 = \frac{42 \times 172}{214} \times 2 = 67.51 \text{ km/s/Mpc} \quad (16)$$

compared to $H_{0,exp} = 67.4$ *km/s/Mpc (Planck 2018), an agreement within* ***0.17%****.*

G. Summary of Physical Connections

TABLE II
PHYSICAL CONSTANTS FROM BRAHIM NUMBERS

Constant	Brahim Formula	Value	Error
	Coupling Constants		
α^{-1}	$B_7 + 1 + 1/(B_1 + 1)$	137.036	2 ppm
$\sin^2\theta_W$	$B_1/(B_7 - 19)$	0.2308	0.19%
α_s^{-1}	$(B_2 - B_1)/2 + 1$	8.50	0.21%
α_w^{-1}	$(B_1 + B_2)/2 - 3$	31.50	0.59%
	Mass Ratios		
m_μ/m_e	$B_4^2/B_7 \times 5$	206.80	0.016%
m_p/m_e	$(B_5 + B_{10}) \cdot \varphi \cdot 4$	1838.09	0.11%
	Cosmology		
H_0	$(B_2 \cdot B_9)/214 \times 2$	67.51	0.17%

VI. Unification Structure

Conjecture 14 (Brahim Unification)**.** *All fundamental coupling constants derive from the Brahim Number hierarchy via:*

$$\alpha_i^{-1} = B_{n_i} + \sum_k c_k \cdot \varphi^{-B_k} \quad (17)$$

where n_i *indexes which Brahim Number governs force* i*, and the* φ*-adic corrections encode quantum effects.*

Remark 2 (GUT Scale). The grand unification scale satisfies:

$$\log_{10}(M_{\text{GUT}}/\text{GeV}) \approx B_1 - 11 = 16 \quad (18)$$

consistent with typical GUT predictions of $M_{\text{GUT}} \sim 10^{16}$ GeV.

The hierarchy of Brahim Numbers suggests the following structure:

- **Level 1**: $B_1 = 27 = \dim(E_6 \text{ fund.})$ — algebraic foundation
- **Level 2**: $B_2, \ldots, B_6$ — intermediate structure
- **Level 3**: $C = 107$ — unification axis
- **Level 4**: $B_7 = 136$ — electromagnetic scale
- **Level 5**: 214 — conservation principle

VII. Discussion

The appearance of identical integers across three disparate mathematical domains—φ-adic analysis, elliptic curve conductors, and exceptional Lie group representations—raises the question of whether Brahim Numbers reflect a deeper mathematical unity.

Several observations support this hypothesis:

1) The 214-symmetry parallels mirror symmetry in Calabi-Yau geometry, where Hodge numbers satisfy $h^{1,1} \leftrightarrow h^{2,1}$.
2) The precision of the α^{-1} formula (2 ppm) far exceeds what would be expected from numerical coincidence.
3) The connection $B_1 = 27 = \dim(\mathbf{27}_{E_6})$ links directly to grand unified theories based on E_6.
4) Bosonic string theory requires 26 dimensions, and $26 = B_1 - 1$.

A. Limitations and Future Work

This work establishes numerical relationships but does not provide:

- A physical derivation explaining *why* these formulas hold
- Predictions for particle masses or other observables
- Extension beyond the first 10 Brahim Numbers

Future research should address:

1) Independent verification of the φ-adic extraction algorithm
2) Investigation of elliptic curves at Brahim conductors for special BSD properties
3) Derivation from string theory compactification
4) Extension to predict additional physical constants

VIII. The Wormhole Hypothesis

The appearance of Brahim Numbers across disparate physical domains—coupling constants, mass ratios, and cosmology—suggests a unifying "wormhole" structure connecting mathematics and physics:

Conjecture 15 (BSD-Physics Wormhole)**.** *The Birch and Swinnerton-Dyer conjecture, applied to elliptic curves at Brahim conductors, provides the mathematical mechanism constraining physical constants. The 214-symmetry acts as a conservation law bridging the two domains.*

The structure can be visualized as:

- **Mathematics side**: Elliptic curves with conductors B_n, L-functions $L(E_{B_n}, s)$, BSD rank predictions
- **Physics side**: Coupling constants, mass ratios, cosmological parameters
- **Wormhole throat**: The 214-symmetry constraint $B_n + B_{N+1-n} = 214$

IX. Conclusion

We have introduced Brahim Numbers, a novel integer sequence satisfying a functional equation with 214-symmetry, and demonstrated connections to elliptic curve conductors, exceptional Lie group representations, and fundamental physical constants across three domains:

1) **Coupling constants**: α^{-1}, α_s^{-1}, α_w^{-1}, $\sin^2\theta_W$
2) **Mass ratios**: m_μ/m_e (0.016% precision), m_p/m_e (0.11%)
3) **Cosmology**: H_0 (0.17% precision)

The formula $\alpha^{-1} \approx B_7 + 1 + 1/(B_1 + 1)$ achieves 2 ppm precision, while the muon-electron mass ratio formula $m_\mu/m_e \approx B_4^2/B_7 \times 5$ achieves 0.016% precision—both far exceeding what would be expected from numerical coincidence.

These results suggest that the fundamental constants of nature may not be arbitrary but instead derive from a mathematical structure rooted in the golden ratio, elliptic curves, and exceptional Lie groups. The "wormhole" connecting BSD conjecture to physics offers a potential path toward deriving these formulas from first principles. If validated, Brahim Numbers could represent a significant step toward answering the long-standing question of why the constants of nature take their observed values.

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Data Availability

Computational code and data are available at DOI: 10.5281/zenodo.18348730. All calculations are reproducible using the provided Python implementation.

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