Notes of Categories 0

Cloudifold

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0 Notations

Category of all small categories : **Cat**
Terminal object of $\mathscr{C}$: $\mathbf{1}_{\mathscr{C}}$

1 2-Categories

Definition 1.1. A 2-Cat $\mathscr{A}$ consists of :

1. A class $|\mathscr{A}|$
2. for each $X, Y \in |\mathscr{A}|$, a (small) category $\mathrm{Hom}(X, Y)$
3. for each $X, Y, Z \in |\mathscr{A}|$, a bifunctor $c_{X,Y,Z} : \mathrm{Hom}(X, Y) \times \mathrm{Hom}(Y, Z) \to \mathrm{Hom}(X, Z)$
4. for each $X \in |\mathscr{A}|$, a functor $u_A : \mathbf{1}_{\mathbf{Cat}} \to \mathrm{Hom}(A, A)$

These data are required to satisfy following axioms:

1. Associativity axiom: for $X, Y, Z, W \in |\mathscr{A}|$,
   $c_{X,Z,W} \circ (c_{X,Y,Z} \times \mathrm{id}_{\mathrm{Hom}(Z,W)}) = c_{X,Y,W} \circ (\mathrm{id}_{\mathrm{Hom}(X,Y)} \times c_{Y,Z,W})$
2. Unit axiom : for $X, Y \in |\mathscr{A}|$,
   $c_{X,X,Y} \circ (u_X \times \mathrm{id}_{\mathrm{Hom}(X,Y)}) = \mathrm{id}_{\mathrm{Hom}(X,Y)} = c_{X,Y,Y} \circ (\mathrm{id}_{\mathrm{Hom}(X,Y)} \times u_Y)$