Coláiste na Tríonóide, Baile Átha Cliath
Trinity College Dublin
Ollscoil Átha Cliath | The University of Dublin

Faculty of Science, Technology, Engineering and Mathematics

School of Computer Science and Statistics

Integrated Computer Science
Year 1
BA (Mod) Computer Science Linguistics and a Language
Year 1
BA (Mod) Computer Science and Business
Year 2
BA (Mod) Computer Science (Joint Honours)
Year 2

Reassessment 2024

Mathematics II

29th August 2024 **Online** **14.00 – 16.00**

Laura Byrne & James O'Malley

Instructions to Candidates:

- You must download and save a local copy of this test paper before you begin work on your solution.
- This is an individual assignment.
- Attempt ALL PARTS of ALL QUESTIONS.
- You must show all of your workings to be eligible for full marks. Answers without any workings will not get any marks.

- Your clearly legible, handwritten work should be submitted as a single pdf file that also includes a signed CSU12002 Assessment Declaration form as the first page of your submission.
- Only one file submission is permitted.
- It is your responsibility to confirm that the file you have uploaded as your solution is the correct file before you hit the submit button.
- It is your responsibility to verify that your solution file is correctly uploaded and suitable for grading.
- All submissions must be through Blackboard. The submission time is recorded in Blackboard.

SECTION A – MATHEMATICAL STATEMENTS (15)

QUESTION A1 (5)

Give the truth table associated with below statement. You must show all steps in the process of building the table.

$$\Big(\big((A \wedge B) \rightarrow C\big) \wedge (A \rightarrow B)\Big) \rightarrow (A \rightarrow C)$$

QUESTION A2 (3)

State whether each of the following statements are an appropriate proposition or not and why.

1. Triangles have three sides.
2. Do all heroes have powers?
3. $5 \times 10 \pmod 3) \equiv 0$

QUESTION A3 (2)

Identify one appropriate subject and predicate pair for each of the following statements.

1. There is rain on the grass.
2. All computer science students know how to code.

QUESTION A4 (5)

In your own words, briefly explain the importance of using parentheses in propositional expressions. Provide an example where they are required.

SECTION B – NUMBER THEORY (35)

QUESTION B1 (10)

Solve the following system of linear congruences, showing each step in the process.

$$x \equiv 1 \ (mod\ 2)$$
$$x \equiv 3 \ (mod\ 5)$$
$$x \equiv 4 \ (mod\ 7)$$

QUESTION B2 (5)

Using Fermat's Little Theorem, find the least residue of $2^{117} + 1$ modulo 17.

QUESTION B3 (10)

In your own words, describe the main flaw of using an affine cipher and how an exponential cipher improves upon this.

Briefly discuss the importance of developing more strong and secure encryption methods in the future.

QUESTION B4 (10)

Check whether the following ISBN-10 is valid.

$$0 - 545 - 01022 - 5$$

Give one example of what could cause this check to fail.

Summarise the advantages and disadvantages of moving from a 10-digit ISBN to a 13-digit ISBN.

SECTION C – PROOFS (25)

Give fully justified reasons for your answers.

Marks will be awarded for notation, clarity, and wording of your proofs in this section.

QUESTION C1 (15)

1. Where n is an integer, prove that if $3n + 2$ is even then n is even using:
   i. Proof by contraposition
   ii. Proof by contradiction (Hint: tautology 20 may help you here)
2. Show that the statement "Every positive integer can be written as the sum of the squares of three integers" is false by finding a counterexample.

Question C2 (10)

Determine whether the following argument is valid. Give a fully justified reason for your answer by using methods we have explored in CSU12002. i.e. by defining suitable propositions, stating the argument using these propositions, then determine whether the argument is valid or not

"Daphne does not like the Euros or Fred likes the Olympics. Shaggy does not like Scooby Snacks or Daphne likes the Euros. Fred does not like the Olympics. Therefore, Shaggy does not like Scooby Snacks."

SECTION D – SETS (25)

QUESTION D1 (10)

For the following two sets A and B, determine whether:

1. $A \subseteq B$
2. $B \subseteq A$
3. $A = B$

You must show all your workings. **Note**: a sketch may help you to visualise the problem, but it will **not** suffice as a solution.

$$A = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = 4\}$$

$$B = \{(2\cos(t), 2\sin(t)) \mid t \in [0, \pi]\}$$

QUESTION D2 (10)

Let the set operator, *, be defined so that:

$$A * B = A \cap \bar{B}$$

Where $\bar{X}$ is the complement of the set X.

Determine, by Veitch diagram, whether:

1. $A * B = B * A$
2. $A * B = \overline{(\bar{A} \cup B)}$
3. $(A * B) * C = A \cap (\bar{B} * C)$

QUESTION D3 (5)

Briefly describe, in your own words, Russel's paradox in the context of naive set theory.

[oOo]

CSU12002 Tautologies

1. $P \vee \neg P$ Law of the excluded middle
2. $\neg(P \wedge \neg P)$ Law of non-contradiction
3. $\neg\neg P \leftrightarrow P$ Law of double negation
4. $(P \wedge Q) \rightarrow P$ Basis for simplification
5. $(P \wedge Q) \rightarrow Q$ Basis for simplification
6. $P \rightarrow P \vee Q$ Basis for addition
7. $Q \rightarrow P \vee Q$ Basis for addition
8. $Q \rightarrow (P \rightarrow Q)$
9. $\neg P \rightarrow (P \rightarrow Q)$
10. $[P \wedge (P \rightarrow Q)] \rightarrow Q$ Modus ponens
11. $[\neg Q \wedge (P \rightarrow Q)] \rightarrow \neg P$ Modus tollens
12. $\neg P \wedge (P \vee Q) \rightarrow Q$
13. $P \rightarrow [Q \rightarrow (P \wedge Q)]$
14. $[(P \rightarrow Q) \wedge (Q \rightarrow R)] \rightarrow (P \rightarrow R)$ Transitivity of implications
15. $(P \rightarrow Q) \rightarrow [(P \vee R) \rightarrow (Q \vee R)]$
16. $(P \rightarrow Q) \rightarrow [(P \wedge R) \rightarrow (Q \wedge R)]$
17. $[(P \leftrightarrow Q) \wedge (Q \leftrightarrow R)] \rightarrow (P \leftrightarrow R)$ Transitivity of equivalences
18. $\neg(P \wedge Q) \leftrightarrow \neg P \vee \neg Q$ De Morgan's law
19. $\neg(P \vee Q) \leftrightarrow \neg P \wedge \neg Q$ De Morgan's law
20. $\neg(P \rightarrow Q) \leftrightarrow P \wedge \neg Q$
21. $(P \rightarrow Q) \leftrightarrow (\neg P \vee Q)$
22. $(P \leftrightarrow Q) \leftrightarrow [(P \rightarrow Q) \wedge (Q \rightarrow P)]$
23. $(P \leftrightarrow Q) \leftrightarrow [(P \wedge Q) \vee (\neg Q \wedge \neg P)]$
24. $(P \rightarrow Q) \leftrightarrow (\neg Q \rightarrow \neg P)$ Law of contraposition (The contrapositive)
25. $[(P \rightarrow Q) \wedge (P \rightarrow R)] \leftrightarrow [P \rightarrow (Q \wedge R)]$
26. $[(P \rightarrow R) \wedge (Q \rightarrow R)] \leftrightarrow [(P \vee Q) \rightarrow R]$ Basis for proof by cases
27. $[P \rightarrow (Q \rightarrow R)] \leftrightarrow [(P \wedge Q) \rightarrow R]$
28. $(P \rightarrow Q \wedge \neg Q) \leftrightarrow \neg P$ Basis for indirect proofs
29. $[(P \wedge (Q \vee R)] \leftrightarrow [(P \wedge Q) \vee (P \wedge R)]$ Law of distributivity
30. $[(P \vee (Q \wedge R)] \leftrightarrow [(P \vee Q) \wedge (P \vee R)]$ Law of distributivity
31. $(P \wedge Q) \leftrightarrow (Q \wedge P)$ Law of commutativity
32. $(P \vee Q) \leftrightarrow (Q \vee P)$ Law of commutativity
33. $[(P \vee (Q \vee R)] \leftrightarrow [(P \vee Q) \vee R]$ Law of associativity
34. $[(P \wedge (Q \wedge R)] \leftrightarrow [(P \wedge Q) \wedge R]$ Law of associativity

CSU12002 Rules of inference involving quantifiers

11. **De Morgan's Laws for Quantifiers (Axioms):**
    a. $\neg(\forall x P(x)) \leftrightarrow \exists x \neg P(x)$;
    b. $\neg(\exists x P(x)) \leftrightarrow \forall x \neg P(x)$;
12. **Universal specification Axiom (US):** If the domain of the variable is not empty, $\forall x P(x) \rightarrow P(t)$ can be asserted, where t is an object in the domain and P is a statement depending upon one free variable.
13. **Universal Generalization Rule of Inference (UG):** If $P(x)$ can be proven, where x is a free variable representing an arbitrary element of a certain domain, $\forall x P(x)$ can be asserted.
14. **Existential Specification Rule of Inference (ES):** If a step of the form $\exists x P(x)$ appears in the proof, $P(c)$ can be asserted, where c is a constant symbol.
15. **Existential Generalisation Axiom (EG):** $\boldsymbol{P(t)} \rightarrow \exists \boldsymbol{x P(x)}$ can be asserted if t is an object in the domain of x (assumed not empty).