Section VII

The Capacitor

Core Topics

(i) The Capacitor

(ii) Power Delivered to a Capacitor

(iii) Uses of Capacitors

(iv) Capacitors in Series and Parallel

The Capacitor:

A capacitor is a two-terminal element that stores energy in an electric field. Its ability to do this is given numerically by its '<u>Capacitance</u>' value –'C'.

The Capacitor:

A capacitor is formed by two conductors (usually parallel conducting plates) separated by a distance. Sometimes a dielectric material is placed between the conductors to provide a means of separating the plates and enhancing the capacitance.

Fabrication methods for capacitors include parallel plates with ceramic or mica dielectric materials rolled with electrolytic paper or foil.

When a voltage is applied across the capacitor, both conductors will possess charge equal in magnitude but opposite in sign.

The Capacitor:

These opposite charges result in an electric field between the conductors.

'Capacitance' is defined as the ratio between the charge on the (positive) conductor and the applied voltage.

i.e.

$$C = \frac{q(t)}{v(t)}$$

The Capacitor:

Although capacitance is the ratio of charge to voltage, it does not depend directly on either of these variables. Capacitance is determined only by the dimensions of the conductors, the separation distance between them and the electrical permittivity if the dielectric material between the conductors:

$$C = \frac{\varepsilon A}{d}$$

‘ε’ is the electrical permittivity.

‘A’ is the common area between the conductors (or plates).

‘d’ is the separation distance between the conductors.

The Capacitor:

What is the capacitance of a system of two circular discs of radius 1cm separated by a distance of 1mm – comprised of plastic (ε_r = 6) ?

$$C = \frac{\varepsilon A}{d} = \frac{\varepsilon_r \varepsilon_o \pi . r}{d} = 16.68\, pF$$

The Capacitor:

In order to obtain the relationship between the terminal voltage and the current in a capacitor, we differentiate:

$$q(t) = Cv(t) \Rightarrow$$

$$\frac{dq(t)}{dt} = c\frac{dv}{dt} \Rightarrow$$

$$\boxed{i(t) = C\frac{dv(t)}{dt}}$$

The Capacitor:

$$i(t) = C\frac{dv(t)}{dt}$$

From this equation it is clear that current will “pass through” a capacitor if and only if the terminal voltage is <u>changing</u> with respect to time.

Hence a constant p.d. across a capacitor will give zero current!

The Capacitor:

It is important to note that when current “passes through” a capacitor, there is no movement of charge between the plates (apart possibly from some small leakage current).

This effective ‘current’ without movement of charge is called displacement current – i.e. charge flow is effected over a displacement.

This occurs because the changing electric field in the dielectric induces charge flow on either side of the plates.

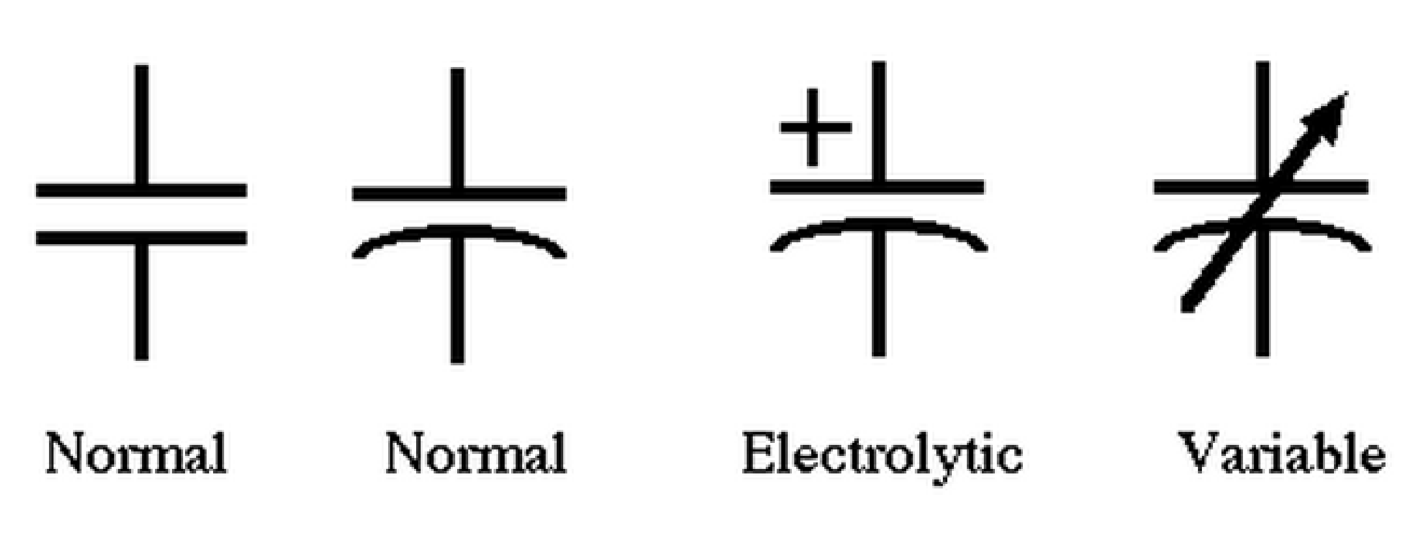
The Capacitor:

Units:

The unit of Capacitance is the Farad (F) – defined as the Capacitance of a capacitor which contains one Coulomb (1C) of charge on its positive plate when a potential difference of 1 Volt is applied.

The Capacitor:

Capacitor Symbols:



Energy stored in a Capacitor:

$$p(t) = v(t)i(t)$$

$$i(t) = C\frac{dv(t)}{dt} \Rightarrow$$
$$p(t) = Cv(t)\frac{dv(t)}{dt}$$

$$= \frac{C}{2}\frac{d(v^2(t))}{dt} \Rightarrow$$

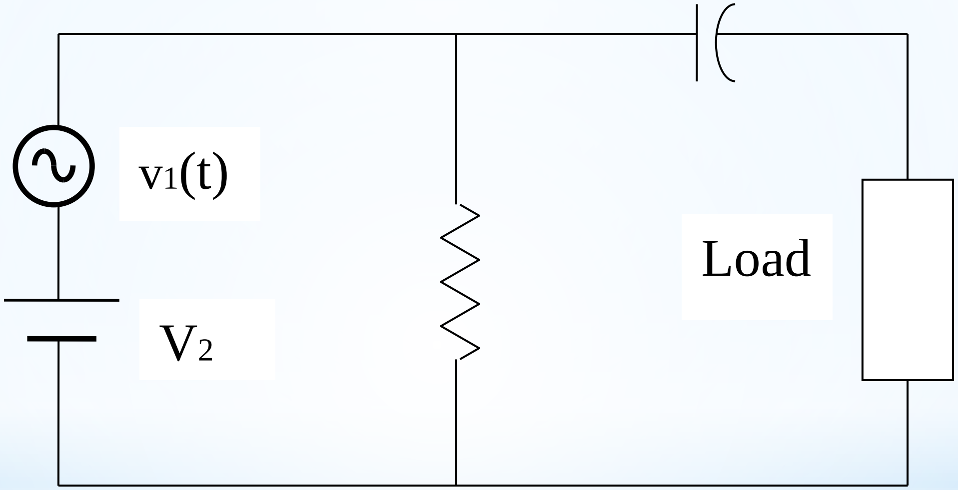
$$\frac{dw(t)}{dt} = \frac{C}{2}\frac{dv^2(t)}{dt} \Rightarrow$$

$$\boxed{w(t) = \frac{C}{2}v^2(t)}$$

$$\left[\begin{array}{c} Aside: \\ The\ Product\ Rule\ for\ derivatives\ tells\ us\ that: \\ \\ \dfrac{du(t)v(t)}{dt} = \dfrac{v(t)du(t)}{dt} + \dfrac{u(t)dv(t)}{dt} \\ \Rightarrow \dfrac{dv(t).v(t)}{dt} = \dfrac{dv^2(t)}{dt} = \dfrac{v(t)dv(t)}{dt} + \dfrac{v(t)dv(t)}{dt} \\ = 2v(t)\dfrac{dv(t)}{dt} \\ Hence: v(t)\dfrac{dv(t)}{dt} = \dfrac{1}{2}\dfrac{d(v^2(t))}{dt} \end{array}\right]$$

Uses of Capacitors:

Capacitors are widely used as filters. Consider the following setup:



Here we have an a.c. supply in series with a d.c. supply.

Uses of Capacitors:

Note that:

$$v_1(t) = V_1 \cos(\omega t)$$

$$v_2(t) = V_2$$

Therefore:

$$v_T(t) = v_1(t) + V_2$$

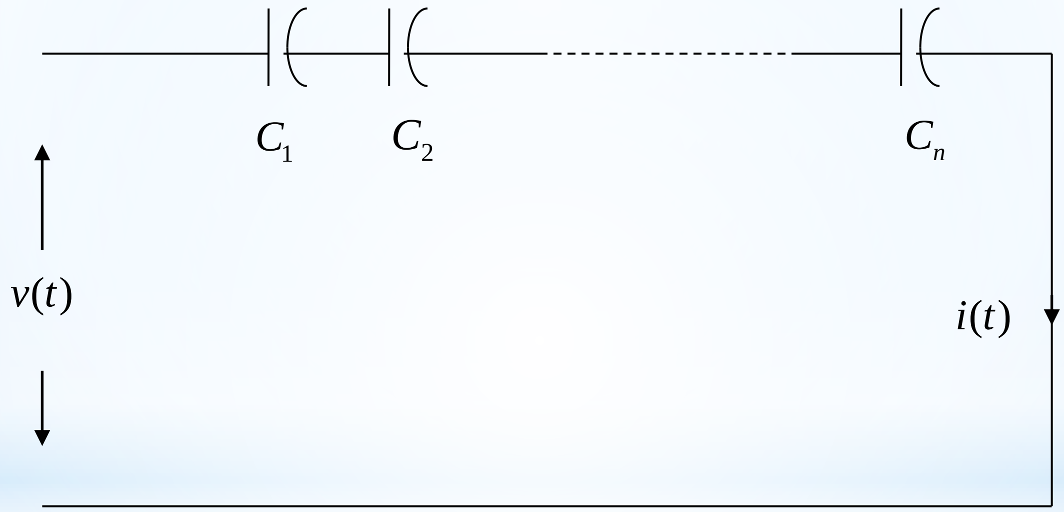
Since d.c. will not travel through a capacitor, the load receives or ‘sees’ only the a.c. signal – i.e. the d.c. component of the signal is filtered out.

Exercise:

What would the load voltage look like if the resistor and the capacitor were interchanged in the above circuit?

Capacitors in Series and Parallel:

Consider the following setup:



$v_1(t)$, $v_2(t)$, ….. , $v_n(t)$ are the voltages across capacitors

C_1, C_2 , …. , C_n.

Capacitors in Series and Parallel:

The charge residing on each capacitor is the same – regardless of the value of its capacitance (why?)

i.e.

$$q(t) = q_1(t) = q_2(t) = \ldots\ldots\ldots = q_n(t)$$

and

$$v(t) = v_1(t) + v_2(t) + \ldots\ldots\ldots + v_n(t)$$

The total capacitance of the system is:

$$C_{Total} = \frac{q(t)}{v(t)}$$

Capacitors in Series and Parallel:

Then:

$$\frac{1}{C_{Total}} = \frac{v(t)}{q(t)} = \frac{v_1(t)}{q(t)} + \frac{v_2(t)}{q(t)} + \ldots.. + \frac{v_n(t)}{q(t)} \Rightarrow$$

$$\boxed{\frac{1}{C_{Total}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots\ldots + \frac{1}{C_n}}$$

- for capacitors in series

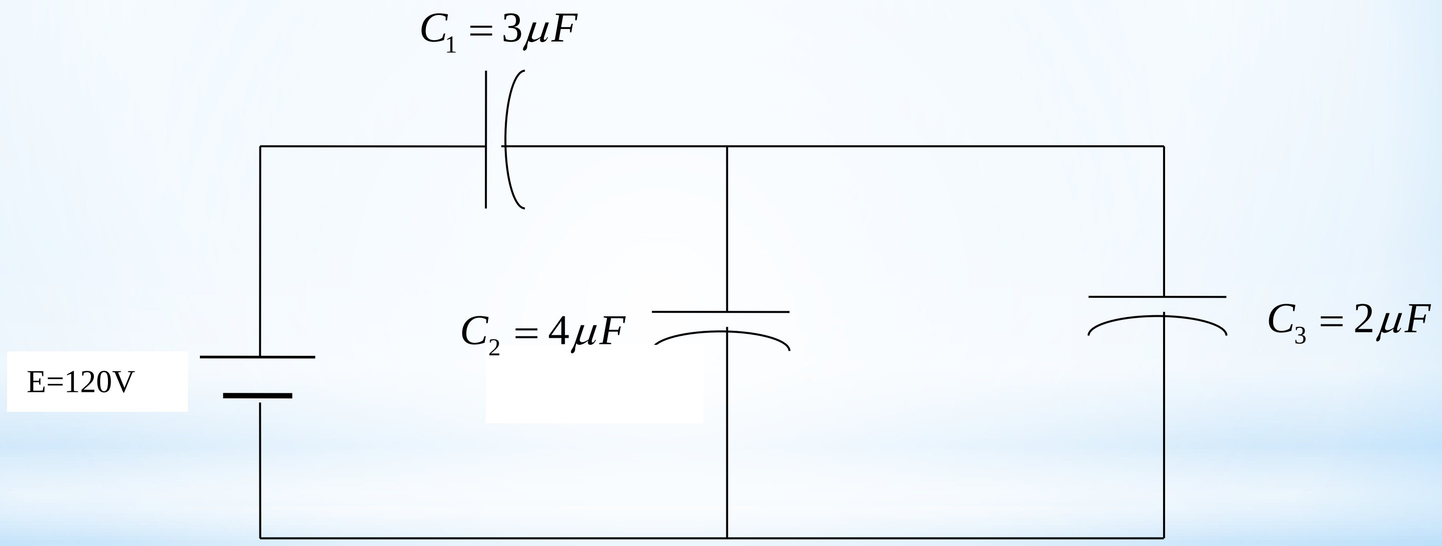
Capacitors in Series and Parallel:

$$C_{Total} = C_1 + C_2 + \ldots\ldots + C_n$$

Exercise: Prove this.

Capacitors in Series and Parallel:

Find the voltage across and charge on each capacitor in the following network:



Capacitors in Series and Parallel:

$$C' = C_2 + C_3 = 4 + 2 = 6\mu F$$

$$C_T = \frac{C_1 C'}{C_1 + C'} = \frac{3.6}{3+6} = 2\mu F$$

$$Q_T = C_T E = 2.10^{-6}.120 = 240\mu C = Q_1 = Q' \Rightarrow$$

$$V_1 = \frac{Q_1}{C_1} = \frac{240.10^{-6}}{3.10^{-6}} = 80V$$

$$V' = \frac{Q'}{C'} = \frac{240.10^{-6}}{6.10^{-6}} = 40V$$

$$or$$

$$V' = E(=120V) - 80V = 40V$$

$$Then:$$

$$Q_2 = C_2 V' = 4.10^{-6}.40 = 160\mu C$$

$$Q_3 = C_3 V' = 2.10^{-6}.40 = 80\mu C$$