Signal Modulation

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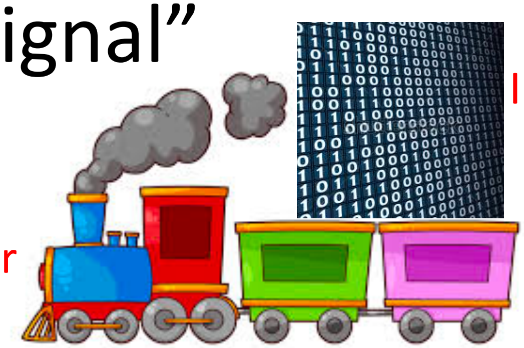
Signal Modulation

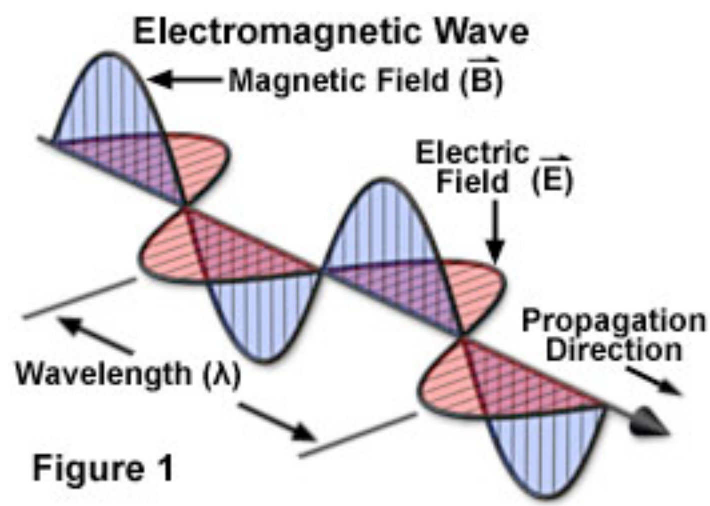
- Signal modulation allows to add information to a “carrier signal”

Information

Carrier



- The carrier signal can be of different type:



Modulation

- Modulating means changing some property of the carrier signal.
- Exmples:
  - For smoke signals I change the presence (Amplitude) of smoke
  - For visual light signal I change the amplitude of light (on-off) to transmit in Morse code
  - For an electromagnetic signal I can change the amplitude, frequency, phase or polarization.

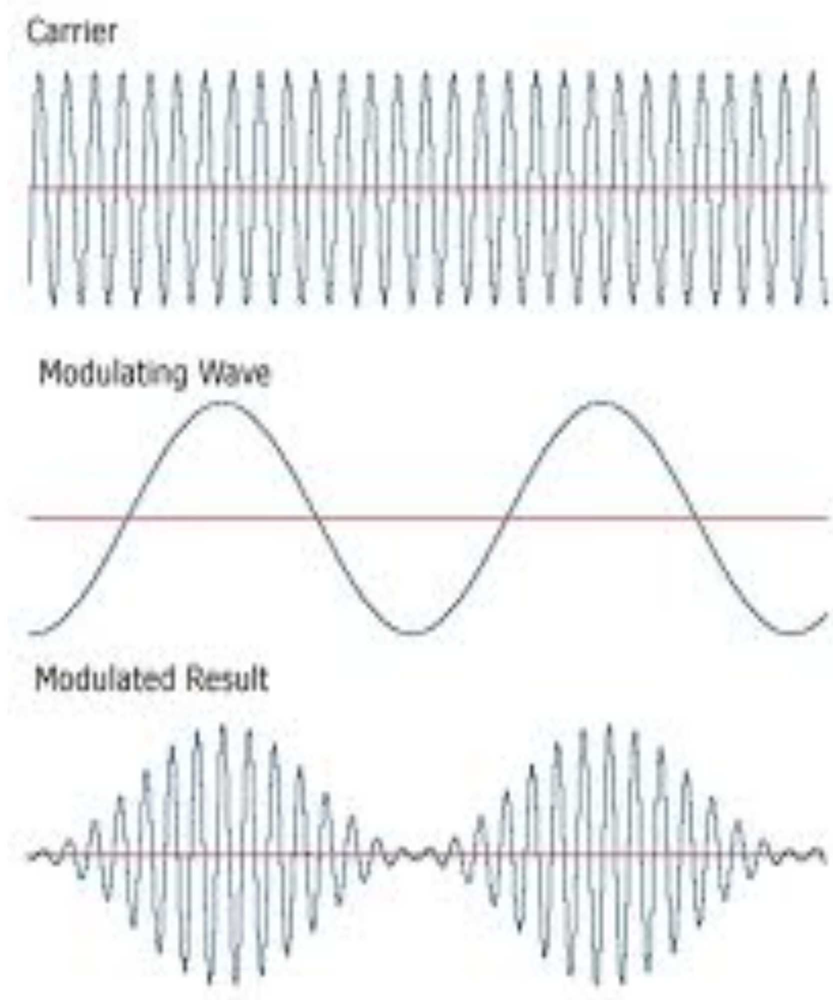
Modulation of electric signal

- A sine wave has three parameters: Amplitude, phase and frequency.
- I can modulate a carrier sine wave by changing one of these parameters.

$$s(t) = A \cdot \sin(2\pi f t + \varphi)$$

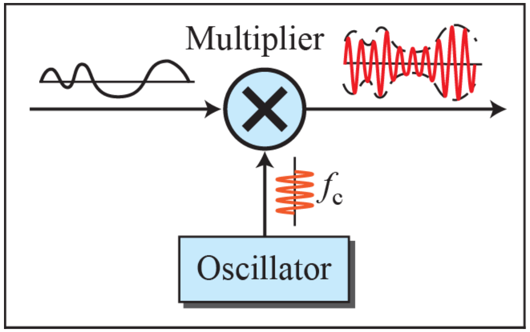
Amplitude modulation

- This is the carrier sine wave
- This is the signal containing information
- This is the modulated signal



Amplitude modulation

- Amplitude modulation is obtained by simply multiplying the information signal by the carrier
- If s(t) is my signal carrying the information, and c(t) is my carrier:
- The modulated signal M(t) = s(t) x c(t)
- This is used in AM radio



Example of amplitude modulation

- My signal is a composite periodic signal, approximating a square wave with two frequencies:
  - a fundamental frequency of 1Hz, and an harmonic of 3 Hz

$$s(t) = 1 \cdot \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t)$$

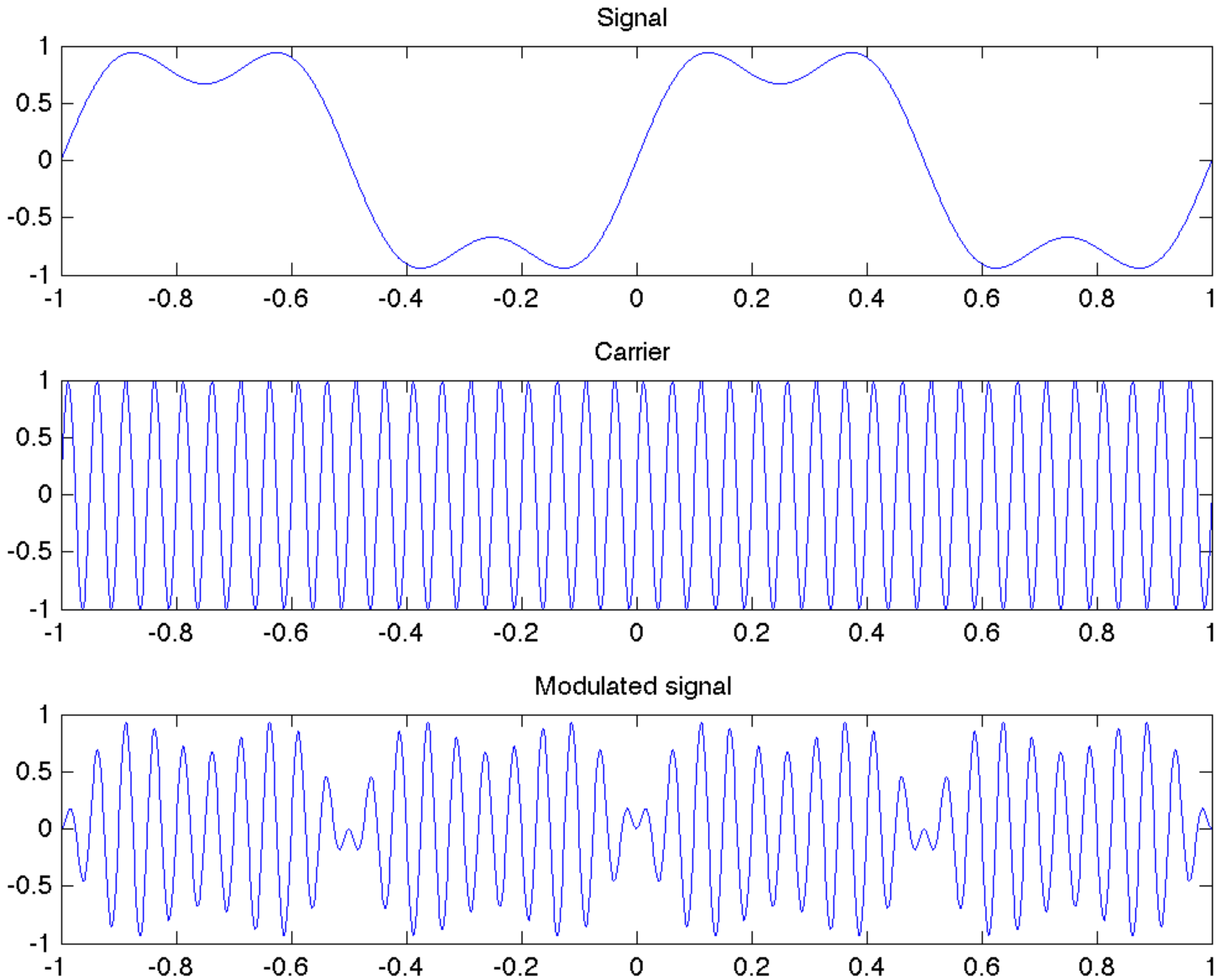
- My carrier is a sine wave of frequency 20Hz

$$c(t) = \sin(2\pi 20t)$$

- My modulated signal is:

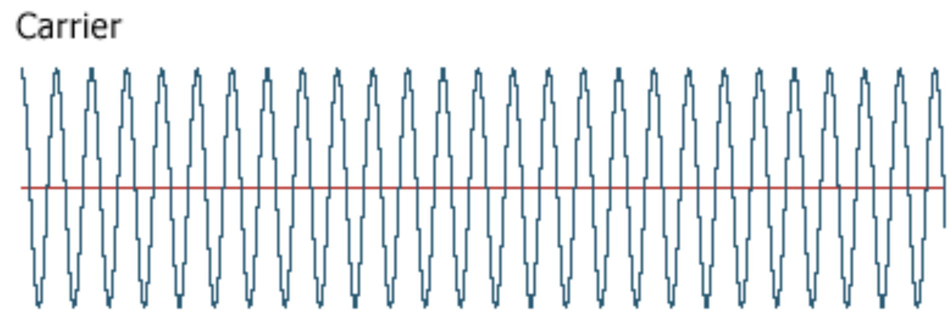
$$m(t) = \left(1 \cdot \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t)\right) \cdot \sin(2\pi 20t)$$

Example of amplitude modulation

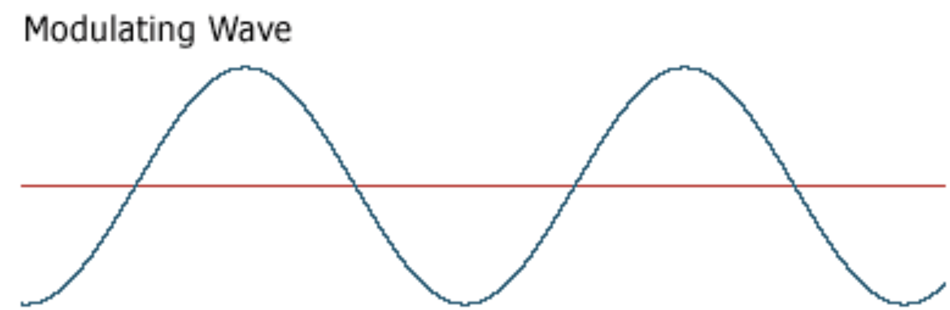


Frequency modulation

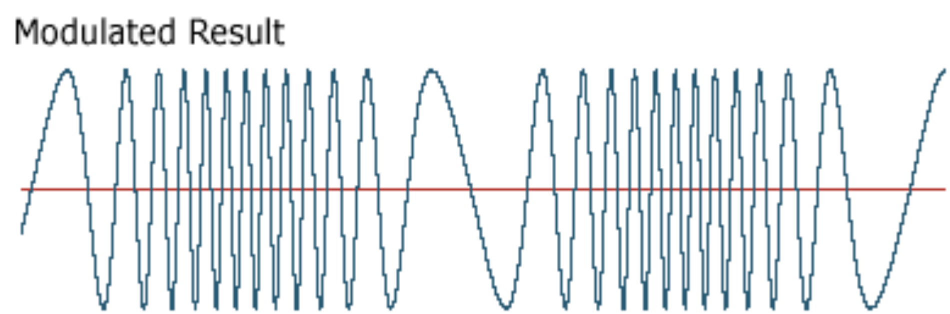
- Another possibility is to change the frequency of the carrier sine wave
  - This is the carrier sine wave



  - This is the signal containing information



  - This is the modulated signal

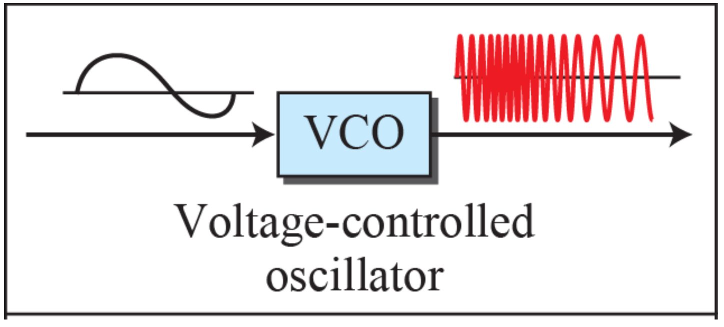


Frequency modulation

- Frequency modulation is obtained by changing the frequency of the carrier using the signal
- Notice that the frequency is the mathematical derivative of the phase, or inversely the phase is the integral of the frequency

- If s(t) is my signal carrying the information, and $\sin(2\pi f_c t)$ is my carrier:
- The modulated signal: $m(t) = \sin\left(2\pi f_c t + \int_0^t s(t)\,dt\right)$

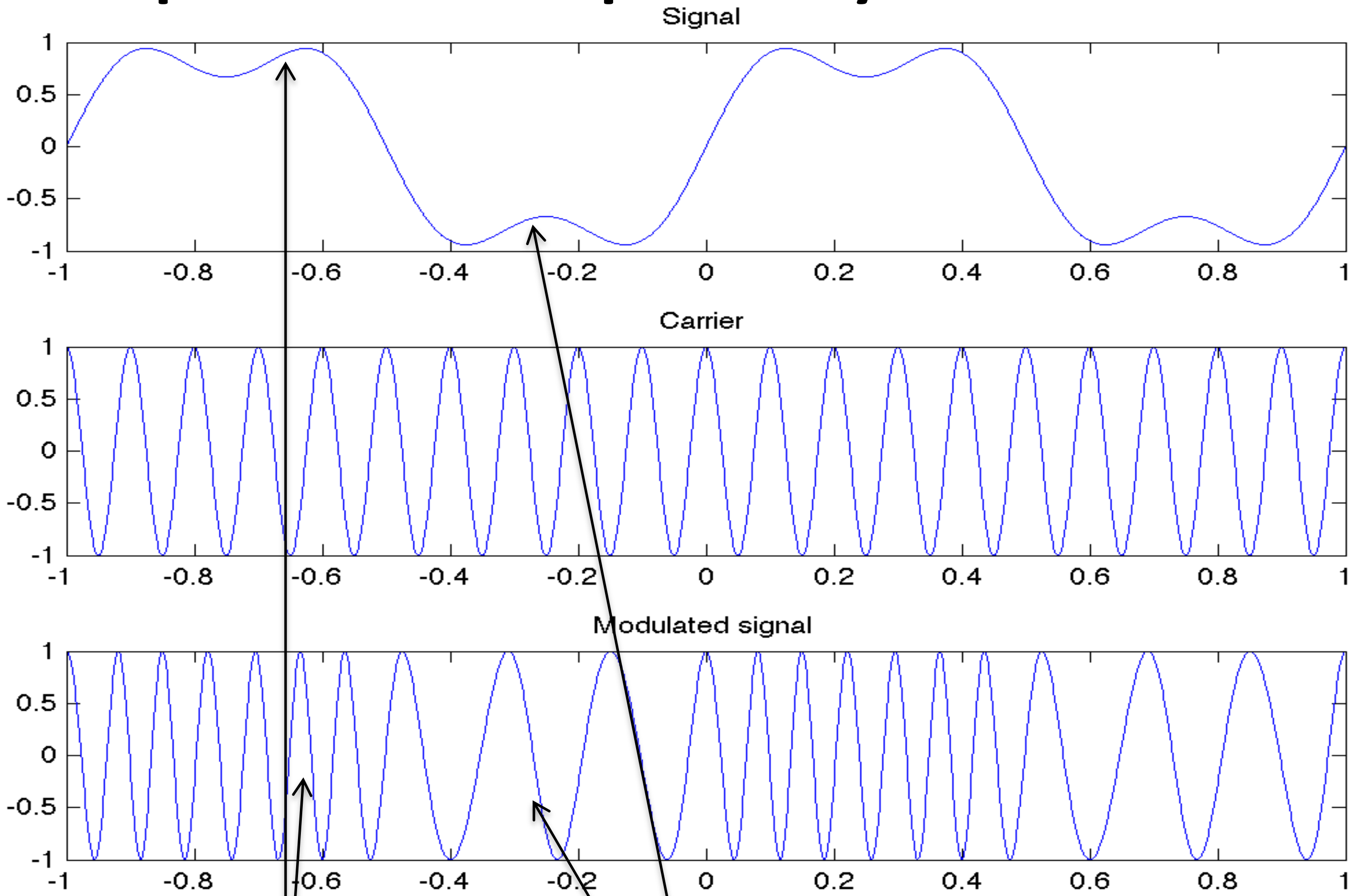
- This is used in FMradio



Example of frequency modulation

- My signal is a composite periodic signal, approximating a square wave with two frequencies:
  - a fundamental frequency of 1Hz, and an harmonic of 3 Hz $$s(t) = 1 \cdot \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t)$$
- My carrier is a sine wave of frequency 10Hz
$$c(t) = \sin(2\pi 10t)$$
- My modulated signal is (β is the frequency modulation index):
$$m(t) = \sin\left(2\pi 10t + \beta \int_0^t \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t) \cdot dt\right)$$

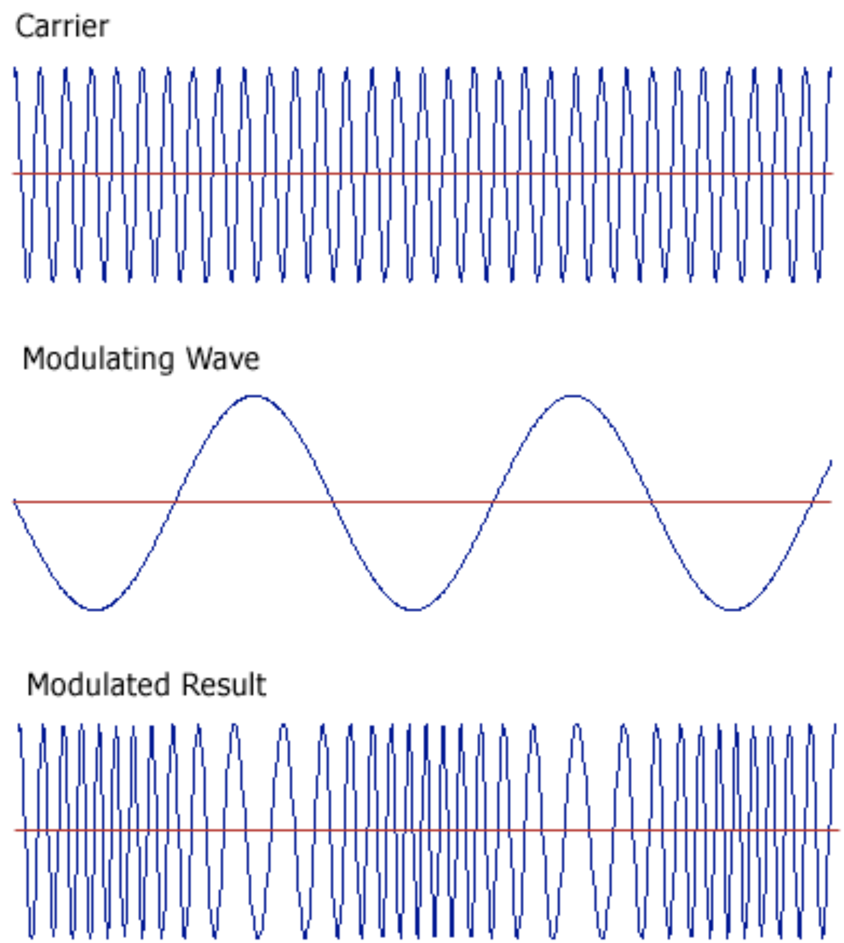
Example of frequency modulation



Notice that the changes occur where the signal is higher (positive or negative)

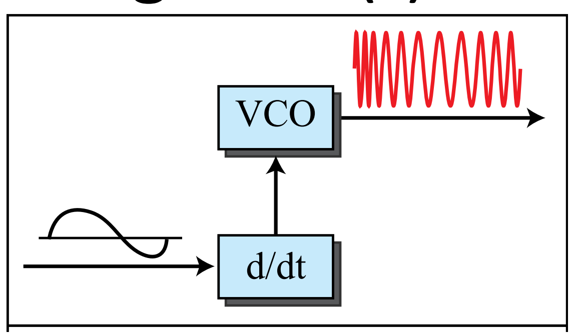
Phase modulation

- Another possibility is to change the phase of the carrier sine wave
  - This is the carrier sine wave
  - This is the signal containing information
  - This is the modulated signal



Phase modulation

- Phase modulation is obtained by changing the phase of the carrier using the signal
- If s(t) is my signal carrying the information, and $\sin(2\pi f_c t)$ is my carrier:
- The modulated signal $M(t) = \sin(2\pi f_c t + s(t))$



- This is not used very often in analog signal, frequency modulation is instead used.

Example of phase modulation

- My signal is a composite periodic signal, approximating a square wave with two frequencies:
  - a fundamental frequency of 1Hz, and an harmonic of 3 Hz

$$s(t) = 1 \cdot \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t)$$

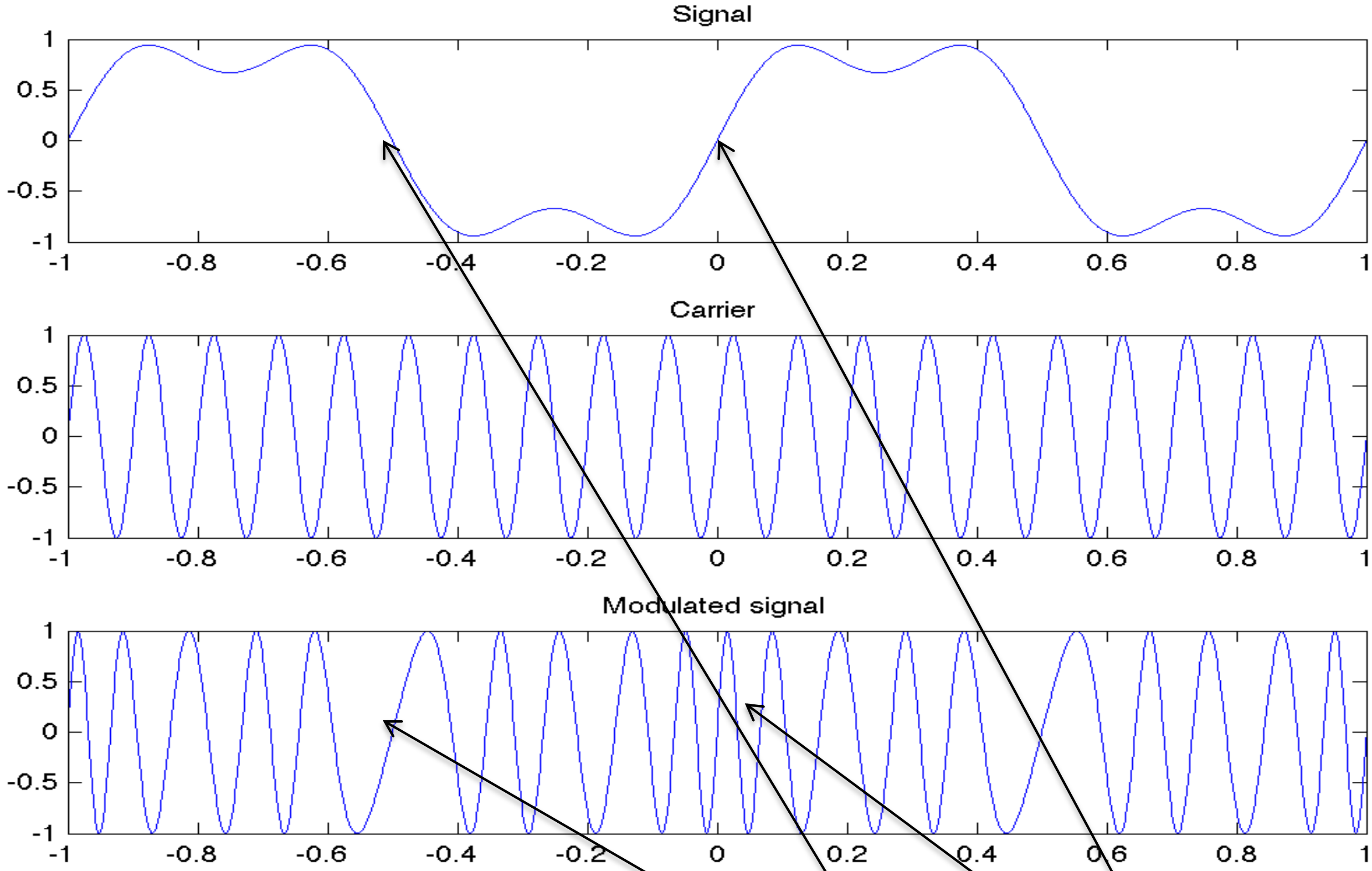
- My carrier is a sine wave of frequency 10Hz

$$c(t) = \sin(2\pi 10t)$$

- My modulated signal is (α is the phase modulation index):

$$m(t) = \sin\left(2\pi 10t + \alpha\left(1 \cdot \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t)\right)\right)$$

Example of phase modulation



Notice that the changes occur where the **rate of change** of the signal is higher (negative or positive)

The frequency of the carrier should be higher than that of the transported signal

A. Agree
B. Somewhat Agree
C. Neutral
D. Somewhat Disagree
E. Disagree

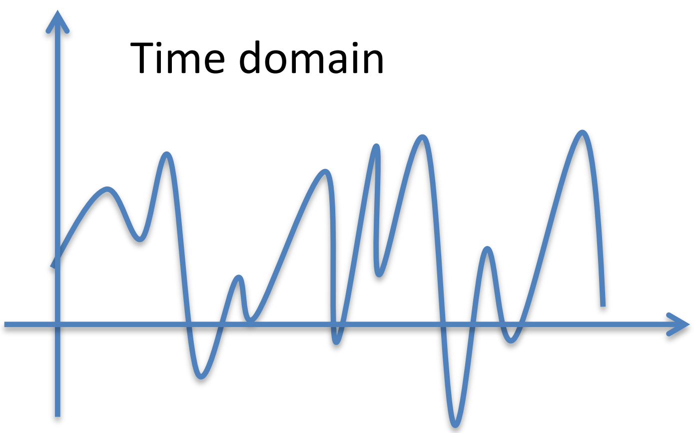
The negative side of the spectrum

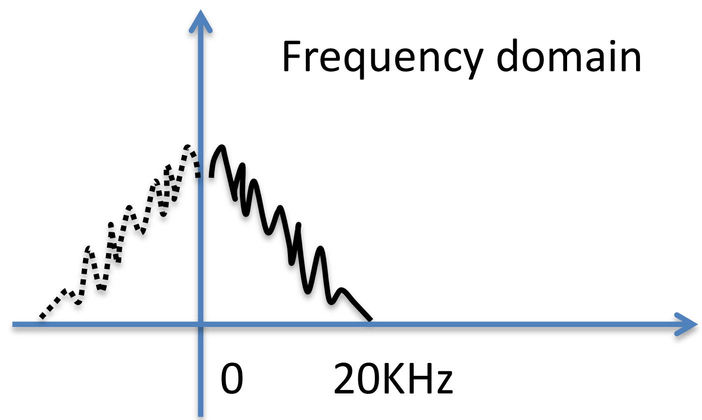
- The Fourier integral creates a mirror image of the positive frequency for the negative frequencies.
- This is true also for the fourier Series, which can be expressed as:

$$F(x) = \sum_{n=-\infty}^{n=\infty} C_n e^{inx}, \qquad e^{inx} = \cos(nx) + isin(nx)$$

Negative spectrum

- Any spectrum will always have a negative side which mirrors the positive side





- This doesn't really matter when we work on baseband, as the negative frequencies don't have a real physical meaning…
- … BUT…

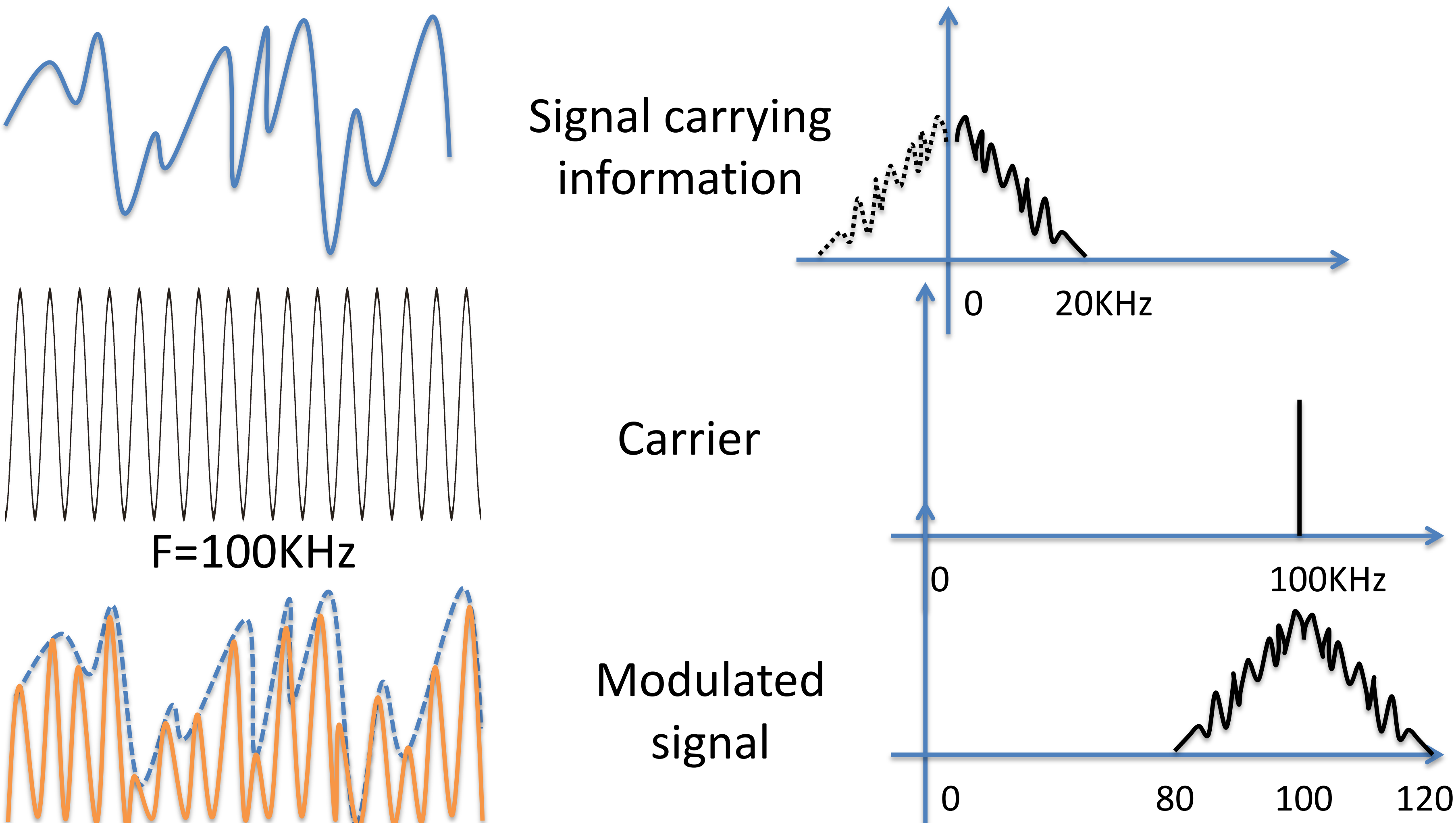
Effects of modulation

- Modulating a signal, shifts its spectrum towards higher frequencies.

- The spectrum of the signal carrying information becomes centered around the frequency of the carrier wave.

Spectrum of a modulated signal

Time domain | **Spectrum domain**



Example of spectrum of modulated signal

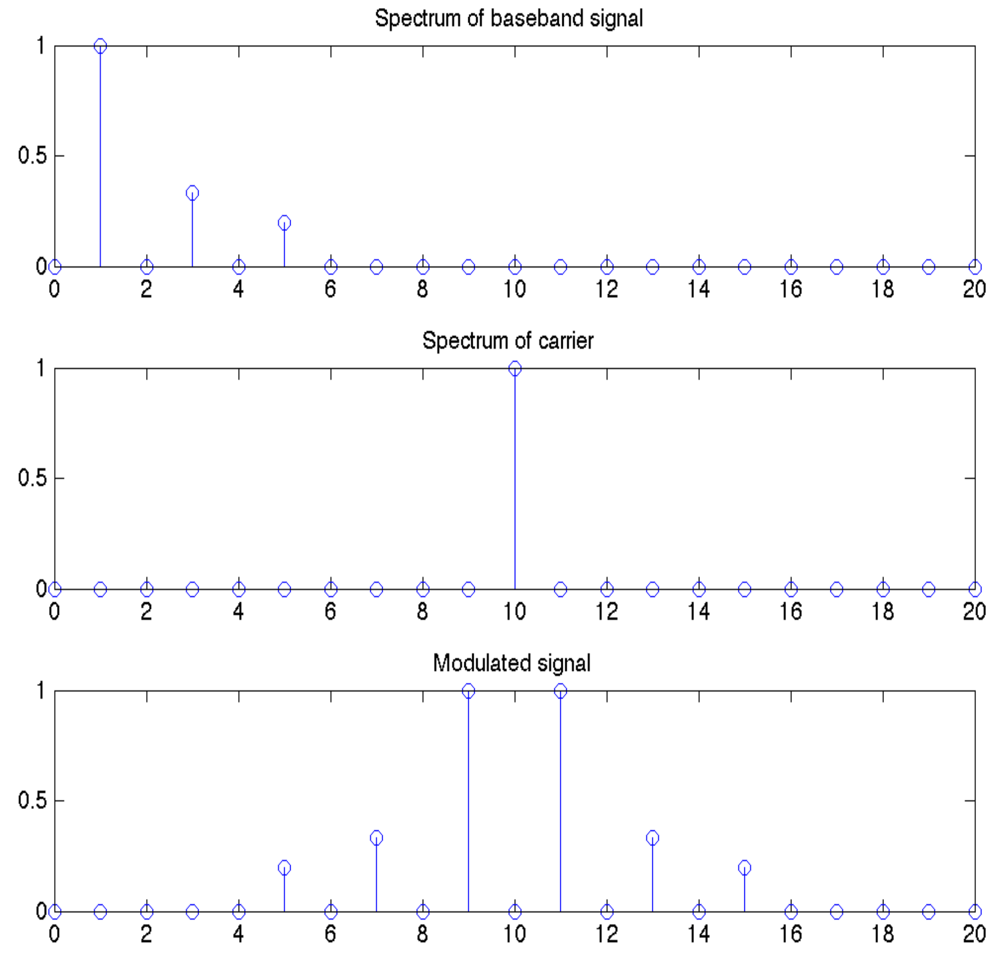
- Signal:

$$s(t) = 1 \cdot \sin(2\pi t) + \frac{1}{3}\sin(2\pi 3t) + \frac{1}{5}\sin(2\pi 5t)$$

- Carrier:

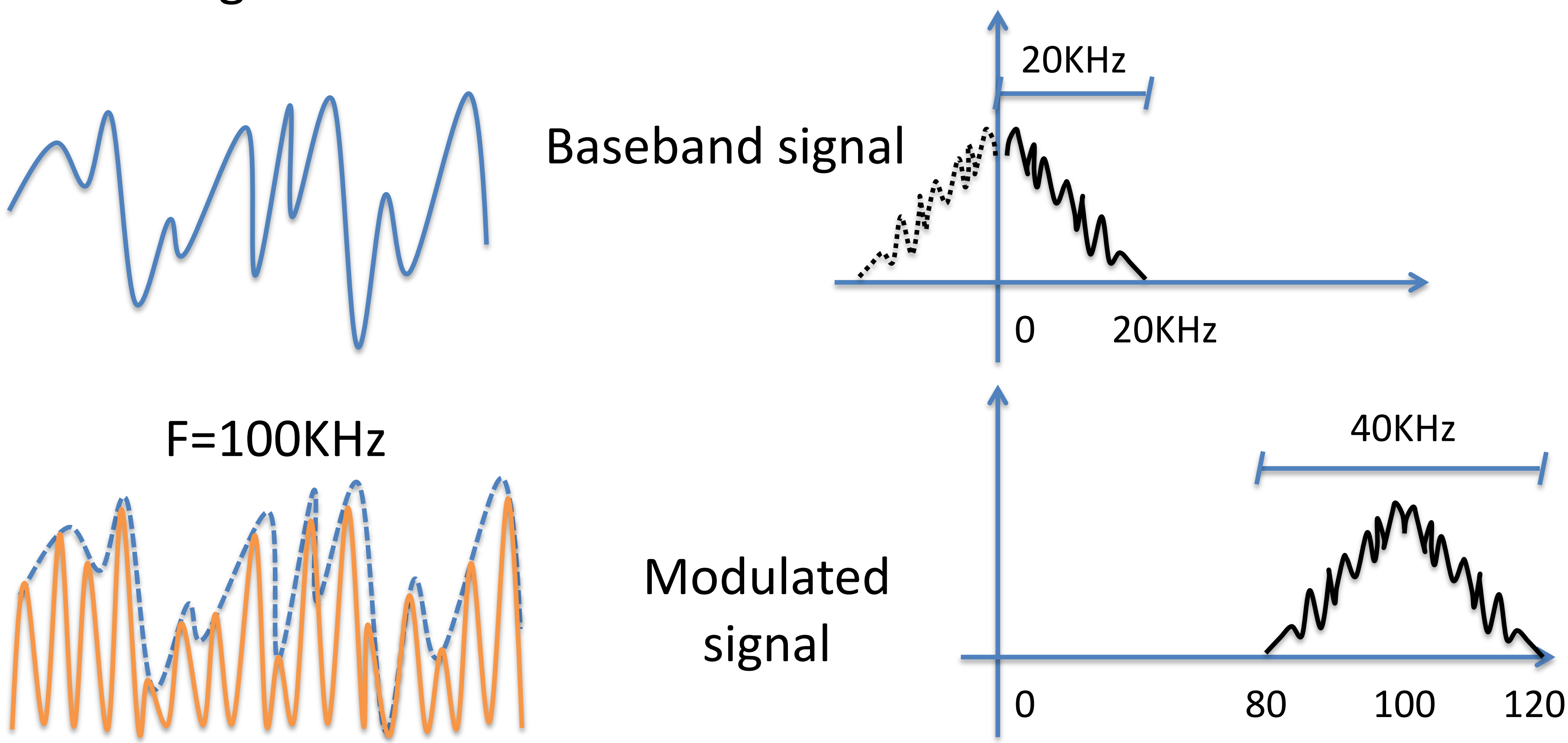
$$c(t) = \sin(2\pi 10t)$$

- The spectrum of the amplitude modulated, is the same as the signal but centered at the carrier frequency



Bandwidth of a modulated signal

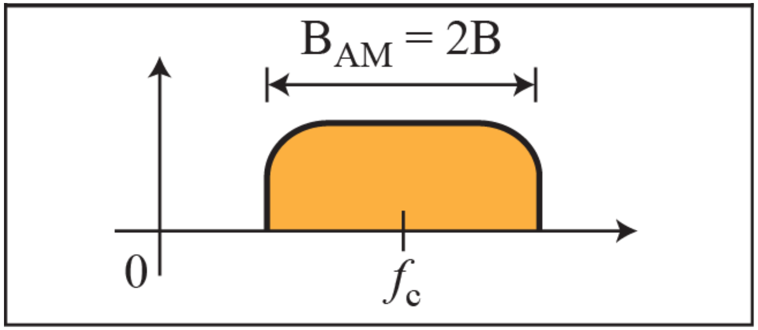
- When a signal is modulated the negative side of the spectrum is moved to the positive side and becomes ‘real’.
- Thus this part also need to be accounted for.
- For example for amplitude modulation, the bandwidth of the signal is the double of that in the baseband



Bandwidth occupied by amplitude modulation

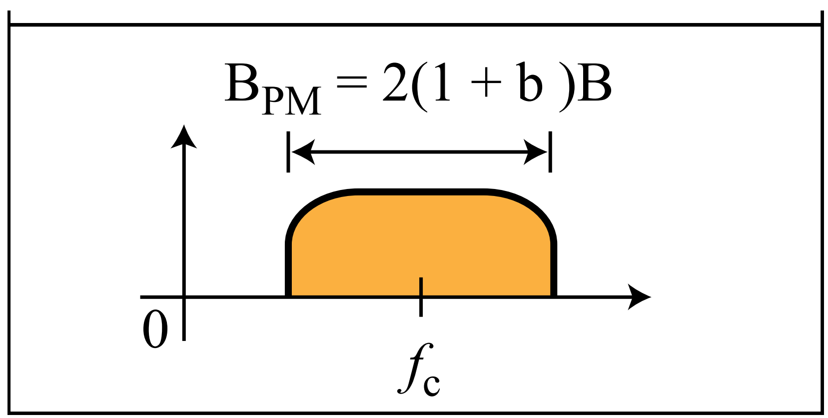
- As in the frequency domain, amplitude modulations simply shifts the baseband signal to the carrier frequency
- The band is simply double of the baseband signal (remember that a baseband signal also has a negative side of the spectrum that is exposed when the signal gets shifted towards higher frequencies).
- IF B_m is the bandwidth of the modulated signal and B_b tat of the baseband signal:

$$\mathbf{B_m=2B_b}$$



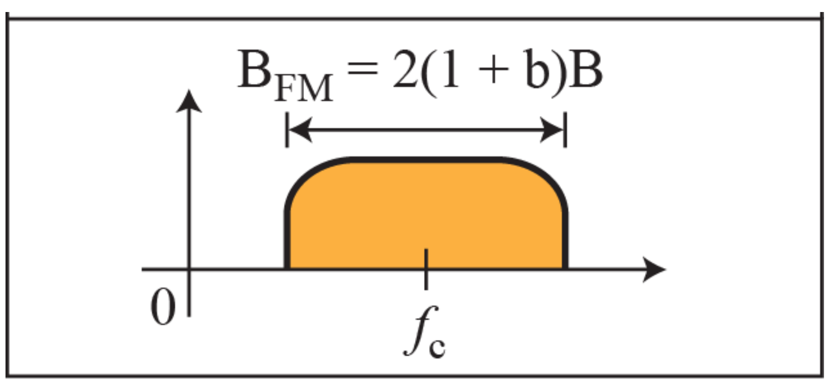
Bandwidth occupied by phase modulation

- Phase modulation shifts the baseband signal to the frequency of the carrier.
- However it also expands the bandwidth:
- Approximately, $\mathbf{B_m=2(1+\beta)B_b}$, where β is between 1 and 3



Bandwidth occupied by frequency modulation

- Frequency modulation shifts the baseband signal to the frequency of the carrier.
- However it also expands the bandwidth:
- Approximately, $\mathbf{B_m=2(1+\beta)B_b}$, where β is about 4



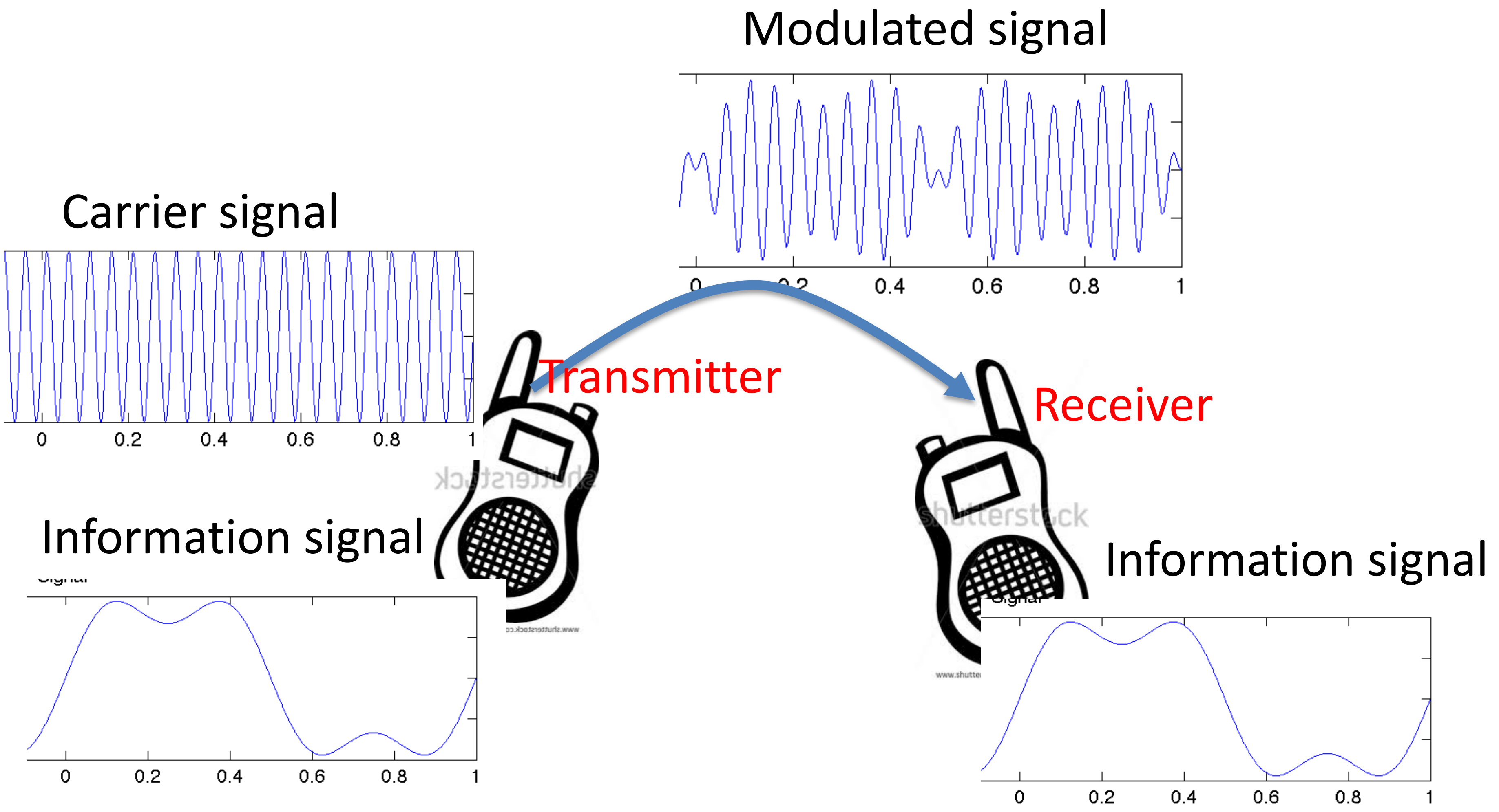
Demodulation

- Demodulation is the process by which a modulated signal is converted back into its original non-modulated version.

- A demodulator extracts the signal from the carrier, converting it back into baseband.

- For example once at the receiver, an FM radio signal is demodulated and converted from 100MHz to baseband. The demodulated signal is then amplified and sent to the speakers.

Demodulation example



We always need to modulate a signal with a carried in order to transmit it

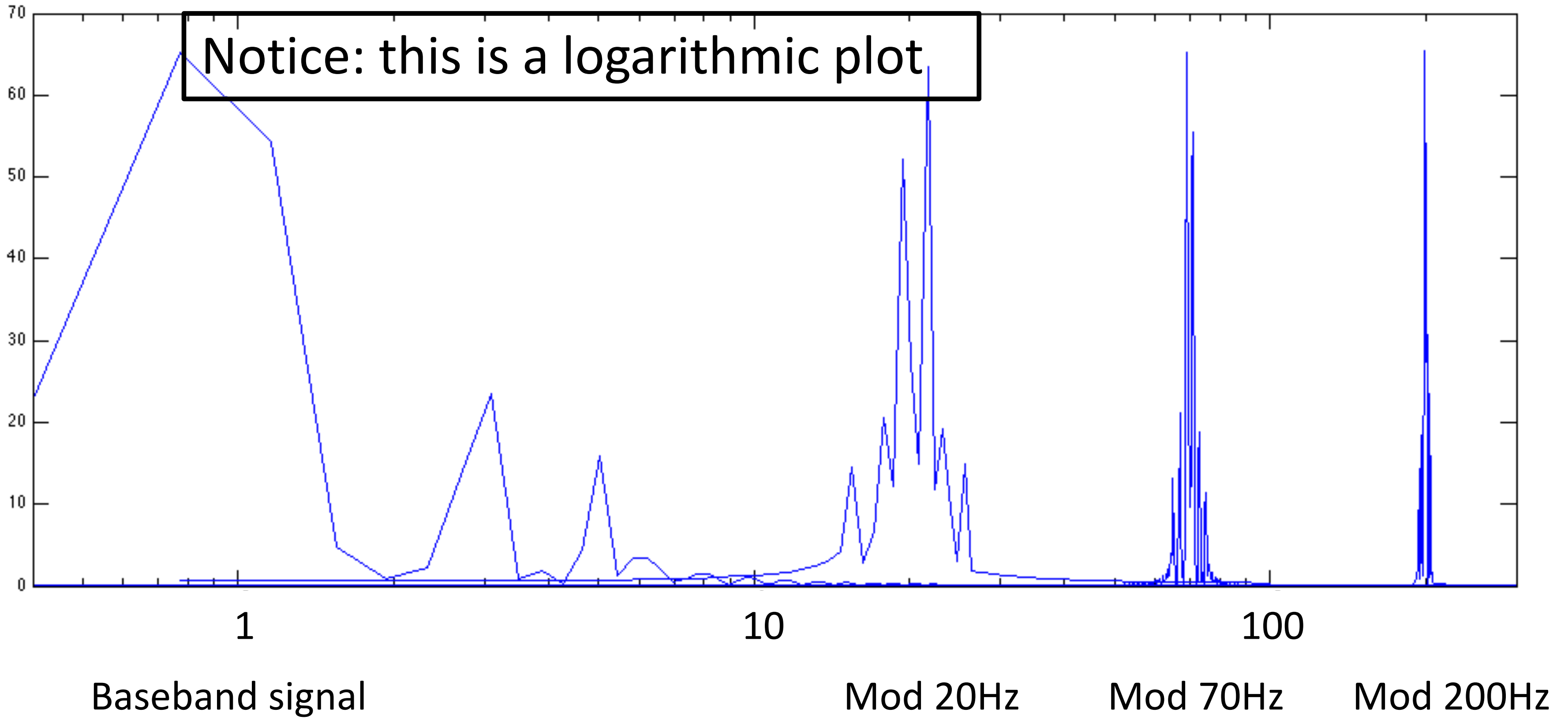
A. Agree
B. Somewhat Agree
C. Neutral
D. Somewhat Disagree
E. Disagree

Why do we need modulation?

- There are a number of reasons to modulate a "baseband" signal into a carrier wave.
  - Higher frequencies have more bandwidth available
  - Transmission over wireless medium is more efficient at high frequency and requires shorter antennas
  - Multiple channels can fit into the same wire or wireless medium (Frequency multiplexing)

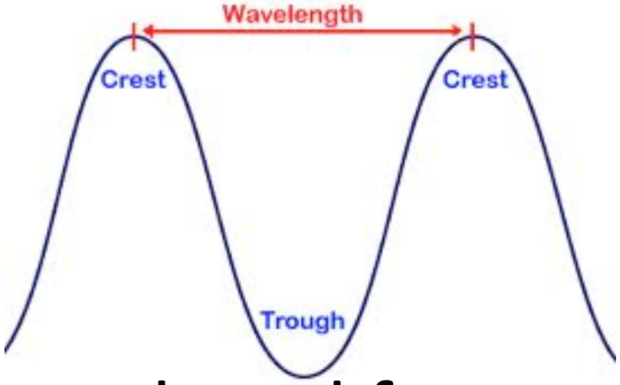
Higher frequencies have more bandwidth

- One of the biggest advantages of using higher frequencies is that there is much more bandwidth available.



Wavelength definition

- Wavelength is the distance in meter between two crests of a sine wave
- In the time domain we called it period, but in the space domain is called wavelength (λ) and is measured in meters.



- Relation between wavelength and frequency:

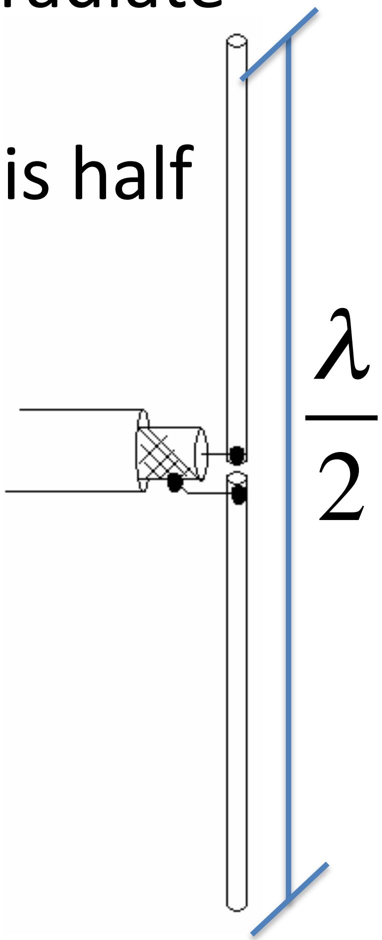
$$\lambda = \frac{c}{f}$$

Where C is the speed of light in the medium considered, and is always slower than the speed of light in the vacuum $C_0 \approx 3x10^8$ m/s

Antennas are shorter at higher frequencies

- Antennas are more practical and tend to radiate more at higher frequencies.
- The ideal size of a simple dipole antenna is half the wavelength.

- So if we consider the vacuum:
  - f=100KHz ➔ $\frac{\lambda}{2} = \frac{3 \cdot 10^8}{2 \cdot 10^5} = 1.5 \cdot 10^3 = 1.5km$
  - F=100MHz ➔ $\frac{\lambda}{2} = \frac{3 \cdot 10^8}{2 \cdot 10^8} = 1.5 \cdot 10^0 = 1.5m$
  - F=10 GHz ➔ $\frac{\lambda}{2} = \frac{3 \cdot 10^8}{2 \cdot 10^{10}} = 1.5 \cdot 10^{-2} = 1.5cm$



Multiplexing !

- Modulating information channels at different frequencies allows to transmit them at the same time over the same medium without interfering.