Topic 9: Recursion

To Understand Recursion You Must First Understand Recursion

Textbook

- Recommended Exercises
  - The Python Workbook: 167, 168, 169, 170 and 171
- Recommend Readings
  - Starting Out with Python
    - Chapter 13 (2^{nd} Ed.) / Chapter 12 (3^{rd} Ed. and 4^{th} Ed.)

Recursion

- Definition:
  - See Recursion
  - Defining something in terms of itself
    - Generally using a smaller or simpler version
- Recursive Function
  - A function that calls itself

A Small Example

- Compute n factorial:
  - Using a loop
    - Initialize result to 1
    - for i ranging from 1 to n (inclusive)
      - Multiply result by i, storing the result back into i
  - Another solution
    - By definition, 0! is 1
    - View n! as n * (n-1)!

A Small Example

Recursion

- A well formed recursive function normally has two cases
  - Base Case:
    - Does not make a recursive call
    - Permits function to terminate
  - Recursive Case:
    - Function calls itself
    - Generally must be a call to a smaller or simpler version of the problem

Useful Examples of Recursion

- Drawing fractals
- Finding a path through a maze
- Flood fill / “paint bucket” tool
- Merge sort, quick sort, binary search
- Finding the total size of all of the files in a directory and its subdirectories
- Parsing / evaluating expressions
- …

Greatest Common Divisor

- Finding the greatest common divisor of two positive integers, x and y:
  - If x can be evenly divided by y, then gcd(x,y) is y
  - Otherwise, gcd(x,y) is gcd(y, remainder of x/y)

Fibonacci Numbers

- A sequence of values:
  - 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …
- Defined recursively:
  - By definition:
    - fib(0) is 0
    - fib(1) is 1
  - Remaining values:
    - Formed by computing the sum of the previous two values in the sequence

Fibonacci Numbers

Advantages of Recursion

- Very well suited to some problems
  - Tree traversals
  - Flood fill
  - Fractal images
  - Quick sort / merge sort
  - …

- Easier to implement for some problems, sometimes faster than iterative

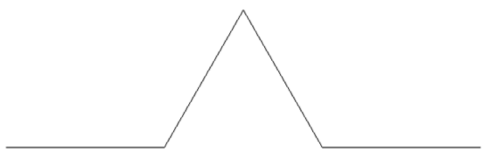
Advantages of Iteration

- Typically
  - Faster (but not always)
  - Requires less memory (most of the time)
- Can be more intuitive for some problems / people

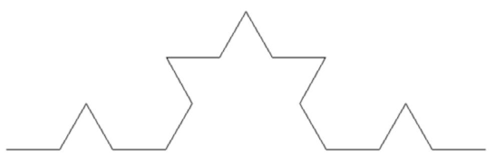
- But some problems are messy to express iteratively

Fractals

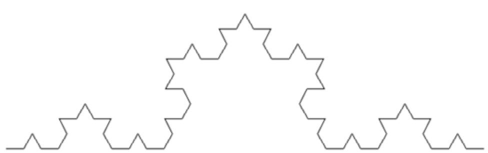
- Self similar images
- Often have reasonably simple recursive definitions
  - 1



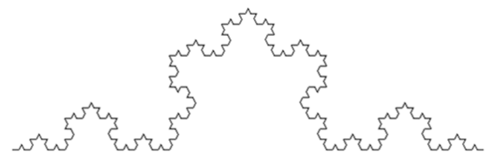
  - 2



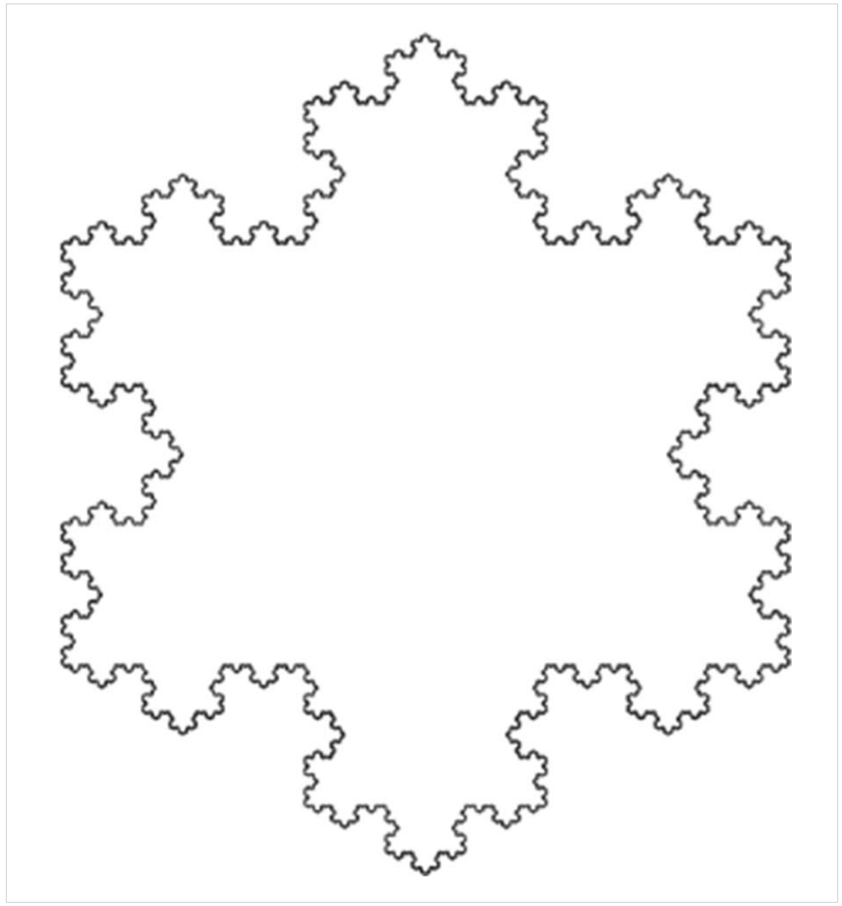
  - 3



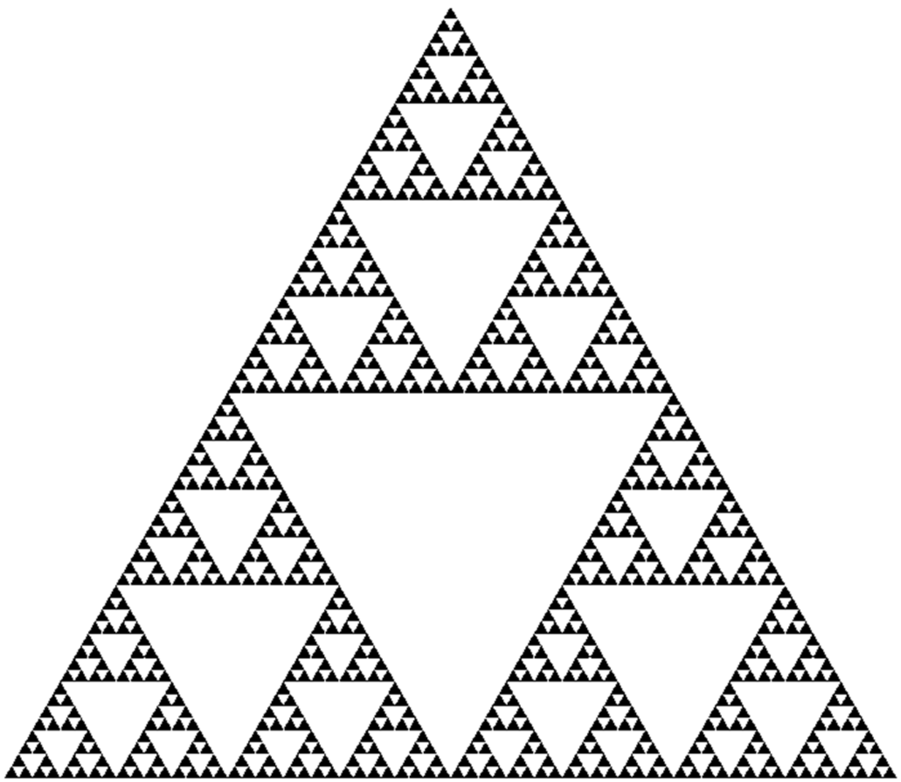
4



Koch Snowflake



Sierpinski Triangle



Sierpinski Trianglge
Source: http://commons.wikimedia.org/wiki/File:Sierpinski-Trigon-7.svg

Fractal Fern

Fractal Fern
Source: http://schools-wikipedia.org/images/67/6740.png.htm

Fractal T-Square

Fractal T-Square

Maze Path Finding

- Consider a two dimensional list containing 4 different values
  - Entrance for the maze
  - Exit for the maze
  - Open spaces
  - Walls
- Assume that the maze is fully enclosed

Maze Path Finding

- Algorithm solve(map, x, y)
  - If the current square is a wall or a space we have already visited, return failure
  - If the current square is the exit point, mark it as part of the solution and return success
  - Mark the current square as part of the solution

  - If solve(map, x, y+1) is successful, return success
  - If solve(map, x, y-1) is successful, return success
  - If solve(map, x+1, y) is successful, return success
  - If solve(map, x-1, y) is successful, return success

  - Mark the current square as visited but not part of the solution
  - Return failure

Maze Path Finding

Recursion

- Recursion: See Recursion
  - Very useful for some problems
  - Caution:
    - Can be inefficient
    - Not a good solution for all problems – Use it when appropriate, don’t abuse it