154Lab10

Jiyoon Clover Jeong

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k-Nearest Neighbors

```
set.seed(10)

data_norm <- function(x) { ((x- min(x)) / (max(x) - min(x))  )}

iris_norm <- as.data.frame(lapply(iris[,-5], data_norm))

my_knn <- function(X_train, X_test, Y_train, k){

  n <- dim(X_test)[1]
  n
  m <- dim(X_train)[1]
  m

  classified <- c()

  for(i in 1:n){
    around <- c()
    dist <- c()

    for(j in 1:m){
      dist[j] <- sqrt(sum((X_test[i,] - X_train[j,])^2))
    }
    around <- Y_train[which(dist %in% sort(dist, decreasing = F)[1:k])]

    classified[i] <- names(which.max(table(around)))
  }

  return(classified)

}



train_idx <- sample(nrow(iris), 90)
train_set <- iris[train_idx, ]
test_set <- iris[-train_idx, ]
my_knn_pred <- my_knn(train_set[, -5], test_set[, -5], train_set$Species, k=1)
knn_pred <- knn(train_set[, -5], test_set[, -5], train_set$Species, k=1)

my_knn_pred
```

```
##  [1] "setosa"     "setosa"     "setosa"     "setosa"     "setosa"
##  [6] "setosa"     "setosa"     "setosa"     "setosa"     "setosa"
## [11] "setosa"     "setosa"     "setosa"     "setosa"     "setosa"
```

```
## [16] "setosa"     "setosa"     "setosa"     "versicolor" "versicolor"
## [21] "versicolor" "versicolor" "versicolor" "versicolor" "versicolor"
## [26] "versicolor" "versicolor" "versicolor" "versicolor" "virginica"
## [31] "versicolor" "versicolor" "versicolor" "versicolor" "virginica"
## [36] "versicolor" "versicolor" "versicolor" "versicolor" "versicolor"
## [41] "virginica"  "versicolor" "virginica"  "virginica"  "virginica"
## [46] "virginica"  "virginica"  "virginica"  "virginica"  "virginica"
## [51] "virginica"  "virginica"  "virginica"  "virginica"  "versicolor"
## [56] "virginica"  "virginica"  "virginica"  "virginica"  "virginica"
```

```
knn_pred
```

```
##  [1] setosa     setosa     setosa     setosa     setosa     setosa
##  [7] setosa     setosa     setosa     setosa     setosa     setosa
## [13] setosa     setosa     setosa     setosa     setosa     setosa
## [19] versicolor versicolor versicolor versicolor versicolor versicolor
## [25] versicolor versicolor versicolor versicolor versicolor virginica
## [31] versicolor versicolor versicolor versicolor virginica  versicolor
## [37] versicolor versicolor versicolor versicolor virginica  versicolor
## [43] virginica  virginica  virginica  virginica  virginica  virginica
## [49] virginica  virginica  virginica  virginica  virginica  virginica
## [55] versicolor virginica  virginica  virginica  virginica  virginica
## Levels: setosa versicolor virginica
```

```
table(my_knn_pred == knn_pred)
```

```
##
## TRUE
##   60
```

k-NN CV

```
set.seed(10)

find_k_CV <- function(X_train, Y_train, k = 1:10, nfold = 10){

  set.seed(11)

  fold <- createFolds(X_train[,1], nfold)
  n <- length(k)

  mse <- matrix(0, n, nfold)

  for(i in 1:n){
    for(j in 1:nfold){
     # my_knn_pred<- my_knn(X_train[-fold[[j]], ], X_train[fold[[j]], ],  Y_train[-fold[[j]] ], i)

      my_knn_pred <- knn(X_train[-fold[[j]], ], X_train[fold[[j]], ],  Y_train[-fold[[j]] ], i)

      mse[i,j] <- mean(my_knn_pred != Y_train[fold[[j]]])

    }
  }
```

```
  return(which.min(rowMeans(mse)))
}
```

```
find_k_CV(train_set[, -5], train_set[, 5])
```

```
## [1] 4
```

Comparisons

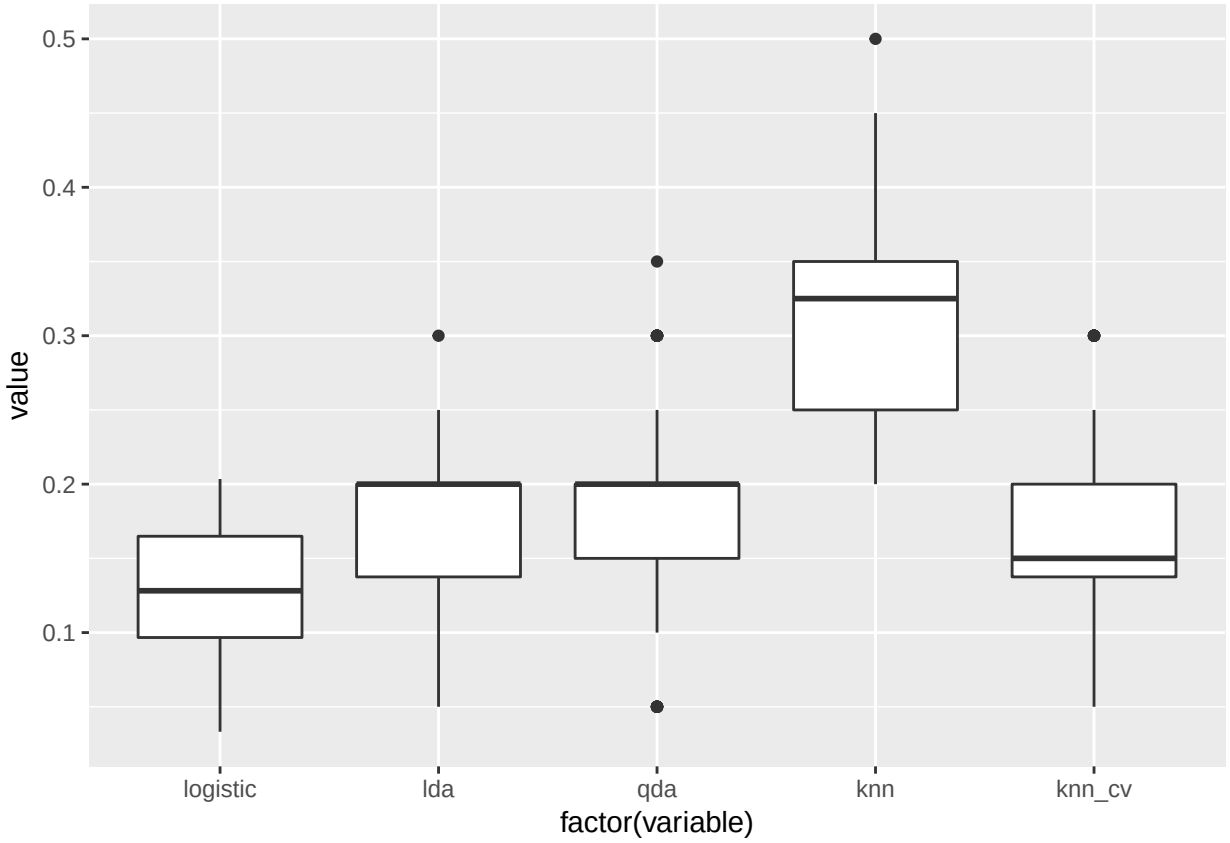
```
set.seed(100)

expit <- function(x) {
  exp(x) / (1 + exp(x))
}


gen_datasets <- function() {

  #1
  id <- diag(c(1, 1))
  df1 <- data.frame(y=factor(rep(c(0, 1), each=50)),
                    rbind(rmvnorm(50, mean=c(0, 0), sigma = id),
                          rmvnorm(50, mean=c(1, 1), sigma = id)))


  #2
  covmat <- matrix(c(1, -0.5, -0.5, 1), nrow=2)
  df2 <- data.frame(y=factor(rep(c(0, 1), each=50)),
                    rbind(rmvnorm(50, mean=c(0, 0), sigma = covmat),
                          rmvnorm(50, mean=c(1, 1), sigma = covmat)))



  #3
  mu <- c(0, 0)
  sigma <- matrix(c(1, 1/2, 1/2, 1), 2)
  nu <- 4
  n <- 50 # Number of draws
  x_first <- t(t(mvrnorm(n, rep(0, length(mu)), sigma) * sqrt(nu / rchisq(n, nu))) + mu)

  mu <- c(1, 1)
  sigma <- matrix(c(1, 1/2, 1/2, 1), 2)
  nu <- 4
  n <- 50 # Number of draws
  x_second <- t(t(mvrnorm(n, rep(0, length(mu)), sigma) * sqrt(nu / rchisq(n, nu))) + mu)
```

```
  df3 <- data.frame(y=factor(rep(c(0, 1), each=50)), rbind(x_first, x_second))



  #4
  covmat2 <- matrix(c(1, 0.5, 0.5, 1), nrow=2)
  df4 <- data.frame(y=factor(rep(c(0, 1), each=50)),
                    rbind(rmvnorm(50, mean=c(0, 0), sigma = covmat2),
                          rmvnorm(50, mean=c(1, 1), sigma = covmat)))



  #5
  x <- matrix(rnorm(200), ncol=2)
  #get x1^2, x2^2, and interaction term...
  df5_temp <- data.frame(x^2, x[, 1] * x[, 2])

  beta <- c(0, 2, -1, -2)
  y <- apply(df5_temp, 1, function(row){
    p <- expit(sum(c(1, row) * beta)) #get pi
    #using pi above, get yi each time...
    sample(x=c(0, 1), size=1, prob=c(1-p, p))
    }
  )
  df5 <- data.frame(y=factor(y), x)



  #6
  x <- matrix(rnorm(200), ncol=2)
  #make true/false...
  y <- 1 * (x[, 1]^2 + x[, 2]^2 > qchisq(p=0.5, df=2))
  df6 <- data.frame(y=factor(y), x)


  return(list(df1, df2, df3, df4, df5, df6))


}
```

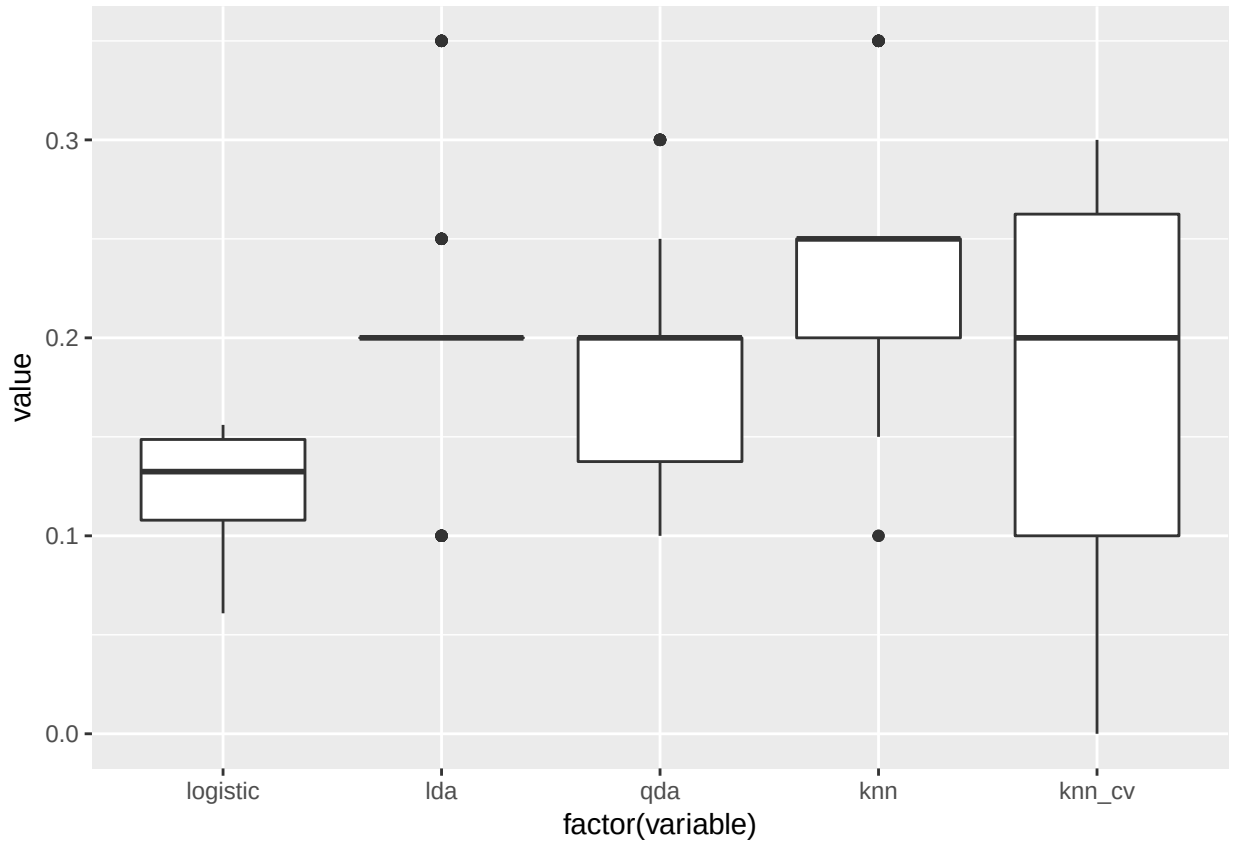
```
set.seed(11)


error <- array(0, dim = c(5,6,100))






for(i in 1:100){
  gendata <- gen_datasets()
  fold <- createDataPartition(gendata[[1]][,1], p = 0.8)$Resample1
  fold
```

```
  for(j in 1:6){


      data <- gendata[[j]]

      # Logistic

      fit <- glm(y ~ ., family = binomial, data = data[fold,] )
      summary(fit)
      error[1,j,i] <- mean((as.numeric(as.character(data[-fold, 1])) - predict(fit, data[-fold, 2:3], ty

      # LDA

      fit <- lda(y ~., data = data[fold, ])
      summary(fit)
      predicted <- as.numeric(as.character(predict(fit, data[-fold, 2:3], type = "response")$class))

      error[2,j,i] <-  mean(((as.numeric(as.character(data[-fold, 1]))) - predicted)^2)


      #QDA

      fit <- qda(y ~., data = data[fold, ])
      summary(fit)
      predicted <- as.numeric(as.character(predict(fit, data[-fold, 2:3], type = "response")$class))

      error[3,j,i] <-  mean(((as.numeric(as.character(data[-fold, 1]))) - predicted)^2)


      #KNN

      predicted <- as.numeric(as.character(knn(data[fold, -1], data[-fold, -1], data[fold,1], k=1)))
      error[4,j,i] <-  mean(((as.numeric(as.character(data[-fold, 1]))) - predicted)^2)

      # KNN CV

      k <- find_k_CV(data[,-1], data[,1])
      predicted <- as.numeric(as.character(knn(data[fold, -1], data[-fold, -1], data[fold,1], k=k)))

      error[5,j,i] <- mean(((as.numeric(as.character(data[-fold, 1]))) - predicted)^2)

  }
}



for(i in 1:6){
  scenario <- data.frame(logistic = error[1,i,], lda = error[2,i,], qda = error[3,i,], knn = error[4,i,]

  melted <- melt(scenario)

  print(ggplot(melted, aes(x = factor(variable), value)) + geom_boxplot() )
}
```

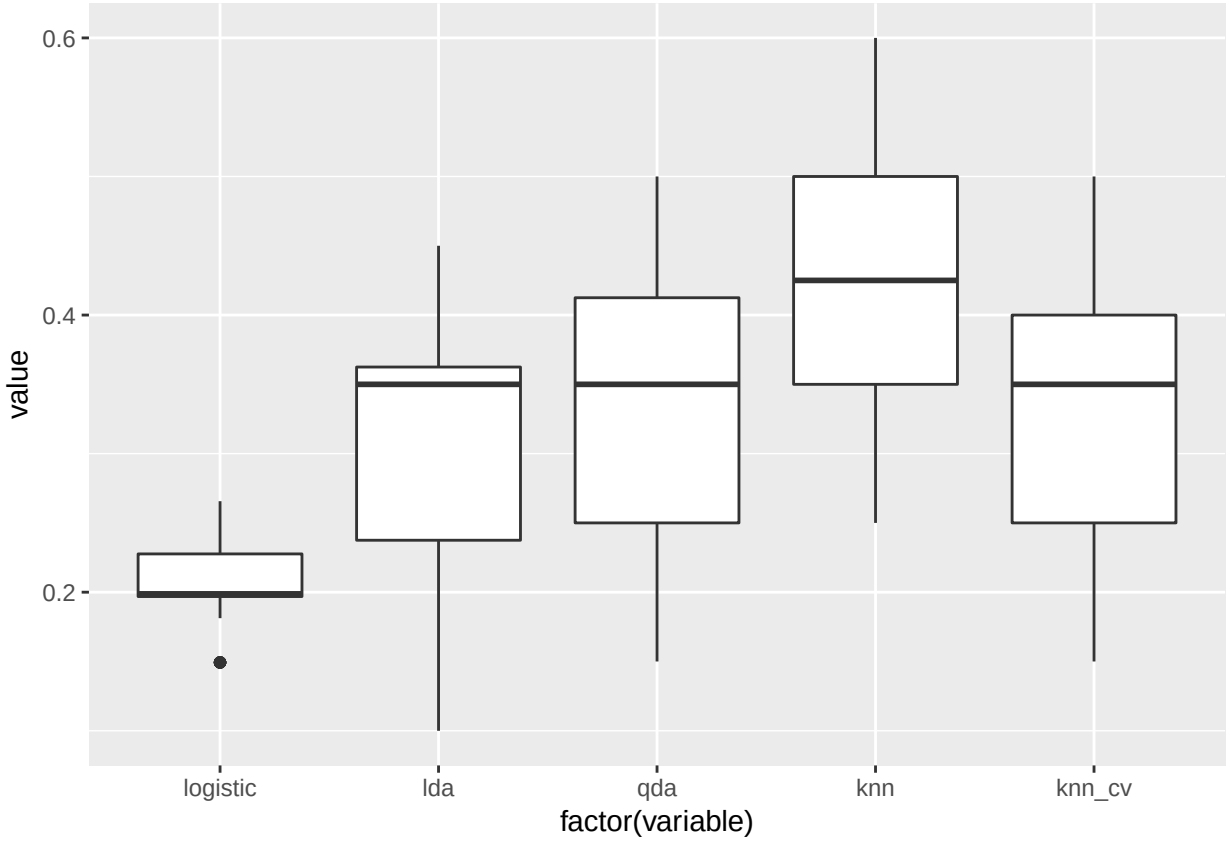
```
## Using  as id variables
## Using  as id variables
```



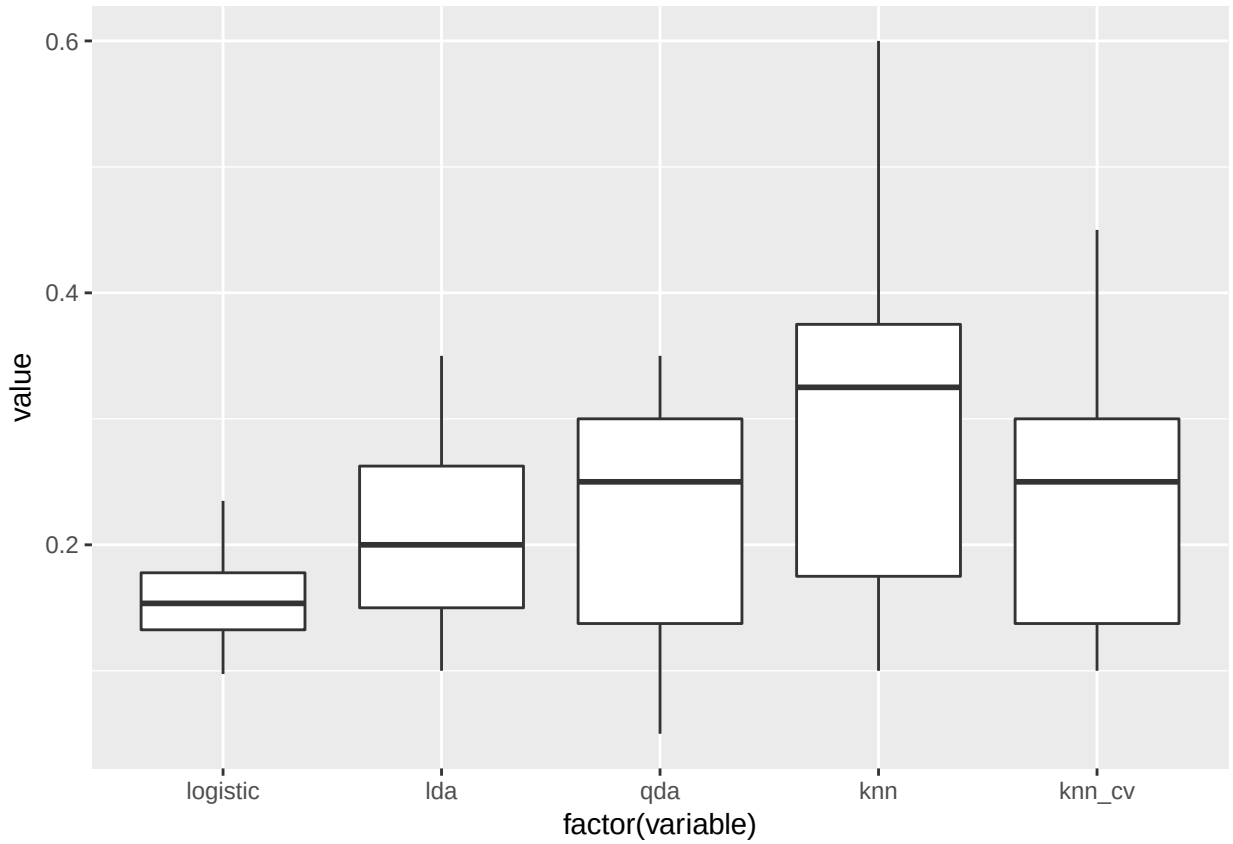

```
## Using  as id variables
```



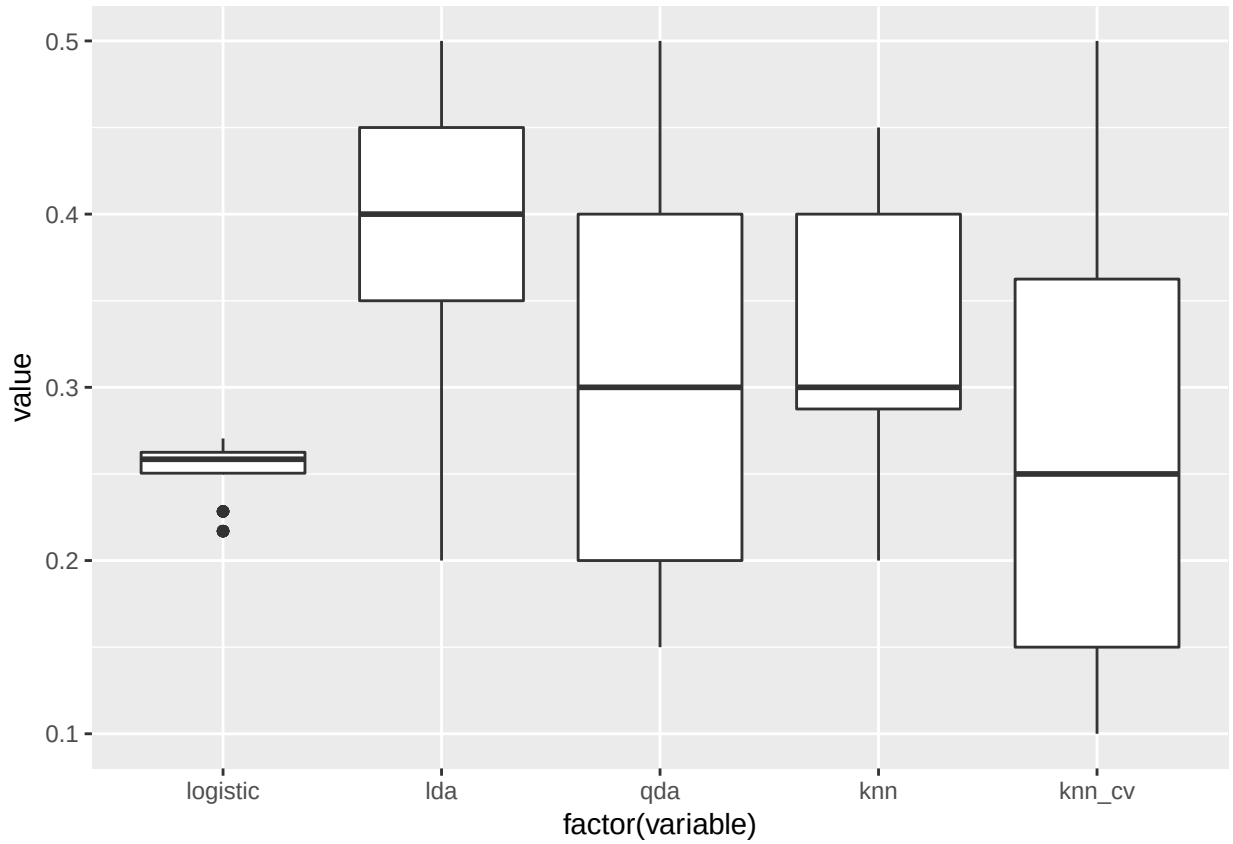

```
## Using  as id variables
```

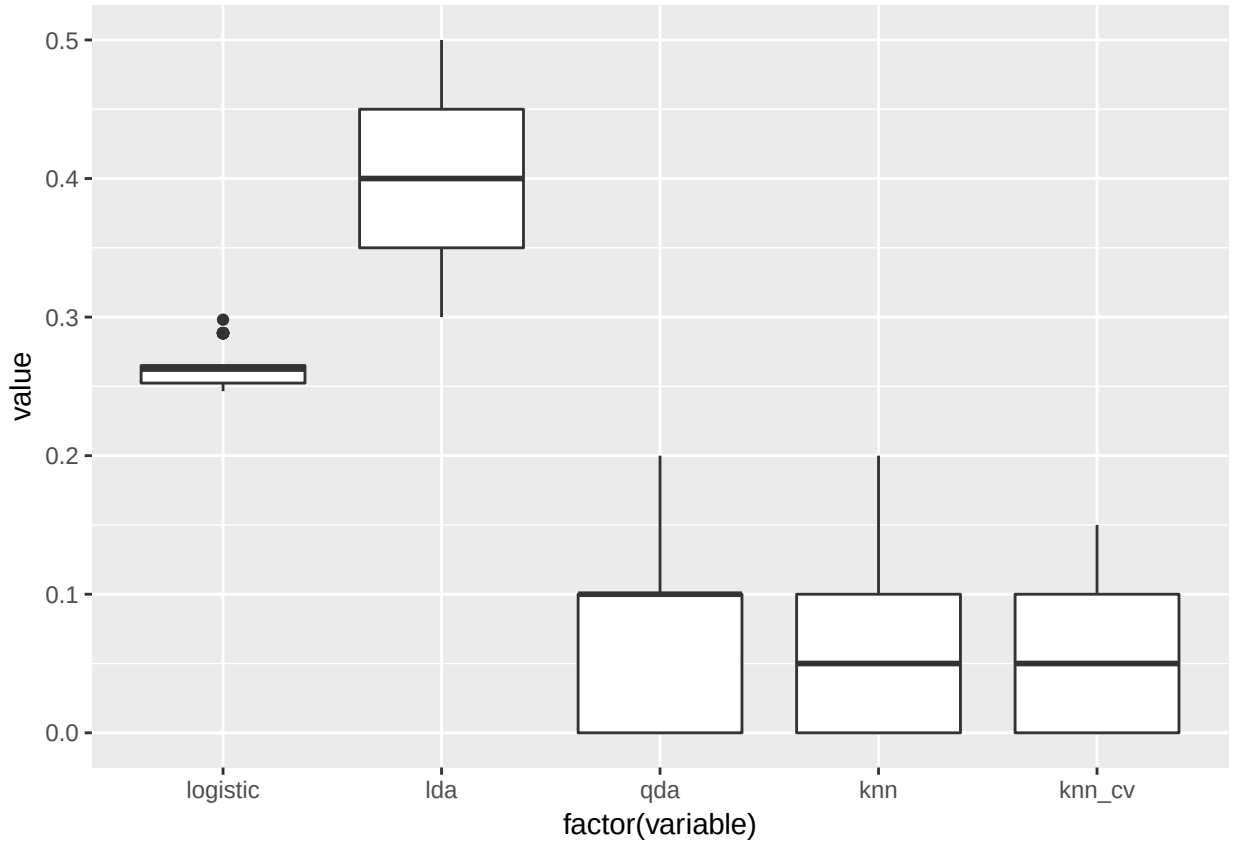



```
## Using  as id variables
```




```
## Using  as id variables
```


0.5
0.4
0.3
0.2
0.1
value
logistic
lda
qda
knn
knn_cv
factor(variable)


0.5
0.4
0.3
0.2
0.1
0.0
value
logistic
lda
qda
knn
knn_cv
factor(variable)

```
error
```

```
## , , 1
##
##           [,1]    [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.2034561 0.13186 0.2137185 0.1410677 0.2601034 0.2474595
## [2,] 0.3000000 0.20000 0.3000000 0.1500000 0.3500000 0.5000000
## [3,] 0.3000000 0.20000 0.3500000 0.1000000 0.3000000 0.0500000
## [4,] 0.2500000 0.10000 0.4000000 0.3500000 0.3000000 0.0000000
## [5,] 0.2500000 0.10000 0.3500000 0.1500000 0.3500000 0.0000000
##
## , , 2
##
##           [,1]       [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.1637631 0.09465499 0.2656727 0.1836247 0.2602845 0.2981205
## [2,] 0.2500000 0.10000000 0.4000000 0.2500000 0.4000000 0.5000000
## [3,] 0.2500000 0.10000000 0.4500000 0.3500000 0.5000000 0.1000000
## [4,] 0.4500000 0.10000000 0.5000000 0.4500000 0.4000000 0.0000000
## [5,] 0.2000000 0.10000000 0.5000000 0.2500000 0.5000000 0.0000000
##
## , , 3
##
##            [,1]      [,2]    [,3]      [,4]      [,5]      [,6]
## [1,] 0.07406691 0.1199204 0.18127 0.1758338 0.2704869 0.2472776
## [2,] 0.10000000 0.1000000 0.10000 0.3000000 0.3500000 0.5000000
## [3,] 0.10000000 0.1500000 0.30000 0.3000000 0.2000000 0.2000000
## [4,] 0.20000000 0.2500000 0.25000 0.2500000 0.3000000 0.2000000
## [5,] 0.10000000 0.1000000 0.40000 0.3000000 0.1500000 0.1500000
##
## , , 4
##
##          [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.183375 0.1423524 0.2474124 0.1535064 0.2504215 0.2884485
## [2,] 0.250000 0.2000000 0.3500000 0.2500000 0.5000000 0.4500000
## [3,] 0.350000 0.2000000 0.3500000 0.3000000 0.3000000 0.1500000
## [4,] 0.500000 0.3500000 0.4500000 0.1000000 0.2000000 0.1000000
## [5,] 0.250000 0.3000000 0.3500000 0.1000000 0.1000000 0.1000000
##
## , , 5
##
##           [,1]     [,2]      [,3]       [,4]      [,5]      [,6]
## [1,] 0.1281649 0.142397 0.1988006 0.09738152 0.2660846 0.2533812
## [2,] 0.2000000 0.350000 0.2500000 0.15000000 0.4500000 0.4000000
## [3,] 0.2000000 0.100000 0.1500000 0.05000000 0.2500000 0.1000000
## [4,] 0.3500000 0.250000 0.4500000 0.10000000 0.2500000 0.1000000
## [5,] 0.1500000 0.300000 0.2500000 0.10000000 0.2500000 0.1000000
##
## , , 6
##
##            [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.09699841 0.06086168 0.1493032 0.148006 0.2702222 0.2623218
## [2,] 0.15000000 0.10000000 0.1500000 0.150000 0.3500000 0.3500000
## [3,] 0.15000000 0.10000000 0.2000000 0.100000 0.4000000 0.1000000
## [4,] 0.35000000 0.20000000 0.3000000 0.200000 0.4500000 0.0000000
```

```
## [5,] 0.15000000 0.00000000 0.1500000 0.250000 0.4000000 0.0000000
##
## , , 7
##
##           [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.1835377 0.09332157 0.2187826 0.234898 0.2584759 0.2465179
## [2,] 0.2500000 0.20000000 0.4500000 0.350000 0.4000000 0.3000000
## [3,] 0.3000000 0.20000000 0.4500000 0.250000 0.4000000 0.0500000
## [4,] 0.3500000 0.15000000 0.4000000 0.600000 0.3000000 0.0500000
## [5,] 0.3000000 0.10000000 0.4500000 0.250000 0.4000000 0.0500000
##
## , , 8
##
##            [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.03319831 0.1123227 0.1985367 0.1923368 0.2613105 0.2523929
## [2,] 0.05000000 0.2000000 0.3500000 0.3000000 0.4500000 0.3500000
## [3,] 0.05000000 0.2000000 0.3000000 0.3000000 0.1500000 0.0000000
## [4,] 0.25000000 0.2500000 0.6000000 0.3500000 0.4000000 0.0000000
## [5,] 0.05000000 0.2000000 0.3500000 0.3000000 0.1500000 0.0000000
##
## , , 9
##
##            [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.09570913 0.1486731 0.2276262 0.1306976 0.2283749 0.265079
## [2,] 0.15000000 0.2000000 0.4000000 0.1000000 0.3500000 0.450000
## [3,] 0.15000000 0.2500000 0.5000000 0.2500000 0.2000000 0.000000
## [4,] 0.30000000 0.2500000 0.5000000 0.5000000 0.3000000 0.000000
## [5,] 0.10000000 0.2000000 0.3500000 0.4500000 0.2000000 0.050000
##
## , , 10
##
##           [,1]      [,2]      [,3]    [,4]     [,5]      [,6]
## [1,] 0.1425956 0.1324262 0.1977104 0.15979 0.217006 0.2630115
## [2,] 0.2000000 0.2500000 0.3500000 0.15000 0.200000 0.5000000
## [3,] 0.2000000 0.2000000 0.4000000 0.30000 0.450000 0.1000000
## [4,] 0.3000000 0.3500000 0.5500000 0.35000 0.450000 0.1000000
## [5,] 0.2000000 0.2500000 0.4000000 0.30000 0.350000 0.0500000
##
## , , 11
##
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.1648849 0.1560302 0.1943324 0.1325304 0.2536756 0.262696
## [2,] 0.2000000 0.2000000 0.2000000 0.2000000 0.4500000 0.300000
## [3,] 0.2000000 0.2000000 0.2500000 0.1500000 0.2000000 0.000000
## [4,] 0.4500000 0.2500000 0.3500000 0.3000000 0.4000000 0.050000
## [5,] 0.2000000 0.2000000 0.2000000 0.1500000 0.2500000 0.150000
##
## , , 12
##
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.1196871 0.1504202 0.2423443 0.1535064 0.2504215 0.2884485
## [2,] 0.1000000 0.2000000 0.3500000 0.2500000 0.5000000 0.4500000
## [3,] 0.1500000 0.3000000 0.3500000 0.3000000 0.3000000 0.1500000
## [4,] 0.2500000 0.2500000 0.3500000 0.1000000 0.2000000 0.1000000
```

```
## [5,] 0.1500000 0.3000000 0.2500000 0.1000000 0.1000000 0.1000000
##
## , , 13
##
##           [,1]     [,2]      [,3]       [,4]      [,5]      [,6]
## [1,] 0.1281649 0.142397 0.1988006 0.09738152 0.2660846 0.2533812
## [2,] 0.2000000 0.350000 0.2500000 0.15000000 0.4500000 0.4000000
## [3,] 0.2000000 0.100000 0.1500000 0.05000000 0.2500000 0.1000000
## [4,] 0.3500000 0.250000 0.4500000 0.10000000 0.2500000 0.1000000
## [5,] 0.1500000 0.300000 0.2500000 0.10000000 0.2500000 0.1000000
##
## , , 14
##
##            [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.09699841 0.06086168 0.1493032 0.148006 0.2702222 0.2623218
## [2,] 0.15000000 0.10000000 0.1500000 0.150000 0.3500000 0.3500000
## [3,] 0.15000000 0.10000000 0.2000000 0.100000 0.4000000 0.1000000
## [4,] 0.35000000 0.20000000 0.3000000 0.200000 0.4500000 0.0000000
## [5,] 0.15000000 0.00000000 0.1500000 0.250000 0.4000000 0.0000000
##
## , , 15
##
##           [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.1835377 0.09332157 0.2187826 0.234898 0.2584759 0.2465179
## [2,] 0.2500000 0.20000000 0.4500000 0.350000 0.4000000 0.3000000
## [3,] 0.3000000 0.20000000 0.4500000 0.250000 0.4000000 0.0500000
## [4,] 0.3500000 0.15000000 0.4000000 0.600000 0.3000000 0.0500000
## [5,] 0.3000000 0.10000000 0.4500000 0.250000 0.4000000 0.0500000
##
## , , 16
##
##            [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.03319831 0.1123227 0.1985367 0.1923368 0.2613105 0.2523929
## [2,] 0.05000000 0.2000000 0.3500000 0.3000000 0.4500000 0.3500000
## [3,] 0.05000000 0.2000000 0.3000000 0.3000000 0.1500000 0.0000000
## [4,] 0.25000000 0.2500000 0.6000000 0.3500000 0.4000000 0.0000000
## [5,] 0.05000000 0.2000000 0.3500000 0.3000000 0.1500000 0.0000000
##
## , , 17
##
##            [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.09570913 0.1486731 0.2276262 0.1306976 0.2283749 0.265079
## [2,] 0.15000000 0.2000000 0.4000000 0.1000000 0.3500000 0.450000
## [3,] 0.15000000 0.2500000 0.5000000 0.2500000 0.2000000 0.000000
## [4,] 0.30000000 0.2500000 0.5000000 0.5000000 0.3000000 0.000000
## [5,] 0.10000000 0.2000000 0.3500000 0.4500000 0.2000000 0.050000
##
## , , 18
##
##           [,1]      [,2]      [,3]    [,4]     [,5]      [,6]
## [1,] 0.1425956 0.1324262 0.1977104 0.15979 0.217006 0.2630115
## [2,] 0.2000000 0.2500000 0.3500000 0.15000 0.200000 0.5000000
## [3,] 0.2000000 0.2000000 0.4000000 0.30000 0.450000 0.1000000
## [4,] 0.3000000 0.3500000 0.5500000 0.35000 0.450000 0.1000000
```

```
## [5,] 0.2000000 0.2500000 0.4000000 0.30000 0.350000 0.0500000
##
## , , 19
##
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.1648849 0.1560302 0.1943324 0.1325304 0.2536756 0.262696
## [2,] 0.2000000 0.2000000 0.2000000 0.2000000 0.4500000 0.300000
## [3,] 0.2000000 0.2000000 0.2500000 0.1500000 0.2000000 0.000000
## [4,] 0.4500000 0.2500000 0.3500000 0.3000000 0.4000000 0.050000
## [5,] 0.2000000 0.2000000 0.2000000 0.1500000 0.2500000 0.150000
##
## , , 20
##
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.1196871 0.1504202 0.2423443 0.1535064 0.2504215 0.2884485
## [2,] 0.1000000 0.2000000 0.3500000 0.2500000 0.5000000 0.4500000
## [3,] 0.1500000 0.3000000 0.3500000 0.3000000 0.3000000 0.1500000
## [4,] 0.2500000 0.2500000 0.3500000 0.1000000 0.2000000 0.1000000
## [5,] 0.1500000 0.3000000 0.2500000 0.1000000 0.1000000 0.1000000
##
## , , 21
##
##           [,1]     [,2]      [,3]       [,4]      [,5]      [,6]
## [1,] 0.1281649 0.142397 0.1988006 0.09738152 0.2660846 0.2533812
## [2,] 0.2000000 0.350000 0.2500000 0.15000000 0.4500000 0.4000000
## [3,] 0.2000000 0.100000 0.1500000 0.05000000 0.2500000 0.1000000
## [4,] 0.3500000 0.250000 0.4500000 0.10000000 0.2500000 0.1000000
## [5,] 0.1500000 0.300000 0.2500000 0.10000000 0.2500000 0.1000000
##
## , , 22
##
##            [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.09699841 0.06086168 0.1493032 0.148006 0.2702222 0.2623218
## [2,] 0.15000000 0.10000000 0.1500000 0.150000 0.3500000 0.3500000
## [3,] 0.15000000 0.10000000 0.2000000 0.100000 0.4000000 0.1000000
## [4,] 0.35000000 0.20000000 0.3000000 0.200000 0.4500000 0.0000000
## [5,] 0.15000000 0.00000000 0.1500000 0.250000 0.4000000 0.0000000
##
## , , 23
##
##           [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.1835377 0.09332157 0.2187826 0.234898 0.2584759 0.2465179
## [2,] 0.2500000 0.20000000 0.4500000 0.350000 0.4000000 0.3000000
## [3,] 0.3000000 0.20000000 0.4500000 0.250000 0.4000000 0.0500000
## [4,] 0.3500000 0.15000000 0.4000000 0.600000 0.3000000 0.0500000
## [5,] 0.3000000 0.10000000 0.4500000 0.250000 0.4000000 0.0500000
##
## , , 24
##
##            [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.03319831 0.1123227 0.1985367 0.1923368 0.2613105 0.2523929
## [2,] 0.05000000 0.2000000 0.3500000 0.3000000 0.4500000 0.3500000
## [3,] 0.05000000 0.2000000 0.3000000 0.3000000 0.1500000 0.0000000
## [4,] 0.25000000 0.2500000 0.6000000 0.3500000 0.4000000 0.0000000
```

```
## [5,] 0.05000000 0.2000000 0.3500000 0.3000000 0.1500000 0.0000000
##
## , , 25
##
##            [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.09570913 0.1486731 0.2276262 0.1306976 0.2283749 0.265079
## [2,] 0.15000000 0.2000000 0.4000000 0.1000000 0.3500000 0.450000
## [3,] 0.15000000 0.2500000 0.5000000 0.2500000 0.2000000 0.000000
## [4,] 0.30000000 0.2500000 0.5000000 0.5000000 0.3000000 0.000000
## [5,] 0.10000000 0.2000000 0.3500000 0.4500000 0.2000000 0.050000
##
## , , 26
##
##           [,1]      [,2]      [,3]    [,4]     [,5]      [,6]
## [1,] 0.1425956 0.1324262 0.1977104 0.15979 0.217006 0.2630115
## [2,] 0.2000000 0.2500000 0.3500000 0.15000 0.200000 0.5000000
## [3,] 0.2000000 0.2000000 0.4000000 0.30000 0.450000 0.1000000
## [4,] 0.3000000 0.3500000 0.5500000 0.35000 0.450000 0.1000000
## [5,] 0.2000000 0.2500000 0.4000000 0.30000 0.350000 0.0500000
##
## , , 27
##
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.1648849 0.1560302 0.1943324 0.1325304 0.2536756 0.262696
## [2,] 0.2000000 0.2000000 0.2000000 0.2000000 0.4500000 0.300000
## [3,] 0.2000000 0.2000000 0.2500000 0.1500000 0.2000000 0.000000
## [4,] 0.4500000 0.2500000 0.3500000 0.3000000 0.4000000 0.050000
## [5,] 0.2000000 0.2000000 0.2000000 0.1500000 0.2500000 0.150000
##
## , , 28
##
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.1196871 0.1504202 0.2423443 0.1535064 0.2504215 0.2884485
## [2,] 0.1000000 0.2000000 0.3500000 0.2500000 0.5000000 0.4500000
## [3,] 0.1500000 0.3000000 0.3500000 0.3000000 0.3000000 0.1500000
## [4,] 0.2500000 0.2500000 0.3500000 0.1000000 0.2000000 0.1000000
## [5,] 0.1500000 0.3000000 0.2500000 0.1000000 0.1000000 0.1000000
##
## , , 29
##
##           [,1]     [,2]      [,3]       [,4]      [,5]      [,6]
## [1,] 0.1281649 0.142397 0.1988006 0.09738152 0.2660846 0.2533812
## [2,] 0.2000000 0.350000 0.2500000 0.15000000 0.4500000 0.4000000
## [3,] 0.2000000 0.100000 0.1500000 0.05000000 0.2500000 0.1000000
## [4,] 0.3500000 0.250000 0.4500000 0.10000000 0.2500000 0.1000000
## [5,] 0.1500000 0.300000 0.2500000 0.10000000 0.2500000 0.1000000
##
## , , 30
##
##            [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.09699841 0.06086168 0.1493032 0.148006 0.2702222 0.2623218
## [2,] 0.15000000 0.10000000 0.1500000 0.150000 0.3500000 0.3500000
## [3,] 0.15000000 0.10000000 0.2000000 0.100000 0.4000000 0.1000000
## [4,] 0.35000000 0.20000000 0.3000000 0.200000 0.4500000 0.0000000
```

```
## [5,] 0.15000000 0.00000000 0.1500000 0.250000 0.4000000 0.0000000
##
## , , 31
##
##           [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.1835377 0.09332157 0.2187826 0.234898 0.2584759 0.2465179
## [2,] 0.2500000 0.20000000 0.4500000 0.350000 0.4000000 0.3000000
## [3,] 0.3000000 0.20000000 0.4500000 0.250000 0.4000000 0.0500000
## [4,] 0.3500000 0.15000000 0.4000000 0.600000 0.3000000 0.0500000
## [5,] 0.3000000 0.10000000 0.4500000 0.250000 0.4000000 0.0500000
##
## , , 32
##
##            [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.03319831 0.1123227 0.1985367 0.1923368 0.2613105 0.2523929
## [2,] 0.05000000 0.2000000 0.3500000 0.3000000 0.4500000 0.3500000
## [3,] 0.05000000 0.2000000 0.3000000 0.3000000 0.1500000 0.0000000
## [4,] 0.25000000 0.2500000 0.6000000 0.3500000 0.4000000 0.0000000
## [5,] 0.05000000 0.2000000 0.3500000 0.3000000 0.1500000 0.0000000
##
## , , 33
##
##            [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.09570913 0.1486731 0.2276262 0.1306976 0.2283749 0.265079
## [2,] 0.15000000 0.2000000 0.4000000 0.1000000 0.3500000 0.450000
## [3,] 0.15000000 0.2500000 0.5000000 0.2500000 0.2000000 0.000000
## [4,] 0.30000000 0.2500000 0.5000000 0.5000000 0.3000000 0.000000
## [5,] 0.10000000 0.2000000 0.3500000 0.4500000 0.2000000 0.050000
##
## , , 34
##
##           [,1]      [,2]      [,3]    [,4]     [,5]      [,6]
## [1,] 0.1425956 0.1324262 0.1977104 0.15979 0.217006 0.2630115
## [2,] 0.2000000 0.2500000 0.3500000 0.15000 0.200000 0.5000000
## [3,] 0.2000000 0.2000000 0.4000000 0.30000 0.450000 0.1000000
## [4,] 0.3000000 0.3500000 0.5500000 0.35000 0.450000 0.1000000
## [5,] 0.2000000 0.2500000 0.4000000 0.30000 0.350000 0.0500000
##
## , , 35
##
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.1648849 0.1560302 0.1943324 0.1325304 0.2536756 0.262696
## [2,] 0.2000000 0.2000000 0.2000000 0.2000000 0.4500000 0.300000
## [3,] 0.2000000 0.2000000 0.2500000 0.1500000 0.2000000 0.000000
## [4,] 0.4500000 0.2500000 0.3500000 0.3000000 0.4000000 0.050000
## [5,] 0.2000000 0.2000000 0.2000000 0.1500000 0.2500000 0.150000
##
## , , 36
##
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.1196871 0.1504202 0.2423443 0.1535064 0.2504215 0.2884485
## [2,] 0.1000000 0.2000000 0.3500000 0.2500000 0.5000000 0.4500000
## [3,] 0.1500000 0.3000000 0.3500000 0.3000000 0.3000000 0.1500000
## [4,] 0.2500000 0.2500000 0.3500000 0.1000000 0.2000000 0.1000000
```

```
## [5,] 0.1500000 0.3000000 0.2500000 0.1000000 0.1000000 0.1000000
##
## , , 37
##
##           [,1]     [,2]      [,3]       [,4]      [,5]      [,6]
## [1,] 0.1281649 0.142397 0.1988006 0.09738152 0.2660846 0.2533812
## [2,] 0.2000000 0.350000 0.2500000 0.15000000 0.4500000 0.4000000
## [3,] 0.2000000 0.100000 0.1500000 0.05000000 0.2500000 0.1000000
## [4,] 0.3500000 0.250000 0.4500000 0.10000000 0.2500000 0.1000000
## [5,] 0.1500000 0.300000 0.2500000 0.10000000 0.2500000 0.1000000
##
## , , 38
##
##            [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.09699841 0.06086168 0.1493032 0.148006 0.2702222 0.2623218
## [2,] 0.15000000 0.10000000 0.1500000 0.150000 0.3500000 0.3500000
## [3,] 0.15000000 0.10000000 0.2000000 0.100000 0.4000000 0.1000000
## [4,] 0.35000000 0.20000000 0.3000000 0.200000 0.4500000 0.0000000
## [5,] 0.15000000 0.00000000 0.1500000 0.250000 0.4000000 0.0000000
##
## , , 39
##
##           [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.1835377 0.09332157 0.2187826 0.234898 0.2584759 0.2465179
## [2,] 0.2500000 0.20000000 0.4500000 0.350000 0.4000000 0.3000000
## [3,] 0.3000000 0.20000000 0.4500000 0.250000 0.4000000 0.0500000
## [4,] 0.3500000 0.15000000 0.4000000 0.600000 0.3000000 0.0500000
## [5,] 0.3000000 0.10000000 0.4500000 0.250000 0.4000000 0.0500000
##
## , , 40
##
##            [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.03319831 0.1123227 0.1985367 0.1923368 0.2613105 0.2523929
## [2,] 0.05000000 0.2000000 0.3500000 0.3000000 0.4500000 0.3500000
## [3,] 0.05000000 0.2000000 0.3000000 0.3000000 0.1500000 0.0000000
## [4,] 0.25000000 0.2500000 0.6000000 0.3500000 0.4000000 0.0000000
## [5,] 0.05000000 0.2000000 0.3500000 0.3000000 0.1500000 0.0000000
##
## , , 41
##
##            [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.09570913 0.1486731 0.2276262 0.1306976 0.2283749 0.265079
## [2,] 0.15000000 0.2000000 0.4000000 0.1000000 0.3500000 0.450000
## [3,] 0.15000000 0.2500000 0.5000000 0.2500000 0.2000000 0.000000
## [4,] 0.30000000 0.2500000 0.5000000 0.5000000 0.3000000 0.000000
## [5,] 0.10000000 0.2000000 0.3500000 0.4500000 0.2000000 0.050000
##
## , , 42
##
##           [,1]      [,2]      [,3]    [,4]     [,5]      [,6]
## [1,] 0.1425956 0.1324262 0.1977104 0.15979 0.217006 0.2630115
## [2,] 0.2000000 0.2500000 0.3500000 0.15000 0.200000 0.5000000
## [3,] 0.2000000 0.2000000 0.4000000 0.30000 0.450000 0.1000000
## [4,] 0.3000000 0.3500000 0.5500000 0.35000 0.450000 0.1000000
```

```
## [5,] 0.2000000 0.2500000 0.4000000 0.30000 0.350000 0.0500000
##
## , , 43
##
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.1648849 0.1560302 0.1943324 0.1325304 0.2536756 0.262696
## [2,] 0.2000000 0.2000000 0.2000000 0.2000000 0.4500000 0.300000
## [3,] 0.2000000 0.2000000 0.2500000 0.1500000 0.2000000 0.000000
## [4,] 0.4500000 0.2500000 0.3500000 0.3000000 0.4000000 0.050000
## [5,] 0.2000000 0.2000000 0.2000000 0.1500000 0.2500000 0.150000
##
## , , 44
##
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.1196871 0.1504202 0.2423443 0.1535064 0.2504215 0.2884485
## [2,] 0.1000000 0.2000000 0.3500000 0.2500000 0.5000000 0.4500000
## [3,] 0.1500000 0.3000000 0.3500000 0.3000000 0.3000000 0.1500000
## [4,] 0.2500000 0.2500000 0.3500000 0.1000000 0.2000000 0.1000000
## [5,] 0.1500000 0.3000000 0.2500000 0.1000000 0.1000000 0.1000000
##
## , , 45
##
##           [,1]     [,2]      [,3]       [,4]      [,5]      [,6]
## [1,] 0.1281649 0.142397 0.1988006 0.09738152 0.2660846 0.2533812
## [2,] 0.2000000 0.350000 0.2500000 0.15000000 0.4500000 0.4000000
## [3,] 0.2000000 0.100000 0.1500000 0.05000000 0.2500000 0.1000000
## [4,] 0.3500000 0.250000 0.4500000 0.10000000 0.2500000 0.1000000
## [5,] 0.1500000 0.300000 0.2500000 0.10000000 0.2500000 0.1000000
##
## , , 46
##
##            [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.09699841 0.06086168 0.1493032 0.148006 0.2702222 0.2623218
## [2,] 0.15000000 0.10000000 0.1500000 0.150000 0.3500000 0.3500000
## [3,] 0.15000000 0.10000000 0.2000000 0.100000 0.4000000 0.1000000
## [4,] 0.35000000 0.20000000 0.3000000 0.200000 0.4500000 0.0000000
## [5,] 0.15000000 0.00000000 0.1500000 0.250000 0.4000000 0.0000000
##
## , , 47
##
##           [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.1835377 0.09332157 0.2187826 0.234898 0.2584759 0.2465179
## [2,] 0.2500000 0.20000000 0.4500000 0.350000 0.4000000 0.3000000
## [3,] 0.3000000 0.20000000 0.4500000 0.250000 0.4000000 0.0500000
## [4,] 0.3500000 0.15000000 0.4000000 0.600000 0.3000000 0.0500000
## [5,] 0.3000000 0.10000000 0.4500000 0.250000 0.4000000 0.0500000
##
## , , 48
##
##            [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.03319831 0.1123227 0.1985367 0.1923368 0.2613105 0.2523929
## [2,] 0.05000000 0.2000000 0.3500000 0.3000000 0.4500000 0.3500000
## [3,] 0.05000000 0.2000000 0.3000000 0.3000000 0.1500000 0.0000000
## [4,] 0.25000000 0.2500000 0.6000000 0.3500000 0.4000000 0.0000000
```

```
## [5,] 0.05000000 0.2000000 0.3500000 0.3000000 0.1500000 0.0000000
##
## , , 49
##
##            [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.09570913 0.1486731 0.2276262 0.1306976 0.2283749 0.265079
## [2,] 0.15000000 0.2000000 0.4000000 0.1000000 0.3500000 0.450000
## [3,] 0.15000000 0.2500000 0.5000000 0.2500000 0.2000000 0.000000
## [4,] 0.30000000 0.2500000 0.5000000 0.5000000 0.3000000 0.000000
## [5,] 0.10000000 0.2000000 0.3500000 0.4500000 0.2000000 0.050000
##
## , , 50
##
##           [,1]      [,2]      [,3]    [,4]     [,5]      [,6]
## [1,] 0.1425956 0.1324262 0.1977104 0.15979 0.217006 0.2630115
## [2,] 0.2000000 0.2500000 0.3500000 0.15000 0.200000 0.5000000
## [3,] 0.2000000 0.2000000 0.4000000 0.30000 0.450000 0.1000000
## [4,] 0.3000000 0.3500000 0.5500000 0.35000 0.450000 0.1000000
## [5,] 0.2000000 0.2500000 0.4000000 0.30000 0.350000 0.0500000
##
## , , 51
##
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.1648849 0.1560302 0.1943324 0.1325304 0.2536756 0.262696
## [2,] 0.2000000 0.2000000 0.2000000 0.2000000 0.4500000 0.300000
## [3,] 0.2000000 0.2000000 0.2500000 0.1500000 0.2000000 0.000000
## [4,] 0.4500000 0.2500000 0.3500000 0.3000000 0.4000000 0.050000
## [5,] 0.2000000 0.2000000 0.2000000 0.1500000 0.2500000 0.150000
##
## , , 52
##
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.1196871 0.1504202 0.2423443 0.1535064 0.2504215 0.2884485
## [2,] 0.1000000 0.2000000 0.3500000 0.2500000 0.5000000 0.4500000
## [3,] 0.1500000 0.3000000 0.3500000 0.3000000 0.3000000 0.1500000
## [4,] 0.2500000 0.2500000 0.3500000 0.1000000 0.2000000 0.1000000
## [5,] 0.1500000 0.3000000 0.2500000 0.1000000 0.1000000 0.1000000
##
## , , 53
##
##           [,1]     [,2]      [,3]       [,4]      [,5]      [,6]
## [1,] 0.1281649 0.142397 0.1988006 0.09738152 0.2660846 0.2533812
## [2,] 0.2000000 0.350000 0.2500000 0.15000000 0.4500000 0.4000000
## [3,] 0.2000000 0.100000 0.1500000 0.05000000 0.2500000 0.1000000
## [4,] 0.3500000 0.250000 0.4500000 0.10000000 0.2500000 0.1000000
## [5,] 0.1500000 0.300000 0.2500000 0.10000000 0.2500000 0.1000000
##
## , , 54
##
##            [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.09699841 0.06086168 0.1493032 0.148006 0.2702222 0.2623218
## [2,] 0.15000000 0.10000000 0.1500000 0.150000 0.3500000 0.3500000
## [3,] 0.15000000 0.10000000 0.2000000 0.100000 0.4000000 0.1000000
## [4,] 0.35000000 0.20000000 0.3000000 0.200000 0.4500000 0.0000000
```

```
## [5,] 0.15000000 0.00000000 0.1500000 0.250000 0.4000000 0.0000000
##
## , , 55
##
##           [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.1835377 0.09332157 0.2187826 0.234898 0.2584759 0.2465179
## [2,] 0.2500000 0.20000000 0.4500000 0.350000 0.4000000 0.3000000
## [3,] 0.3000000 0.20000000 0.4500000 0.250000 0.4000000 0.0500000
## [4,] 0.3500000 0.15000000 0.4000000 0.600000 0.3000000 0.0500000
## [5,] 0.3000000 0.10000000 0.4500000 0.250000 0.4000000 0.0500000
##
## , , 56
##
##            [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.03319831 0.1123227 0.1985367 0.1923368 0.2613105 0.2523929
## [2,] 0.05000000 0.2000000 0.3500000 0.3000000 0.4500000 0.3500000
## [3,] 0.05000000 0.2000000 0.3000000 0.3000000 0.1500000 0.0000000
## [4,] 0.25000000 0.2500000 0.6000000 0.3500000 0.4000000 0.0000000
## [5,] 0.05000000 0.2000000 0.3500000 0.3000000 0.1500000 0.0000000
##
## , , 57
##
##            [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.09570913 0.1486731 0.2276262 0.1306976 0.2283749 0.265079
## [2,] 0.15000000 0.2000000 0.4000000 0.1000000 0.3500000 0.450000
## [3,] 0.15000000 0.2500000 0.5000000 0.2500000 0.2000000 0.000000
## [4,] 0.30000000 0.2500000 0.5000000 0.5000000 0.3000000 0.000000
## [5,] 0.10000000 0.2000000 0.3500000 0.4500000 0.2000000 0.050000
##
## , , 58
##
##           [,1]      [,2]      [,3]    [,4]     [,5]      [,6]
## [1,] 0.1425956 0.1324262 0.1977104 0.15979 0.217006 0.2630115
## [2,] 0.2000000 0.2500000 0.3500000 0.15000 0.200000 0.5000000
## [3,] 0.2000000 0.2000000 0.4000000 0.30000 0.450000 0.1000000
## [4,] 0.3000000 0.3500000 0.5500000 0.35000 0.450000 0.1000000
## [5,] 0.2000000 0.2500000 0.4000000 0.30000 0.350000 0.0500000
##
## , , 59
##
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.1648849 0.1560302 0.1943324 0.1325304 0.2536756 0.262696
## [2,] 0.2000000 0.2000000 0.2000000 0.2000000 0.4500000 0.300000
## [3,] 0.2000000 0.2000000 0.2500000 0.1500000 0.2000000 0.000000
## [4,] 0.4500000 0.2500000 0.3500000 0.3000000 0.4000000 0.050000
## [5,] 0.2000000 0.2000000 0.2000000 0.1500000 0.2500000 0.150000
##
## , , 60
##
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.1196871 0.1504202 0.2423443 0.1535064 0.2504215 0.2884485
## [2,] 0.1000000 0.2000000 0.3500000 0.2500000 0.5000000 0.4500000
## [3,] 0.1500000 0.3000000 0.3500000 0.3000000 0.3000000 0.1500000
## [4,] 0.2500000 0.2500000 0.3500000 0.1000000 0.2000000 0.1000000
```

```
## [5,] 0.1500000 0.3000000 0.2500000 0.1000000 0.1000000 0.1000000
##
## , , 61
##
##           [,1]     [,2]      [,3]       [,4]      [,5]      [,6]
## [1,] 0.1281649 0.142397 0.1988006 0.09738152 0.2660846 0.2533812
## [2,] 0.2000000 0.350000 0.2500000 0.15000000 0.4500000 0.4000000
## [3,] 0.2000000 0.100000 0.1500000 0.05000000 0.2500000 0.1000000
## [4,] 0.3500000 0.250000 0.4500000 0.10000000 0.2500000 0.1000000
## [5,] 0.1500000 0.300000 0.2500000 0.10000000 0.2500000 0.1000000
##
## , , 62
##
##            [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.09699841 0.06086168 0.1493032 0.148006 0.2702222 0.2623218
## [2,] 0.15000000 0.10000000 0.1500000 0.150000 0.3500000 0.3500000
## [3,] 0.15000000 0.10000000 0.2000000 0.100000 0.4000000 0.1000000
## [4,] 0.35000000 0.20000000 0.3000000 0.200000 0.4500000 0.0000000
## [5,] 0.15000000 0.00000000 0.1500000 0.250000 0.4000000 0.0000000
##
## , , 63
##
##           [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.1835377 0.09332157 0.2187826 0.234898 0.2584759 0.2465179
## [2,] 0.2500000 0.20000000 0.4500000 0.350000 0.4000000 0.3000000
## [3,] 0.3000000 0.20000000 0.4500000 0.250000 0.4000000 0.0500000
## [4,] 0.3500000 0.15000000 0.4000000 0.600000 0.3000000 0.0500000
## [5,] 0.3000000 0.10000000 0.4500000 0.250000 0.4000000 0.0500000
##
## , , 64
##
##            [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.03319831 0.1123227 0.1985367 0.1923368 0.2613105 0.2523929
## [2,] 0.05000000 0.2000000 0.3500000 0.3000000 0.4500000 0.3500000
## [3,] 0.05000000 0.2000000 0.3000000 0.3000000 0.1500000 0.0000000
## [4,] 0.25000000 0.2500000 0.6000000 0.3500000 0.4000000 0.0000000
## [5,] 0.05000000 0.2000000 0.3500000 0.3000000 0.1500000 0.0000000
##
## , , 65
##
##            [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.09570913 0.1486731 0.2276262 0.1306976 0.2283749 0.265079
## [2,] 0.15000000 0.2000000 0.4000000 0.1000000 0.3500000 0.450000
## [3,] 0.15000000 0.2500000 0.5000000 0.2500000 0.2000000 0.000000
## [4,] 0.30000000 0.2500000 0.5000000 0.5000000 0.3000000 0.000000
## [5,] 0.10000000 0.2000000 0.3500000 0.4500000 0.2000000 0.050000
##
## , , 66
##
##           [,1]      [,2]      [,3]    [,4]     [,5]      [,6]
## [1,] 0.1425956 0.1324262 0.1977104 0.15979 0.217006 0.2630115
## [2,] 0.2000000 0.2500000 0.3500000 0.15000 0.200000 0.5000000
## [3,] 0.2000000 0.2000000 0.4000000 0.30000 0.450000 0.1000000
## [4,] 0.3000000 0.3500000 0.5500000 0.35000 0.450000 0.1000000
```

```
## [5,] 0.2000000 0.2500000 0.4000000 0.30000 0.350000 0.0500000
##
## , , 67
##
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.1648849 0.1560302 0.1943324 0.1325304 0.2536756 0.262696
## [2,] 0.2000000 0.2000000 0.2000000 0.2000000 0.4500000 0.300000
## [3,] 0.2000000 0.2000000 0.2500000 0.1500000 0.2000000 0.000000
## [4,] 0.4500000 0.2500000 0.3500000 0.3000000 0.4000000 0.050000
## [5,] 0.2000000 0.2000000 0.2000000 0.1500000 0.2500000 0.150000
##
## , , 68
##
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.1196871 0.1504202 0.2423443 0.1535064 0.2504215 0.2884485
## [2,] 0.1000000 0.2000000 0.3500000 0.2500000 0.5000000 0.4500000
## [3,] 0.1500000 0.3000000 0.3500000 0.3000000 0.3000000 0.1500000
## [4,] 0.2500000 0.2500000 0.3500000 0.1000000 0.2000000 0.1000000
## [5,] 0.1500000 0.3000000 0.2500000 0.1000000 0.1000000 0.1000000
##
## , , 69
##
##           [,1]     [,2]      [,3]       [,4]      [,5]      [,6]
## [1,] 0.1281649 0.142397 0.1988006 0.09738152 0.2660846 0.2533812
## [2,] 0.2000000 0.350000 0.2500000 0.15000000 0.4500000 0.4000000
## [3,] 0.2000000 0.100000 0.1500000 0.05000000 0.2500000 0.1000000
## [4,] 0.3500000 0.250000 0.4500000 0.10000000 0.2500000 0.1000000
## [5,] 0.1500000 0.300000 0.2500000 0.10000000 0.2500000 0.1000000
##
## , , 70
##
##            [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.09699841 0.06086168 0.1493032 0.148006 0.2702222 0.2623218
## [2,] 0.15000000 0.10000000 0.1500000 0.150000 0.3500000 0.3500000
## [3,] 0.15000000 0.10000000 0.2000000 0.100000 0.4000000 0.1000000
## [4,] 0.35000000 0.20000000 0.3000000 0.200000 0.4500000 0.0000000
## [5,] 0.15000000 0.00000000 0.1500000 0.250000 0.4000000 0.0000000
##
## , , 71
##
##           [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.1835377 0.09332157 0.2187826 0.234898 0.2584759 0.2465179
## [2,] 0.2500000 0.20000000 0.4500000 0.350000 0.4000000 0.3000000
## [3,] 0.3000000 0.20000000 0.4500000 0.250000 0.4000000 0.0500000
## [4,] 0.3500000 0.15000000 0.4000000 0.600000 0.3000000 0.0500000
## [5,] 0.3000000 0.10000000 0.4500000 0.250000 0.4000000 0.0500000
##
## , , 72
##
##            [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.03319831 0.1123227 0.1985367 0.1923368 0.2613105 0.2523929
## [2,] 0.05000000 0.2000000 0.3500000 0.3000000 0.4500000 0.3500000
## [3,] 0.05000000 0.2000000 0.3000000 0.3000000 0.1500000 0.0000000
## [4,] 0.25000000 0.2500000 0.6000000 0.3500000 0.4000000 0.0000000
```

```
## [5,] 0.05000000 0.2000000 0.3500000 0.3000000 0.1500000 0.0000000
##
## , , 73
##
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.09570913 0.1486731 0.2276262 0.1306976 0.2283749 0.265079
## [2,] 0.15000000 0.2000000 0.4000000 0.1000000 0.3500000 0.450000
## [3,] 0.15000000 0.2500000 0.5000000 0.2500000 0.2000000 0.000000
## [4,] 0.30000000 0.2500000 0.5000000 0.5000000 0.3000000 0.000000
## [5,] 0.10000000 0.2000000 0.3500000 0.4500000 0.2000000 0.050000
##
## , , 74
##
##           [,1]      [,2]      [,3]    [,4]     [,5]      [,6]
## [1,] 0.1425956 0.1324262 0.1977104 0.15979 0.217006 0.2630115
## [2,] 0.2000000 0.2500000 0.3500000 0.15000 0.200000 0.5000000
## [3,] 0.2000000 0.2000000 0.4000000 0.30000 0.450000 0.1000000
## [4,] 0.3000000 0.3500000 0.5500000 0.35000 0.450000 0.1000000
## [5,] 0.2000000 0.2500000 0.4000000 0.30000 0.350000 0.0500000
##
## , , 75
##
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.1648849 0.1560302 0.1943324 0.1325304 0.2536756 0.262696
## [2,] 0.2000000 0.2000000 0.2000000 0.2000000 0.4500000 0.300000
## [3,] 0.2000000 0.2000000 0.2500000 0.1500000 0.2000000 0.000000
## [4,] 0.4500000 0.2500000 0.3500000 0.3000000 0.4000000 0.050000
## [5,] 0.2000000 0.2000000 0.2000000 0.1500000 0.2500000 0.150000
##
## , , 76
##
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.1196871 0.1504202 0.2423443 0.1535064 0.2504215 0.2884485
## [2,] 0.1000000 0.2000000 0.3500000 0.2500000 0.5000000 0.4500000
## [3,] 0.1500000 0.3000000 0.3500000 0.3000000 0.3000000 0.1500000
## [4,] 0.2500000 0.2500000 0.3500000 0.1000000 0.2000000 0.1000000
## [5,] 0.1500000 0.3000000 0.2500000 0.1000000 0.1000000 0.1000000
##
## , , 77
##
##           [,1]     [,2]      [,3]       [,4]      [,5]      [,6]
## [1,] 0.1281649 0.142397 0.1988006 0.09738152 0.2660846 0.2533812
## [2,] 0.2000000 0.350000 0.2500000 0.15000000 0.4500000 0.4000000
## [3,] 0.2000000 0.100000 0.1500000 0.05000000 0.2500000 0.1000000
## [4,] 0.3500000 0.250000 0.4500000 0.10000000 0.2500000 0.1000000
## [5,] 0.1500000 0.300000 0.2500000 0.10000000 0.2500000 0.1000000
##
## , , 78
##
##           [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.09699841 0.06086168 0.1493032 0.148006 0.2702222 0.2623218
## [2,] 0.15000000 0.10000000 0.1500000 0.150000 0.3500000 0.3500000
## [3,] 0.15000000 0.10000000 0.2000000 0.100000 0.4000000 0.1000000
## [4,] 0.35000000 0.20000000 0.3000000 0.200000 0.4500000 0.0000000
```

```
## [5,] 0.15000000 0.00000000 0.1500000 0.250000 0.4000000 0.0000000
##
## , , 79
##
##           [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.1835377 0.09332157 0.2187826 0.234898 0.2584759 0.2465179
## [2,] 0.2500000 0.20000000 0.4500000 0.350000 0.4000000 0.3000000
## [3,] 0.3000000 0.20000000 0.4500000 0.250000 0.4000000 0.0500000
## [4,] 0.3500000 0.15000000 0.4000000 0.600000 0.3000000 0.0500000
## [5,] 0.3000000 0.10000000 0.4500000 0.250000 0.4000000 0.0500000
##
## , , 80
##
##            [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.03319831 0.1123227 0.1985367 0.1923368 0.2613105 0.2523929
## [2,] 0.05000000 0.2000000 0.3500000 0.3000000 0.4500000 0.3500000
## [3,] 0.05000000 0.2000000 0.3000000 0.3000000 0.1500000 0.0000000
## [4,] 0.25000000 0.2500000 0.6000000 0.3500000 0.4000000 0.0000000
## [5,] 0.05000000 0.2000000 0.3500000 0.3000000 0.1500000 0.0000000
##
## , , 81
##
##            [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.09570913 0.1486731 0.2276262 0.1306976 0.2283749 0.265079
## [2,] 0.15000000 0.2000000 0.4000000 0.1000000 0.3500000 0.450000
## [3,] 0.15000000 0.2500000 0.5000000 0.2500000 0.2000000 0.000000
## [4,] 0.30000000 0.2500000 0.5000000 0.5000000 0.3000000 0.000000
## [5,] 0.10000000 0.2000000 0.3500000 0.4500000 0.2000000 0.050000
##
## , , 82
##
##           [,1]      [,2]      [,3]    [,4]     [,5]      [,6]
## [1,] 0.1425956 0.1324262 0.1977104 0.15979 0.217006 0.2630115
## [2,] 0.2000000 0.2500000 0.3500000 0.15000 0.200000 0.5000000
## [3,] 0.2000000 0.2000000 0.4000000 0.30000 0.450000 0.1000000
## [4,] 0.3000000 0.3500000 0.5500000 0.35000 0.450000 0.1000000
## [5,] 0.2000000 0.2500000 0.4000000 0.30000 0.350000 0.0500000
##
## , , 83
##
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.1648849 0.1560302 0.1943324 0.1325304 0.2536756 0.262696
## [2,] 0.2000000 0.2000000 0.2000000 0.2000000 0.4500000 0.300000
## [3,] 0.2000000 0.2000000 0.2500000 0.1500000 0.2000000 0.000000
## [4,] 0.4500000 0.2500000 0.3500000 0.3000000 0.4000000 0.050000
## [5,] 0.2000000 0.2000000 0.2000000 0.1500000 0.2500000 0.150000
##
## , , 84
##
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.1196871 0.1504202 0.2423443 0.1535064 0.2504215 0.2884485
## [2,] 0.1000000 0.2000000 0.3500000 0.2500000 0.5000000 0.4500000
## [3,] 0.1500000 0.3000000 0.3500000 0.3000000 0.3000000 0.1500000
## [4,] 0.2500000 0.2500000 0.3500000 0.1000000 0.2000000 0.1000000
```

```
## [5,] 0.1500000 0.3000000 0.2500000 0.1000000 0.1000000 0.1000000
##
## , , 85
##
##           [,1]     [,2]      [,3]       [,4]      [,5]      [,6]
## [1,] 0.1281649 0.142397 0.1988006 0.09738152 0.2660846 0.2533812
## [2,] 0.2000000 0.350000 0.2500000 0.15000000 0.4500000 0.4000000
## [3,] 0.2000000 0.100000 0.1500000 0.05000000 0.2500000 0.1000000
## [4,] 0.3500000 0.250000 0.4500000 0.10000000 0.2500000 0.1000000
## [5,] 0.1500000 0.300000 0.2500000 0.10000000 0.2500000 0.1000000
##
## , , 86
##
##            [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.09699841 0.06086168 0.1493032 0.148006 0.2702222 0.2623218
## [2,] 0.15000000 0.10000000 0.1500000 0.150000 0.3500000 0.3500000
## [3,] 0.15000000 0.10000000 0.2000000 0.100000 0.4000000 0.1000000
## [4,] 0.35000000 0.20000000 0.3000000 0.200000 0.4500000 0.0000000
## [5,] 0.15000000 0.00000000 0.1500000 0.250000 0.4000000 0.0000000
##
## , , 87
##
##           [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.1835377 0.09332157 0.2187826 0.234898 0.2584759 0.2465179
## [2,] 0.2500000 0.20000000 0.4500000 0.350000 0.4000000 0.3000000
## [3,] 0.3000000 0.20000000 0.4500000 0.250000 0.4000000 0.0500000
## [4,] 0.3500000 0.15000000 0.4000000 0.600000 0.3000000 0.0500000
## [5,] 0.3000000 0.10000000 0.4500000 0.250000 0.4000000 0.0500000
##
## , , 88
##
##            [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.03319831 0.1123227 0.1985367 0.1923368 0.2613105 0.2523929
## [2,] 0.05000000 0.2000000 0.3500000 0.3000000 0.4500000 0.3500000
## [3,] 0.05000000 0.2000000 0.3000000 0.3000000 0.1500000 0.0000000
## [4,] 0.25000000 0.2500000 0.6000000 0.3500000 0.4000000 0.0000000
## [5,] 0.05000000 0.2000000 0.3500000 0.3000000 0.1500000 0.0000000
##
## , , 89
##
##            [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.09570913 0.1486731 0.2276262 0.1306976 0.2283749 0.265079
## [2,] 0.15000000 0.2000000 0.4000000 0.1000000 0.3500000 0.450000
## [3,] 0.15000000 0.2500000 0.5000000 0.2500000 0.2000000 0.000000
## [4,] 0.30000000 0.2500000 0.5000000 0.5000000 0.3000000 0.000000
## [5,] 0.10000000 0.2000000 0.3500000 0.4500000 0.2000000 0.050000
##
## , , 90
##
##           [,1]      [,2]      [,3]    [,4]     [,5]      [,6]
## [1,] 0.1425956 0.1324262 0.1977104 0.15979 0.217006 0.2630115
## [2,] 0.2000000 0.2500000 0.3500000 0.15000 0.200000 0.5000000
## [3,] 0.2000000 0.2000000 0.4000000 0.30000 0.450000 0.1000000
## [4,] 0.3000000 0.3500000 0.5500000 0.35000 0.450000 0.1000000
```

```
## [5,] 0.2000000 0.2500000 0.4000000 0.30000 0.350000 0.0500000
##
## , , 91
##
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.1648849 0.1560302 0.1943324 0.1325304 0.2536756 0.262696
## [2,] 0.2000000 0.2000000 0.2000000 0.2000000 0.4500000 0.300000
## [3,] 0.2000000 0.2000000 0.2500000 0.1500000 0.2000000 0.000000
## [4,] 0.4500000 0.2500000 0.3500000 0.3000000 0.4000000 0.050000
## [5,] 0.2000000 0.2000000 0.2000000 0.1500000 0.2500000 0.150000
##
## , , 92
##
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.1196871 0.1504202 0.2423443 0.1535064 0.2504215 0.2884485
## [2,] 0.1000000 0.2000000 0.3500000 0.2500000 0.5000000 0.4500000
## [3,] 0.1500000 0.3000000 0.3500000 0.3000000 0.3000000 0.1500000
## [4,] 0.2500000 0.2500000 0.3500000 0.1000000 0.2000000 0.1000000
## [5,] 0.1500000 0.3000000 0.2500000 0.1000000 0.1000000 0.1000000
##
## , , 93
##
##           [,1]     [,2]      [,3]       [,4]      [,5]      [,6]
## [1,] 0.1281649 0.142397 0.1988006 0.09738152 0.2660846 0.2533812
## [2,] 0.2000000 0.350000 0.2500000 0.15000000 0.4500000 0.4000000
## [3,] 0.2000000 0.100000 0.1500000 0.05000000 0.2500000 0.1000000
## [4,] 0.3500000 0.250000 0.4500000 0.10000000 0.2500000 0.1000000
## [5,] 0.1500000 0.300000 0.2500000 0.10000000 0.2500000 0.1000000
##
## , , 94
##
##            [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.09699841 0.06086168 0.1493032 0.148006 0.2702222 0.2623218
## [2,] 0.15000000 0.10000000 0.1500000 0.150000 0.3500000 0.3500000
## [3,] 0.15000000 0.10000000 0.2000000 0.100000 0.4000000 0.1000000
## [4,] 0.35000000 0.20000000 0.3000000 0.200000 0.4500000 0.0000000
## [5,] 0.15000000 0.00000000 0.1500000 0.250000 0.4000000 0.0000000
##
## , , 95
##
##           [,1]       [,2]      [,3]     [,4]      [,5]      [,6]
## [1,] 0.1835377 0.09332157 0.2187826 0.234898 0.2584759 0.2465179
## [2,] 0.2500000 0.20000000 0.4500000 0.350000 0.4000000 0.3000000
## [3,] 0.3000000 0.20000000 0.4500000 0.250000 0.4000000 0.0500000
## [4,] 0.3500000 0.15000000 0.4000000 0.600000 0.3000000 0.0500000
## [5,] 0.3000000 0.10000000 0.4500000 0.250000 0.4000000 0.0500000
##
## , , 96
##
##            [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.03319831 0.1123227 0.1985367 0.1923368 0.2613105 0.2523929
## [2,] 0.05000000 0.2000000 0.3500000 0.3000000 0.4500000 0.3500000
## [3,] 0.05000000 0.2000000 0.3000000 0.3000000 0.1500000 0.0000000
## [4,] 0.25000000 0.2500000 0.6000000 0.3500000 0.4000000 0.0000000
```

```
## [5,] 0.05000000 0.2000000 0.3500000 0.3000000 0.1500000 0.0000000
##
## , , 97
##
##            [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.09570913 0.1486731 0.2276262 0.1306976 0.2283749 0.265079
## [2,] 0.15000000 0.2000000 0.4000000 0.1000000 0.3500000 0.450000
## [3,] 0.15000000 0.2500000 0.5000000 0.2500000 0.2000000 0.000000
## [4,] 0.30000000 0.2500000 0.5000000 0.5000000 0.3000000 0.000000
## [5,] 0.10000000 0.2000000 0.3500000 0.4500000 0.2000000 0.050000
##
## , , 98
##
##           [,1]      [,2]      [,3]    [,4]     [,5]      [,6]
## [1,] 0.1425956 0.1324262 0.1977104 0.15979 0.217006 0.2630115
## [2,] 0.2000000 0.2500000 0.3500000 0.15000 0.200000 0.5000000
## [3,] 0.2000000 0.2000000 0.4000000 0.30000 0.450000 0.1000000
## [4,] 0.3000000 0.3500000 0.5500000 0.35000 0.450000 0.1000000
## [5,] 0.2000000 0.2500000 0.4000000 0.30000 0.350000 0.0500000
##
## , , 99
##
##           [,1]      [,2]      [,3]      [,4]      [,5]     [,6]
## [1,] 0.1648849 0.1560302 0.1943324 0.1325304 0.2536756 0.262696
## [2,] 0.2000000 0.2000000 0.2000000 0.2000000 0.4500000 0.300000
## [3,] 0.2000000 0.2000000 0.2500000 0.1500000 0.2000000 0.000000
## [4,] 0.4500000 0.2500000 0.3500000 0.3000000 0.4000000 0.050000
## [5,] 0.2000000 0.2000000 0.2000000 0.1500000 0.2500000 0.150000
##
## , , 100
##
##           [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
## [1,] 0.1196871 0.1504202 0.2423443 0.1535064 0.2504215 0.2884485
## [2,] 0.1000000 0.2000000 0.3500000 0.2500000 0.5000000 0.4500000
## [3,] 0.1500000 0.3000000 0.3500000 0.3000000 0.3000000 0.1500000
## [4,] 0.2500000 0.2500000 0.3500000 0.1000000 0.2000000 0.1000000
## [5,] 0.1500000 0.3000000 0.2500000 0.1000000 0.1000000 0.1000000
```