


# Formulas

Stat 151A, Fall 2017

December 6, 2017

1.

$$\frac{RSS}{\sigma^2} \sim \chi_{n-p-1}^2.$$

2. If  $\hat{\beta}$  is the ols estimator then (under assumptions that you must know) 

$$\frac{\hat{\beta}_j - \beta_j}{s.e(\hat{\beta}_j)} \sim t_{n-p-1}$$

3. For testing a reduced model  $m$  against the full model  $M$ :

$$T := \frac{(RSS(m) - RSS(M))/(p - q)}{RSS(M)/(n - p - 1)} \sim F_{p-q, n-p-1},$$

$q$  is the number of variables in the reduced model without counting the intercept.

4. In one way ANOVA, if  $\mu_1 = \dots = \mu_t$ , then

$$T = \frac{\sum_{i=1}^t n_i (\bar{y}_i - \bar{y})^2 / (t - 1)}{\sum_{i=1}^t \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_i)^2 / (n - t)}$$

has  $F$ -distribution with  $t - 1$  and  $n - t$  degrees of freedom.

5. Let  $L \in \mathbb{R}^{q \times (p+1)}$  be a full rank matrix with  $\text{rank } q \leq p + 1$ . If  $L\beta = c$ , for  $c \in \mathbb{R}^q$ , then

$$\frac{(L\hat{\beta} - c)^T [L(X^T X)^{-1} L^T]^{-1} (L\hat{\beta} - c)}{q \hat{\sigma}^2} \sim F_{q, n-p-1}.$$

6.

$$RSS_{[i]} = RSS - \frac{\hat{e}_i^2}{1 - h_i}.$$

7. Suppose  $Z$  is a random vector with mean  $\mu$  and covariance matrix  $\Sigma$ . Then

$$\mathbb{E}(Z^T A Z) = \text{tr}(A \Sigma) + \mu^T A \mu. \quad (1)$$

8.

$$\hat{\beta}_{[i]} = \hat{\beta} - \frac{\hat{e}_i}{1 - h_i} (X^T X)^{-1} x_i. \quad (2)$$

9.

$$\hat{e}_{[i]} = \frac{\hat{e}_i}{1 - h_i}. \quad (3)$$

10.

$$t_i = \frac{\hat{e}_{[i]} \sqrt{1 - h_i}}{\sqrt{RSS_{[i]} / (n - p - 2)}} = \frac{\hat{e}_i}{\sqrt{RSS_{[i]} / (n - p - 2)} \sqrt{1 - h_i}}.$$

11.

$$r_i = \frac{\hat{e}_i}{\sqrt{RSS / (n - p - 1)} \sqrt{1 - h_i}}.$$

12.

$$C_i = r_i^2 \frac{h_i}{(1 - h_i)(p + 1)} = \frac{(\hat{\beta} - \hat{\beta}_{[i]})^T X^T X (\hat{\beta} - \hat{\beta}_{[i]})}{(p + 1) \hat{\sigma}^2}.$$

13.

$$\text{cov}(AZ) = A \text{cov}(Z) A^T$$



14.

$$f(x; \theta_i, \phi_i) := h(x, \phi_i) \exp \left( \frac{x \theta_i - b(\theta_i)}{a(\phi_i)} \right).$$

15.

$$\text{var}(y_i) = b''(\theta_i) a(\phi_i).$$

16.

$$\beta^{(m+1)} = \beta^{(m)} - (H\ell(\beta^{(m)}))^{-1} \nabla \ell(\beta^{(m)})$$

17. Logistic

$$\begin{aligned} \nabla \ell(\beta) &= \sum_{i=1}^n (y_i - p_i) (1, x_{i1}, \dots, x_{ip})^T \\ H\ell(\beta) &= -\sum_{i=1}^n p_i (1 - p_i) (1, x_{i1}, \dots, x_{ip})^T (1, x_{i1}, \dots, x_{ip}). \end{aligned}$$

18.

$$\begin{aligned} \beta^{(m+1)} &= (X^T W X)^{-1} X^T W Z \\ Z &= X \beta^{(m)} + W^{-1} (Y - p). \end{aligned}$$

19.

$RSS(j, c)$	$= n_1 \bar{p}_1 (1 - \bar{p}_1) + n_2 \bar{p}_2 (1 - \bar{p}_2),$	
Cross-entropy or Deviance	: $-2n_1 (\bar{p}_1 \log \bar{p}_1 + (1 - \bar{p}_1) \log(1 - \bar{p}_1))$	
Misclassification Error	: $n_1 \min(\bar{p}_1, 1 - \bar{p}_1).$	

20.

$$\begin{aligned} \nabla_x (b^T x) &= b \\ \nabla_x (\tfrac{1}{2} x^T A x) &= A x. \end{aligned}$$