Homework 2

Stat 151A, Fall 2017

Due: September 26

1. (1.0 points) Problem 9.6 in Fox

2. (1.0 points) Problem 11.2 in Fox

3. (a) (0.5 points) Show that the mean of the response values is equal to the mean of the fitted values,
$$\bar{y} = \bar{\hat{y}}$$
   (b) (0.5 points) Show that $\boldsymbol{y}^T\hat{\boldsymbol{y}} = \hat{\boldsymbol{y}}^T\hat{\boldsymbol{y}}$.
   (c) (0.5 points) Use these facts to show that $\widehat{Cov}(y, \hat{y})$ is equal to $\widehat{Var}(\hat{y})$, where
$$\widehat{Cov}(x, y) = \frac{1}{n}\sum_i (x_i - \bar{x})(y_i - \bar{y})$$

4. (1.5 points) In the following regression output, the value of the F-statistic (last line) and its p-value are missing. Fill them in, providing proper reasoning, based on the available information.

```
Call:
lm(formula = y ~ x1 + x2 + x3 + x4 + x5, data)

Residuals:
Min       1Q   Median       3Q      Max
-10.9307  -2.8923  -0.3829   3.1778   9.5804

Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.63014    5.95379   0.274    0.784
x1           0.85682    0.05065  16.916  < 2e-16 ***
x2          -2.02587    0.39720  -5.100 6.77e-07 ***
x3           0.04083    0.14899   0.274    0.784
x4          -0.33431    0.08191  -4.082 6.05e-05 ***
x5           0.24481    0.18236   1.342    0.181
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Signif. codes:  0 âĂŸ***âĂŹ 0.001 âĂŸ**âĂŹ 0.01 âĂŸ*âĂŹ 0.05 âĂŸ.âĂŹ 0.1 âĂŸ âĂŹ
```

```
Residual standard error: 4.122 on 246 degrees of freedom
Multiple R-squared: 0.7228,Adjusted R-squared: 0.7172
F-statistic: XXXXX on X and 246 DF,  p-value: XXXXX
```

5. (2.5 points) Problem 9.14 in Fox. Note that all the values you need for (c) and (d) are given in the section 5.2.

6. In the Bodyfat dataset, see here `https://rstudio-pubs-static.s3.amazonaws.com/65314_c0d1e5696cdd4e93a3784ea67f9e3d34.html`, consider the linear model:

$$\text{BODYFAT} = \beta_0 + \beta_1 \text{KNEE} + \beta_2 \text{THIGH} + \beta_3 \text{HIP} + \beta_4 \text{ANKLE} + e$$

Assume that the errors are i.i.d normal.

(a) (1 points) Construct an F-test for testing $H_0 : \beta_1 + \beta_2 = \beta_3 + \beta_4$. Describe your method and report the value of the F-statistic, its degrees of freedom and the p-value.

(b) (1 points) Construct a t-test for testing $H_0 : \beta_1 + \beta_2 = \beta_3 + \beta_4$. Describe your method and report the value of the t-statistic, its degrees of freedom and the p-value.

(c) (0.5 points) How is the value of your t-test statistic related to the value of the F-test statistic?