# Math 32B - Fall 2019
# Final Exam

**Full Name:** ______________________________

**UID:** ______________________________

**Circle the name of your TA and the day of your discussion:**

Steven Gagniere    Jason Snyder    Ryan Wilkinson

Tuesday    Thursday

**Instructions:**

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- Simplify your answers as much as possible.
- Include units with your answer where applicable.
- Calculators are not allowed but you may have a $3 \times 5$ inch notecard.

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| Page | Points | Score |
|---|---|---|
| 1 | 15 | |
| 2 | 10 | |
| 3 | 17 | |
| 4 | 12 | |
| 5 | 10 | |

| Page | Points | Score |
|---|---|---|
| 6 | 15 | |
| 7 | 11 | |
| 8 | 10 | |
| Bonus | | |
| Total: | 100 | |

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1. (5 points) Find the average height of a point in the region $\mathcal{W}$ in $\mathbb{R}^3$ with boundary $z = \sqrt{x^2 + y^2}$ for $0 \leq z \leq 1$ and $z = 1$ for $x^2 + y^2 \leq 1$. That is, find the average height of a point in the solid cone.

2. (10 points) Let $\mathcal{W}$ be the region in $\mathbb{R}^3$ bounded by the surfaces $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$. Find the volume of $\mathcal{W}$.



/15

3. (10 points) Let $\mathcal{R}$ be the parallelogram with vertices $(2, 0)$, $(0, 1)$, $(-2, 0)$, and $(0, -1)$. Use the change of variables $u = x + 2y$ and $v = x - 2y$ to evaluate $\iint_{\mathcal{R}} (3x + 6y)^2 \, dA$.



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4. (7 points) Let $\mathcal{C}$ be the triangle with vertices $(0,0)$, $(1,0)$, and $(1,2)$ oriented clockwise. Use Green's Theorem to evaluate $\displaystyle\int_{\mathcal{C}} \left(e^{x^2} + 2y\right)\, dx + \left(\sin(y^3) + x^2 - x\right)\, dy$.

5. (10 points) Let $\mathcal{S}$ be the part of the surface $z = 4 - x^2 - y^2$ for $z \geq 0$ oriented upward. Let $\mathbf{F}(x,y,z) = \langle x^2 \sin z, y^2, xy\rangle$. Use Stokes' Theorem to evaluate $\displaystyle\iint_{\mathcal{S}} \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$.



/17

6. (12 points) Find the surface area of the surface $\mathcal{S}$ given by the portion of $z = \sqrt{x^2 + y^2}$ contained within the elliptic cylinder $2y^2 + z^2 \leq 1$.



/12

7. (10 points) Let $\mathcal{C}$ be the triangle with vertices $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$ oriented counterclockwise from above and let $\mathbf{F}(x, y, z) = \langle x + y^2, y + z^2, x^2 + z\rangle$. Use Stokes' Theorem to evaluate $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.



/10

8. (5 points) Find the work done by the force field $\mathbf{F}(x, y, z) = \langle x^2, xy, z^2 \rangle$ in moving a particle along the line segment from $(1, 0, 0)$ to $(2, 1, 2)$.

9. (10 points) Suppose $\mathbf{F}(x, y, z) = \langle 3xz^2 + 2yz^2, 3x^2z^3 - yz^2, 3x^2y^2 + z^3 \rangle$. Let $\mathcal{W}$ be the solid defined by $x^2 + y^2 + z^2 \leq 1$ and let $\mathcal{S}$ be its boundary with positive orientation. Evaluate $\displaystyle\iint_{\mathcal{S}} \mathbf{F} \cdot d\mathbf{S}$.



/15

10. (6 points) Consider the vector field $\mathbf{F}(x, y, z) = \langle x^2y, xy^2z, xe^z \rangle$.

    (a) Is $\mathbf{F}$ conservative? Why or why not?

    (b) Is there a vector field $\mathbf{G}$ defined on $\mathbb{R}^3$ such that $\operatorname{curl} \mathbf{G} = \mathbf{F}$?

11. (5 points) Let $\mathcal{C}$ be the curve parameterized by $x = \sqrt{t}$, $y = t + 1$, $z = t^2$ for $0 \leq t \leq 1$. Show the vector field $\mathbf{F}(x, y, z) = \langle y^2z + 2xz^2, 2xyz, xy^2 + 2x^2z + e^z \rangle$ is conservative and use this fact to evaluate $\displaystyle\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.



/11

12. (10 points) Let $\mathcal{C}$ be the curve parameterized by $\mathbf{r}(t) = \langle 2\cos t, 2\sin t\rangle$ for $0 \le t \le \pi$. Find $\displaystyle\int_{\mathcal{C}} \left(e^x + y^2\cos x\right)\,dx + \left(e^{2y} + 2y\sin x + x\right)\,dy$. *Hint:* Complete $\mathcal{C}$ to form a closed curve and use Green's Theorem.



/10