

All problems are to be written up clearly and thoroughly, using complete sentences. This assignment is due in discussion at 2pm on Tuesday, February 4th.

For all T/F problems on the homework, provide a brief justification for your answer. That may be citing an appropriate theorem or providing a counterexample.

1. From the book:

Section 6.3 problems 9, 12.

Section 6.4 problems 1, 7, 14.

Section 6.5 problems 1, 5.

An *isometry* is a linear operator $T : V \rightarrow V$ on an inner product space such that $\langle T(x), T(y) \rangle = \langle x, y \rangle$ for all pairs $x, y \in V$.

2. Let V be a finite-dimensional inner product space over $\mathbb{F} = \mathbb{R}$ or \mathbb{C} .
 - (a) Fix $y \in V$ and suppose $\langle x, y \rangle = 0$ for all $x \in V$. Show that $y = 0$.
 - (b) Let $T : V \rightarrow V$ be an isometry. Prove that T is an isomorphism.
 - (c) Find all isometries $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that have $\det T = 1$.