# Math 32A - Winter 2019
# Final Exam

**Full Name:** ______________________________

**UID:** ______________________________

**Circle the name of your TA and the day of your discussion:**

Qi Guo | Talon Stark | Tianqi (Tim) Wu

Tuesday | Thursday

**Instructions:**

- Read each problem carefully.
- Show all work clearly and circle or box your final answer where appropriate.
- Justify your answers. A correct final answer without valid reasoning will not receive credit.
- Simplify your answers as much as possible.
- Include units with your answer where applicable.
- Calculators are not allowed but you may have a $3 \times 5$ inch notecard.

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| Page | Points | Score |
|---|---|---|
| 1 | 12 | |
| 2 | 9 | |
| 3 | 10 | |
| 4 | 10 | |
| 5 | 12 | |
| 6 | 10 | |

| Page | Points | Score |
|---|---|---|
| 7 | 10 | |
| 8 | 6 | |
| 9 | 10 | |
| 10 | 5 | |
| 11 | 6 | |
| Total: | 100 | |

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1. (12 points) Consider the points $P = (1, 0, 1)$, $Q = (1, 1, 0)$, and $R = (0, 1, 3)$.

   (a) Find cosine of the angle $\angle PQR$.

   (b) Find the area of the triangle formed by $P$, $Q$, and $R$.

   (c) Let $S = (3, 6, 1)$. Find the volume of the parallelepiped with edges $QP$, $QR$, and $QS$.



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2. (2 points) Describe in words and sketch a picture of the region in $\mathbb{R}^3$ represented by the following inequality. In addition to sketching the region in $\mathbb{R}^3$, sketch the $y = 0$ trace.

$$1 \leq x^2 + \frac{z^2}{4} \leq 4.$$

3. (4 points) Consider the plane $3x + 6y + 9z = 17$ and the line $\mathbf{L}(t) = \langle 1 + 4t, 8t, 5 + 12t \rangle$. Are they parallel, perpendicular, or neither? Be sure to justify your answer.

4. (3 points) Show the following limit does not exist.

$$\lim_{(x,y,z)\to(0,0,0)} \frac{xy + yz^2 + xz^2}{x^2 + y^2 + z^2}$$



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5. (6 points) Let $f(x, y, z) = xy + z^2$.

   (a) Find the directional derivative of $f(x, y, z)$ at $(1, 1, 1)$ in the direction of $\mathbf{v} = \langle 4, -4, 2 \rangle$.

   (b) Give a unit vector in the direction in which $f$ decreases most rapidly at $(1, 1, 1)$.

6. (4 points) Find the equation of the tangent plane to the surface $x^2 = xy + z^2 + \ln(z) - 6$ at the point $P = (-1, -6, 1)$.



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7. (10 points) Consider the function $f(x, y) = x^2 e^y$.

(a) Use linear approximation to $f(x, y)$ at the point $(1, 0)$ to estimate the value of $f(0.98, 0.01)$.

(b) Find the maximum rate of change of $f$ at the point $(1, 0)$.

(c) Find all directions $\mathbf{u}$ such that the directional derivative of $f$ at $(1, 0)$ is 2.



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8. (12 points) Consider the curve $\mathbf{r}(t) = \langle \cos t, \cos t, \sqrt{2}\sin t \rangle$, where $0 \leq t \leq 2\pi$.

(a) Find a parametrization of the tangent line to the curve at the point $\left(\frac{1}{2}, \frac{1}{2}, \sqrt{\frac{3}{2}}\right)$.

(b) Reparametrize the curve with respect to arc length.

(c) Find the maximum value of $f(x, y, z) = \frac{1}{2}x^2 + \frac{1}{2}y^2 - z$ on the curve $\mathbf{r}(t)$.



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9. (10 points) Find and classify all critical points of the function

$$f(x, y) = 2x^3 - 3x^2y - 12x^2 - 3y^2$$



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10. (10 points) Let $f(x, y) = x^2 + y^2 + 4x - 4y$. Use Lagrange multipliers to find the absolute maximum and absolute minimum values of $f$ on the domain $x^2 + y^2 \leq 18$.

11. (6 points) Consider the surface $y^2 = xz + 9$ and the origin.

(a) Find the points on the surface that are closest to the origin.

(b) Show the points you found in part (a) do indeed give the minimum distance.

(c) Show there are no points on the surface for which the distance is a maximum. That is, show there are points on the surface that are aribtrarily far from the origin.

(d) Why does this not contradict the Extreme Value Theorem?



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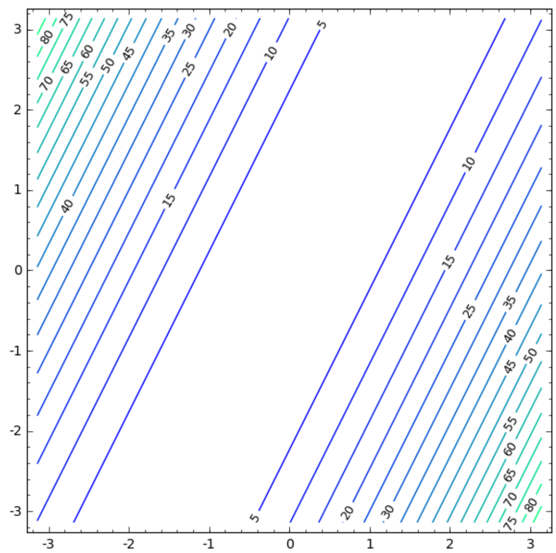
12. (4 points) Suppose $z = f(x, y)$ where $x = g(s, t)$ and $y = h(s, t)$. Use the chain rule to find $\dfrac{\partial f}{\partial s}(1, 2)$ given the following values

$$\begin{aligned} g(1,2) &= 3, \quad g_s(1,2) = -1, \quad g_t(1,2) = 5 \\ h(1,2) &= 6, \quad h_s(1,2) = -5, \quad h_t(1,2) = 11 \\ f_x(3,6) &= 7, \quad f_y(3,6) = 8. \end{aligned}$$

13. (6 points) Consider the contour plot for $f(x, y)$ below.



(a) Determine the sign of each of the following derivatives.

$f_x(-2, 1)$ ________ $f_y(-2, 1)$ ________

$f_{xx}(-2, 1)$ ________ $f_{xy}(-2, 1)$ ________ $f_{yy}(-2, 1)$ ________

(b) Give the components of a unit vector in the direction of $\nabla f$ at the point $(-2, 1)$. (You may estimate as necessary.)



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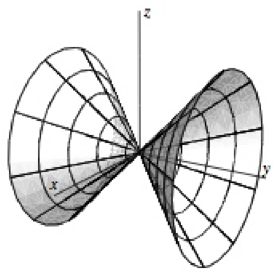
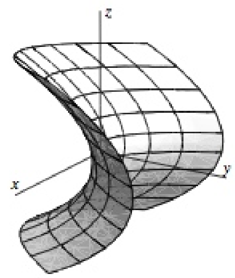
14. (5 points) Choose the picture that each equation describes.

1. $x^2 - y + z^2 = 1$ __________
2. $x^2 - y - z^2 = 0$ __________
3. $x^2 - y^2 + z^2 = -1$ __________
4. $z = \cos(x - y)$ __________
5. $x^2 - y^2 + z^2 = 0$ __________



(A)



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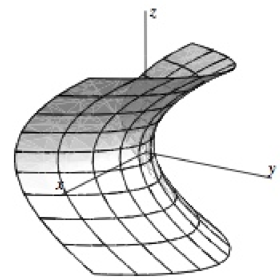

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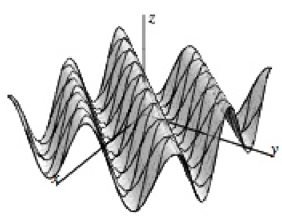

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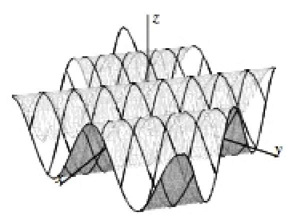

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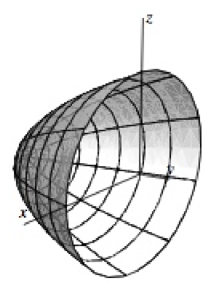

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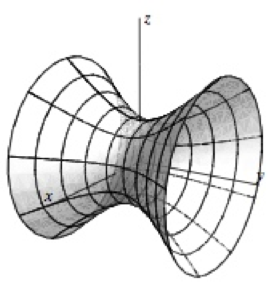

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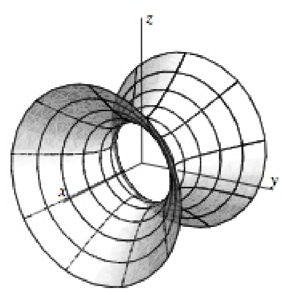

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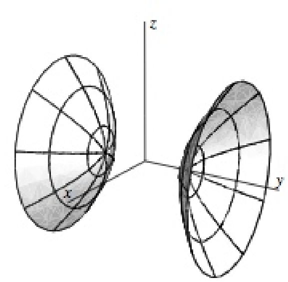

(I)



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15. (6 points) Match each function with its contour plot below.

1. $f(x, y) = \sin(xy)$ ____________

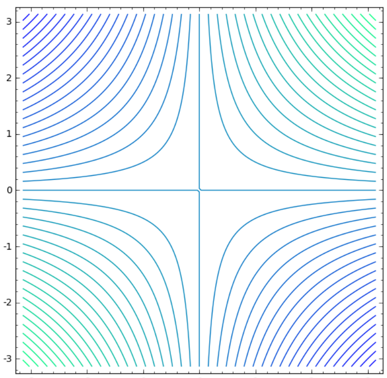
2. $f(x, y) = \arctan(xy)$ ____________

3. $f(x, y) = x^2 + y^2$ ____________

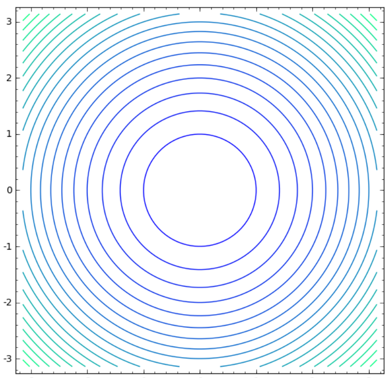
4. $f(x, y) = \sqrt{x^2 + y^2}$ ____________

5. $f(x, y) = xy$ ____________

6. $f(x, y) = \dfrac{1}{1 + x^2 + y^2}$ ____________



(i)



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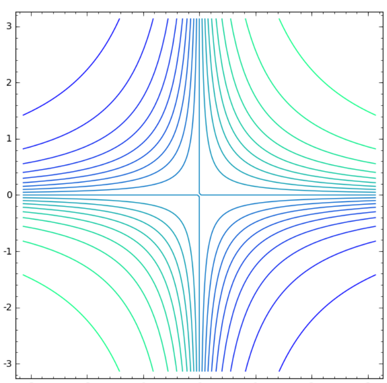

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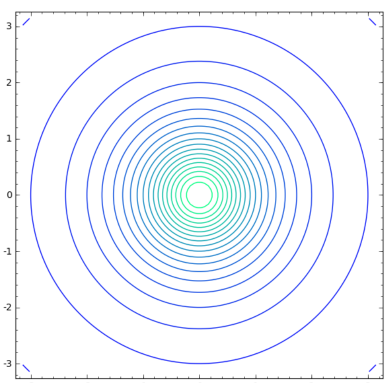

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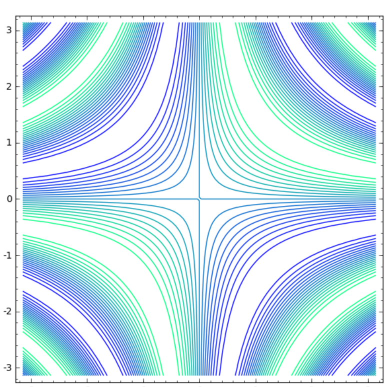

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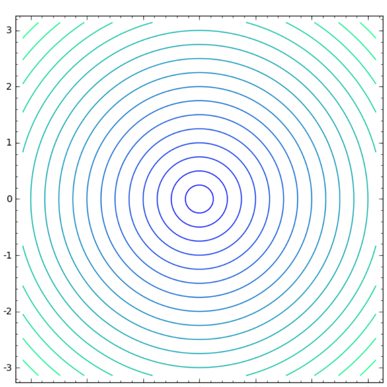

(vi)



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