

Math 32A - Winter 2019

Exam 2

Full Name: _____

UID: _____

Circle the name of your TA and the day of your discussion:

Qi Guo

Talon Stark

Tianqi (Tim) Wu

Tuesday

Thursday

Instructions:

- Read each problem carefully.
 - Show all work clearly and circle or box your final answer where appropriate.
 - Justify your answers. A correct final answer without valid reasoning will not receive credit.
 - Simplify your answers as much as possible.
 - Include units with your answer where applicable.
 - Calculators are not allowed but you may have a 3×5 inch notecard.
-

Page	Points	Score
1	32	
2	26	
3	22	
4	20	
Total:	100	

1. (20 points) Consider the surface defined by $ze^{2x} + x^2y + y = 3 + 2e^{z+1}$.
- (a) Find an equation of the tangent plane to the surface at the point $P = (0, 5, -1)$.
- (b) Find a vector equation for the line passing through the surface at $P = (0, 5, -1)$ orthogonal to the plane found in part (a).
2. (12 points) Either give an example of a function $f(x, y)$ with $f_x(x, y) = 2x + y^2e^x$ and $f_y(x, y) = x^2 + y^2e^x$ or show that no such function f can exist.

3. (16 points) Reparameterize the curve $\mathbf{r}(t) = \langle \sqrt{15} t^2, \cos(t^2), \sin(t^2) \rangle$ where $t \geq 0$ with respect to arc length.

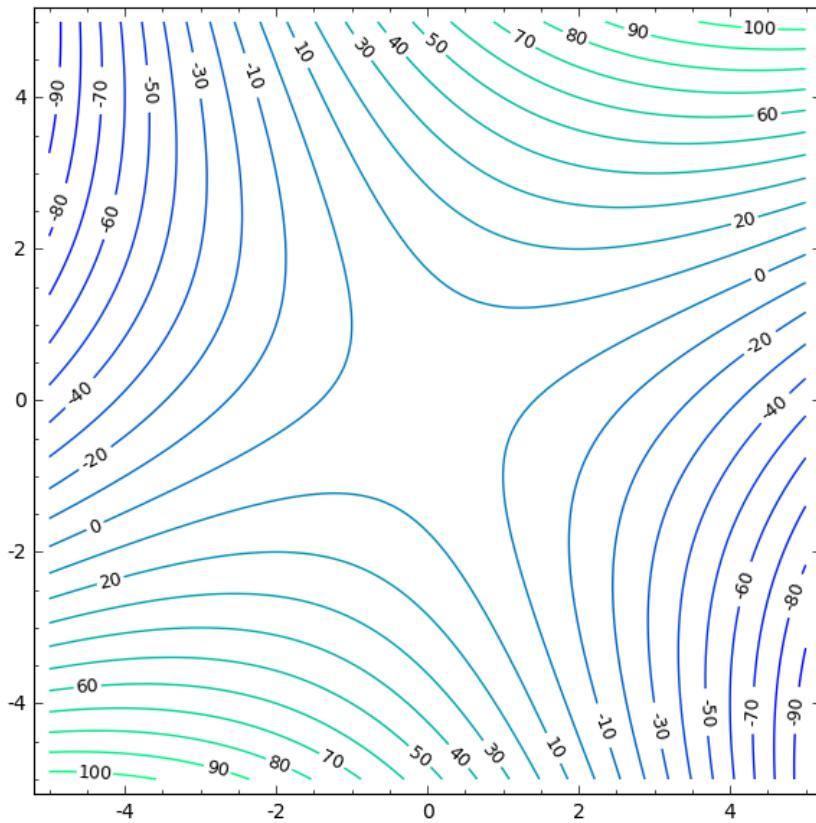
4. (10 points) Show the following limit does not exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{xy^2}{x^2 + 3y^4}$$

5. (22 points) Consider the function $f(x, y) = \sqrt{10 - x^2 - 5y^2}$.

- (a) Use a linear approximation to $f(x, y)$ at the point $(2, 1)$ to estimate the value of $f(1.95, 1.04)$.
- (b) Find the directional derivative of f at the point $(2, 1)$ in the direction of $\langle 4, 3 \rangle$.
- (c) Find the maximum rate of change of f at the point $(2, 1)$.

6. (20 points) Consider the contour plot for $f(x, y)$ below.



- (a) Determine the sign of each of the following derivatives.

$$f_x(3, 0) \text{ _____}$$

$$f_y(3, 0) \text{ _____}$$

$$f_{xx}(3, 0) \text{ _____}$$

$$f_{xy}(3, 0) \text{ _____}$$

$$f_{yy}(3, 0) \text{ _____}$$

- (b) Give the components of a unit vector in the direction of the steepest decline at the point $(-1, 0)$. (You may estimate as necessary.)

- (c) Give the components of a unit vector orthogonal to $\nabla f(2, 2)$. (You may estimate as necessary.)

THIS PAGE LEFT INTENTIONALLY BLANK

You may use this page for scratch work. Work found on this page will not be graded unless clearly indicated in the exam.