COMP9020

Foundations of Computer Science

Week 2 Recap

Assignment 1

Assignment 1: due **12 noon (AEDST), Monday 11th October**.

- Lateness penalty: 10% (of raw mark) every 12 hours or part thereof
- Unable to meet deadline through illness/injury/misadventure: apply for Special Consideration
- Submission via webCMS or give – single pdf file, 2Mb maximum size.
- Submissions should be typed, not handwritten.
- Typing math symbols: unicode, LaTeX

Assignment 1: Marking

- Looking for **ability** and **understanding**
  - Don't do too little, don't do too much (if in doubt, ask!)
  - External resources can be used, but should not be necessary
  - Can (and do) use results from lectures/assignments/quizzes
- Marks do not necessarily reflect difficulty
- Marking generally done on a 4 or 5 point percentage scale:
  - 0%: No reasonable attempt
  - 20-25%: Shows promise
  - 50%: Major errors; 3+ minor errors – not demonstrating one of understanding or ability
  - 75-80%: Minor errors
  - 100%: Excellent answer, clearly demonstrating understanding and ability

Assignment 1: Comments

- Q1: Bézout's Identity
- Q2: Use (a) for (b) and use (b) for (c).
- Q3 & Q7: "Difficult" questions - though partial marks are available.
- Q4:
  - Laws of set operations; Principle of duality; Uniqueness of complement – ok
  - Limit use of derived rules to those proven (idempotence, double complement, any self-proven result)
  - Online tool
- Q5 and Q8:
  - Proofs need to be general (true for arbitrary sets)
  - Counterexamples need to be concrete (specific example)
- Q6: (b) & (c) Any connection you observe; why if possible

Week 2 Recap

Sets:

- Set notation: $\in$, $\emptyset$, $\mathcal{U}$, $\subseteq$, $\{\dots\}$, $[\dots]$, $(\dots)$
- Set operations: $\cap$, $\cup$, ^{c}, $\setminus$, $\oplus$, $\mathrm{Pow}()$, $\times$
- Defining sets:
  - Explicitly listing elements
  - Subsets of a given set
  - Constructed using set operations
- Cardinality: $|X| = \#(X) = \mathrm{card}(X)$
- Venn diagrams
- Laws of set operations

Formal languages:

- Symbols, words, languages
- Language definitions: Σ^*, $\mathrm{length}()$, concatenation, Kleene star

Laws of Set Operations

For all sets A, B, C:

Commutativity	$A \cup B = B \cup A$
	$A \cap B = B \cap A$
Associativity	$(A \cup B) \cup C = A \cup (B \cup C)$
	$(A \cap B) \cap C = A \cap (B \cap C)$
Distribution	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity	$A \cup \emptyset = A$
	$A \cap \mathcal{U} = A$
Complementation	$A \cup (A^c) = \mathcal{U}$
	$A \cap (A^c) = \emptyset$

Other useful set laws

Idempotence	$A \cap A = A$
	$A \cup A = A$
Double complementation	$(A^c)^c = A$
Annihilation	$A \cap \emptyset = \emptyset$
	$A \cup \mathcal{U} = \mathcal{U}$
de Morgan's Laws	$(A \cap B)^c = A^c \cup B^c$
	$(A \cup B)^c = A^c \cap B^c$

Theorem (Principle of Duality)

If you can prove $A_1 = A_2$ *using the Laws of Set Operations then you can prove* $\mathit{dual}(A_1) = \mathit{dual}(A_2)$

Theorem (Uniqueness of complement)

$A \cap B = \emptyset$ *and* $A \cup B = \mathcal{U}$ *if, and only if,* $B = A^c$.

Exercises

Exercises

Show the following for all sets A, B, C:

- $B \cup (A \cap \emptyset) = B$
- $(C \cup A) \cap (B \cup A) = A \cup (B \cap C)$
- $(A \cap B) \cup (A \cup B^c)^c = B$

Exercises

Give counterexamples to show the following do not hold for all sets:

- $A \setminus (B \setminus C) = (A \setminus B) \setminus C$
- $(A \cup B) \setminus C = A \cup (B \setminus C)$
- $(A \setminus B) \cup B = A$

Need to know for this course

- How to define sets
- Cartesian product
- Cardinality computations
- Proofs using Laws of Set Operations
- How to define languages