



# COMP9020

Foundations of Computer Science

Lecture 1: Course Introduction

## Acknowledgement of Country

I would like to acknowledge and pay my respect to the Bedegal people who are the Traditional Custodians of the land on which UNSW is built, and of Elders past and present.

# Outline

## Technical details

What you can expect from me

What I will expect from you

# COMP9020 21T3 Staff

Lecturer: Paul Hunter  
Email: paul.hunter@unsw.edu.au  
Lectures: Mondays 2-4pm and Fridays 11am-1pm  
Consults: Tuesdays 1-2pm and Thursdays 8-9pm  
Research: Theoretical CS: Algorithms, Formal verification

# Online arrangements

## Lectures:

- Zoom: <https://unsw.zoom.us/j/96612524105> (passcode: comp9020)
- Recordings available on echo360 (through [Moodle](#))

## Consultations:

- Zoom: <https://unsw.zoom.us/j/87192636642> (passcode:  $1+1=2$ )
- Group-based, student-driven
- Wiki for questions

## Other points of contact:

- Course forums
- Email

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# What is this course about?

## What is Computer Science?

*“Computer science no more about computers than astronomy is about telescopes”*

– E. Dijkstra

# Course Aims

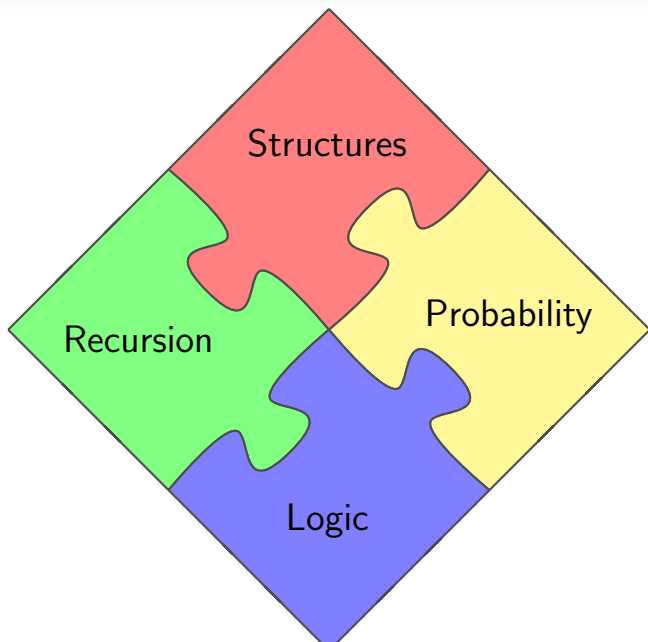
Computer Science is about exploring the ability, and limitation, of computers to solve problems. It covers:

- **What** are computers capable of solving?
- **How** can we get computers to solve problems?
- **Why** do these approaches work?

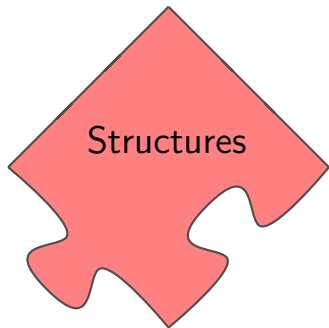
This course aims to increase your level of mathematical maturity to assist with the fundamental problem of **finding, formulating, and proving** properties of programs.



## Course Topics

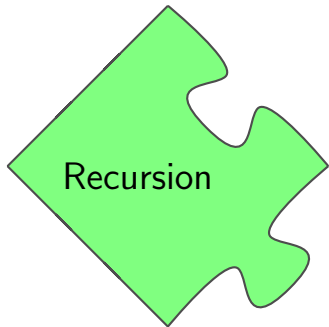


# Course Topics



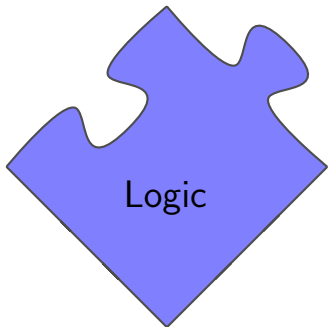
- Week 2: Set Theory
- Week 2: Formal Languages
- Week 3: Relations
- Week 4: Functions
- Week 5: Graph Theory

# Course Topics



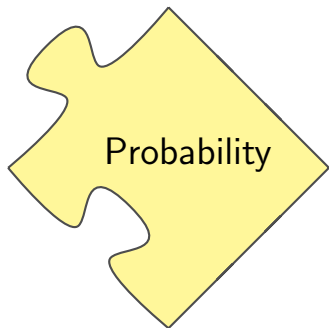
- Week 6: Recursion
- Week 7: Algorithmic Analysis
- Week 7: Induction

# Course Topics



- Week 8: Boolean Logic
- Week 8: Propositional Logic

# Course Topics



- Week 9: Combinatorics
- Week 9: Probability
- Week 10: Statistics

# Course Material

All course information is placed on the course website

[www.cse.unsw.edu.au/~cs9020/](http://www.cse.unsw.edu.au/~cs9020/)

Content includes:

- Lecture slides and recordings
- Quizzes and Assignments
- Course Forums
- Practice questions
- Challenge questions
- Study groups

# Course Material

## Textbooks:

- KA Ross and CR Wright: Discrete Mathematics
- E Lehman, FT Leighton, A Meyer:  
Mathematics for Computer Science

## Alternatives:

- K Rosen: Discrete Mathematics and its Applications

# Assessment Summary

50% exam, 40% assignments, 10% quizzes:

- 9 quizzes, worth up to 1.67 marks each
- 3 assignments, worth up to 13.33 marks each
- final exam (2 hours) worth up to 50 marks

Quizzes are available for 48 hours before the first lecture of the week. Assignments due on Mondays of weeks 5, 8 and 11.

**You must achieve 40% on the final exam to pass**

Your final score will be taken from your 6 best quiz results, 3 assignments and final exam.



# Late policy and Special Consideration

All assessments are submitted through the course website

## Lateness policy

- Assignments: 10% off raw mark per 12 hours or part thereof
- Quizzes: Late submissions not accepted
- Exam: Late submissions not accepted

If you cannot meet a deadline through illness or misadventure you need to apply for [Special Consideration](#).

## More information

View the course outline at:

<https://webcms3.cse.unsw.edu.au/COMP9020/21T3/outline>

Particularly the sections on **Student conduct** and **Plagiarism**.

# Outline

Technical details

What you can expect from me

What I will expect from you

# Assessments

To achieve good marks in this course you need to demonstrate:

- Your understanding of the material
- Your ability to work with the material

## NB

*How you get an answer is as, if not more important than what the answer is.*

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# Mathematical communication

## Guidelines for good mathematical writing

Mathematical writing should be:

- Clear
- Logical
- Sensical

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- Sensical

### NB

*All submitted work must be typeset. Diagrams may be hand drawn.*

# Examples

## Example (Bad)

Ex 1 a) ~~300~~ 51 b) 72 c) 12

$$\begin{aligned}\text{Ex 2: } (A \setminus B) \cup (B \setminus A) &= (A \cap B^c) \cup (B \cap A^c) = (A \cup B) \cap (A \cup A^c) \cap (B \cup B^c) \cap (\cancel{A \cup A^c}) \\ &= (A \cup B) \cap (A^c \cup B^c) = (A \cup B) \cap (A \cap B)^c = (A \cup B) \setminus (A \cap B) \text{ by DeM, Dist}\end{aligned}$$

Ex 3 a) Yes b) No c) Yes d) No e) Yes Ex 4 a) True b) False

~~Ex 4 a) True b) False~~



# Examples

## Example (Good)

Ex. 2

$$\begin{aligned}(A \setminus B) \cup (B \setminus A) &= (A \cap B^c) \cup (B \cap A^c) && \text{(Def.)} \\ &= ((A \cap B^c) \cup B) \cap ((A \cap B^c) \cup A^c) && \text{(Dist.)} \\ &= (A \cup B) \cap (B^c \cup B) \\ &\quad \cap (A \cup A^c) \cap (B^c \cup A^c) && \text{(Dist.)} \\ &= (A \cup B) \cap (A^c \cup B^c) && \text{(Ident.)} \\ &= (A \cup B) \cap (A \cap B)^c && \text{(DeM.)} \\ &= (A \cup B) \setminus (A \cap B) && \text{(Def.)}\end{aligned}$$

# Examples

## Example (Good)

Ex. 4a

We will show that if  $R_1$  and  $R_2$  are symmetric, then  $R_1 \cap R_2$  is symmetric.

Suppose  $(a, b) \in R_1 \cap R_2$ .

Then  $(a, b) \in R_1$  and  $(a, b) \in R_2$ .

Because  $R_1$  is symmetric,  $(b, a) \in R_1$ ; and because  $R_2$  is symmetric,  $(b, a) \in R_2$ .

Therefore  $(b, a) \in R_1 \cap R_2$ .

Therefore  $R_1 \cap R_2$  is symmetric.

# Proofs

A large component of your work in this course is giving **proofs** of **propositions**.

A **proposition** is a statement that is either true or false.

## Example

Propositions:

- $3 + 5 = 8$
- All integers are either even or odd
- There exist  $a, b, c$  such that  $1/a + 1/b + 1/c = 4$

Not propositions:

- $3 + 5$
- $x$  is even or  $x$  is odd
- $1/a + 1/b + 1/c = 4$

# Proposition structure

Common proposition structures include:

If A then B  $(A \Rightarrow B)$

A if and only if B  $(A \Leftrightarrow B)$

For all x, A  $(\forall x.A)$

There exists x such that A  $(\exists x.A)$

$\forall$  and  $\exists$  are known as **quantifiers**.

# Proofs

A large component of your work in this course is giving **proofs** of **propositions**.

A proof of a proposition is an argument to convince the reader/marker that the proposition is true.

A **proof** of a proposition is a finite sequence of logical steps, starting from base assumptions (**axioms** and **hypotheses**), leading to the proposition in question.

# Proofs

## Example

Prove:  $3 \times 2 = 2 \times 3$

$$3 \times 2 = (2 + 1) \times 2$$

# Proofs

## Example

Prove:  $3 \times 2 = 2 \times 3$

$$\begin{aligned} 3 \times 2 &= (2 + 1) \times 2 \\ &= (2 \times 2) + (1 \times 2) \end{aligned}$$

# Proofs

## Example

Prove:  $3 \times 2 = 2 \times 3$

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# Proofs

## Example

Prove:  $3 \times 2 = 2 \times 3$

$$\begin{aligned} 3 \times 2 &= (2 + 1) \times 2 \\ &= (2 \times 2) + (1 \times 2) \\ &= (1 \times 2) + (2 \times 2) \\ &= 2 + (2 \times 2) \end{aligned}$$

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# Proofs

## Example

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# Proofs

## Example

Prove:  $3 \times 2 = 2 \times 3$

$$\begin{aligned} 3 \times 2 &= (2 + 1) \times 2 \\ &= (2 \times 2) + (1 \times 2) \\ &= (1 \times 2) + (2 \times 2) \\ &= 2 + (2 \times 2) \\ &= (2 \times 1) + (2 \times 2) \\ &= 2 \times (1 + 2) \\ &= 2 \times 3. \end{aligned}$$

## Proofs: How much detail?

- Depends on the context (question, expectation, audience, etc)
- Each **step** should be justified (excluding basic algebra and arithmetic)

# Proofs: pitfalls

Starting from the proposition and deriving true **is not valid**.

## Example

Prove:  $0 = 1$

	$0$	$=$	$1$	
So (mult. by 2)	$0$	$=$	$2$	
So (subtract 1)	$-1$	$=$	$1$	
So	$(-1)^2$	$=$	$(1)^2$	
So	$1$	$=$	$1$	which is true.

Does this mean that  $0 = 1$ ?

# Proofs: pitfalls

Make sure each step is logically valid

## Example

$$-20 = -20$$

# Proofs: pitfalls

Make sure each step is logically valid

## Example

$$-20 = -20$$

So

$$25 - 45 = 16 - 36$$



# Proofs: pitfalls

Make sure each step is logically valid

## Example

$$-20 = -20$$

So  $25 - 45 = 16 - 36$

So  $5^2 - 2 \cdot 5 \cdot \frac{9}{2} = 4^2 - 2 \cdot 4 \cdot \frac{9}{2}$

# Proofs: pitfalls

Make sure each step is logically valid

## Example

$$-20 = -20$$

$$\text{So} \quad 25 - 45 = 16 - 36$$

$$\text{So} \quad 5^2 - 2 \cdot 5 \cdot \frac{9}{2} = 4^2 - 2 \cdot 4 \cdot \frac{9}{2}$$

$$\text{So} \quad 5^2 - 2 \cdot 5 \cdot \frac{9}{2} + \left(\frac{9}{2}\right)^2 = 4^2 - 2 \cdot 4 \cdot \frac{9}{2} + \left(\frac{9}{2}\right)^2$$

## Proofs: pitfalls

Make sure each step is logically valid

### Example

$$-20 = -20$$

$$\text{So} \quad 25 - 45 = 16 - 36$$

$$\text{So} \quad 5^2 - 2 \cdot 5 \cdot \frac{9}{2} = 4^2 - 2 \cdot 4 \cdot \frac{9}{2}$$

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$$\text{So} \quad \left(5 - \frac{9}{2}\right)^2 = \left(4 - \frac{9}{2}\right)^2$$

# Proofs: pitfalls

Make sure each step is logically valid

## Example

$$-20 = -20$$

$$\text{So} \quad 25 - 45 = 16 - 36$$

$$\text{So} \quad 5^2 - 2 \cdot 5 \cdot \frac{9}{2} = 4^2 - 2 \cdot 4 \cdot \frac{9}{2}$$

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$$\text{So} \quad 5 - \frac{9}{2} = 4 - \frac{9}{2}$$

Does this mean that  $5 = 4$ ?

# Proofs: pitfalls

Make sure each step is logically valid

## Example

Suppose  $a = b$ . Then,

$$\begin{aligned} & a^2 = ab \\ \text{So } & a^2 - b^2 = ab - b^2 \\ \text{So } & (a - b)(a + b) = (a - b)b \\ \text{So } & a + b = b \\ \text{So } & a = 0 \end{aligned}$$

This is true no matter what value  $a$  is given at the start, so does that mean everything is equal to 0?

## Proofs: pitfalls

For propositions of the form  $\forall x.A$  where  $x$  can have infinitely many values:

- You cannot enumerate infinitely many cases in a proof.
- Only considering a finite number of cases is not sufficient.

### Example

For all  $n$ ,  $n^2 + n + 41$  is prime

True for  $n = 0, 1, 2, \dots, 39$ . Not true for  $n = 40$ .

# Proofs: pitfalls

The order of quantifiers matters when it comes to propositions:

## Example

- For every number  $x$ , there is a number  $y$  such that  $y$  is larger than  $x$
- There is a number  $y$  such that for every number  $x$ ,  $y$  is larger than  $x$

## Proof strategies: direct proof

Proposition form	You need to do this
$A \Rightarrow B$	Assume A and prove B
$A \Leftrightarrow B$	Prove “If A then B” and “If B then A”
$\forall x.A$	Show A holds for every possible value of x
$\exists x.A$	Find a value of x that makes A true



## Proof strategies: contradiction

To prove  $A$  is true, assume  $A$  is false and derive a contradiction.  
That is, start from the negation of the proposition and derive false.

### Example

Prove:  $\sqrt{2}$  is irrational

Proof: Assume  $\sqrt{2}$  is rational ...

## Negating propositions

Proposition form	Its negation
$A \text{ and } B$	
$A \text{ or } B$	
$A \Rightarrow B$	
$A \Leftrightarrow B$	
$\forall x.A$	
$\exists x.A$	

## Negating propositions

Proposition form	Its negation
$A \text{ and } B$	not $A$ or not $B$
$A \text{ or } B$	
$A \Rightarrow B$	
$A \Leftrightarrow B$	
$\forall x.A$	
$\exists x.A$	

## Negating propositions

Proposition form	Its negation
$A \text{ and } B$	not $A$ or not $B$
$A \text{ or } B$	not $A$ and not $B$
$A \Rightarrow B$	
$A \Leftrightarrow B$	
$\forall x.A$	
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## Negating propositions

Proposition form	Its negation
$A \text{ and } B$	not $A$ or not $B$
$A \text{ or } B$	not $A$ and not $B$
$A \Rightarrow B$	$A$ and not $B$
$A \Leftrightarrow B$	$A$ and not $B$ , or $B$ and not $A$
$\forall x.A$	
$\exists x.A$	

## Negating propositions

Proposition form	Its negation
$A \text{ and } B$	not $A$ or not $B$
$A \text{ or } B$	not $A$ and not $B$
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$A \Leftrightarrow B$	$A$ and not $B$ , or $B$ and not $A$
$\forall x.A$	$\exists x. \text{ not } A$
$\exists x.A$	

## Negating propositions

Proposition form	Its negation
$A \text{ and } B$	not $A$ or not $B$
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$A \Leftrightarrow B$	$A$ and not $B$ , or $B$ and not $A$
$\forall x. A$	$\exists x. \text{ not } A$
$\exists x. A$	$\forall x. \text{ not } A$



## Proof strategies: contrapositive

To prove a proposition of the form “If A then B” you can prove “If not B then not A”

### Example

Prove: If  $m + n \geq 73$  then  $m \geq 37$  or  $n \geq 37$ .

## Proof strategies: dealing with $\forall$

How can we check infinitely many cases?

- Choose an **arbitrary** element: an object with no assumptions about it (may have to check several cases)
- Induction (see week 5)

### Example

Prove: For every integer  $n$ ,  $n^2$  will have remainder 0 or 1 when divided by 4.

**Note:** “Arbitrary” is not the same as “random”.