

COMP9517

Computer Vision

2022 Term 3 Week 1

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SYDNEY

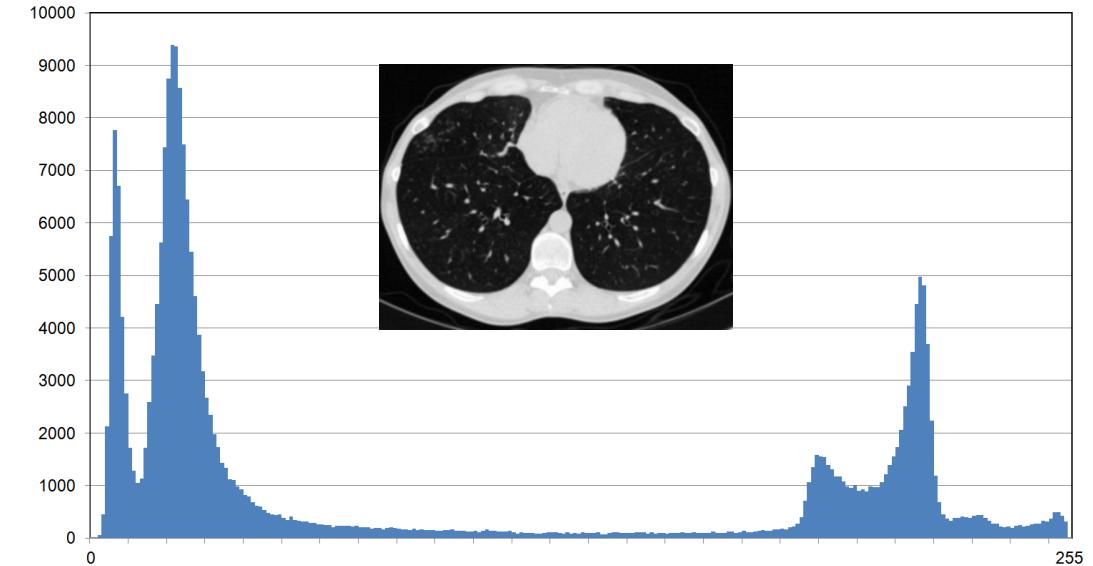


Image Processing

Part 1

What is image processing?

- **Image processing** = image in > image out
- Aims to **suppress distortions** and **enhance relevant information**
- Used to **prepare images for further analysis** and interpretation
- **Image analysis** = image in > features out
- **Computer vision** = image in > interpretation out

Types of image processing

- Two main types of image processing operations:
 - **Spatial domain operations** (in image space) Next week
 - **Transform domain operations** (mainly in Fourier space)
- Two main types of spatial domain operations: Today
 - **Point operations** (intensity transformations on individual pixels)
 - **Neighbourhood operations** (spatial filtering on groups of pixels)

Topics and learning goals

- Describe the workings of **basic point operations**
Contrast stretching, thresholding, inversion, log/power transformations
- Understand and use the **intensity histogram**
Histogram specification, equalization, matching
- Define **arithmetic and logical operations**
Summation, subtraction, AND/OR, averaging

Spatial domain operations

- General form of spatial domain operations

$$g(x, y) = T[f(x, y)]$$

where

$f(x, y)$ is the input image

$g(x, y)$ is the processed image

$T[\cdot]$ is the operator applied at (x, y)

Spatial domain operations

- Point operations: T operates on individual pixels

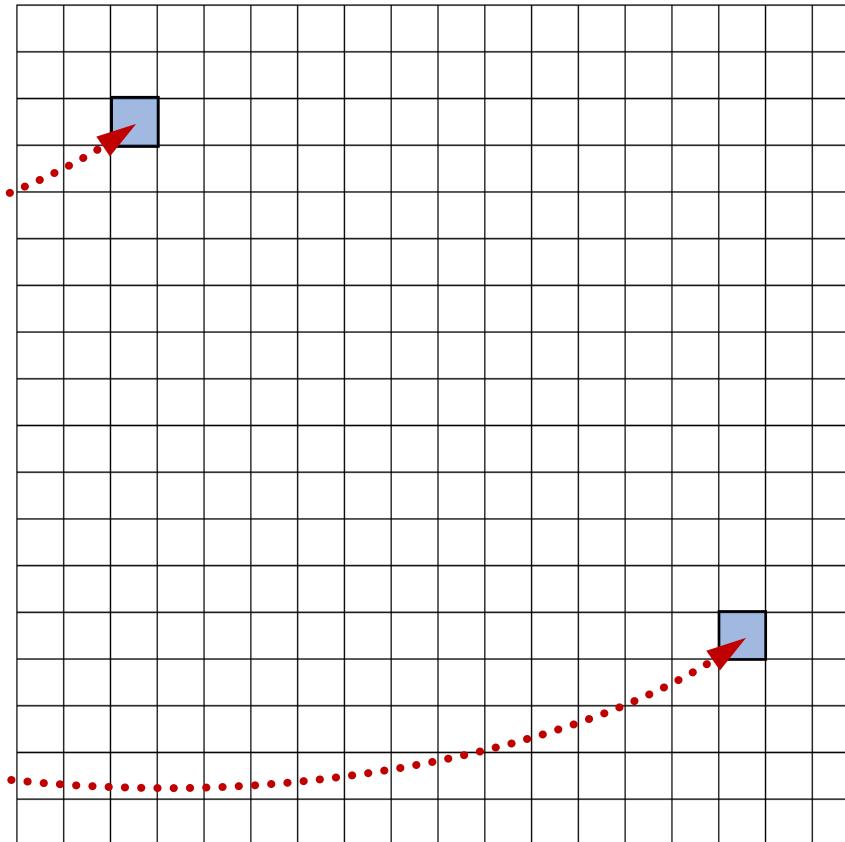
$$T: \mathbb{R} \rightarrow \mathbb{R} \quad g(x, y) = T(f(x, y))$$

- Neighbourhood operations: T operates on multiple pixels

$$T: \mathbb{R}^2 \rightarrow \mathbb{R} \quad g(x, y) = T(f(x, y), f(x + 1, y), f(x - 1, y), \dots)$$

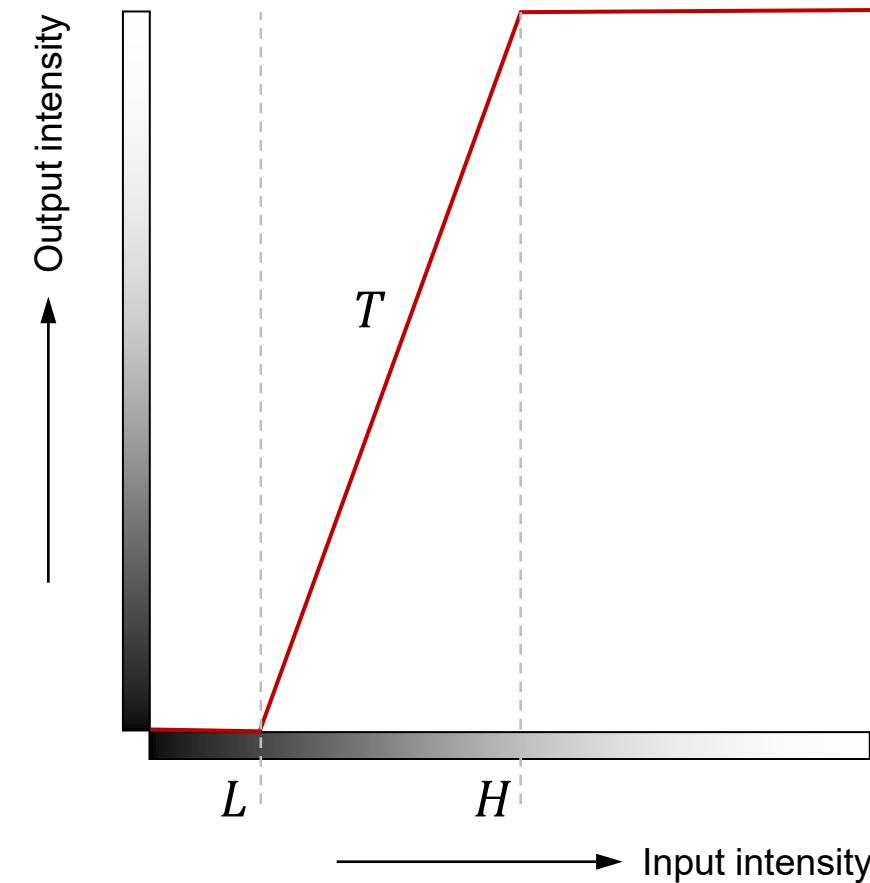
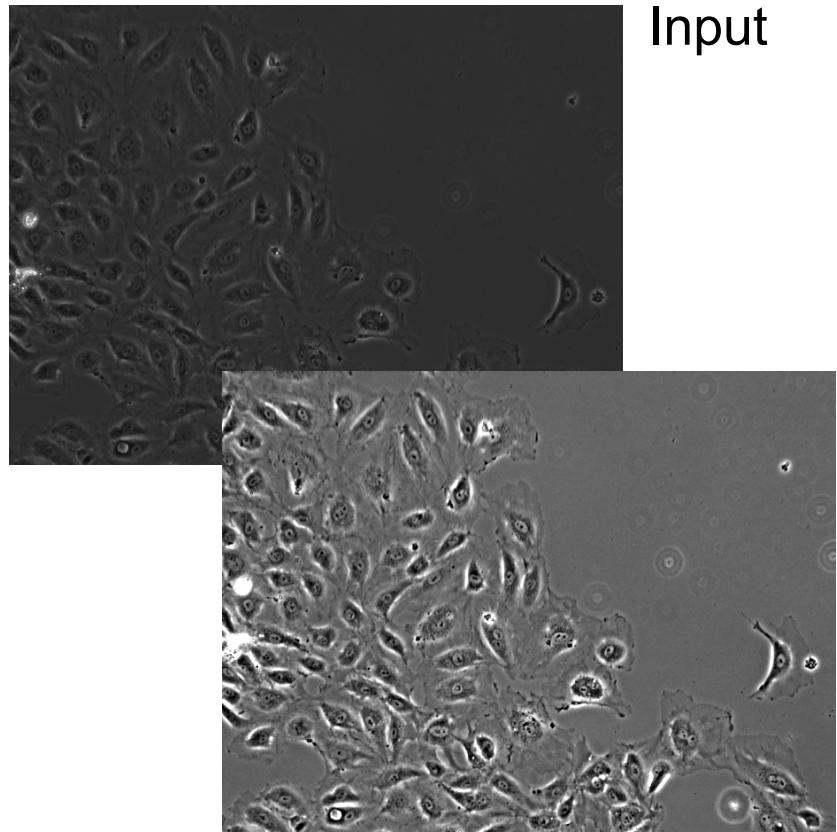
Point operations

Input image



Output image

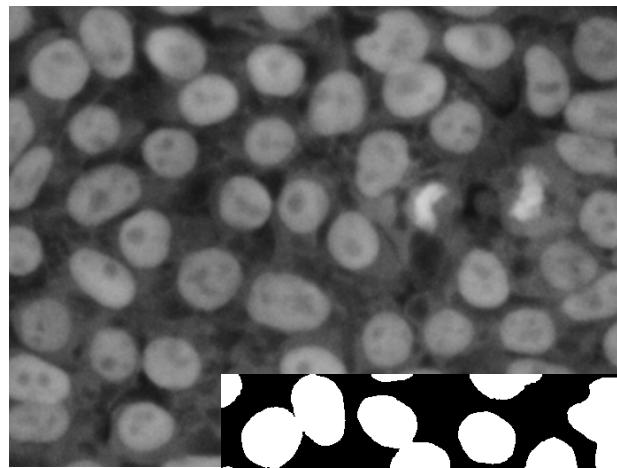
Contrast stretching



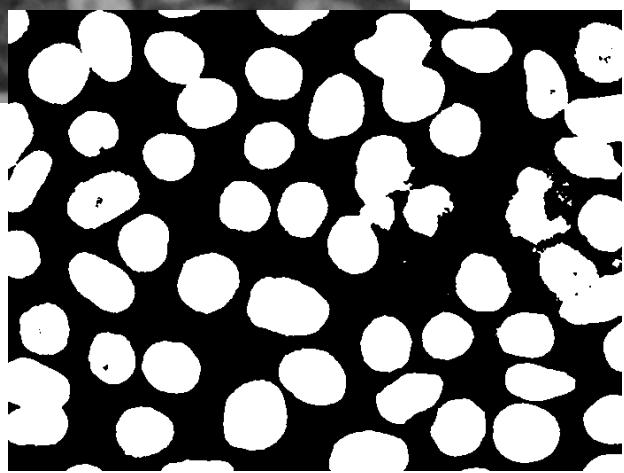
Contrast stretching

- Produces images of higher contrast
- Puts values below L in the input to the minimum (black) in the output
- Puts values above H in the input to the maximum (white) in the output
- Linearly scales values between L and H (inclusive) in the input to between the minimum (black) and the maximum (white) in the output

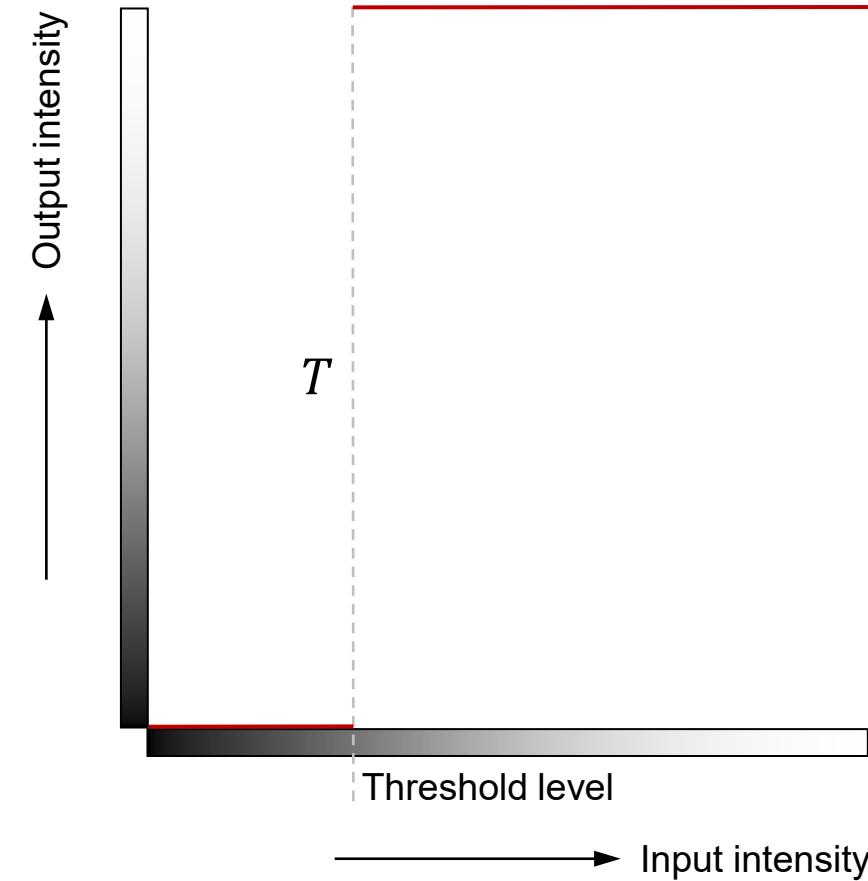
Intensity thresholding



Input



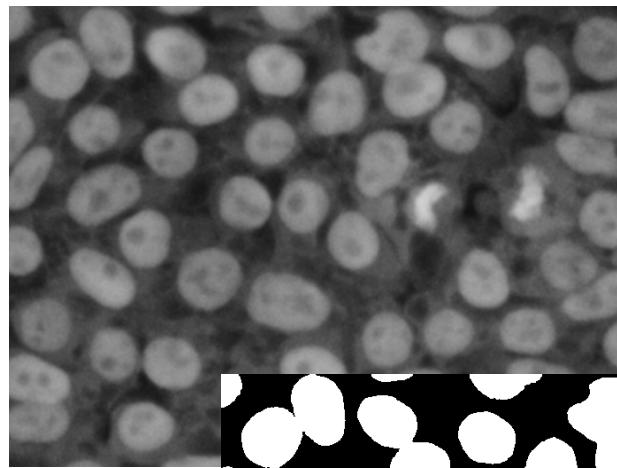
Output



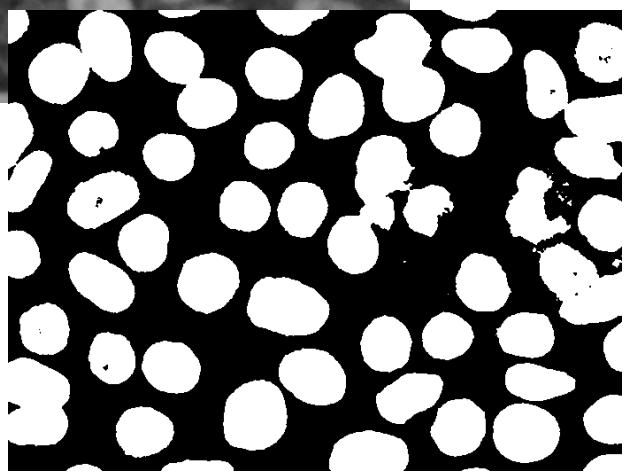
Intensity thresholding

- Limiting case of contrast stretching
- Produces binary images of gray-scale images
- Puts values below the threshold to black in the output
- Puts values equal/above the threshold to white in the output
- Popular method for image segmentation (discussed later)
- Useful only if object and background intensities are very different
- Result depends strongly on the threshold level (user parameter)

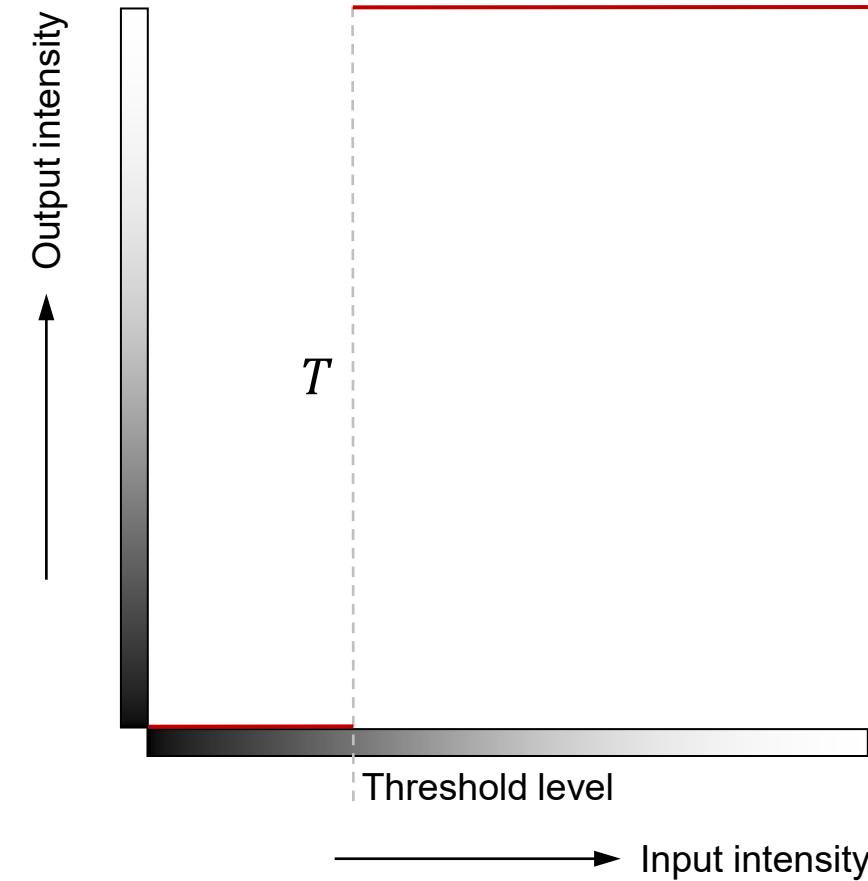
Intensity thresholding



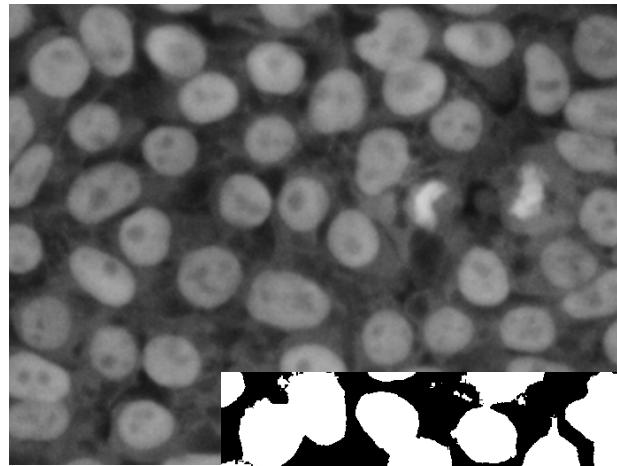
Input



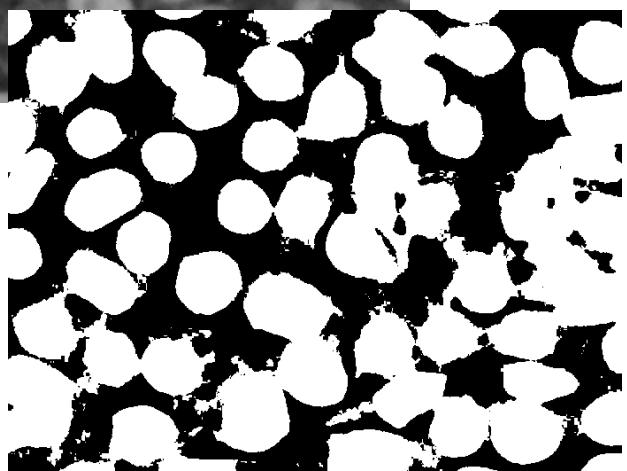
Output



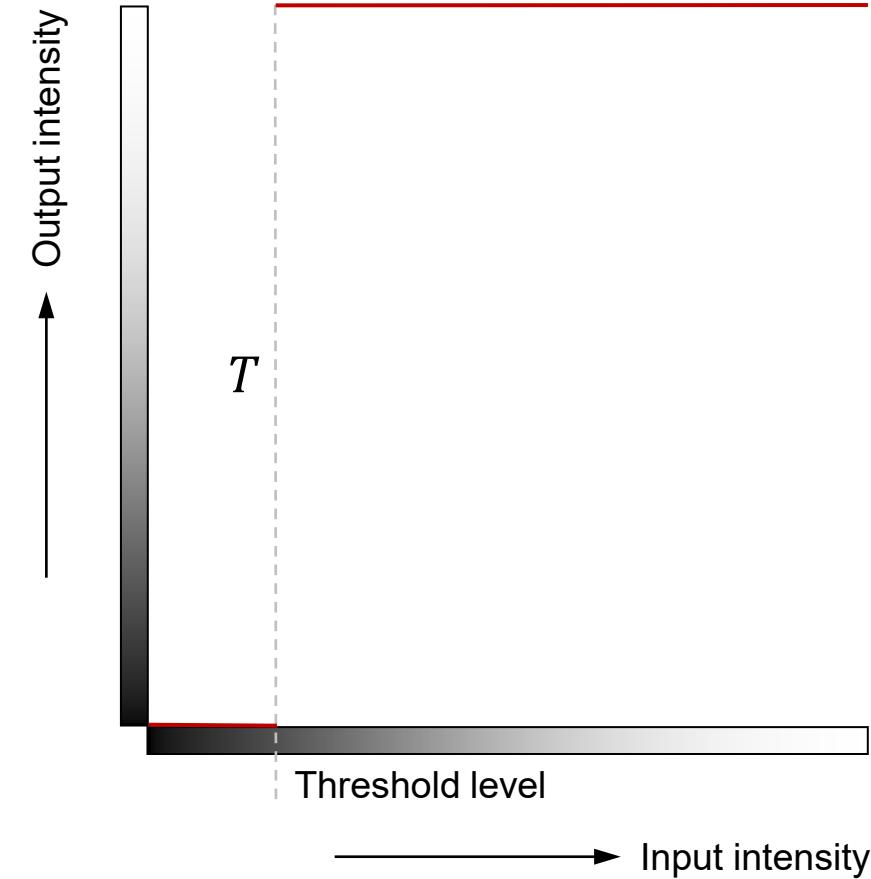
Intensity thresholding



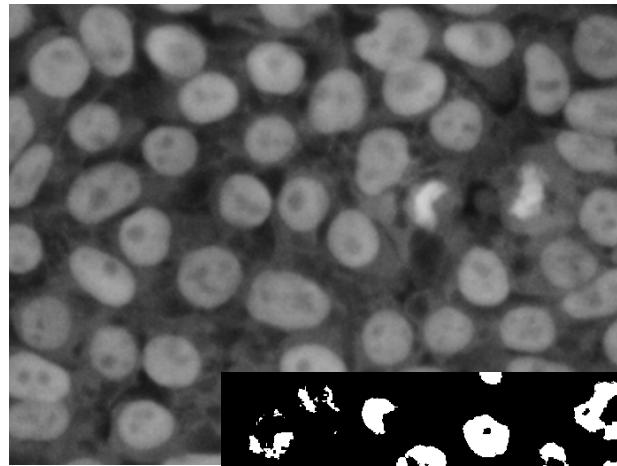
Input



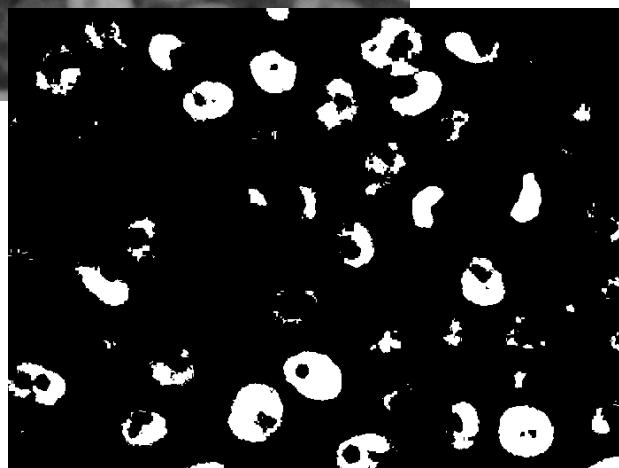
Output



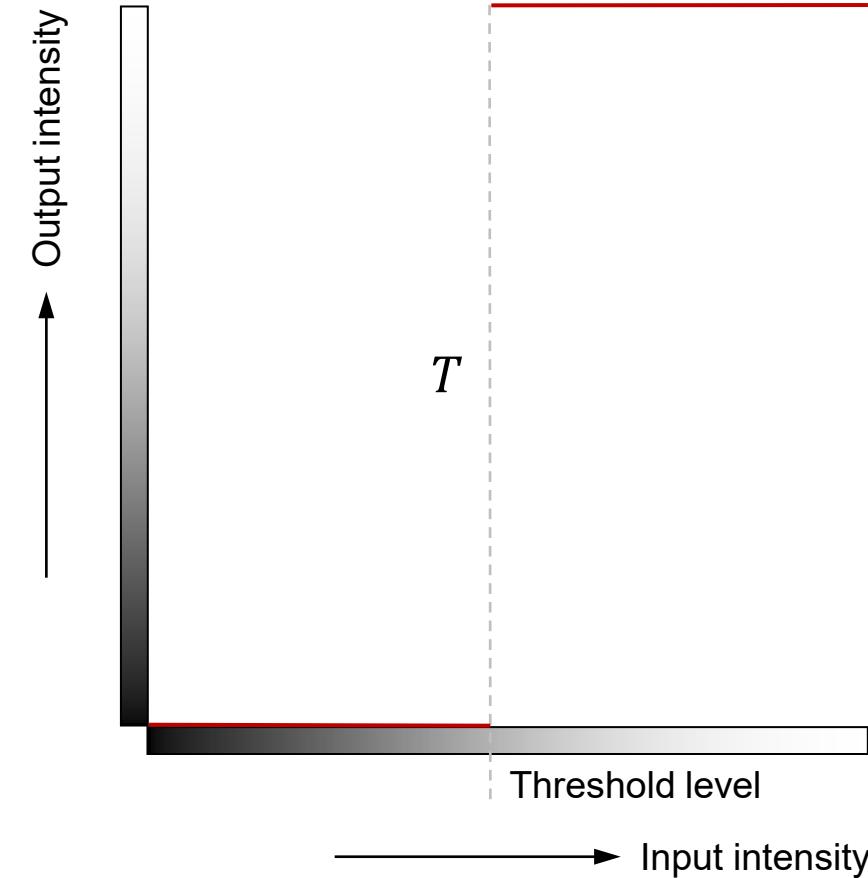
Intensity thresholding



Input



Output



Automatic intensity thresholding

- Otsu's method for computing the threshold automatically <https://doi.org/10.1109/TSMC.1979.4310076>
Exhaustively searches for the threshold **minimising the intra-class variance**

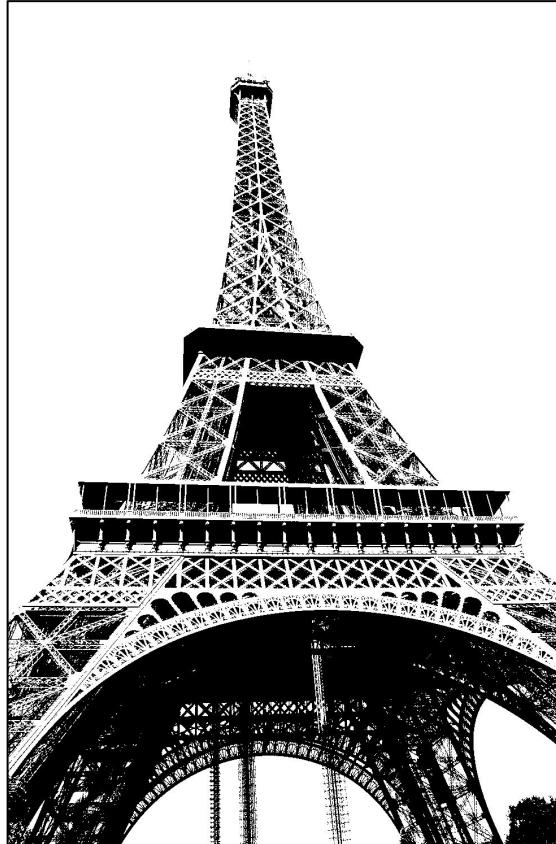
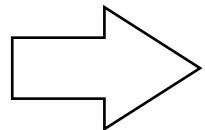
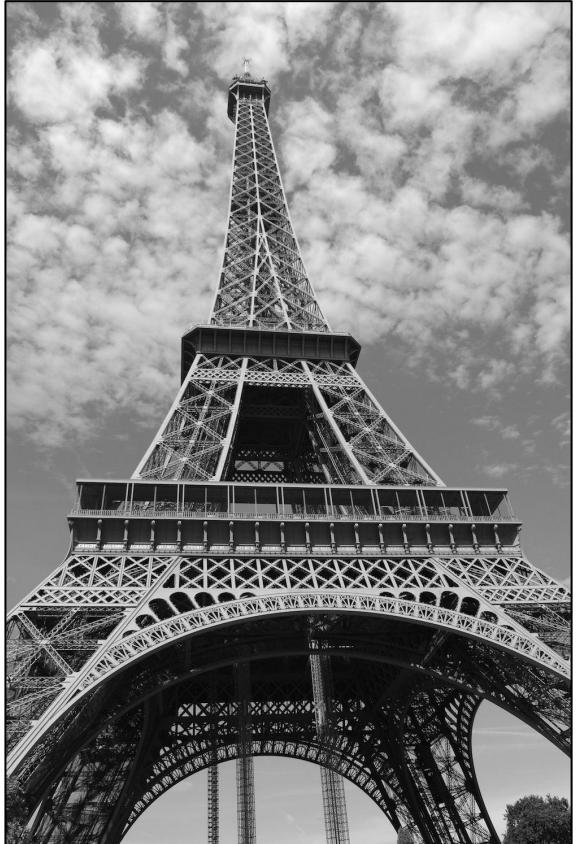
$$\sigma_W^2 = p_0 \sigma_0^2 + p_1 \sigma_1^2$$

Equivalent to **maximising the inter-class variance** (much faster to compute)

$$\sigma_B^2 = p_0 p_1 (\mu_0 - \mu_1)^2$$

Here, p_0 is the fraction of pixels below the threshold (class 0), p_1 is the fraction of pixels equal to or above the threshold (class 1), μ_0 and μ_1 are the mean intensities of pixels in class 0 and class 1, σ_0^2 and σ_1^2 are the intensity variances, and $p_0 + p_1 = 1$ and $\sigma_0^2 + \sigma_1^2 = \sigma^2$

Otsu thresholding example



Automatic intensity thresholding

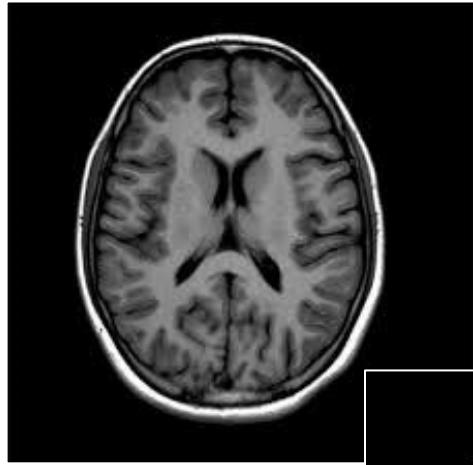
- Iso-data method for computing the threshold automatically
 1. Select an arbitrary initial threshold t
 2. Compute μ_0 and μ_1 with respect to the threshold
 3. Update the threshold to the mean of the means: $t = (\mu_0 + \mu_1)/2$
 4. If the threshold changed in Step 3, go to Step 2

Upon convergence, the threshold is midway between the two class means

Iso-data thresholding example



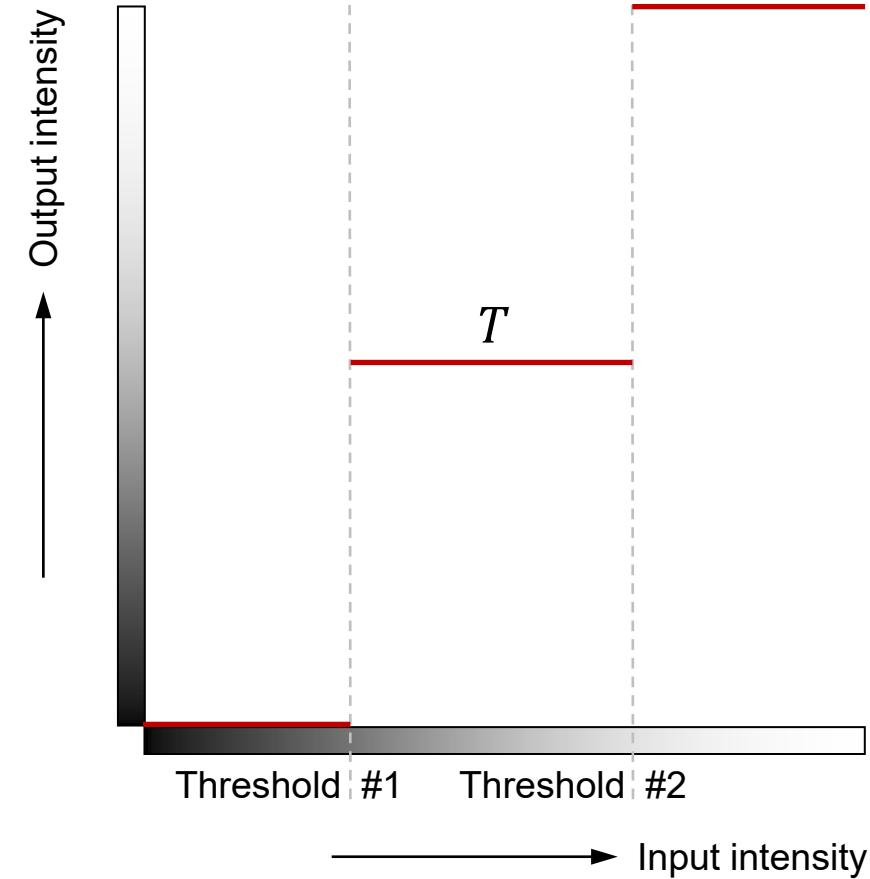
Multilevel thresholding



Input



Output



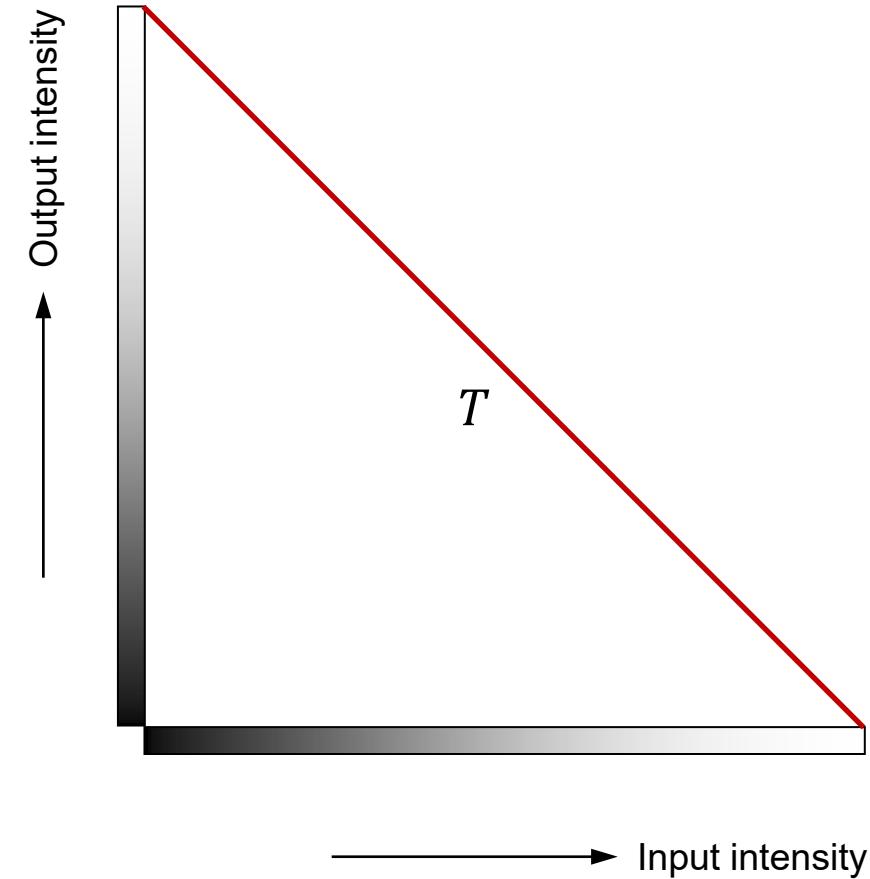
Intensity inversion



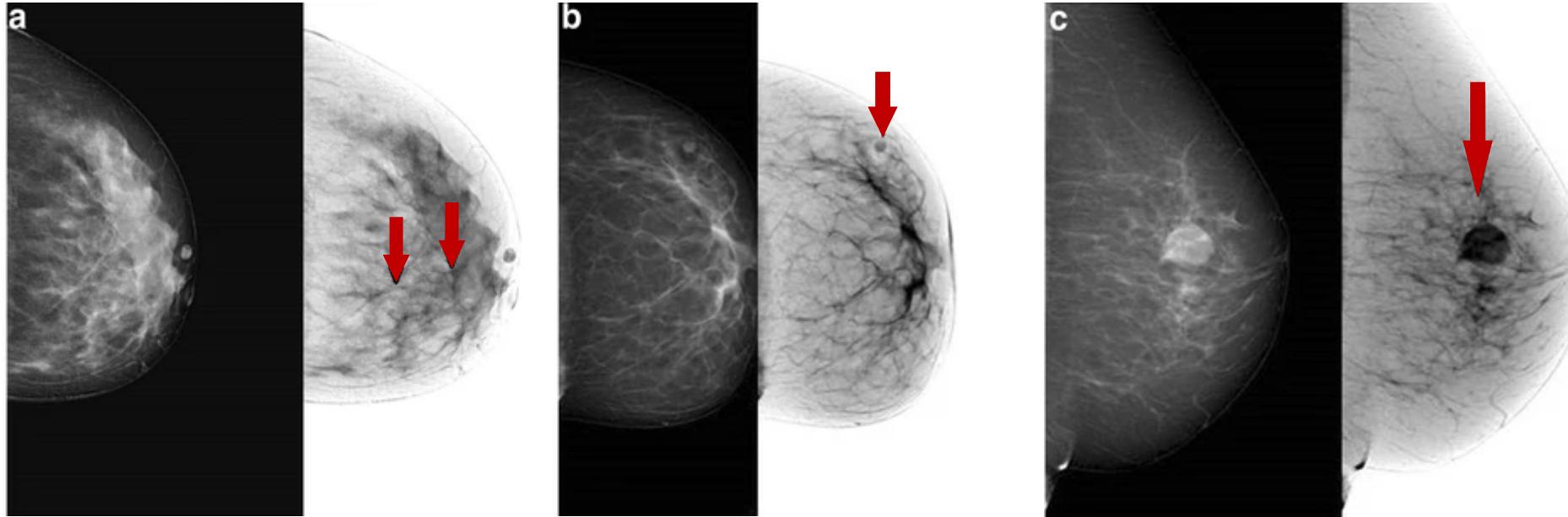
Input



Output



Intensity inversion examples



“Assessment of grayscale inverted images in addition to standard images facilitates the detection of microcalcification.” <https://doi.org/10.1186/s12880-017-0196-6>

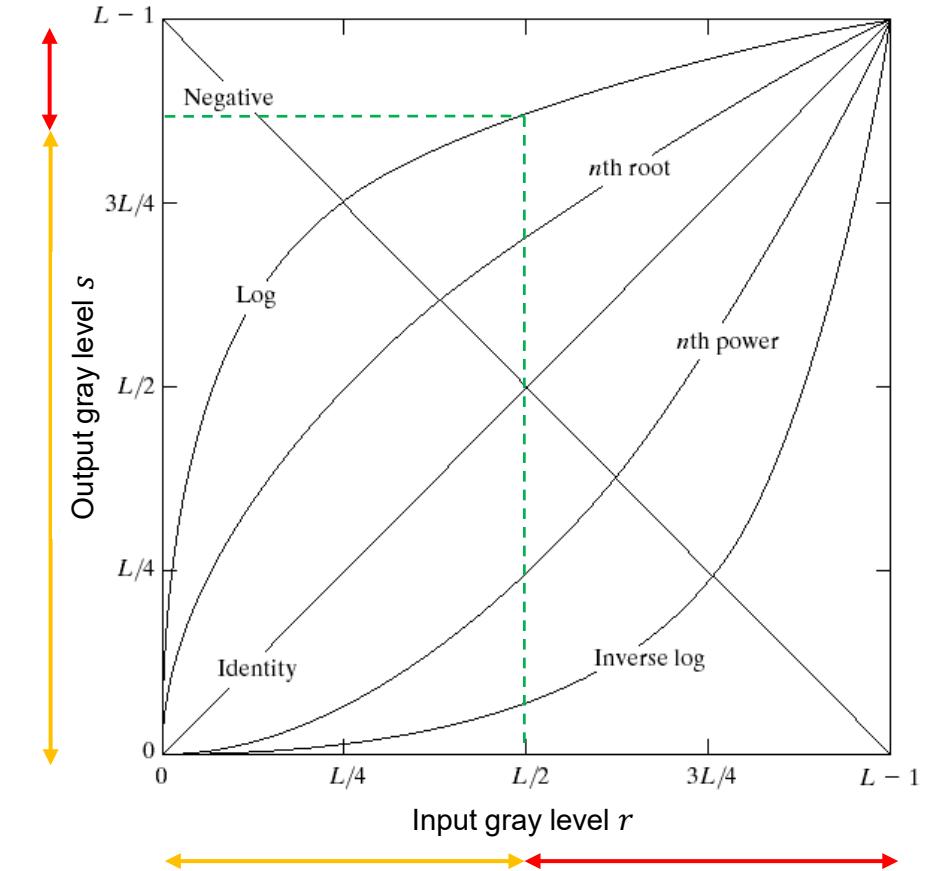
Log transformation

- Definition of log transformation

$$s = c \log(1 + r)$$

where r is the input intensity, s is the output intensity, and c is a constant

- Maps a narrow input range of low gray-level values into a wider range of output values, and vice versa for higher gray-level values
- Also compresses the dynamic range of images with large variations in pixel values (such as Fourier spectra, to be discussed later)



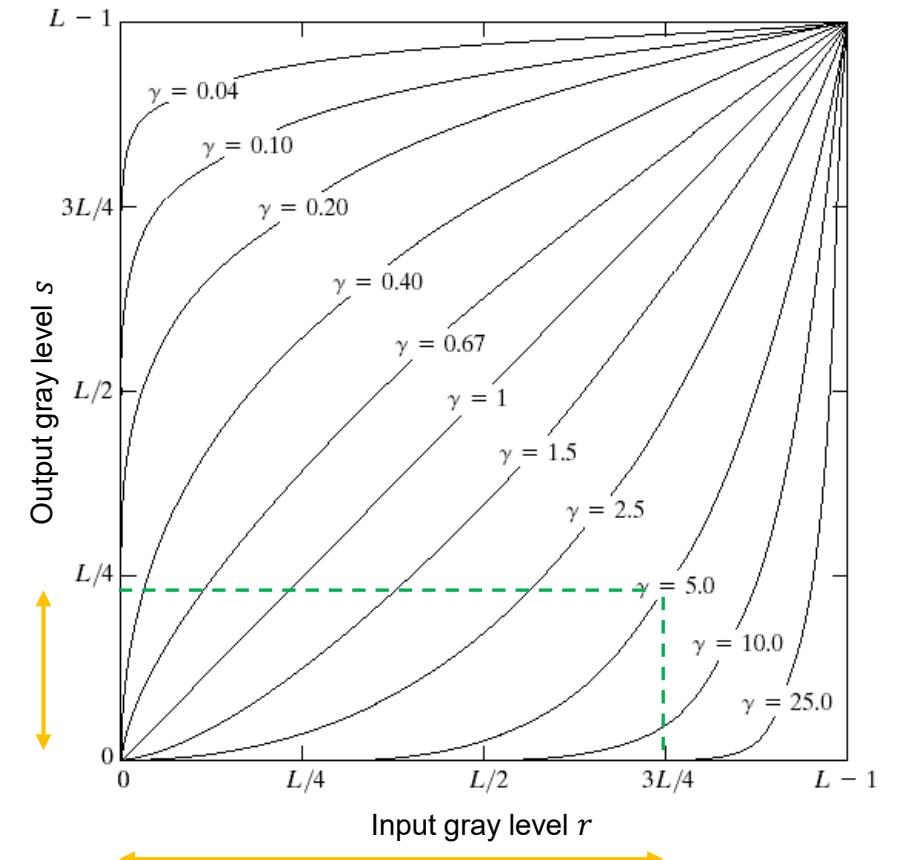
Power transformation

- Definition of power transformation

$$s = c r^\gamma$$

where c and γ are constants

- Similar to log transformation
- Represents a family of transformations by varying γ
- Many devices respond according to a power law
- Example power transformation: gamma correction
- Useful for general-purpose contrast manipulation



Power transformation examples

$$c = 1$$

Input



$$\gamma = 3$$



$$\gamma = 4$$

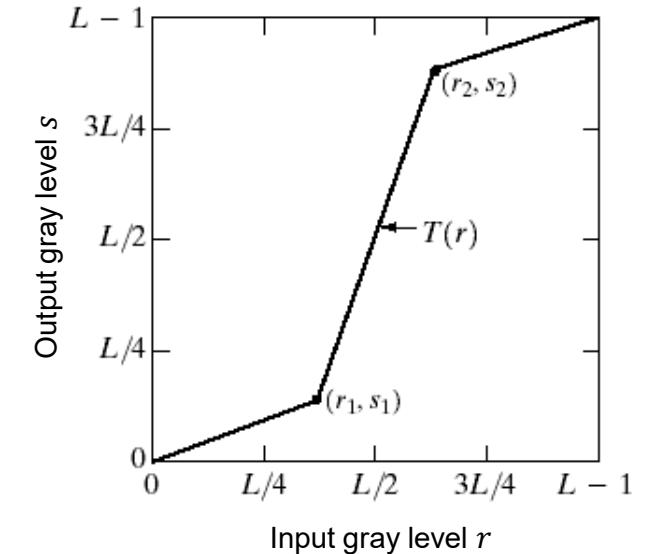


$$\gamma = 5$$



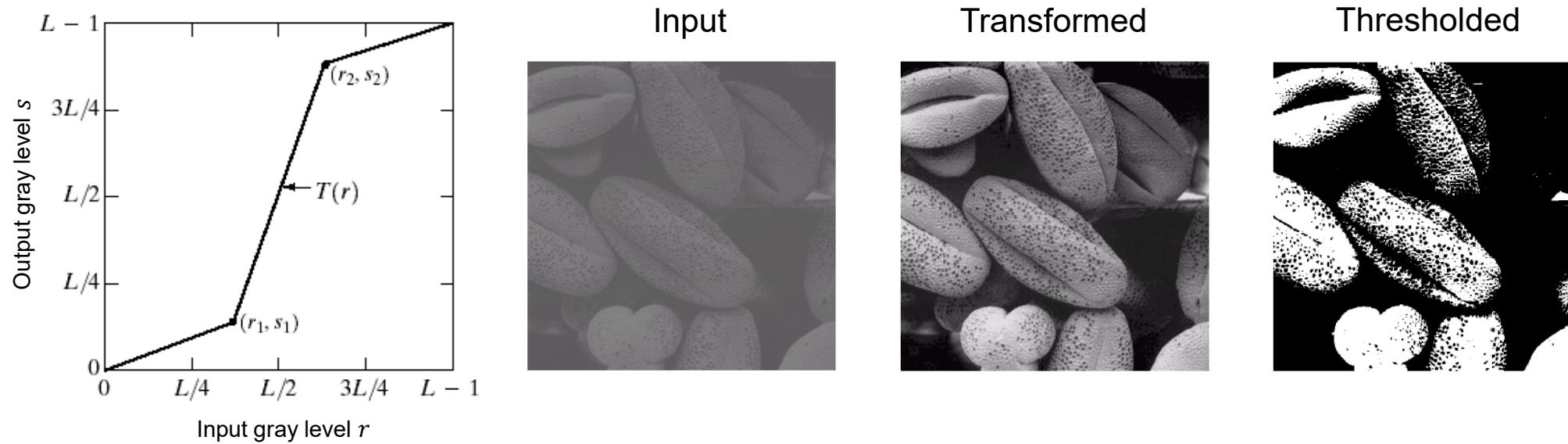
Piecewise linear transformations

- Complementary to other transformation methods
- Enable more fine-tuned design of transformations
- Can have very complex shapes
- Requires more user input



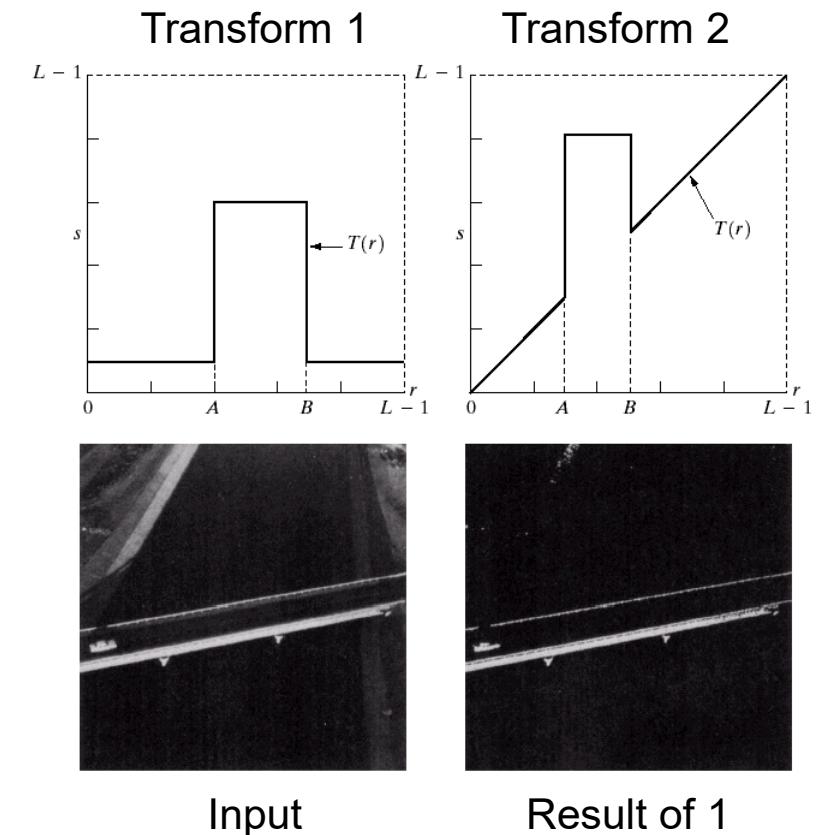
Piecewise contrast stretching

- One of the simplest piecewise linear transformations
- Increases the dynamic range of gray levels in images
- Used in display devices or recording media to span full range



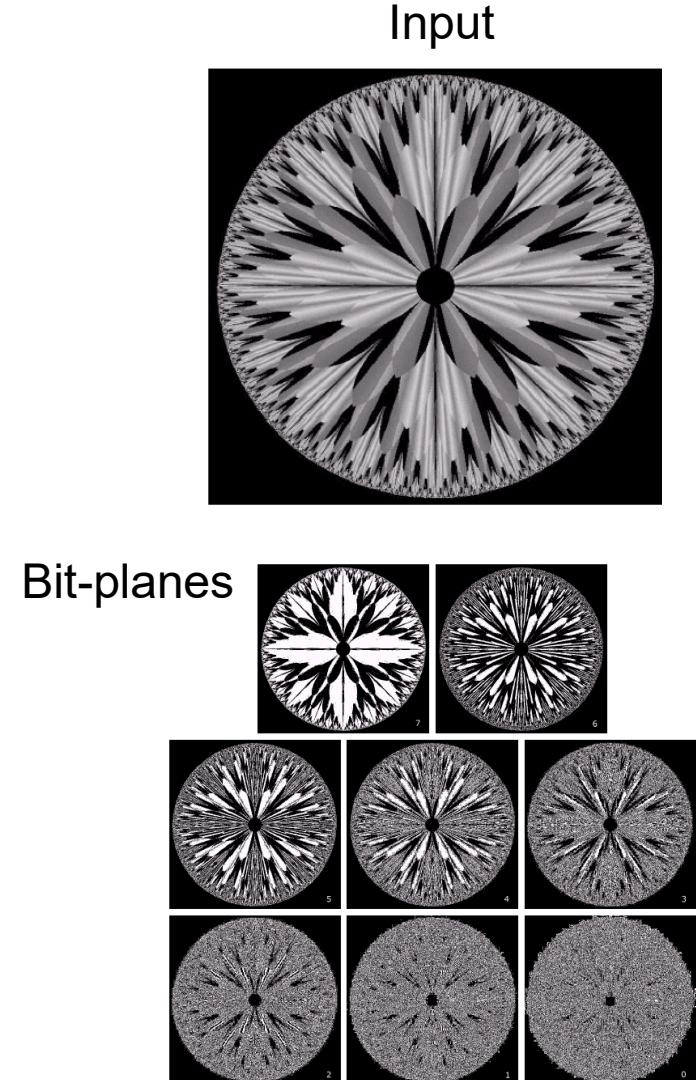
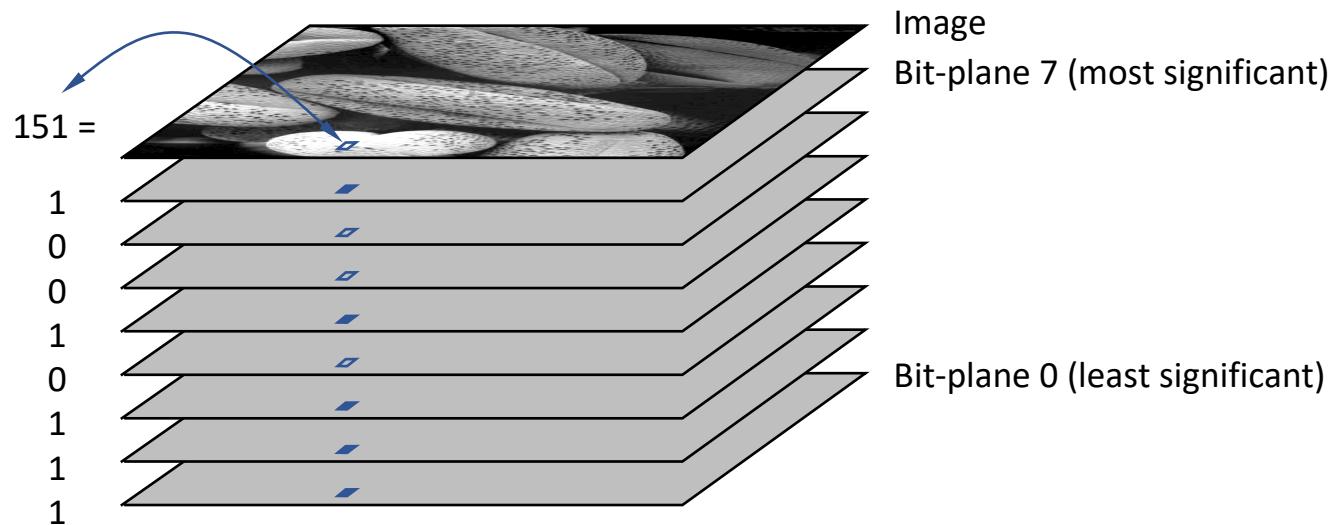
Gray-level slicing

- Used to highlight a specific range of gray levels
- Two different slicing approaches:
 - 1) High value for all gray levels in a range of interest and low value for all others (produces a binary image)
 - 2) Brighten a desired range of gray levels while preserving background and other gray-scale tones of the image



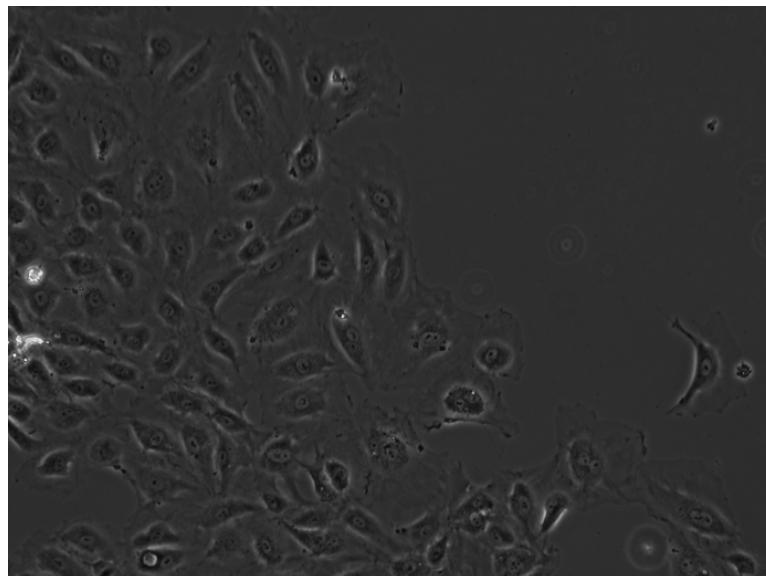
Bit-plane slicing

- Highlights contribution to total image by specific bits
- An image with n bits/pixel has n bit-planes
- Can be useful for image compression

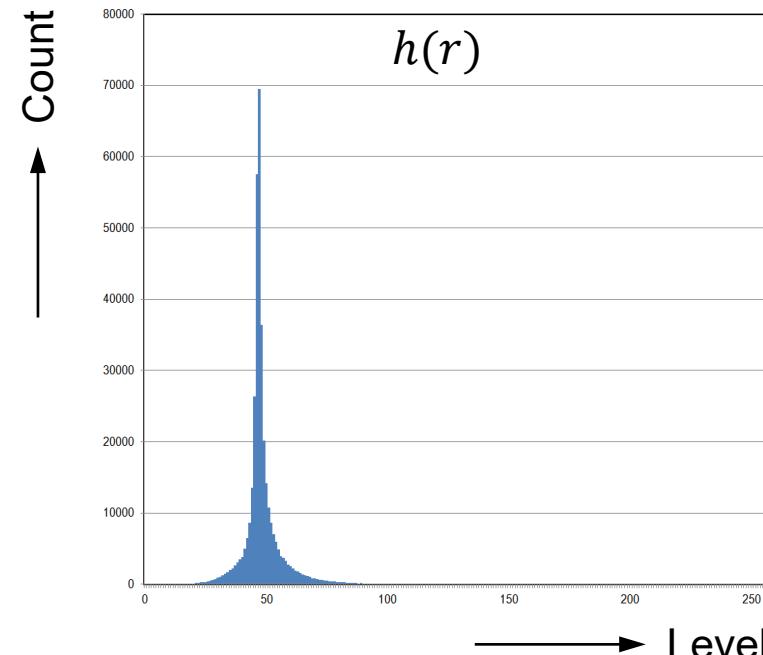


Histogram of pixel values

- For every possible gray-level value, count the number of pixels having that value, and plot the pixel counts as a function of gray-level



8-bit image



$$L = 2^8 = 256$$

$$N = \#\text{pixels}$$

$$\sum_{r=0}^{L-1} h(r) = N$$

Normalized histogram
= probability function

$$\frac{1}{N} h(r) = p(r)$$

Histogram processing

- **Histogram equalization**

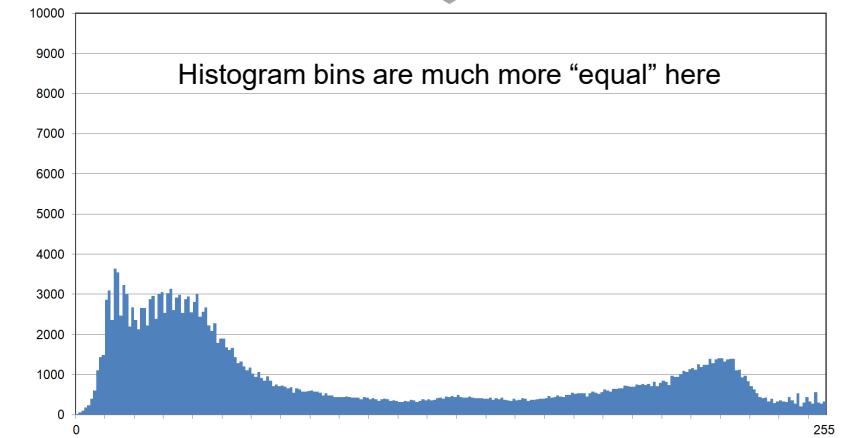
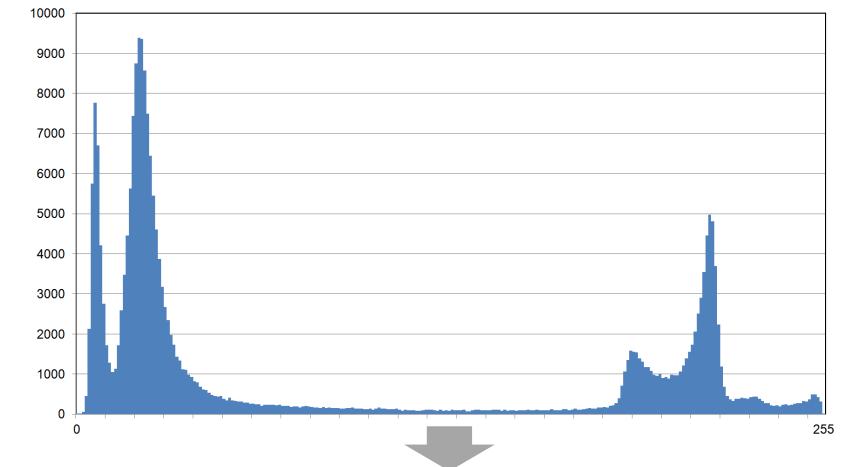
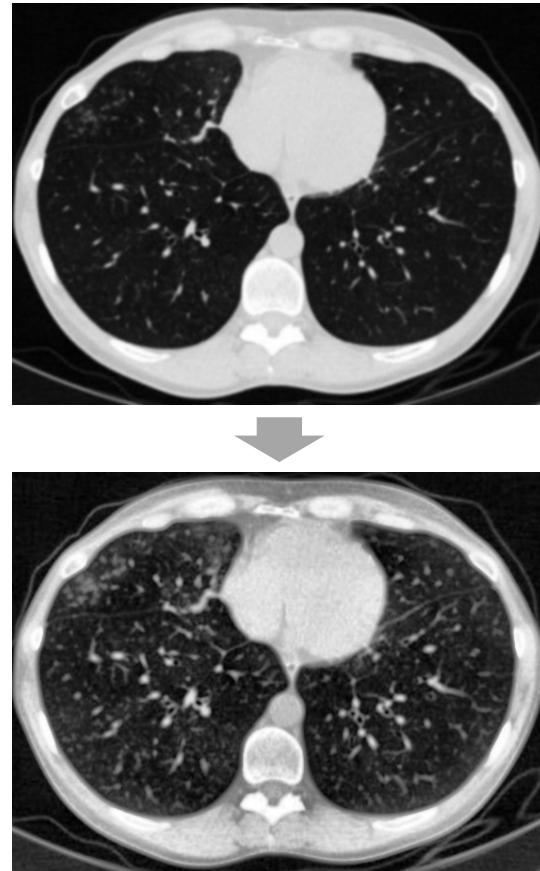
Aim: To get an image with equally distributed intensity levels over the full intensity range

- **Histogram specification** (also called **histogram matching**)

Aim: To get an image with a specified intensity distribution, determined by the shape of the histogram

Histogram equalization

Enhances contrast for intensity values near histogram maxima and decreases contrast near histogram minima



Histogram equalization

- Let $r \in [0, L - 1]$ represent pixel values (intensities, gray levels)
 $r = 0$ represents black and $r = L - 1$ represents white
- Consider transformations $s = T(r)$, $0 \leq r \leq L - 1$, satisfying
 - 1) $T(r)$ is single-valued and monotonically increasing in $0 \leq r \leq L - 1$
This guarantees that the inverse transformation $T^{-1}(s)$ exists
 - 2) $0 \leq T(r) \leq L - 1$ for $0 \leq r \leq L - 1$
This guarantees that the input and output ranges will be the same

Histogram equalization (continuous case)

Consider r and s as continuous random variables over $[0, L - 1]$ with PDFs $p_r(r)$ and $p_s(s)$

If $p_r(r)$ and $T(r)$ are known and $T^{-1}(s)$ satisfies monotonicity, then, from probability theory

$$p_s(s) = p_r(r) \left| \frac{dr}{ds} \right| \quad \text{<https://www.cl.cam.ac.uk/teaching/2003/Probability/prob11.pdf>}$$

Let us choose: $s = T(r) = (L - 1) \int_0^r p_r(\xi) d\xi$

This is the CDF (cumulative distribution function) of r which satisfies conditions (1) and (2)

Now: $\frac{ds}{dr} = \frac{dT(r)}{dr} = (L - 1) \frac{d}{dr} \left[\int_0^r p_r(\xi) d\xi \right] = (L - 1)p_r(r)$

Therefore: $p_s(s) = p_r(r) \left| \frac{1}{(L-1)p_r(r)} \right| = \frac{1}{L-1}$ for $0 \leq s \leq L - 1$ (uniform distribution)

Histogram equalization (discrete case)

For discrete values we get probabilities and summations instead of PDFs and integrals:

$$p_r(r_k) = n_k / MN \text{ for } k = 0, 1, \dots, L - 1$$

where MN is total number of pixels in image, n_k is the number of pixels with gray level r_k and L is the total number of gray levels in the image

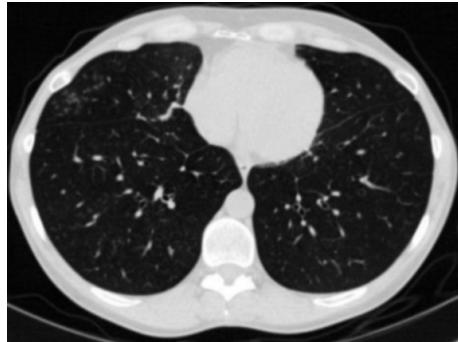
$$\text{Thus: } s_k = T(r_k) = (L - 1) \sum_{j=0}^k p_r(r_j) = \frac{L-1}{MN} \sum_{j=0}^k n_j \text{ for } k = 0, 1, \dots, L - 1$$

This transformation is called *histogram equalization*

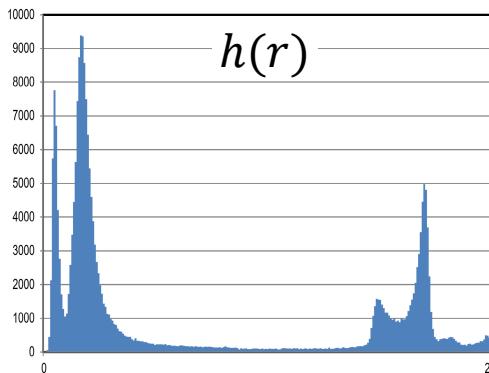
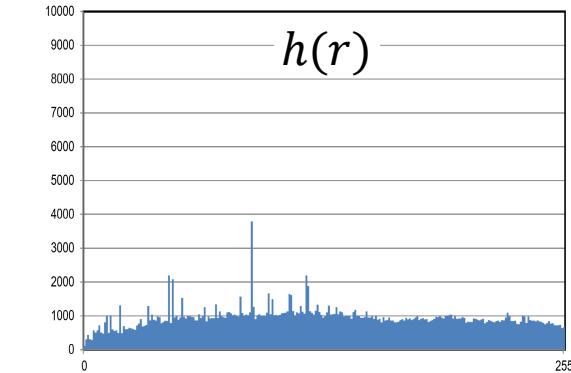
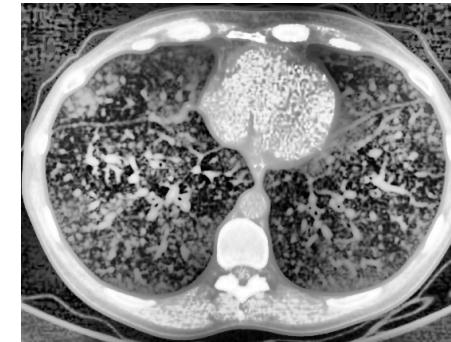
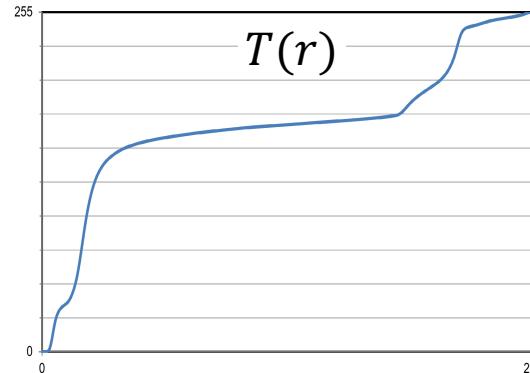
However, in practice, getting a perfectly uniform distribution for discrete images is rare

Constrained histogram equalization

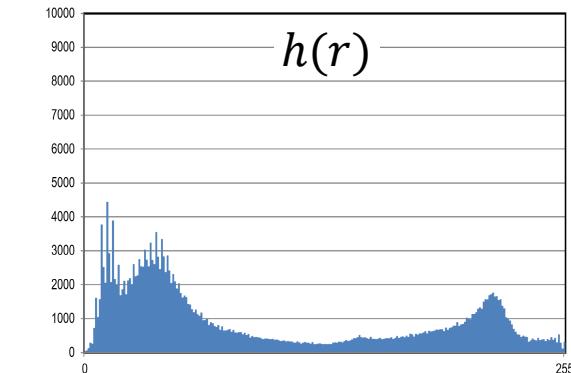
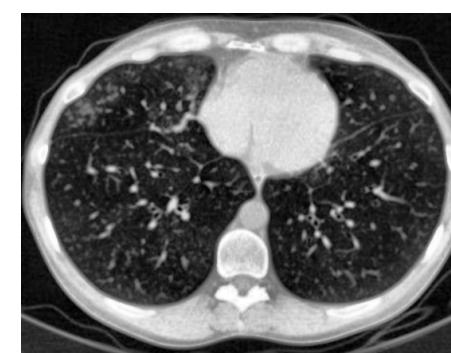
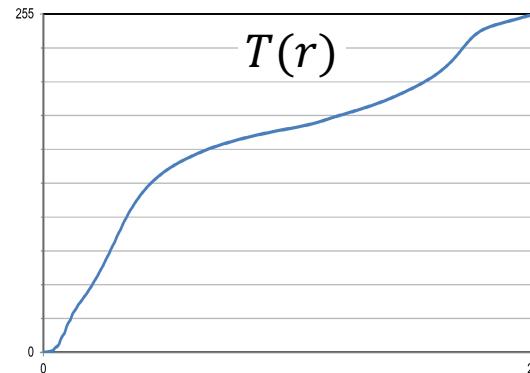
Input



Full histogram equalization (slope of $T(r)$ is unconstrained)



Constrained histogram equalization (slope of $T(r)$ is constrained)



Histogram matching (continuous case)

Assume that r and s are continuous and $p_z(z)$ is the target distribution for the output image

From our previous analysis we know the following transformation results in a uniform distribution:

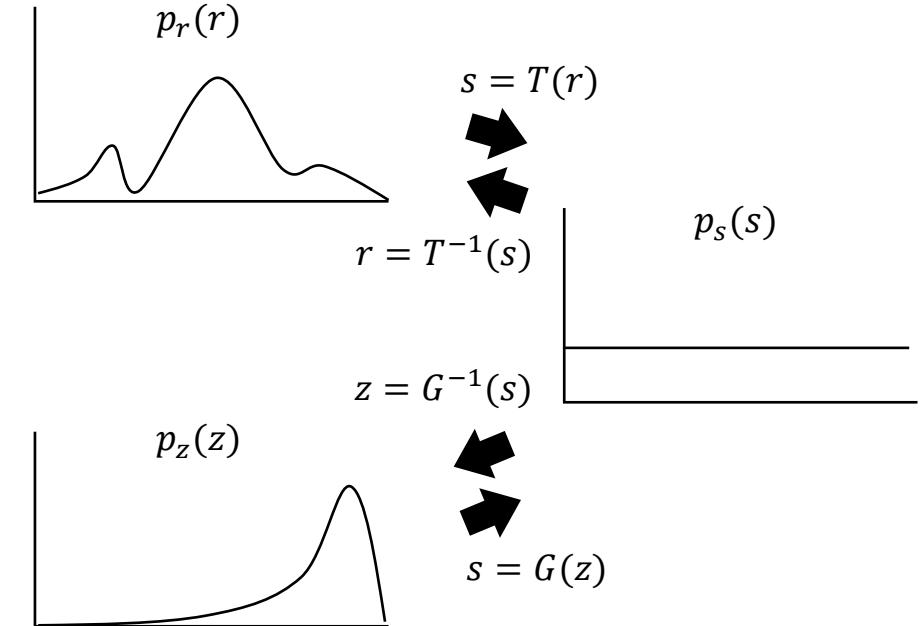
$$s = T(r) = (L - 1) \int_0^r p_r(\xi) d\xi$$

Now we can define a function $G(z)$ as:

$$G(z) = (L - 1) \int_0^z p_z(\xi) d\xi = s$$

Therefore:

$$z = G^{-1}(s) = G^{-1}[T(r)]$$



Histogram matching (discrete case)

For discrete image values we can write:

$$s_k = T(r_k) = (L - 1) \sum_{j=0}^k p_r(r_j) = \frac{L-1}{MN} \sum_{j=0}^k n_j$$

$$k = 0, 1, \dots, L - 1$$

And: $G(z_q) = (L - 1) \sum_{i=0}^q p_z(z_i)$

Therefore: $z_q = G^{-1}(s_k)$

Histogram matching example

<https://automaticaddison.com/tag/image-processing/page/3/>

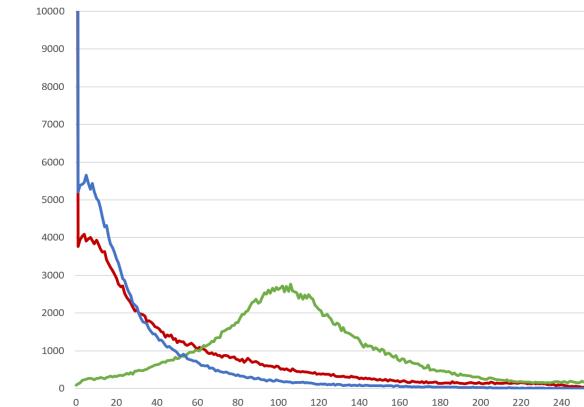
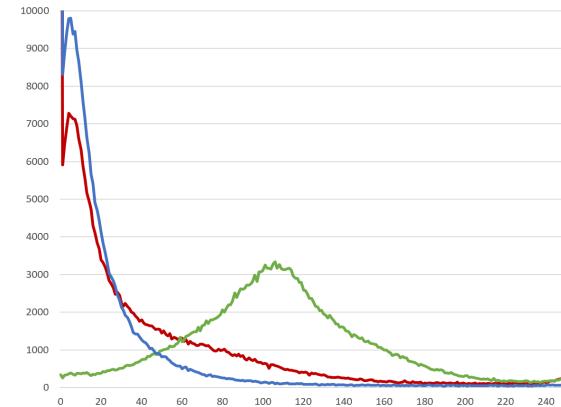
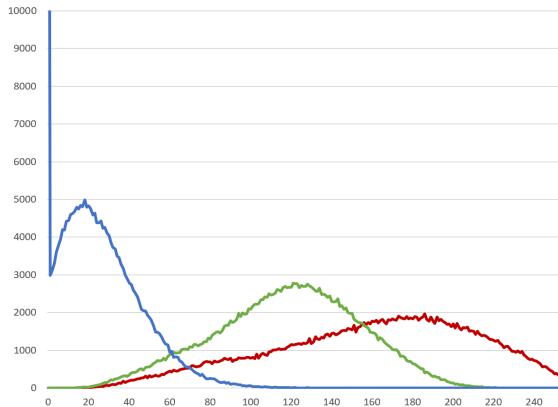
Input



Target



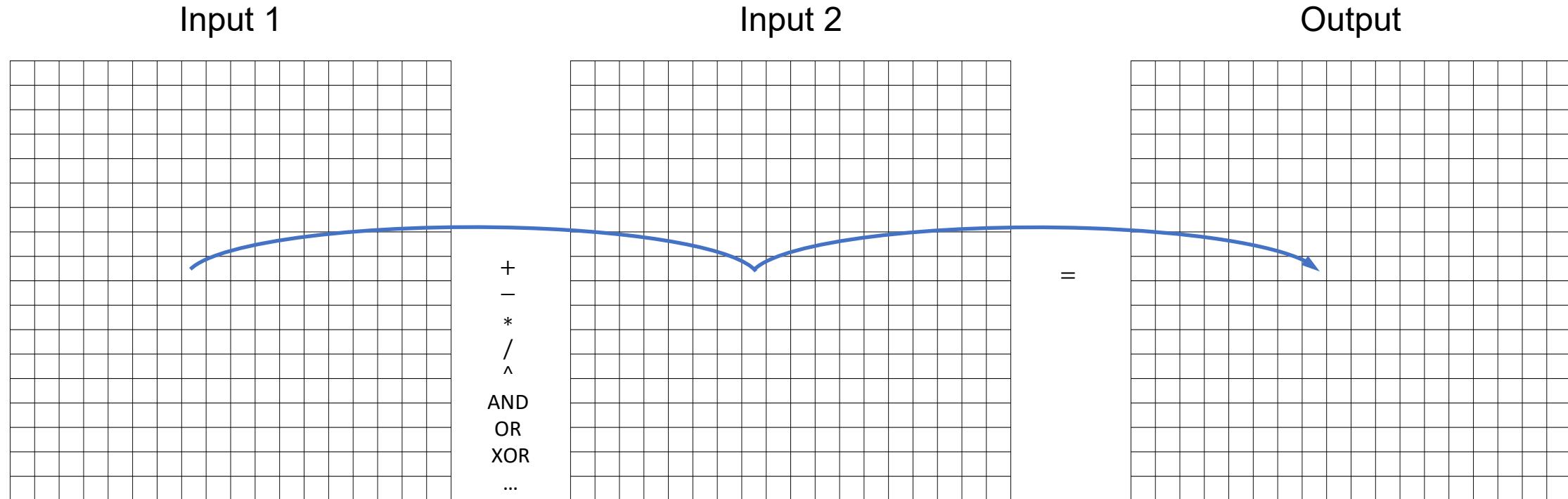
Output



Matching
done for
each colour
channel
(R,G,B)

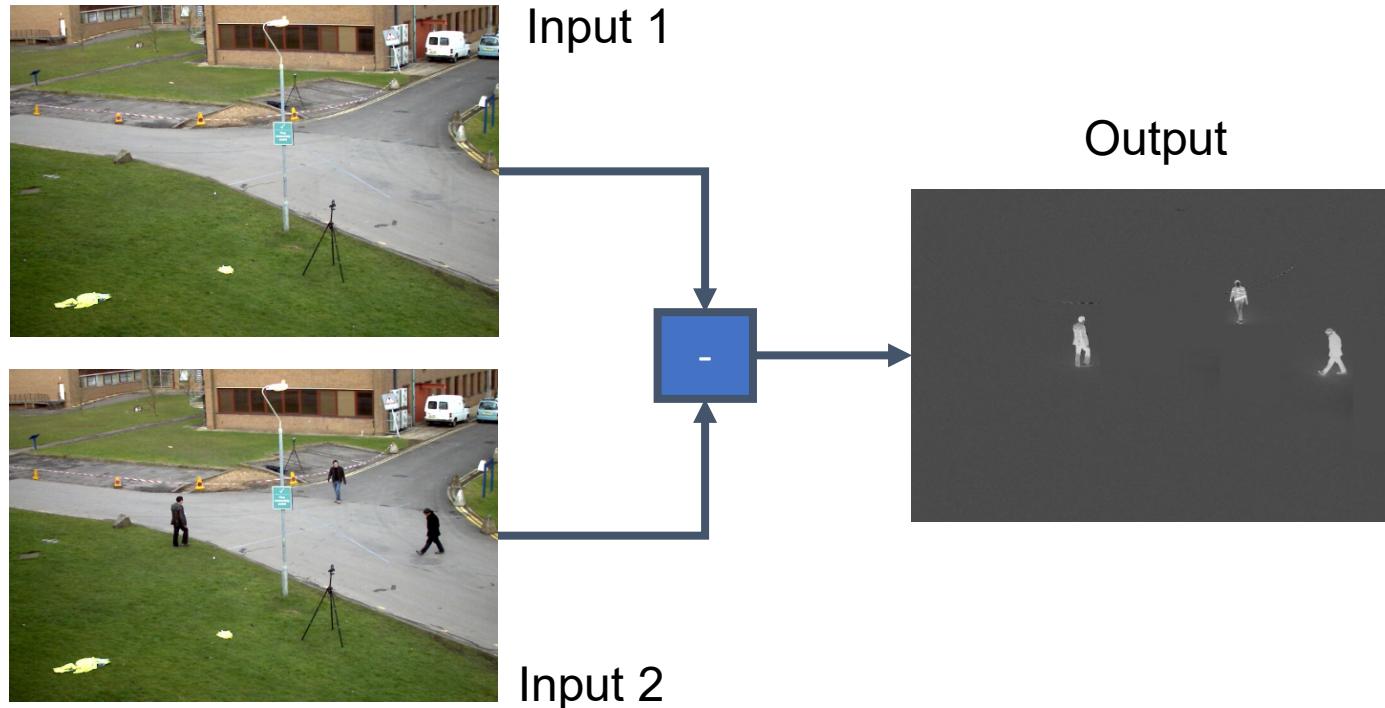
Arithmetic and logical operations

- Defined on a pixel-by-pixel basis between two images



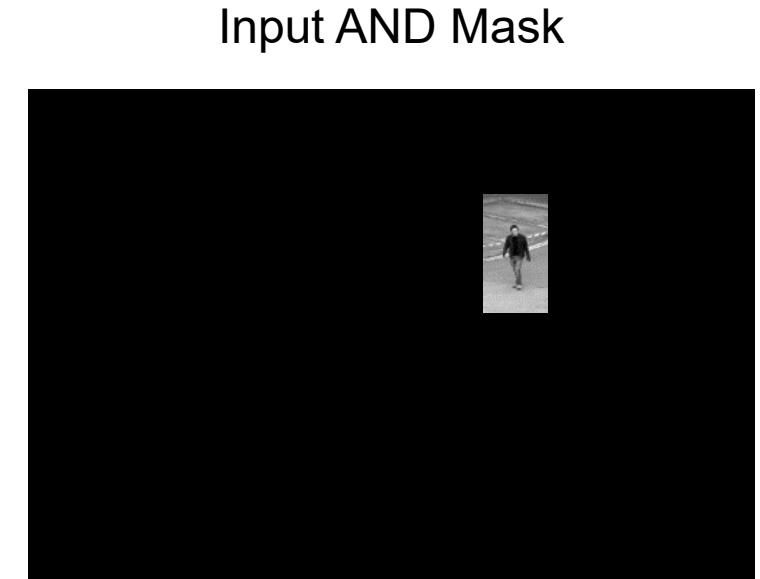
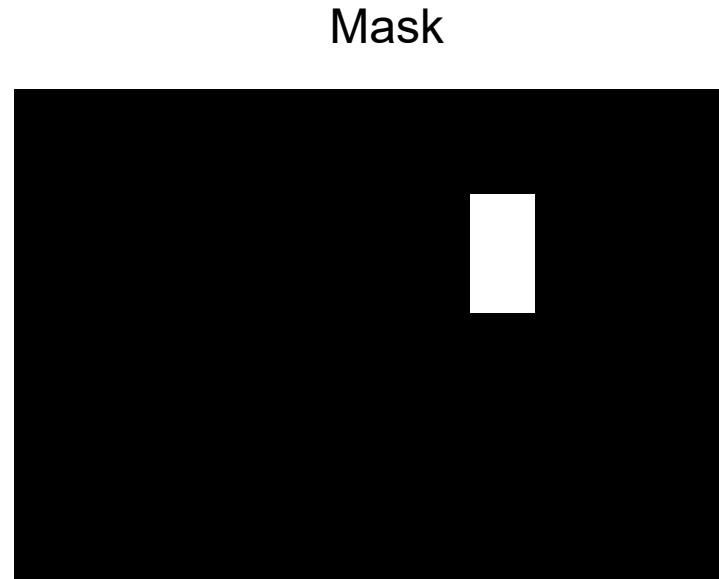
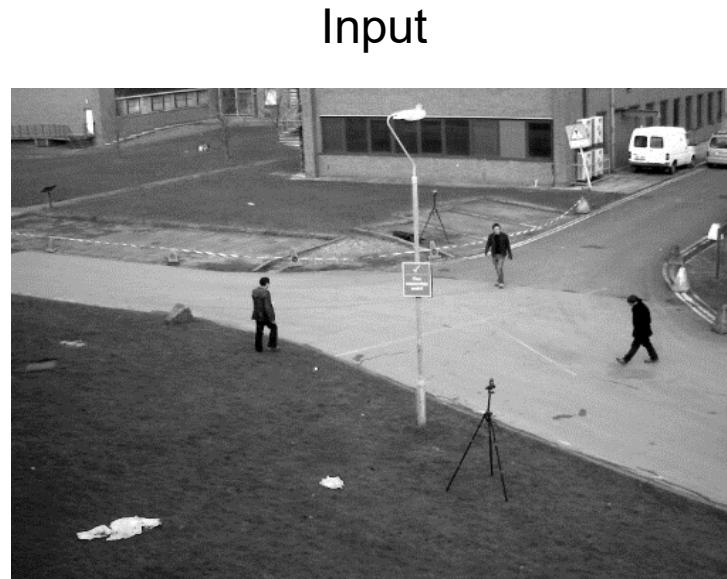
Arithmetic and logical operations

- Useful arithmetic operations include addition and subtraction



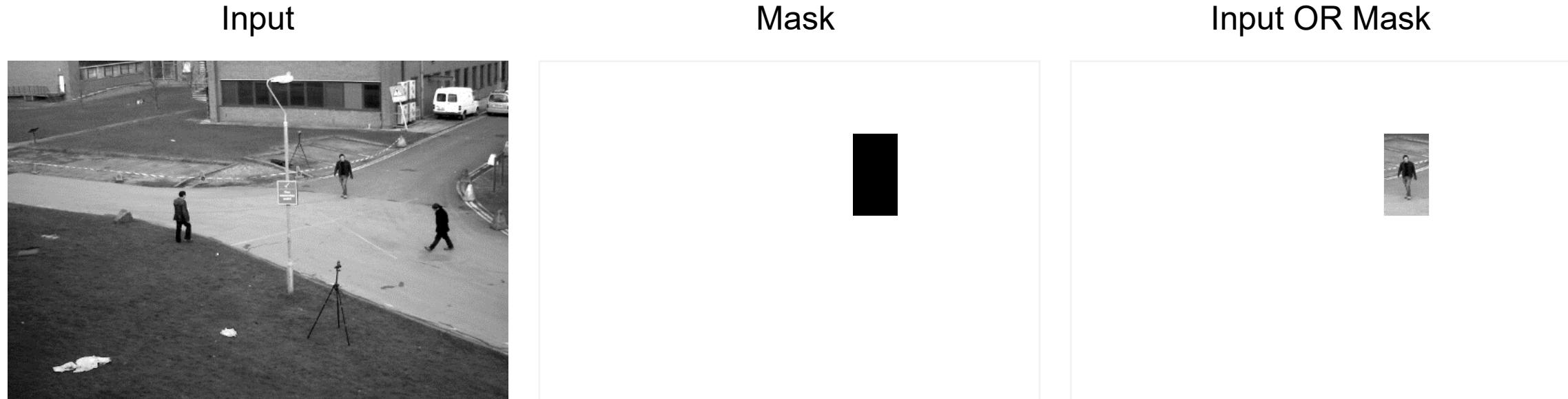
Arithmetic and logical operations

- Useful logical operations include bitwise AND and OR



Arithmetic and logical operations

- Useful logical operations include bitwise AND and OR



Averaging

- Useful for example to reduce noise in images

Assume the true noise-free image is $g(x, y)$ and the actual observed images are $f_i(x, y) = g(x, y) + n_i(x, y)$ for $i = 1, \dots, N$, where the n_i are zero-mean, independent and identically distributed (i.i.d.) noise images, then we have $E[f_i(x, y)] = g(x, y)$ and $\text{VAR}[f_i(x, y)] = \text{VAR}[n_i(x, y)] = \sigma^2(x, y)$

$$\rightarrow \bar{f}(x, y) = \frac{1}{N} \sum_{i=1}^N f_i(x, y) = \frac{1}{N} \sum_{i=1}^N [g(x, y) + n_i(x, y)] = g(x, y) + \frac{1}{N} \sum_{i=1}^N n_i(x, y)$$

$$\rightarrow \text{VAR}\left[\frac{1}{N} \sum_{i=1}^N n_i(x, y)\right] = \frac{1}{N^2} \sum_{i=1}^N \text{VAR}[n_i(x, y)] = \frac{1}{N^2} N \sigma^2(x, y) = \frac{\sigma^2(x, y)}{N}$$

Averaging

- Useful for example to reduce noise in images



$N = 1$

σ

$N = 8$

$\sigma/2.8$

$N = 16$

$\sigma/4$

$N = 64$

$\sigma/8$

$N = 128$

$\sigma/11.3$

Further reading on discussed topics

- Sections 3.1-3.3 of Szeliski
- Chapter 3 of Gonzalez and Woods 2002

Acknowledgement

- Some images drawn from the mentioned resources