Filter and its Applications

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日期不对

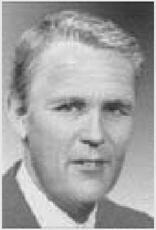
Quick Overview

为啥这里啥都没有

仍然没有

R. E. Kalman

我不明白，为什么非要放在这里，跟前边衔接不上，后边也接不上



Born 1930 in Hungry
Studied at MIT/Columbia
Developed filter in 1960/61

His passing not only brought about personal loss but also a sad reminder of the passing of a golden era in systems and control.

还是应该加上是谁说的，然后加个框吧

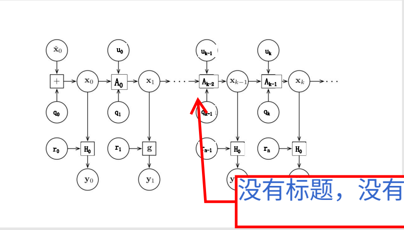
State Space Model

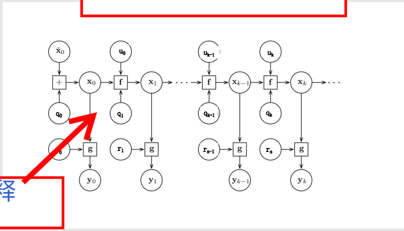
后边对应的Nonlinear Process这里应该写Linear process吧

不是说我错了吗，为啥没有改过来啊

process	Linear Gaussian	Nonlinear Gaussian
motion model	$\mathbf{A}_k\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{q}_k$	$\mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{q}_k)$
observation model	$\mathbf{H}_k\mathbf{x}_k + \mathbf{r}_k$	$\mathbf{g}(\mathbf{x}_k, \mathbf{r}_k)$
Estimation	kalman Filter	EKF, particle Filter

还有这里





没有标题，没有解释

State Space Model

从这里到后边都是线性模型，这个方程放在introduction有问题啊

$$\begin{cases} \mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \\ \mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{r}_k \end{cases} \tag{1}$$

State Space Model

这里跟下边明显是同一个方程，然而标号不同，去掉吧

应该解释k

$$\begin{cases} \mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \\ \mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{r}_k \end{cases} \quad (1)$$

这个下标应该是k

system state: $\mathbf{x}_k \in \mathbb{R}^N$

input state: $\mathbf{u}_k \in \mathbb{R}^N$

process noise: $\mathbf{q}_k \in \mathbb{R}^N \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_0)$

measurement: $\mathbf{y}_k \in \mathbb{R}^M$

measurement noise: $\mathbf{r}_k \in \mathbb{R}^M \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_0)$

这个下标应该是k

State Space Model

$$\begin{cases} \mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \\ \mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{r}_k \end{cases} \tag{2}$$

State Space Model

$$
\begin{cases}
\mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \\
\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{r}_k
\end{cases}
\tag{2}
$$

$$
\begin{aligned}
\text{transition matrix:} \quad & \mathbf{A}_k \in \mathbb{R}^{N \times N} \\
\text{control-input matrix:} \quad & \mathbf{B}_k \in \mathbb{R}^{N \times N} \\
\text{observation matrix:} \quad & \mathbf{H}_k \in \mathbb{R}^{M \times N}
\end{aligned}
$$

Filter Hypothesis

这里跟第五页有什么区别？而且放在这里也不合适啊

1. Kalman filter hypothesis:
   - The dynamic system is linear.
   - The noise is a Gaussian distribution.
   - Posterior probability is a Gaussian distribution.
2. The expansion of Kalman filter:
   - Non-linear: EKF, UKF.
   - Non-linear, Non-Gaussian: Particle Filter.

Kalman Filter

Predict

$$
\begin{aligned}
\text{Predicted state estimate} \quad & \hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \\
\text{Predicted estimate covariance} \quad & \mathbf{P}_{k|k-1} = \mathbf{A}_k \mathbf{P}_{k-1|k-1} \mathbf{A}_k^{\mathrm{T}} + \mathbf{Q}_k \\
\text{measurement residual} \quad & \tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}
\end{aligned}
$$

P没有解释，主要是针对P0来说

Kalman Filter

Predict

$$
\begin{aligned}
\text{Predicted state estimate} \quad & \hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \\
\text{Predicted estimate covariance} \quad & \mathbf{P}_{k|k-1} = \mathbf{A}_k \mathbf{P}_{k-1|k-1} \mathbf{A}_k^{\mathrm{T}} + \mathbf{Q}_k \\
\text{measurement residual} \quad & \tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1}
\end{aligned}
$$

Update

$$
\begin{aligned}
\text{residual covariance} \quad & \mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k \\
\text{"Optimal" Kalman gain} \quad & \mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^{\mathrm{T}} \mathbf{S}_k^{-1} \\
\text{Updated state estimate} \quad & \hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k \\
\text{Updated estimate covariance} \quad & \mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1}
\end{aligned}
$$

Nonlinear Process

$$\begin{cases} \mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{q}_k) \\ \mathbf{y}_k = \mathbf{g}(\mathbf{x}_k, \mathbf{r}_k) \end{cases} \tag{3}$$

$$\text{transition model:} \quad \mathbf{f}$$

$$\text{observation model:} \quad \mathbf{g}$$

Nonlinear Process

$$
\begin{cases}
\mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{q}_k) \\
\mathbf{y}_k = \mathbf{g}(\mathbf{x}_k, \mathbf{r}_k)
\end{cases} \tag{3}
$$

$$
\begin{aligned}
\text{transition model:} &\quad \mathbf{f} \\
\text{observation model:} &\quad \mathbf{g}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{q}_k) &\approx \check{\mathbf{x}}_k + \mathbf{A}_{k-1}(\mathbf{x}_{k-1} - \check{\mathbf{x}}_{k-1}) + \mathbf{q}'_k \\
\mathbf{g}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{r}_k) &\approx \check{\mathbf{y}}_k + \mathbf{H}_k(\mathbf{x}_k - \check{\mathbf{x}}_k) + \mathbf{r}'_k
\end{aligned} \tag{4}
$$

Extended Kalman Filter

Predict

$$\text{Predicted state estimate} \quad \hat{\mathbf{x}}_{k|k-1} = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{q}_k)$$

$$\text{Predicted estimate covariance} \quad \mathbf{P}_{k|k-1} = \mathbf{A}_k \mathbf{P}_{k-1|k-1} \mathbf{A}_k^{\mathrm{T}} + \mathbf{Q}_k'$$

$$\text{measurement residual} \quad \tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{g}(\mathbf{x}_k, \mathbf{r}_k)$$

这里为啥加上一个撇，虽然是我写的，但是你应该补充一下吧

Extended Kalman Filter

Predict

$$\begin{aligned} \text{Predicted state estimate} \quad \hat{\mathbf{x}}_{k|k-1} &= \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{q}_k) \\ \text{Predicted estimate covariance} \quad \mathbf{P}_{k|k-1} &= \mathbf{A}_k \mathbf{P}_{k-1|k-1} \mathbf{A}_k^{\mathrm{T}} + \mathbf{Q}'_k \\ \text{measurement residual} \quad \tilde{\mathbf{y}}_k &= \mathbf{z}_k - \mathbf{g}(\mathbf{x}_k, \mathbf{r}_k) \end{aligned}$$

Update

$$\begin{aligned} \text{residual covariance} \quad \mathbf{S}_k &= \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k \\ \text{"Optimal" Kalman gain} \quad \mathbf{K}_k &= \mathbf{P}_{k|k-1} \mathbf{H}_k^{\mathrm{T}} \mathbf{S}_k^{-1} \\ \text{Updated state estimate} \quad \hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k \\ \text{Updated estimate covariance} \quad \mathbf{P}_{k|k} &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} \end{aligned}$$

Particle Filter

Other names of particle filters:

写这里有啥意义？有些以偏概全，有些只是类似，用other name有问题吧

- Condensation Algorithms
- Sequential sampling-importance re-sampling(SIR)
- Bootstrap Filtering
- Interacting particle approximations
- Survival of the fittest
- ......

Particle Filter-History

- **First attempts-simulations of growing polymers**
  - M. N. Rosenbluth and A.W. Rosenbluth, "Monte Carlo calculation of the average extension of molecular chains," Journal of Chemical Physics, vol. 23, no. 2, pp. 356-359, 1956.

另外一个E去哪了？

这个会议就叫F吗？再确认一下吧。

- **First application in signal processing-1993**
  - N. J. Gordon, D. J. Salmond, and A. F. M. Smith, "Novel approach to nonlinear/non-Gaussian Bayesian state estimation,"IEE Proceedings-F, vol. 140, no. 2, pp. 107-113, 1993.

这个横杠是干啥的

Particle Filter

- Sequential Monte Carlo Algorithm Based on Bayesian Criterion
- Looking for a set of discrete sample points to approximate the probability density function

Particle Filter-Monte Carlo

- Estimate integral values (p is a gaussian distribution)

$$v = \int_{-\infty}^{+\infty} x^2 p(x)\,dx$$

这时高斯分布吗，为啥要挑个高斯分布作为例子？多容易混淆啊！

- Monte Carlo:
  1. Simulate M random variables

$$x^{(m)} \sim N(0, \sigma^2)$$

  2. Calculate the mean

$$v = \frac{1}{M}\sum_{m=1}^{M} (x^{(m)})^2$$

Particle Filter-Importance sampling

- Integral calculation

$$E(f(x)) = \int_{-\infty}^{+\infty} f(x)p(x)dx = \int_{-\infty}^{+\infty} f(x)\frac{p(x)}{\pi(x)}\pi(x)dx$$

- Monte Carlo:
  1. Simulate M random variables $(\pi(x))$

$$x^{(m)} \sim \pi(x)$$

  2. Calculate the mean

这那是mean，不是期望吗

$$E(f(x)) \approx \frac{1}{M}\sum_{1}^{M} f(x^{(m)}) \underbrace{\frac{p(x^{(m)})}{\pi(x^{(m)})}}_{w^{(m)}}$$

Particle Filter-Importance sampling

突然出现了g，g是什么

- Integral calculation

$$E(g_t(x_{0:t})) = \int g_t(x_{0:t})p(x_{0:t}|y_{1:t})dx_{0:t}$$

$$E(g_t(x_{0:t})) = \int g_t(x_{0:t})\frac{p(x_{0:t}|y_{1:t})}{q(x_{0:t}|y_{1:t})}q(x_{0:t}|y_{1:t})dx_{0:t}$$

- Monte Carlo:

突然出现一个Sigma

1. Simulate M random variables $(q(x))$
2. Calculate the mean

波浪号是干啥的

$$E(g_t(x_{0:t})) = \frac{\frac{1}{M}\sum g_t(x_{0:t}^{(i)})w_t(x_{0:t}^{(i)})}{\frac{1}{M}\sum w_t(x_{0:t}^{(i)})} = \sum^{M} g_t(x^{(i)}_{0:t})\tilde{w}(x^{(i)}_t)$$

为什么会有一个d

这个公式没有显示完整，不能缩小一点吗

$$\hat{p}(x_{0:t}|y_{1:t}) = \sum^{M}\tilde{w}_t^{(i)}\sigma_{x_{0:t}^{(i)}}(dx_{0:t})$$

Particle Filter-Sequential importance sampling

$$p(x_{0:t}) = p(x_0)\prod_{j=1}^{t} p(x_j|x_{j-1}),\quad p(y_{1:t}|x_{0:t}) = \prod_{j=1}^{t} p(y_j|x_j)$$

为啥要用粗体，

1. **Predicted state estimate** $x_k^{(m)} = p(x_k|x_{k-1})$

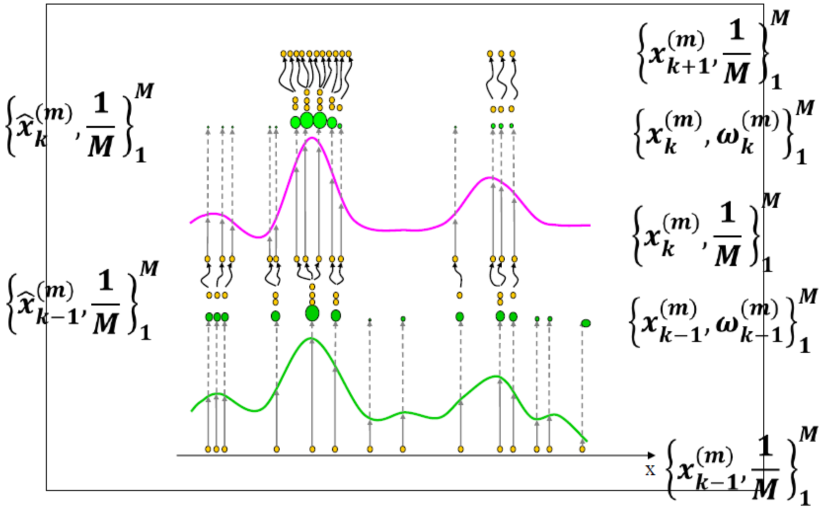
2a. **Weight computation** $w_k^{*(m)} = w_{k-1}^{*(m)} p(y_k|x_k^{(m)})$

2b. **Weight normalization** $w_k^{(m)} = \dfrac{w_k^{*(m)}}{\sum_{m=1}^{M} w_k^{*(m)}}$

3. **Estimate computation** $E(g(x_k|y_{1:k})) = \sum_{m=1}^{M} g(x_k^{(m)}) w_k^{(m)}$

什么叫做估计计算？

Particle Filter-Resampling



Extended Kalman Filter

这里为啥是EKF，是PF吧

Advantages

- Wide adaptability

Extended Kalman Filter

这里为啥是EKF，是PF吧

Advantages

就一个优点，说的过去吗，还放在最后讲

- Wide adaptability

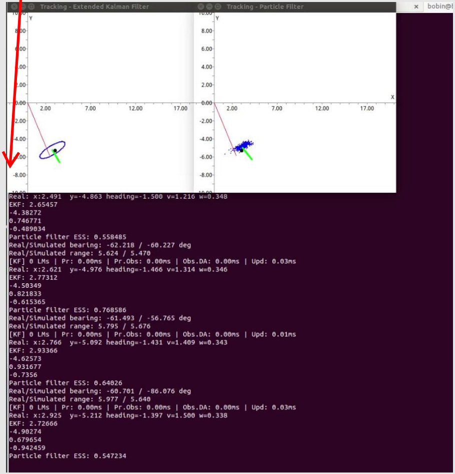
Disadvantages

讲pf的优缺点，跟谁对比呢？前边讲了为什么不跟kalman、EKF对比

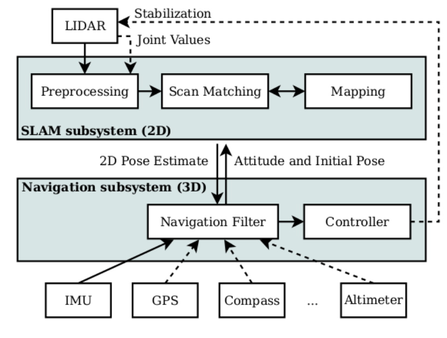
- High computational complexity
- The number of particles
- The selection of important density function

Tracking

没有标题，没有解释



Hector SLAM



- Kohlbrecher S, Von Stryk O, Meyer J, et al. A flexible and scalable slam system with full 3d motion estimation[C]//Safety, Security, and Rescue Robotics (SSRR), 2011 IEEE International Symposium on. IEEE, 2011: 155-160.

Hector SLAM

3D state

$$\mathbf{x} = \begin{bmatrix} \Omega^T & \mathbf{p}^T & \mathbf{v}^T \end{bmatrix}$$

where

$$\begin{aligned} \Omega &= [\phi, \theta, \varphi] & \text{roll, pitch and yaw Euler angles} \\ \mathbf{p} &= [\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z] & \text{posotion} \\ \mathbf{v} &= [\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z] & \text{velocity} \end{aligned}$$

- Kohlbrecher S, Von Stryk O, Meyer J, et al. A flexible and scalable slam system with full 3d motion estimation[C]//Safety, Security, and Rescue Robotics (SSRR), 2011 IEEE International Symposium on. IEEE, 2011: 155-160.

Hector SLAM

Dynamic system

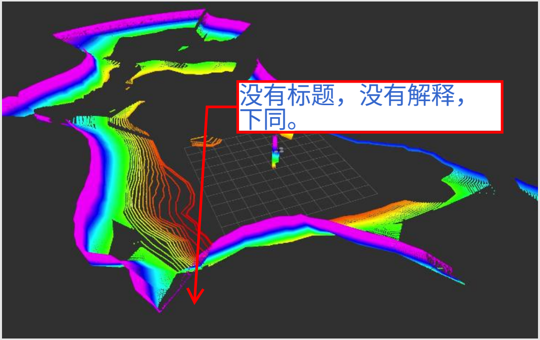
$$\dot{\Omega} = \mathbf{E}_\omega \cdot \omega$$
$$\dot{\mathbf{p}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = \mathbf{R}_\omega \cdot \mathbf{a} + \mathbf{g}$$

where

$\mathbf{R}_\omega$	Rotation matrix from Sensor to world
$\mathbf{E}_\omega$	maps angular rates to the derivatives of the Euler angles
$\mathbf{g}$	constant gravity vector

- Kohlbrecher S, Von Stryk O, Meyer J, et al. A flexible and scalable slam system with full 3d motion estimation[C]//Safety, Security, and Rescue Robotics (SSRR), 2011 IEEE International Symposium on. IEEE, 2011: 155-160.

Hector SLAM



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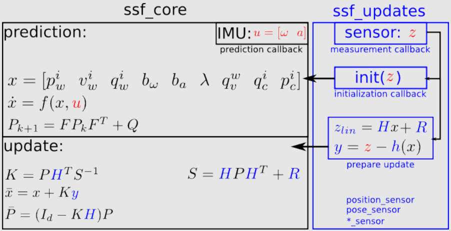
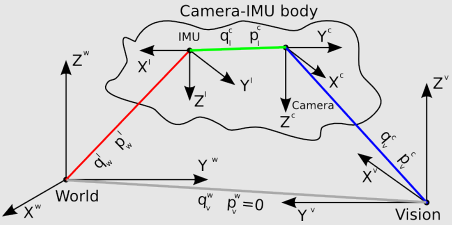
Multi-Sensor Fusion





Stephan Weiss, Markus W. Achtelik, Margarita Chli and Roland Siegwart. Versatile Distributed Pose Estimation and Sensor Self-Calibration for Autonomous MAVs. in IEEE

Multi-Sensor Fusion

- Stephan Weiss, Markus W. Achtelik, Margarita Chli and Roland Siegwart. Versatile Distributed Pose Estimation and Sensor Self-Calibration for Autonomous MAVs. in IEEE International Conference on Robotics and Automation (ICRA), 2012. pdf
- Simon Lynen, Markus Achtelik, Stephan Weiss, Margarita Chli and Roland Siegwart, A Robust and Modular Multi-Sensor Fusion Approach Applied to MAV Navigation. in Proc. of the IEEE/RSJ Conference on Intelligent Robots and Systems (IROS), 2013.

Thank you