Filter and its Applications

Tian Chen

Institute of Automation, Chinese Academy of Sciences
University of Chinese Academy of Sciences

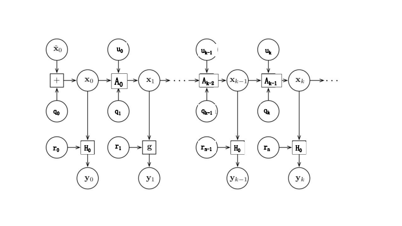
2017.05.13

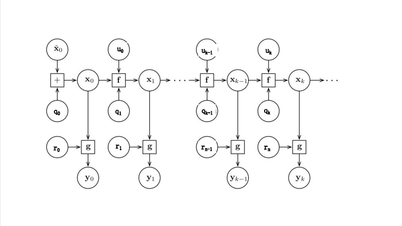
Quick Overview

1. Introduction
2. Filter
   - Linear Gaussian Process
   - Nonlinear Gaussian Process
3. Applications
   - SLAM (Simultaneous Localization And Mapping)
   - Inertial Navigation
   - Tracking

Introduction

process	Linear Gaussian	Nonline Gaussian
motion model	$\mathbf{A}_k\mathbf{x}_{k-1} + \mathbf{B}_k\mathbf{u}_k + \mathbf{q}_k$	$\mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{q}_k)$
observation model	$\mathbf{H}_k\mathbf{x}_k + \mathbf{r}_k$	$\mathbf{g}(\mathbf{x}_k, \mathbf{r}_k)$
Estimation	kalman Filter	EKF, particle Filter





1. Introduction

2. Filter
   - Linear Gaussian Process
   - Nonlinear Gaussian Process

3. Applications
   - SLAM (Simultaneous Localization And Mapping)
   - Inertial Navigation
   - Tracking

State Space Model

$$
\begin{cases}
\mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \\
\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{r}_k
\end{cases}
\tag{1}
$$

State Space Model

$$
\begin{cases}
\mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \\
\mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{r}_k
\end{cases} \tag{1}
$$

$$
\begin{aligned}
\text{system state:} \quad & \mathbf{x}_k \in \mathbb{R}^N \\
\text{input state:} \quad & \mathbf{u}_k \in \mathbb{R}^N \\
\text{process noise:} \quad & \mathbf{q}_k \in \mathbb{R}^N \sim \mathcal{N}(\mathbf{0}, \mathbf{Q}_0) \\
\text{measurement:} \quad & \mathbf{y}_k \in \mathbb{R}^M \\
\text{measurement noise:} \quad & \mathbf{r}_k \in \mathbb{R}^M \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_0)
\end{aligned}
$$

State Space Model

$$\begin{cases} \mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \\ \mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{r}_k \end{cases} \tag{2}$$

State Space Model

$$
\begin{cases} \mathbf{x}_k = \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \\ \mathbf{y}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{r}_k \end{cases} \tag{2}
$$

$$
\begin{aligned}
\text{transition matrix:} \quad & \mathbf{A}_k \in \mathbb{R}^{N \times N} \\
\text{control-input matrix:} \quad & \mathbf{B}_k \in \mathbb{R}^{N \times N} \\
\text{observation matrix:} \quad & \mathbf{H}_k \in \mathbb{R}^{M \times N}
\end{aligned}
$$

Kalman Filter

Predict

$$\begin{aligned} \text{Predicted state estimate} \quad & \hat{\mathbf{x}}_{k|k-1} = \mathbf{A}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \\ \text{Predicted estimate covariance} \quad & \mathbf{P}_{k|k-1} = \mathbf{A}_k \mathbf{P}_{k-1|k-1} \mathbf{A}_k^{\mathrm{T}} + \mathbf{Q}_k \\ \text{measurement residual} \quad & \tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1} \end{aligned}$$

Kalman Filter

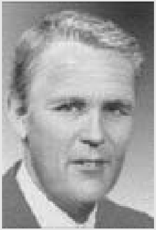
Predict

$$\begin{aligned} \text{Predicted state estimate} \quad \hat{\mathbf{x}}_{k|k-1} &= \mathbf{A}_k \hat{\mathbf{x}}_{k-1|k-1} + \mathbf{B}_k \mathbf{u}_k + \mathbf{q}_k \\ \text{Predicted estimate covariance} \quad \mathbf{P}_{k|k-1} &= \mathbf{A}_k \mathbf{P}_{k-1|k-1} \mathbf{A}_k^{\mathrm{T}} + \mathbf{Q}_k \\ \text{measurement residual} \quad \tilde{\mathbf{y}}_k &= \mathbf{z}_k - \mathbf{H}_k \hat{\mathbf{x}}_{k|k-1} \end{aligned}$$

Update

$$\begin{aligned} \text{residual covariance} \quad \mathbf{S}_k &= \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k \\ \text{"Optimal" Kalman gain} \quad \mathbf{K}_k &= \mathbf{P}_{k|k-1} \mathbf{H}_k^{\mathrm{T}} \mathbf{S}_k^{-1} \\ \text{Updated state estimate} \quad \hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k \\ \text{Updated estimate covariance} \quad \mathbf{P}_{k|k} &= (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1} \end{aligned}$$

R. E. Kalman



Born 1930 in Hungry
Studied at MIT/Columbia
Developed filter in 1960/61

His passing not only brought about personal loss but also a sad reminder of the passing of a golden era in systems and control.

Nonlinear Gaussian Process

$$\begin{cases} \mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{q}_k) \\ \mathbf{y}_k = \mathbf{g}(\mathbf{x}_k, \mathbf{r}_k) \end{cases} \tag{3}$$

$$\begin{aligned} \text{transition model:} \quad & \mathbf{f} \\ \text{observation model:} \quad & \mathbf{g} \end{aligned}$$

Nonlinear Gaussian Process

$$\begin{cases} \mathbf{x}_k = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{q}_k) \\ \mathbf{y}_k = \mathbf{g}(\mathbf{x}_k, \mathbf{r}_k) \end{cases} \tag{3}$$

$$\text{transition model:} \quad \mathbf{f}$$

$$\text{observation model:} \quad \mathbf{g}$$

$$\begin{aligned} \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{q}_k) &\approx \check{\mathbf{x}}_k + \mathbf{A}_{k-1}(\mathbf{x}_{k-1} - \check{\mathbf{x}}_{k-1}) + \mathbf{q}'_k \\ \mathbf{g}(\mathbf{x}_k, \mathbf{u}_k, \mathbf{r}_k) &\approx \check{\mathbf{y}}_k + \mathbf{H}_k(\mathbf{x}_k - \check{\mathbf{x}}_k) + \mathbf{r}'_k \end{aligned} \tag{4}$$

Extended Kalman Filter

Predict

$$
\begin{aligned}
\text{Predicted state estimate} \quad & \hat{\mathbf{x}}_{k|k-1} = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{q}_k) \\
\text{Predicted estimate covariance} \quad & \mathbf{P}_{k|k-1} = \mathbf{A}_k \mathbf{P}_{k-1|k-1} \mathbf{A}_k^{\mathrm{T}} + \mathbf{Q}'_k \\
\text{measurement residual} \quad & \tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{g}(\mathbf{x}_k, \mathbf{r}_k)
\end{aligned}
$$

Extended Kalman Filter

Predict

$$
\begin{aligned}
\text{Predicted state estimate} \quad & \hat{\mathbf{x}}_{k|k-1} = \mathbf{f}(\mathbf{x}_{k-1}, \mathbf{u}_k, \mathbf{q}_k) \\
\text{Predicted estimate covariance} \quad & \mathbf{P}_{k|k-1} = \mathbf{A}_k \mathbf{P}_{k-1|k-1} \mathbf{A}_k^{\mathrm{T}} + \mathbf{Q}'_k \\
\text{measurement residual} \quad & \tilde{\mathbf{y}}_k = \mathbf{z}_k - \mathbf{g}(\mathbf{x}_k, \mathbf{r}_k)
\end{aligned}
$$

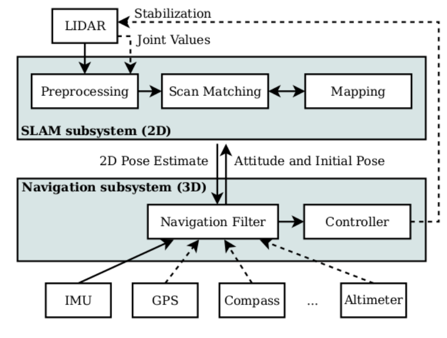
Update

$$
\begin{aligned}
\text{residual covariance} \quad & \mathbf{S}_k = \mathbf{H}_k \mathbf{P}_{k|k-1} \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k \\
\text{"Optimal" Kalman gain} \quad & \mathbf{K}_k = \mathbf{P}_{k|k-1} \mathbf{H}_k^{\mathrm{T}} \mathbf{S}_k^{-1} \\
\text{Updated state estimate} \quad & \hat{\mathbf{x}}_{k|k} = \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k \tilde{\mathbf{y}}_k \\
\text{Updated estimate covariance} \quad & \mathbf{P}_{k|k} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_{k|k-1}
\end{aligned}
$$

Particle Filter

1. Introduction
2. Filter
   - Linear Gaussian Process
   - Nonlinear Gaussian Process
3. Applications
   - SLAM (Simultaneous Localization And Mapping)
   - Inertial Navigation
   - Tracking

Hector SLAM



- Kohlbrecher S, Von Stryk O, Meyer J, et al. A flexible and scalable slam system with full 3d motion estimation[C]//Safety, Security, and Rescue Robotics (SSRR), 2011 IEEE International Symposium on. IEEE, 2011: 155-160.

Hector SLAM

3D state

$$\mathbf{x} = \begin{bmatrix} \Omega^T & \mathbf{p}^T & \mathbf{v}^T \end{bmatrix}$$

where

$$\begin{aligned} \Omega &= [\phi, \theta, \varphi] && \text{roll, pitch and yaw Euler angles} \\ \mathbf{p} &= [\mathbf{p}_x, \mathbf{p}_y, \mathbf{p}_z] && \text{posotion} \\ \mathbf{v} &= [\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z] && \text{velocity} \end{aligned}$$

- Kohlbrecher S, Von Stryk O, Meyer J, et al. A flexible and scalable slam system with full 3d motion estimation[C]//Safety, Security, and Rescue Robotics (SSRR), 2011 IEEE International Symposium on. IEEE, 2011: 155-160.

Hector SLAM

Dynamic system

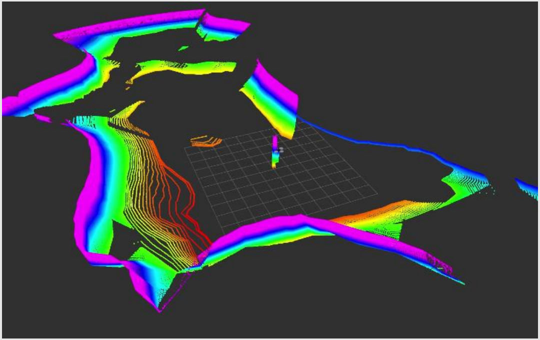
$$\dot{\Omega} = \mathbf{E}_\omega \cdot \omega$$
$$\dot{\mathbf{p}} = \mathbf{v}$$
$$\dot{\mathbf{v}} = \mathbf{R}_\omega \cdot \mathbf{a} + \mathbf{g}$$

where

$\mathbf{R}_\omega$	Rotation matrix from Sensor to world
$\mathbf{E}_\omega$	maps angular rates to the derivatives of the Euler angles
$\mathbf{g}$	constant gravity vector

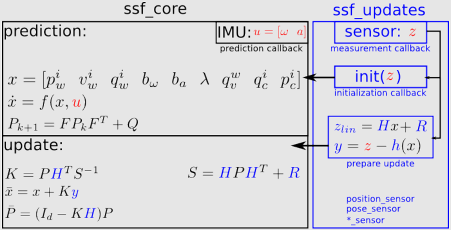
- Kohlbrecher S, Von Stryk O, Meyer J, et al. A flexible and scalable slam system with full 3d motion estimation[C]//Safety, Security, and Rescue Robotics (SSRR), 2011 IEEE International Symposium on. IEEE, 2011: 155-160.

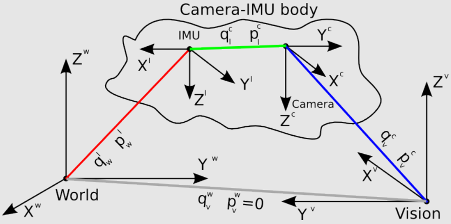
Hector SLAM



- Kohlbrecher S, Von Stryk O, Meyer J, et al. A flexible and scalable slam system with full 3d motion estimation[C]//Safety, Security, and Rescue Robotics (SSRR), 2011 IEEE International Symposium on. IEEE, 2011: 155-160.

Multi-Sensor Fusion



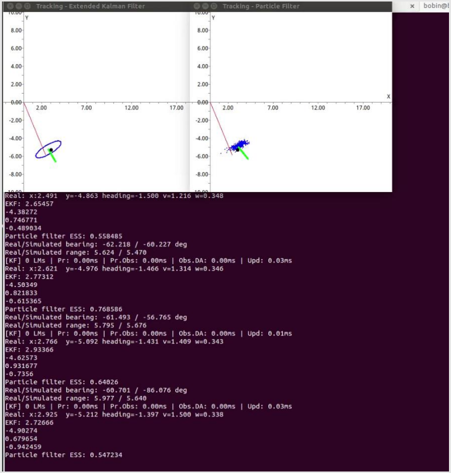


- Stephan Weiss, Markus W. Achtelik, Margarita Chli and Roland Siegwart. Versatile Distributed Pose Estimation and Sensor Self-Calibration for Autonomous MAVs. in IEEE

Multi-Sensor Fusion

- Stephan Weiss, Markus W. Achtelik, Margarita Chli and Roland Siegwart. Versatile Distributed Pose Estimation and Sensor Self-Calibration for Autonomous MAVs. in IEEE International Conference on Robotics and Automation (ICRA), 2012. pdf
- Simon Lynen, Markus Achtelik, Stephan Weiss, Margarita Chli and Roland Siegwart, A Robust and Modular Multi-Sensor Fusion Approach Applied to MAV Navigation. in Proc. of the IEEE/RSJ Conference on Intelligent Robots and Systems (IROS), 2013.

Tracking


Tracking - Extended Kalman Filter
Tracking - Particle Filter
bobin@
Real: x:2.491 y=-4.863 heading=-1.500 v=1.216 w=0.348
EKF: 2.65457
-4.38272
0.746771
-0.489034
Particle filter ESS: 0.558485
Real/Simulated bearing: -62.218 / -60.227 deg
Real/Simulated range: 5.624 / 5.470
[KF] 0 LMs | Pr: 0.00ms | Pr.Obs: 0.00ms | Obs.DA: 0.00ms | Upd: 0.03ms
Real: x:2.621 y=-4.976 heading=-1.466 v=1.314 w=0.346
EKF: 2.77312
-4.50349
0.821833
-0.615365
Particle filter ESS: 0.768586
Real/Simulated bearing: -61.493 / -56.765 deg
Real/Simulated range: 5.795 / 5.676
[KF] 0 LMs | Pr: 0.00ms | Pr.Obs: 0.00ms | Obs.DA: 0.00ms | Upd: 0.01ms
Real: x:2.766 y=-5.092 heading=-1.431 v=1.409 w=0.343
EKF: 2.93366
-4.62573
0.931677
-0.7356
Particle filter ESS: 0.64026
Real/Simulated bearing: -60.701 / -86.076 deg
Real/Simulated range: 5.977 / 5.640
[KF] 0 LMs | Pr: 0.00ms | Pr.Obs: 0.00ms | Obs.DA: 0.00ms | Upd: 0.03ms
Real: x:2.925 y=-5.212 heading=-1.397 v=1.500 w=0.338
EKF: 2.72666
-4.90274
0.679654
-0.942459
Particle filter ESS: 0.547234

Thank you