Information retrieval

Embeddings and ranking

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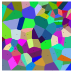
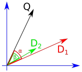
Today's outline

- Short summary of last lecture
- Embeddings
- Ranking

IR main steps





The tf-idf matrix

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Definition

The matrix M which rows – corresponding to each document – are:

$$D_t = \frac{\text{\# t in D}}{\text{\# tokens in D}} \times I(t)$$

is called the **tf-idf** (term frequency-inverse document frequency) representation.

Question

What are the advantages of the vector model ?

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Question

What are the advantages of the vector model ?

- Have a direct weightening by information carried by tokens
- Framework for latent semantics

Latent semantics: low rank approximation

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Theorem

Let M be the tf matrix: M_{ij} is the frequency of token j in document i. $M^\top M$ is symmetric and its eigenvectors are orthogonal and form a basis of the token space.

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What IR latent semantics tackles?

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Recent techniques (well, mostly since 2013)

Machine learning techniques can be used to **learn better vector representation**[a] **of tokens**, and more generally of any data (document, sentence, word, image, etc.).

[a]aka embeddings

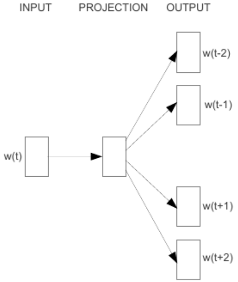
Embeddings: a general technique with many derivatives

Many models have been developed for representing various type of data. Here is a small list of freely available models:

Model	Data represented
`word2vec`	Tokens
`GloVe`	Tokens
`fastText`	Tokens
`doc2vec`	Documents
`dna2vec`	Genomic sequences

Word2vec: predict the context of a token

The core idea of word2vec is to learn a vector representation allows to predict the context of the token. Thereby, tokens appearing in similar context will be encoded closely in the vector space.



Skip-gram

[Mikolov, Tomas; et al. (2013)]

word2vec's latent semantics

The word2vec embeddings have interesting semantic features[1].

Table 8: *Examples of the word pair relationships, using the best word vectors from Table 4 (Skip-gram model trained on 783M words with 300 dimensionality).*

Relationship	Example 1	Example 2	Example 3
France - Paris	Italy: Rome	Japan: Tokyo	Florida: Tallahassee
big - bigger	small: larger	cold: colder	quick: quicker
Miami - Florida	Baltimore: Maryland	Dallas: Texas	Kona: Hawaii
Einstein - scientist	Messi: midfielder	Mozart: violinist	Picasso: painter
Sarkozy - France	Berlusconi: Italy	Merkel: Germany	Koizumi: Japan
copper - Cu	zinc: Zn	gold: Au	uranium: plutonium
Berlusconi - Silvio	Sarkozy: Nicolas	Putin: Medvedev	Obama: Barack
Microsoft - Windows	Google: Android	IBM: Linux	Apple: iPhone
Microsoft - Ballmer	Google: Yahoo	IBM: McNealy	Apple: Jobs
Japan - sushi	Germany: bratwurst	France: tapas	USA: pizza

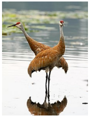
[1] Note that `GloVe` is better at this

Vector model: bright and dark side

The tf-idf vector model is good…

- Similarity based on information carried by tokens
- Flexible querying (latent semantics)
- Naturally rank documents
- Works well in practice

…but still not perfect:

- ignore polysemy  vs. 
- ignore the *truth* of the information 

Dealing with the *truth*

How to deal with the truth?

It is almost impossible to deal with truth judgment only from the document data.

However, we can assume that we trust information coming from *authorities* (well-known newspaper, official website, etc.).

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Idea

Rank the results of the querying system according to their authority.

How do we know who is the authority ?

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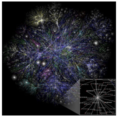
Idea

Rank the results of the querying system according to their authority.

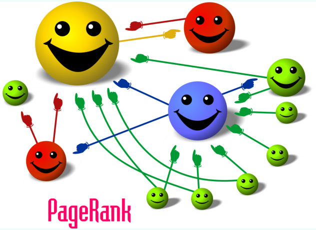
How do we know who is the authority ?

$\rightarrow$ We extract it from the web structure  !

Authority and web structure

Who is the authority?

If you only represent the web by a graph where each node is a web page and each directed edge is an HTML link.



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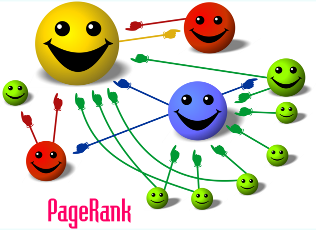

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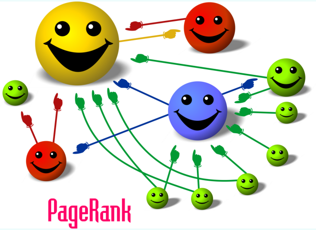

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Imagine an algorithm able to detect/rank authorities.

PageRank formalization (simple version)

Random surfer model

Imagine a user having the following behavior clicking on random links on the Internet.

The more links leading to a page, the more chance (and the more times) the user visits the page.

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The more links leading to a page, the more chance (and the more times) the user visits the page.
After a loooong time, we measure the average number of times the user visited a given page P, we denote R_P.

Definition of the rank according to PageRank

We define the authority/ranking of a page by the R_P value.

PageRank algorithm (simple version)

Data: $A :=$ graph of the WWW $\quad A_{ij} \quad = \quad \begin{cases} \frac{1}{N_j} & \text{if link from } j \text{ to } i \\ 0 & \text{else} \end{cases}$

Result: Ranking of web pages

$R_0 := S$;

repeat

$\quad R^{(i+1)} \leftarrow AR^{(i)}$

$\quad \delta \leftarrow ||R^{(i)} - R^{(i+1)}||_1$

until $\delta \leq \epsilon$;

Algorithm 1: simplified PageRank

Milestone of Google (algo designed by L. Page, Google co-founder), and drove the initial success of Google.

PageRank without sink effect

Sink effect

What if a page does not have any outgoing connection?

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The random eager surfer

Imagine the user having now the following behavior[a]

- click on a random link on the current web page with probability $p(t)$
- or jump to a random web page on the Internet with probability $1 - p(t)$

[a] In the original paper by Page, the balance between the two events is given by its trap feeling: the more trapped it gets, the more likely the user will jump somewhere else.

Full PageRank

To avoid a *sink* effect, we introduce random jumps to a set of pages encoded in E.

Data: Graph of the WWW

Result: Ranking of web pages

$R_0 := S$;

repeat

$R^{(i+1)} \leftarrow AR_i$

$d \leftarrow ||R^{(i)}||_1 - ||R^{(i+1)}||_1$

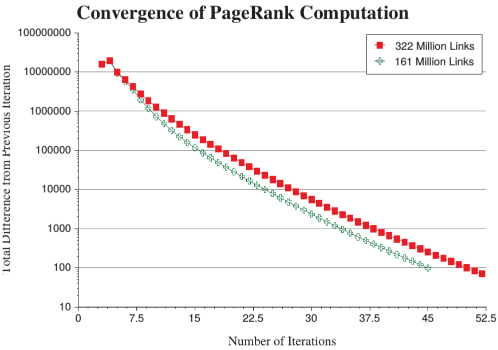
$R^{(i+1)} \leftarrow R^{(i+1)} + d.E$

$\delta \leftarrow ||R^{(i)} - R^{(i+1)}||_1$

until $\delta \leq \epsilon$;

Algorithm 2: PageRank

PageRank convergence



[L. Page, 98]

Full PageRank

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6 Personalized PageRank

An important component of the PageRank calculation is E – a vector over the Web pages which is used as a source of rank to make up for the rank sinks such as cycles with no outedges (see Section 2.4). However, aside from solving the problem of rank sinks, E turns out to be a powerful parameter to adjust the page ranks. Intuitively the E vector corresponds to the distribution of web pages that a random surfer periodically jumps to. As we see below, it can be used to give broad general views of the Web or views which are focussed and personalized to a particular individual.

...

Such personalized page ranks may have a number of applications, including personal search engines. These search engines could save users a great deal of trouble by efficiently guessing a large part of their interests given simple input such as their bookmarks or home page. We show an example of this in Appendix A with the "Mitchell" query. In this example, we demonstrate that while there are many people on the web named Mitchell, the number one result is the home page of a colleague of John McCarthy named John Mitchell.

Summary

- Tf-Idf vector representation of a document
- Flexible vector queries (cosine similarity)
- Latent semantics (lower rank projection of the tf matrix)
- PageRank

Next lecture

- Machine learning in IR
- Hamds-on (Finalizing your search engine)

Information function is unique up to a $\times$ constant

Let $a \in \mathbb{R}_+$ and $p \in \mathbb{N}$.
$f(a) = f(a^{\frac{q}{q}}) = f((a^{\frac{1}{q}})^q) = q.f(a^{\frac{1}{q}})$.
So for any $p, q \in \mathbb{N}$,

$$f(a^{\frac{p}{q}}) = \frac{p}{q} f(a)$$

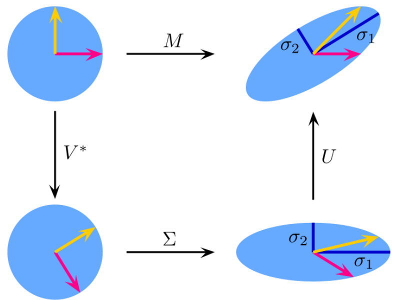
By density of $\mathbb{Q}$ in $\mathbb{R}$ and continuity of f, $f(a^x) = x.f(a)$.
If $f \neq 0$, there is a b such that $f(b) = 1$, so that $\forall x \in \mathbb{R}_+, f(b^x) = x$ so that $f = \log_b$

$||R_i||_1 < ||R_{i+1}||_1$ comes from sinks

If $\forall j$ there exists at least a page i and a link $j \rightarrow i$, then:

$$\begin{aligned} ||R_{i+1}||_1 &= ||A.R_i||_1 \\ &= \sum_i \sum_{j \rightarrow i} \frac{R_j}{N_j} \\ &= \ldots \\ &= 1 \end{aligned}$$

We can decompose a matrix as a composition of orthogonal operation, scaling and again orthogonal operation.



$$M = U \cdot \Sigma \cdot V^*$$

This decomposition is coined the Singular Value Decomposition (SVD).

Low rank approximation of the tf-idf matrix

Eckart-Young-Mirsky Theorem

Let $M \in \mathbb{R}^{d \times t}, t < d$. If $M = U\Sigma V^\top$ is the SVD decomposition of M with $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_t$, then the best[a] r-rank approximation of M is $(r < t)$:

$$\hat{M} := U_r \Sigma_{r,r} V_r^\top$$

where X_r is the restriction of X to the first r columns, and $\Sigma_{r,r}$ to the first r lines and columns.

[a] In the sense minimizing $||M - \hat{M}||_F = \sum_{i,j} (m_{i,j} - \hat{m}_{i,j})^2$