1a. $y = e^{-4x}$ $y' = -4e^{-4x}$ → $e^{ax} \to ae^{ax}$

b. $y = \sin(3x-1)$ $y' = 3\cos(3x-1)$ → $\sin x \to \cos x$ + chain rule

c $y = \frac{1}{\sqrt{x}}$ $y' = \frac{1}{2x^{\frac{3}{2}}}$ $\sqrt{x} \to x^{\frac{1}{2}} \to y' \frac{3}{2x^2}$

d $y = \pi^3$ $y' = 0$ π^3 is a constant

e. $y = \tan^{-1}(2x+1)$ $y' = $ ~~a tan (2x+1)~~ $\frac{1}{2x^2+2x+1}$ → $\tan^{-1}(ax+b) = \frac{a}{1+(ax+b)^2}$

f $y = \frac{1}{\sin(2x)} = \csc(2x)$ $\cot(2x)\csc(2x) \times 2$ + chain rule

↳ Simplify to $\csc(2x)$

2. a $y = 2x^4 - 2x^3$ $y' = 8x^3 - 6x^2$

b $x = 3\cos(8t) + 5e^{6t}$ $y' = -24\sin(8t) + 30e^{6t}$

c. $y = \sqrt{x} + \ln(x^{1/3})$ $y' = \frac{3x^{\frac{1}{2}}+2}{6x}$

$(fg)' = f'g + gf'$

3. a $y = \sin(3t)\sin(2t)$ $y' = \cos(3x)\,3\sin(2x) + \cos(2x)\sin(3x)$

b $y = \ln_2(2z)\sin z$ $y' = \frac{\sin(x)}{x} + \cos(x)\ln(2x)$

c $y = x^2 e^{3x}$ $y' = 2x^{3x} + e^{3x}3x^2$

d $y = 7t^2e^t\sin(2t)$ $y' = 7(2xe^x\sin(2x) + (e^x\sin(2x) + \cos(2x)\cdot 2e^x)x^2)$

4. a $y = \cos t/\sin t$ $y' = -\cos(x) - \cot(x)\csc(x)$

b $y = \ln x/\tan x$ $y' = \frac{\cot(x)}{x} - \csc^2(x)\ln(x)$

c $y = (x^3+x+2)/(x^2-x)$ $y' = -\frac{2(2x-1)}{x^2(x-1)^2}$

d $y = \cosh 2x / \sinh(x-1)$ $y' = 2\sinh(2x)\operatorname{csch}(x-1) - \coth(x-1)\operatorname{csch}(x-1)\cosh(2x)$

5. a $y = (t^3+1)^{111}$ $y' = 333(3x+1)^{110}$

b $y = 3e^{\cos t}$ $y' = -3e^{\cos(x)}\sin(x)$

c $y = e^{t^2}$ $y' = e^{t^2}2x$

d $y = \ln(x^3+2)$ $y' = \frac{3x^2}{x^3+2}$

6 $y' = 0$ for $y = e^{-t}t^2$ $y' = -e^{-t}t^2 + 2e^{-t}t$

$y' = 0$ at $t = 0$ or $t = 2$

7 $y = t^3 + t^2$, $y' = 3t^2 + 2t$ $y'' = 6t + 2$

8 $y = -t^3$, $y' = -3t^2$ $y'' = -6t$ $0 = -6t$ at $t = 0$

concave up at $y'' > 0$ at $x < 0$ } ~~...~~ in local domain

concave down at $y'' < 0$ at $x > 0$ }

9 max at $y'' < 0$ min at $y'' > 0$ → $y = x^4 - 3x^2$ $y' = 4x^3 - 6x$

min at $(-\sqrt{\frac{3}{2}}, -\frac{9}{4})$ and $(\sqrt{\frac{3}{2}}, -\frac{9}{4})$. max at $(0,0)$ $y'' = 12x^2 - 6$

$y' = 0$ at $0, \pm\sqrt{\frac{3}{2}}$

10 a. $\int (\sqrt{x})^7 dx = \sqrt{x}^6 \sqrt{x} \to x^3\sqrt{x} \to x^{\frac{7}{2}}$ Power rule $\frac{x^{\frac{7}{2}+1}}{\frac{7}{2}+1} + C$

b. $\int \frac{1}{3x^3} dx = \frac{1}{3} \cdot \int \frac{1}{x^3} dx = \frac{1}{3} \cdot \int x^{-3} = \frac{1}{3} \cdot \frac{x^{-3+1}}{-3+1} = \frac{-1}{6x^2} + C$

c: $\int e^{2x} dx = u = 2x \to \int e^u \frac{1}{2} du \to \frac{1}{2} e^u = \frac{1}{2} e^{2x} + C$

d. $\int (x^5 + 3x^4 + 4x) dx = \int x^5 dx + \int 3x^4 dx + \int 4 dx = \frac{x^6}{6} + \frac{3x^5}{5} + 2x^2 + C$

e. $\int 4\sin(5x+1) dx = u = 5x+1 \to 4\int \sin(u) \frac{1}{5} = -\frac{4}{5}\cos(5x+1) + C$

f. $\int \tan \frac{2t}{T} dx =$

$u = \frac{2t}{T} \to \int \frac{T\tan(u)}{2} du = \frac{T}{2} \cdot \int \tan(u) du$

$= \frac{T}{2} \cdot \int \frac{\sin(u)}{\cos(u)} du$

$v = \cos(u) \to \frac{T}{2} \left(\int -\frac{1}{v} dv \right)$

$= \frac{T}{2} \cdot -\left(\int \frac{1}{v} dv \right)$

$= \frac{T}{2} (-\ln|v|)$

$= \frac{T}{2} \left(-\ln\left|\cos\left(\frac{2t}{T}\right)\right| \right)$

$= -\frac{T}{2} \ln\left|\cos\left(\frac{2t}{T}\right)\right| + C$

11 a $\int_4^6 f(x) dx$

b $\int_1^4 4f(x) dx$

c $\int_1^6 (3f(x) - 2g(x)) dx$

d $\int_6^1 2f(x) dx$

$\int_1^4 f(x) dx = 3 = \mathrm{I}$ $\int_1^6 f(x) dx = 6 = \mathrm{II}$

$\int_1^6 g(x) dx = 5 = \mathrm{III}$

a. $= \mathrm{II} - \mathrm{I} = 6 - 3 = 3$ units

b. $= 4(\mathrm{II} - \mathrm{I}) = 4(6-3) = 12$ units

c. $= 3(\mathrm{II}) - 2(\mathrm{III}) = 3(6) - 2(5) = 18 - 10 = 8$ units

d. $= \cancel{18}\ 6$