Joneselis1 ENGR123 A6

1. $f_X(x) = \begin{cases} kx^2, & 1 \leq x \leq 4 \\ 0, & \text{otherwise} \end{cases}$

a. $k = \frac{1}{21} \rightarrow \int_1^4 kx^2\,dx$

$= k\left[\frac{x^{2+1}}{2+1}\right]_1^4 = k\left[\frac{x^3}{3}\right]_1^4$

$= k\left(\frac{4^3}{3} - \frac{1^3}{3}\right)$

$= k \times 21$

$k = \frac{1}{21}$

b. $P(x \leq 2)$

$1 \leq 2 \leq 4$

$f_X(2) = \frac{2^2}{21}$

$= \frac{4}{21}$

c. $P(x = 2)$

$1 \leq 2 \leq 4$

$f_X(2) = \frac{2^2}{21}$

$= \frac{4}{21}$

d. $E(x), E(x^2), Var(x)$

$E(x) = \int_1^4 x\,kx^2\,dx$ see

$= \int_1^4 kx^3\,dx$

$= k\left[\frac{x^4}{4}\right]_1^4 = k\left(\frac{4^4}{4} - \frac{1^4}{4}\right)$

$= k\left(256 - \frac{1}{4}\right)$

$= k\,\frac{255}{4}$

$= \frac{1}{21} \times \frac{255}{4} = \frac{85}{28}$

$\frac{85}{28} = x$

$E(x^2) = \int_1^4 x^2\,kx^2\,dx$

$= \int_1^4 kx^4$ see $\rightarrow$

$= k\left[\frac{x^5}{5}\right]_1^4 = k\left(\frac{4^5}{5} - \frac{1^5}{5}\right)$

$= k\,\frac{1023}{5}$

$= \frac{1}{21} \times \frac{1023}{5} = \frac{341}{35}$

$Var(x) = E(x^2) - E(x)^2 = \frac{341}{35} - \left(\frac{85}{28}\right)^2 = \frac{2067}{3920}$

2 $f_X(x) = \begin{cases} \frac{A}{x^4}, & x \geq 1 \\ 0, & x < 1 \end{cases}$

a. $a = 3$? $\int_1^\infty \frac{A}{x^4}\,dx \to \frac{A}{3x^3} \cancel{\ell} = 0 - \left(-\frac{A}{3}\right)$

$= 0 + \frac{A}{3}$

$= \frac{A}{3} \to A = 3$

b. $P(2 \leq x \leq 3)$

$= P(3) - P(2)$

$= \frac{A}{3^4} - \frac{A}{2^4}$

$= \frac{A}{81} - \frac{A}{16}$

$= -\frac{65}{1296}$

$\frac{-65}{1296} \times 4 = \frac{65}{1296} \times 3$

$= -\frac{65}{432}$

c. $E(x) = \int_1^\infty x \frac{A}{x^4}$

$= \int_1^\infty \frac{A}{x^3}$

$= 0 - \left(\frac{9}{2}\right)$

$= 0 + \frac{9}{2}$

$= \frac{9}{2}$

$E(x) = \frac{9}{2} = \frac{3}{2}$

d. $E(x^3)$ Can't solve like ↗ as it diverges for $\int_1^\infty x^3 \frac{A}{x^4}\,dx$

$\frac{A}{\left(\left(\frac{A}{2}\right)^3\right)^4} = \frac{A}{\frac{A^{12}}{2^{12}}} = \frac{A \cdot 2^{12}}{A^{12}} = \frac{2^{12}}{A^{13}} = \frac{4096}{A^{13}} \to A = 3 \to \frac{4096}{3^{13}}$

$= \frac{4096}{1594323} = \frac{2^{12}}{3^{13}}$

3. $\lambda = 5$ $\quad f_X(x) = \begin{cases} 5e^{-5x} & x \geq 0 \\ 0, & x < 0 \end{cases}$

a. find CDF $\int_0^{\infty} 5e^{-5x}\, dx$

b. $P(x \geq 10) = 5e^{-50} = \frac{5}{e^{50}} = 9.64E\text{-}22$

c. $E(x) = \int_0^{\infty} 5e^{-5x}\, dx = 0 - (-1) = \underline{1}$

4. $Y \sim Exp(\frac{1}{3}) \rightarrow f_x(x) = \begin{cases} 0, & x < 0 \\ 1 - e^{-\frac{1}{3}x}, & x \geq 0 \end{cases}$

$\cancel{1-e^{\frac{3}{3}} = 0.95021}$

a. $P(y \leq 3) = \cancel{1 - e^{\frac{3}{3}} = 1 - e^{-3} = 1 - \frac{1}{e^3} = 0.95021}$

b. $P(y > 4) = 1 - F(4) =$

$1 - (\frac{1}{3} e^{-\frac{4}{3}}) = 0.9121$

c. $P(2 < y < 6) = (1 - e^{-\frac{6}{3}}) - (1 - e^{-\frac{2}{3}}) = 0.37808$

d. Mean $E(x) = \int_0^{\infty} x \frac{1}{3} e^{-\frac{1}{3}x}\, dx = 3$

5. $x$ is normal $\mu = 20 =$ mean $\quad Var = \sigma^2 = 25$

$Z = \frac{x - \mu}{\sigma} \quad \sigma^2 = 25$ so $\sigma = 5$

a. $P(x \leq 16) = P(Z \leq \frac{16-20}{5}) = P(\frac{-4}{5}) = P(-0.8) =$ 0.7881

b. $P(x \leq 24) = P(Z \leq \frac{24-20}{5}) = P(Z \leq 0.8) =$ 0.2119

c. $P(17 \leq x \leq 23) = P(x \leq 23) - P(x \leq 17) = P(Z \leq \frac{23-20}{5}) - P(Z \leq \frac{17-20}{5})$

$= P(Z \leq \frac{3}{5}) - P(Z \leq -\frac{3}{5})$

$= P(Z \leq 0.6) - P(Z \leq -0.6)$

$(0.5 + 0.2257) - (0.5 - 0.2257) = 0.7257 - 0.2743 = 0.4514$

~~4c 8b~~

5D. $x$ when $p(\cancel{X} \geq x) = 0.4$

$p(Z \geq z) = 0.1$ where $Z = \frac{x - \mu}{\sigma}$

from ~~tables~~ inverse table where ~~p~~ = 0.1

$z = 0.2533$

$x = 0.2533 \times 5 + 20 = \underline{\underline{25.33}}$