ENGR121 Lab 1 2021
Introduction to Wireless Communications Engineering

A well engineered digital communications system enables:

1. A high data rate transmission over a communication channel between the transmitter and the receiver (think: high resolution youtube video on your smartphone).
2. The fewest possible bit errors at the receiver (think: no pixelated or choppy youtube videos).

The first goal is a high data transmission rate. Improvements in data rates have been astonishing. Consider the five generations of cell phone technology for example:

Cell phone data rates				
Generation	2G	3G	4G	5G
Introduction (approx)	1990	2000	2010	2020 expected
Max data rate (approx)	14.4 Kbps	2 Mbps	200 Mbps +	10 Gbps +

Note that's very roughly a factor of 100 each decade, much faster improvement than processor speeds.

Will 5G be "fast enough?" Of course not. We will always be seeking faster data transmission.

But is there an upper limit to the rate at which data can be transmitted over a communications channel? It turns out there is. Claude Shannon published a paper in 1948 that derived the channel capacity, or maximum theoretical data rate, for a communication link. This work can be thought of as the '$E = mc^2$ of communications,' as for the last 60+ years communications engineers have strived to design systems with channel capacity coming closer and closer to this limit. If you are enrolled in ENGR 101, you may recall discussion of the pioneering work of Claude Shannon in information theory, and the idea that information is the amount of surprise delivered. In this lab we are again using his ideas. Digital communication technology depends heavily on Shannon's work. And no, the year 1948 is not a typo. He thought through these ideas when most of your grandparents were young-ish and cell phones were not even science fiction yet.

The second goal, one of minimizing bit errors, is a consequence of the noise present in every communications system. System designers have come up with clever coding algorithms, where the data bits are scrambled and modified in order to reduce and recover the bit errors at the receiver[1].

This lab gives a sneak preview of the main results governing the above two communications system metrics. You will study the impact of key system parameters on the channel capacity and the bit error rate of a wireless communication system. More advanced topics in this subject area are covered in courses such as NWEN 243 and ECEN 310.

First Some Work on Logs

We will need to use logs in this lab - the mathematical ones, not the ones you see on those big trucks. We will first have a review of base-10 logs and see how they work. Take some notes on a

[1] This is the subject of Coding Theory, for example MATH 324.

sheet of paper, take a photo, and upload it to the script. You can refer to these notes later and you work on the lab, and they will likely prove useful later in the term as well.

CORE [12 marks each question]

1. What are common logs all about? Give a brief description of what logs are, show how to invert logs, and list a few rules for working with logs.

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Logarithms are used to find the power to which a number must be raised to in order to get another number.

Common logs are logarithmic equations with a base 10. So this finds *N* in regards to $M = 10^N$ based on *M* being a real number.

Logs can be inverted using the formula $Log_b(M) = N \rightarrow M = b^N$.

LogAB = LogA + LogB

LogA/B = LogA - LogB

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The Basic Idea: Sending 1s and 0s on an Electromagnetic Wave

The basic idea of wireless digital communications systems is to begin with an electromagnetic wave (a radio wave) and encode the ones and zeros on the wave as variations in the amplitude, frequency, or phase of the signal. The following schematic shows the idea roughly for digital frequency modulation (digital FM), with high frequencies representing 1's a low frequencies representing 0's. The original wave is called the *carrier wave* as it carries the data.

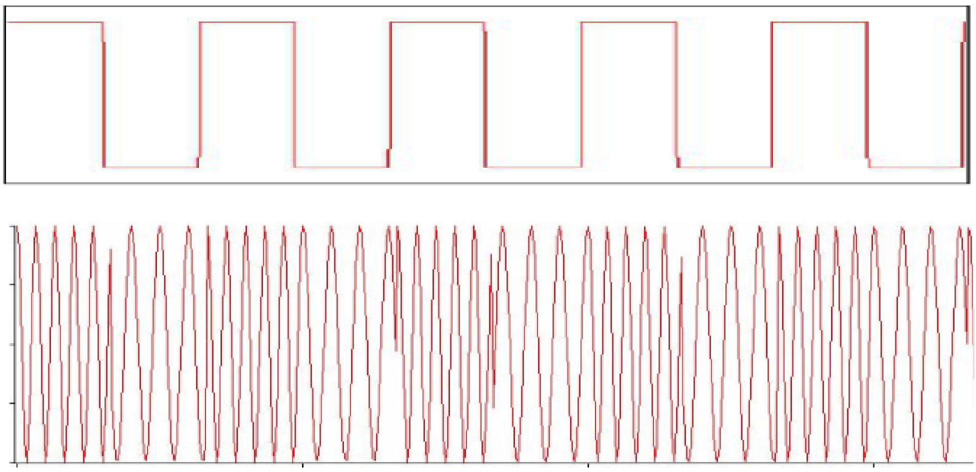

Figure 1: Bits encoded on a carrier wave by varying the frequency.

Channel Capacity

Bandwidth in Physics and Engineering

A wave that has a single frequency is described by $y = A\sin(kx \pm \omega t)$. It extends from $t = -\infty$ to $+\infty$. Any signal that does not do that is not a single frequency - it contains a range of frequencies. In particular, a signal such, as the one above actually contains a significant range of frequencies. The key idea is this: the more 1s and 0s you want to put on the wave, the wider the range of frequencies you need in your signal. This links the concepts of bandwidth in physics and capacity (often colloquially called bandwidth) in digital communications technology.

Physics: Bandwidth is the range of frequencies in the signal.

Digital Comm: Bandwidth is the number of ones and zeros per transmitted per second.

These two ideas fit very closely: BW in digital communications is a consequence of BW in physics.

Noise Again

There is a catch. If there is a lot of noise on the signal, the number of 1s and 0s you can reliably transmit per second is reduced. You need more time to make sure whether you are reading the 1s and 0s correctly if you have a lot of noise and/or correct misread bits.

Shannon Again

Claude Shannon studied this in detail. Shannon's famous result is remarkably simple: the capacity (maximum data rate possible), C, of a single antenna transmitter-receiver link is [2]

$$C = W Log_2(1 + SNR_R) \qquad \text{bits/second (bps)} \qquad (1)$$

It is a function of two fundamental system parameters:

- bandwidth, W, which is the frequency range (in Hz) used for transmission
- received signal-to-noise ratio, SNR_R, which is a ratio of the received signal power to the noise power :

$$SNR_R = \frac{P_{signal\ received}}{P_{noise}} \qquad (2)$$

The received SNR (SNR_R) can be computed from the SNR at the transmitter (SNR_T) and the gain of the wireless channel over which the signal is sent:

$$SNR_R = SNR_T |h|^2 \qquad (3)$$

where SNR_T is the ratio of the transmitted signal power to the noise power and $|h|^2$ is the wireless channel attenuation or gain. The gain h is actually a complex number (more in later classes) but we just need its magnitude which is a real number.

The result in eq. (1) is intuitively satisfying - increasing SNR_R by either increasing signal power or operating with lower noise will increase the maximum possible data rate. So will transmitting data over a wider frequency bandwidth.

Note that Shannon's formula uses log in base 2. That will be convenient later, but you can translate to log base 10 if you wish:

$$C = \frac{1}{Log_{10} 2} W Log_{10}(1 + SNR_R) \; = \; 3.32 * W Log_{10}(1 + SNR_R) \qquad (4)$$

2. Let's see what the maximum C might be in a somewhat realistic example for a cell signal. Suppose you have a physics BW of W = 3 MHz, a gain of $|h| = 0.5$, and a transmitter signal to noise ratio of SNR_T = 1.4. Find C. Show your calculations. Hint: remember to square $|h|$.

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[2] The derivation of this result is quite involved — it is covered in ECEN 410.

W = 3 MHz = 3000000Hz
$|h| = 0.5$
SNR_T = 1.4.
$3.32 * WLog_{10}(1 + SNR_R) = C$
$SNR_R = SNR_T|h|^2$

$SNR_R = 1.4 * 0.5^2 = 0.35$
$C = 3.32 * 3000000Log_{10}(1 + 0.35)$ = 1298124.334210261bps

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Decibels!
You've heard of decibels probably. But what is a Decibel or dB? Basically it is a way of handling very large and very small numbers conveniently. More precisely, it is a way of working with variables that can range from the very small to the very large. We will need dB in the next step so we introduce it here.

A good example application of dB is human hearing. The human ear can process sounds with intensities ranging over 12 orders of magnitude (no kidding). The human ear can handle sound intensities (watts per square metre) ranging from 1x10^{-12} W/m^2 to about 1 W/m^2, or a range of a million millions. 1x10^{-12} W/m^2 is barely audible to someone with good hearing while 1 W/m^2 is the threshold of pain for most people. It is hard to process data over such a wide range, so we define the sound ***intensity level*** β in dB as follows. Note I is intensity and β is intensity level.

$$\beta = (10dB)log(\frac{I}{1x10^{-12}}) \qquad (5)$$

3. Find the intensity level in dB for sound with intensity 1×10^{-12}, 3×10^{-6}, and 1 W/m^2.
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$\beta = (10dB)log(\frac{I}{1*10^{-12}})$

1. I = 1*10^(-12)
   $\beta = (10dB)log(\frac{1*10^{(-12)}}{1*10^{(-12)}})$ = 0 db
2. I = 3*10^(-6)
   $\beta = (10dB)log(\frac{3*10^{(-6)}}{1*10^{-12}})$ = 6.477121254719663 db
3. I = 1
   $\beta = (10dB)log(\frac{1}{1*10^{-12}})$ = 12 db

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We use dB or similar concepts in many applications including earthquakes and the like.

COMPLETION [10 marks each question]
Let us look at a practical example of channel capacity:

1. Use **WiFi** on your smartphone to measure and record the download and upload data rates (in bits/second, bps) you are achieving. There are many apps available for this, for example speedtest.net. **Note: Be careful! Use WiFi. If you use your cellular data and you are on a limited data plan, this can be an expensive experiment. Multiple speed tests can quickly consume a lot of data — speedtest.net will use ~100 MB over a good connection in < 30 s! You can use data from another student or ask for some taken by a tutor if you do not want to use your own cell phone.**

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Download = 13.1 mbps = 1.31e+7 bps
Upload = 21.3 mbps = 2.13e+7 bps

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2. The bandwidth that your device uses depends on the WiFi system. Let's assume that you are using 40 MHz of bandwidth on WiFi. Calculate the minimum SNR_R required at the receiver to achieve the two data rates you measured. Since SNR can vary greatly, it is typically given in decibels, i.e., $SNR_{dB} = 10dB * Log(SNR)$ where the SNR can be either the transmit or receive SNR. Express your answers both as numbers and in dB.

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$SNR_{dB} = 10dB * Log(SNR)$
$3.32 * WLog_{10}(1 + SNR_R) \ = \ C$
$SNR_R = SNR_T|h|^2$
W = 40mhz = 4*10^(7)hz
C(Download) = 1.31*10^(7)bps
C(Upload) = 2.13*10^(7)bps

C(Download)

$3.32 * (4 * 10^7)\, Log_{10}(1 + SNR_R) \ = \ 1.31 * 10^7 pbs$
$SNR_R \ = \ 10^{131/1328}$ *-1 = 0.25500246349*
$SNR_{dB} = 10dB * Log(SNR)$
$SNR_{dB} = 10 * Log(0.25500246349) \ = \ -5.93455623978db$

C(Upload)

$3.32 * (4 * 10^7)\, Log_{10}(1 + SNR_R) \ = \ 2.13 * 10^7 pbs$
$SNR_R \ = \ 10^{213/1328}$ *-1 = 0.44674358809*
$SNR_{dB} = 10 * Log(0.44674358809) \ = \ -3.49941672011db$

Minimum required would be the greater of the 2. So in this case the minimum would be roughly *0.44674358809 or* $-3.49941672011db$

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3. Assuming that your wireless channel is characterised by $|h| = 0.5$, what is the transmit SNR (SNR_T) in each case (downlink and uplink)? Express as a number and in dB. Hint: you cannot multiply the SNR in dB by $|h|^2$. You have to use equation 3 and then convert to dB.

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$SNR_R = SNR_T|h|^2$
$SNR_{dB} = 10dB * Log(SNR)$
|h| = 0.5

$SNR_R = SNR_T|h|^2$

Divide both sides by $|h|^2$

$SNR_R / |h|^2 = SNR_T$

C(Download)

= *0.25500246349 /* $|0.5|^2$ = *1.02000985396*

$10 * Log(1.02000985396)$ = *0.08604367349 bd*

C(Upload)

= *0.44674358809 / 0.25 = 1.78697435236*

$10 * Log(1.78697435236)$ = *2.52118319317 bd*

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2 Bit Error Rate

For a simple communications channel, the *bit error rate* (BER), which is the ratio of incorrectly received bits to the total number of bits transmitted, is a function of only one system parameter - the signal to noise ratio.

4. Which SNR is relevant for the BER - the received or transmitted SNR? Explain.

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Received SNR, SNR_R, as the bit error rate measures the received bits to then compare it to the transmitted bits.

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5. Which SNR larger - the received or transmitted SNR? Explain.

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$SNR_R = SNR_T|h|^2$

SNR_R will be larger if $|h|$ is greater than 1, as per the equation you have to multiply SNR_T to $|h|^2$ in order to get SNR_R.

If $|h|$ is less than 1, SNR_T will be greater as SNR_R would be a fraction of SNR_T. (ie $|h|$ = 0.5 so $|h|^2$ = 0.25 so in this case SNR_R would be a quarter of SNR_T).

As well as that $|h|$ is squared within the equation so following basic multiplication laws, SNR_T must be exponentially smaller or greater depending if $|h|$ is greater than or less than 1.

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CHALLENGE [7 marks each question]

1. We assumed $|h| = 0.5$ in our calculations, and that meant that the received SNR_R was one quarter of the transmitted SNR_T since $|h|^2 = 0.25$. Calculate how much SNR_T we can save by designing a better system, one with perfect gain. Express your results in dB. Remember that lower transmit power translates to longer battery life. If for each dB reduction in SNR_T your phone battery would last an extra 30 minutes, in theory, how much longer should your phone survive on a single charge?

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$SNR_{dB} = 10dB * Log(SNR)$

$SNR_R = SNR_T|h|^2$

Perfect gain will be when $|h|$ = 1. So in that context, $SNR_R = SNR_T$ as $|h|^2$ = 1.

C(Download)

0.08604367349 bd = SNRt before perfect

$-5.93455623978db$ = SNRt after perfect as $SNR_R = SNR_T$.

Difference = 0.08604367349 - -5.93455623978 = *6.02059991327 so that equates to roughly 3 more hours of charge for download*

C(Upload)

2.52118319317db = SNRt before perfect

$-3.49941672011db$ = SNRt after perfect as $SNR_R = SNR_T$.

Difference = 2.52118319317 - -3.49941672011 = 6.02059991328 so roughly another 3 hours more charge for upload

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2. Explain the connection between computer science BW and physics BW. Use this idea to explain why fibre cables carry data at such a high rate. Hint: roughly what is the frequency of the infrared light used in a fibre cable? Google it!

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