```
> with(DEtools); with(plots)
```

[*AreSimilar*, *Closure*, *DEnormal*, *DEplot*, *DEplot3d*, *DEplot_polygon*, *DFactor*, *DFactorLCLM*, *DFactorsols*, *Dchangevar*, *Desingularize*, *FindODE*, *FunctionDecomposition*, *GCRD*, *Gosper*, *Heunsols*, *Homomorphisms*, *IVPsol*, *IsHyperexponential*, *LCLM*, *MeijerGsols*, *MultiplicativeDecomposition*, *ODEInvariants*, *PDEchangecoords*, *PolynomialNormalForm*, *RationalCanonicalForm*, *ReduceHyperexp*, *RiemannPsols*, *Xchange*, *Xcommutator*, *Xgauge*, *Zeilberger*, *abelsol*, *adjoint*, *autonomous*, *bernoullisol*, *buildsol*, *buildsym*, *canoni*, *caseplot*, *casesplit*, *checkrank*, *chinisol*, *clairautsol*, *constcoeffsols*, *convertAlg*, *convertsys*, *dalembertsol*, *dcoeffs*, *de2diffop*, *dfieldplot*, *diff_table*, *diffop2de*, *dperiodic_sols*, *dpolyform*, *dsubs*, *eigenring*, *endomorphism_charpoly*, *equinv*, *eta_k*, *eulersols*, *exactsol*, *expsols*, *exterior_power*, *firint*, *firtest*, *formal_sol*, *gen_exp*, *generate_ic*, *genhomosol*, *gensys*, *hamilton_eqs*, *hypergeometricsols*, *hypergeomsols*, *hyperode*, *indicialeq*, *infgen*, *initialdata*, *integrate_sols*, *intfactor*, *invariants*, *kovacicsols*, *leftdivision*, *liesol*, *line_int*, *linearsol*, *matrixDE*, *matrix_riccati*, *maxdimsystems*, *moser_reduce*, *muchange*, *mult*, *mutest*, *newton_polygon*, *normalG2*, *ode_int_y*, *ode_y1*, *odeadvisor*, *odepde*, *parametricsol*, *particularsol*, *phaseportrait*, *poincare*, *polysols*, *power_equivalent*, *rational_equivalent*, *ratsols*, *redode*, *reduceOrder*, *reduce_order*, *regular_parts*, *regularsp*, *remove_RootOf*, *riccati_system*, *riccatisol*, *rifread*, *rifsimp*, *rightdivision*, *rtaylor*, *separablesol*, *singularities*, *solve_group*, *super_reduce*, *symgen*, *symmetric_power*, *symmetric_product*, *symtest*, *transinv*, *translate*, *untranslate*, *varparam*, *zoom*]

[*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *shadebetween*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*] **(1)**

```
> f:=x->x^2-2*x
```

$$f := x \mapsto x^2 - 2 \cdot x \qquad \textbf{(2)}$$

```
> ec:=diff(x(t),t)=f(x(t))
```

$$ec := \frac{d}{dt} x(t) = x(t)^2 - 2\,x(t) \tag{3}$$

```
> sol:=solve(f(x)=0,x)
```

$$sol := 0, 2 \tag{4}$$

```
> D(f)(sol[1])
```

$$-2 \tag{5}$$

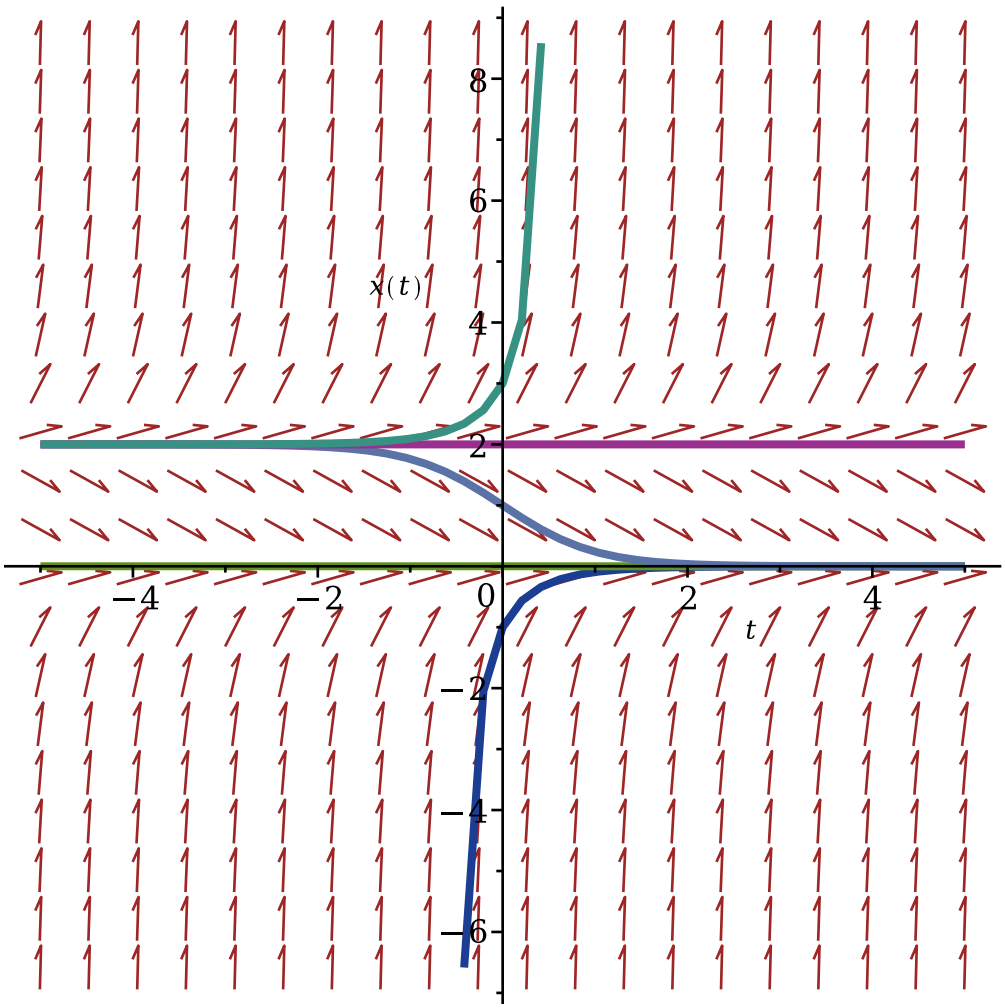
```
> D(f)(sol[2])
```

$$2 \tag{6}$$

```
> # x = 0 -> stabil, x = 2 -> instabil
> DEplot(ec,x(t), t = -5..5, [[x(0)=-1],[x(0)=0],[x(0)=1],[x(0)=2],
  [x(0)=3]])
```

Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further left of -.54930616, probably a singularity

Warning, plot may be incomplete, the following errors(s) were issued:
cannot evaluate the solution further right of .54930618, probably a singularity




```
> f:=x->x*(x-1)*(x-2)
```

$$f := x \mapsto x \cdot (x-1) \cdot (x-2) \quad (7)$$

```
> ec:=diff(x(t),t)=f(x(t))
```

$$ec := \frac{d}{dt} x(t) = x(t)\ (x(t)-1)\ (x(t)-2) \quad (8)$$

```
> sol:=solve(f(x)=0,x)
```

$$sol := 0, 1, 2 \quad (9)$$

```
> D(f)(sol[1])
```

$$2 \quad (10)$$

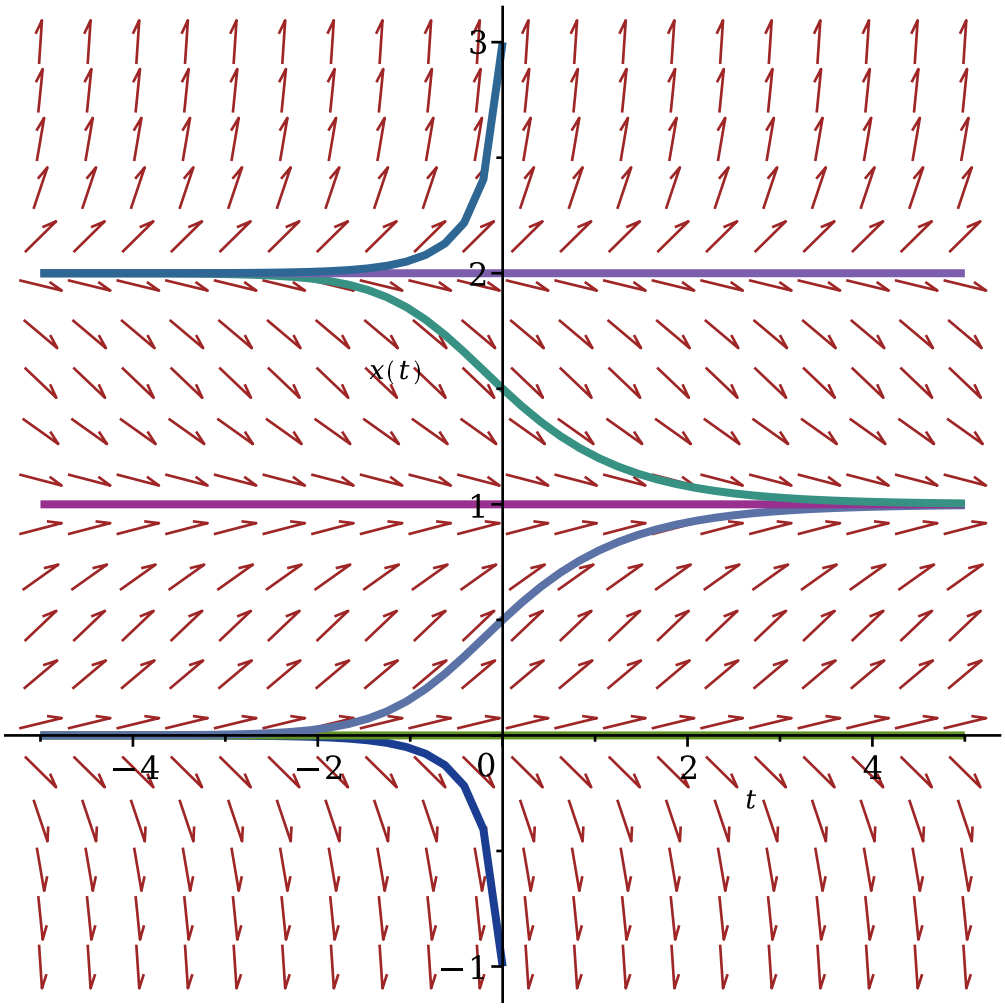
```
> D(f)(sol[2])
```

$$-1 \quad (11)$$

```
> D(f)(sol[3])
```

$$2 \quad (12)$$

```
> # x=0 si x=2 instabile, x=1 stabil
> DEplot(ec,x(t), t = -5..5, [[x(0)=-1],[x(0)=0],[x(0)=1/2],[x(0)=
  1],[x(0)=3/2],[x(0)=2],[x(0)=3]])
Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further right of .14384102, probably
a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further right of .14384100, probably
a singularity
```




```
> _EnvAllSolutions:=true
```

$$_EnvAllSolutions := true \quad (13)$$

```
> f:=x->sin(x)
```

$$f := x \mapsto \sin(x) \quad (14)$$

```
> ec:=diff(x(t),t)=f(x(t))
```

$$ec := \frac{d}{dt} x(t) = \sin(x(t)) \quad (15)$$

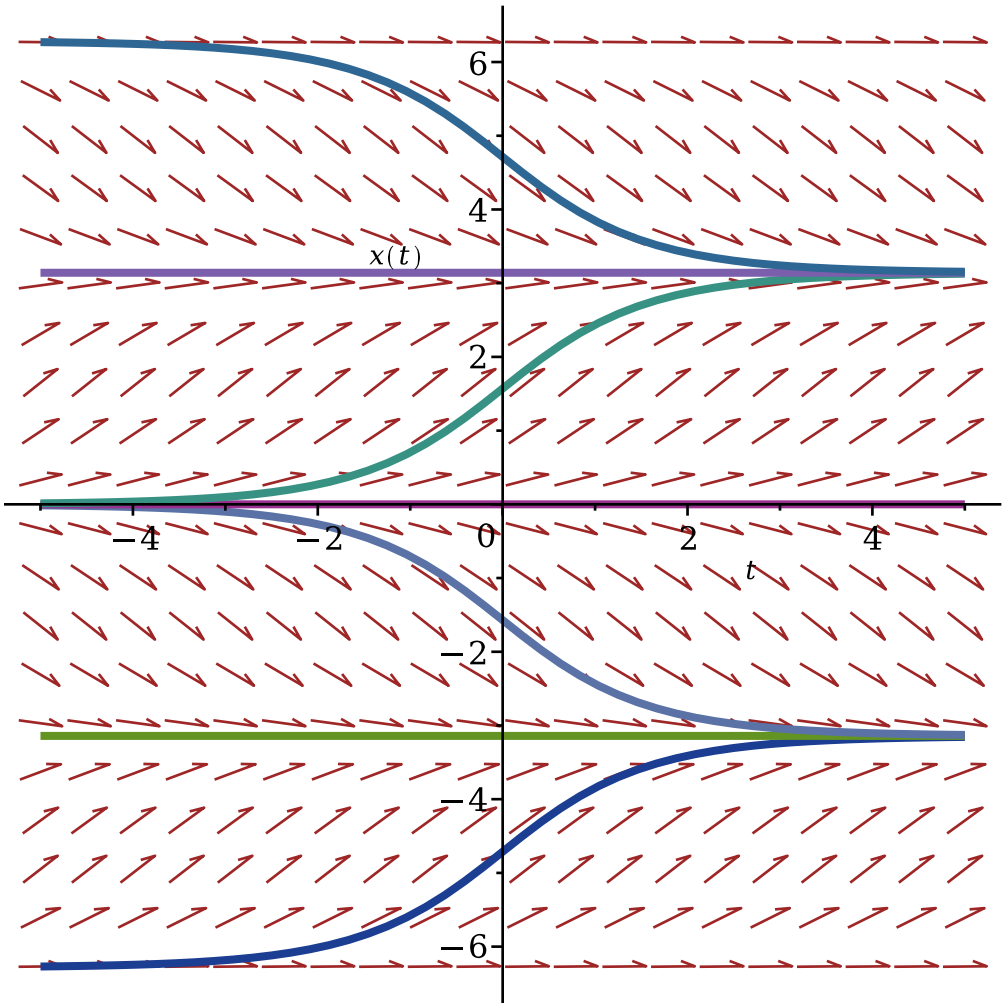
```
> sol:=solve(f(x)=0,x)
```

$$sol := \pi _Z1\sim \quad (16)$$

```
> D(f)(sol)
```

$$(-1)^{_Z1\sim} \quad (17)$$

```
> # solutiile sunt de forma k*Pi. Daca k e par, punctul e instabil,
  daca k e impar, punctul e stabil
> DEplot(ec,x(t), t = -5..5, [[x(0)=-3*Pi/2],[x(0)=-Pi],[x(0)=-
  Pi/2],[x(0)=0],[x(0)=Pi/2],[x(0)=Pi],[x(0)=3*Pi/2]])
```




```
> restart
> with(DEtools); with(plots); with(linalg)
```

[*AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeometricsols, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor,*

invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom]

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*]

[*BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian*]

(18)

```
> ec1:=diff(x(t),t)=2*x(t)+y(t)
```

$$ec1 := \frac{d}{dt} x(t) = 2\,x(t) + y(t) \quad (19)$$

```
> ec2:=diff(y(t),t)=x(t)+2*y(t)
```

$$ec2 := \frac{d}{dt} y(t) = x(t) + 2\,y(t) \quad (20)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = 2\,x(t) + y(t),\ \frac{d}{dt} y(t) = x(t) + 2\,y(t) \quad (21)$$

```
> A:=matrix([[2,1],[1,2]])
```

$$A := \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad (22)$$

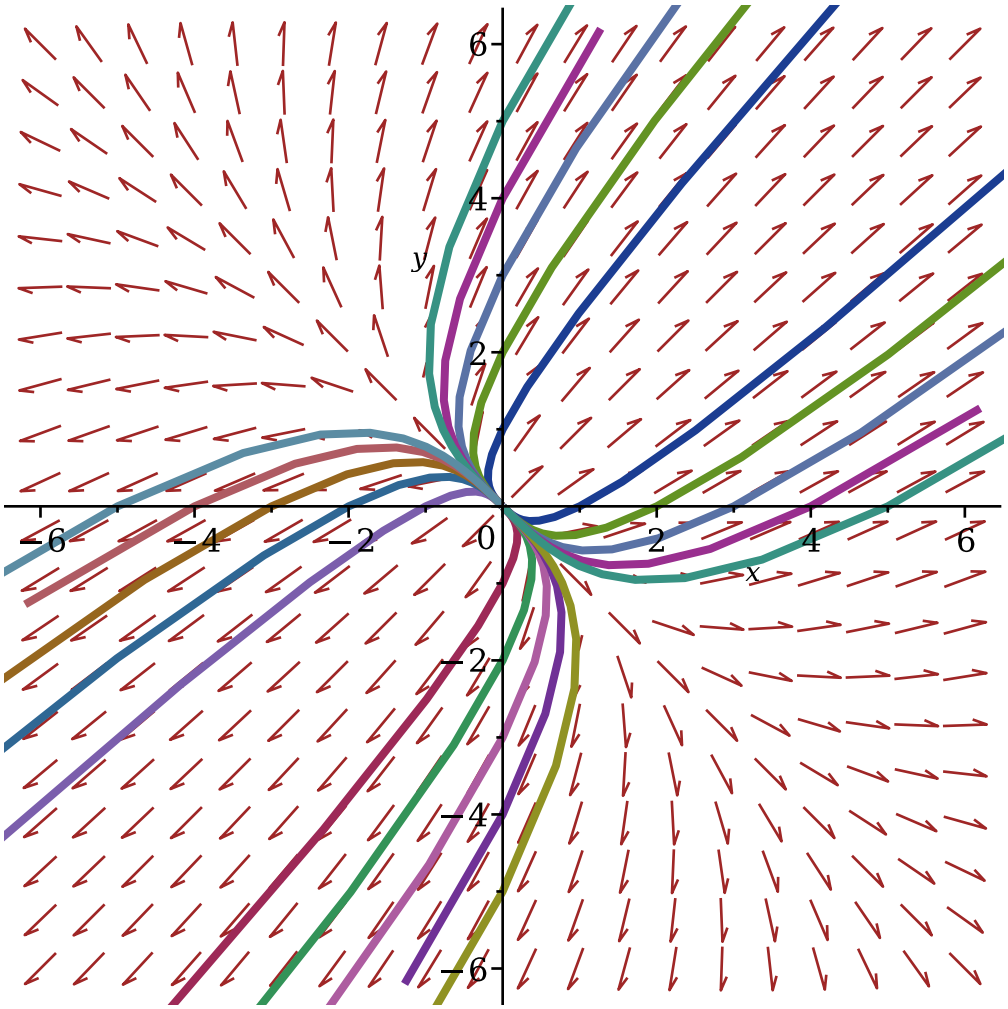
```
> eigenvals(A)
```

$$3,\ 1 \quad (23)$$

```
> # (0,0) e nod instabil
> cond_in:=[x(0)=0,y(0)=i]$i=1..5,[x(0)=-i,y(0)=0]$i=1..5,[x(0)=0,y
  (0)=-i]$i=1..5,[x(0)=i,y(0)=0]$i=1..5
```

$$cond_in := [x(0) = 0, y(0) = 1], [x(0) = 0, y(0) = 2], [x(0) = 0, y(0) = 3], [x(0) = 0, y(0) = 4], [x(0) = 0, y(0) = 5], [x(0) = -1, y(0) = 0], [x(0) = -2, y(0) = 0], [x(0) = -3, y(0) = 0], [x(0) = -4, y(0) = 0], [x(0) = -5, y(0) = 0], [x(0) = 0, y(0) = -1], [x(0) = 0, y(0) = -2], [x(0) = 0, y(0) = -3], [x(0) = 0, y(0) = -4], [x(0) = 0, y(0) = -5], [x(0) = 1, y(0) = 0], [x(0) = 2, y(0) = 0], [x(0) = 3, y(0) = 0], [x(0) = 4, y(0) = 0], [x(0) = 5, y(0) = 0] \quad (24)$$

```
> DEplot([sist],[x(t),y(t)],t=-5..5,x=-6..6,y=-6..6,[cond_in])
```




```
> ec1:=diff(x(t),t)=-3*x(t)+4*y(t)
```

$$ec1 := \frac{d}{dt} x(t) = -3\,x(t) + 4\,y(t) \qquad (25)$$

```
> ec2:=diff(y(t),t)=-2*x(t)+3*y(t)
```

$$ec2 := \frac{d}{dt} y(t) = -2\,x(t) + 3\,y(t) \qquad (26)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = -3\,x(t) + 4\,y(t),\ \frac{d}{dt} y(t) = -2\,x(t) + 3\,y(t) \qquad (27)$$

```
> A:=matrix([[-3,4],[-2,3]])
```

$$A := \begin{bmatrix} -3 & 4 \\ -2 & 3 \end{bmatrix} \qquad (28)$$

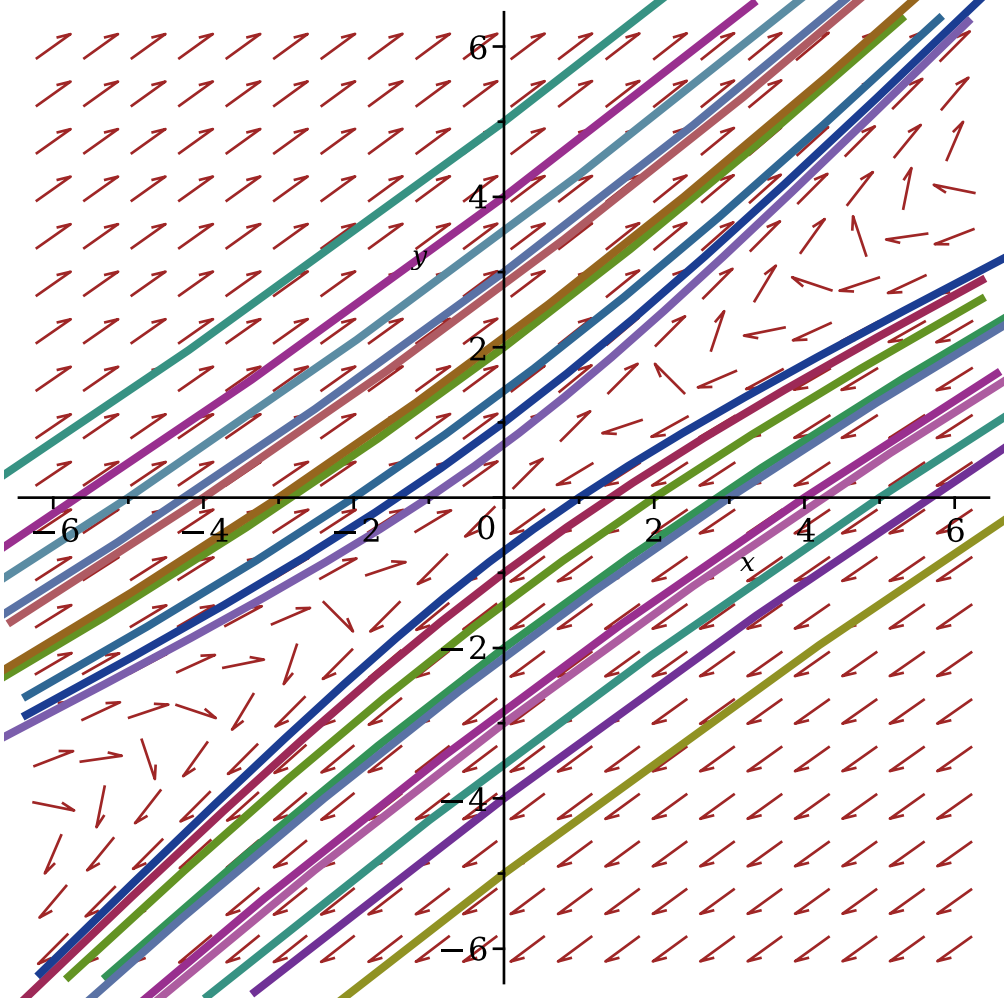
```
> eigenvals(A)
```

$$1, -1 \qquad (29)$$

```
> # (0,0) e nod instabil de tip sa
> cond_in:=[x(0)=0,y(0)=i]$i=1..5,[x(0)=-i,y(0)=0]$i=1..5,[x(0)=0,y
  (0)=-i]$i=1..5,[x(0)=i,y(0)=0]$i=1..5
```

$$cond_in := [x(0)=0, y(0)=1], [x(0)=0, y(0)=2], [x(0)=0, y(0)=3], [x(0)=0, y(0)=4], [x(0)=0, y(0)=5], [x(0)=-1, y(0)=0], [x(0)=-2, y(0)=0], [x(0)=-3, y(0)=0], [x(0)=-4, y(0)=0], [x(0)=-5, y(0)=0], [x(0)=0, y(0)=-1], [x(0)=0, y(0)=-2], [x(0)=0, y(0)=-3], [x(0)=0, y(0)=-4], [x(0)=0, y(0)=-5], [x(0)=1, y(0)=0], [x(0)=2, y(0)=0], [x(0)=3, y(0)=0], [x(0)=4, y(0)=0], [x(0)=5, y(0)=0] \quad (30)$$

```
> DEplot([sist],[x(t),y(t)],t=-5..5,x=-6..6,y=-6..6,[cond_in])
```




```
> ec1:=diff(x(t),t)=x(t)+4*y(t)
```

$$ec1 := \frac{d}{dt}x(t) = x(t) + 4\,y(t) \quad (31)$$

```
> ec2:=diff(y(t),t)=x(t)+y(t)
```

$$ec2 := \frac{d}{dt}y(t) = x(t) + y(t) \quad (32)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt}x(t) = x(t) + 4\,y(t),\ \frac{d}{dt}y(t) = x(t) + y(t) \quad (33)$$

```
> A:=matrix([[1,4],[1,1]])
```

$$A := \begin{bmatrix} 1 & 4 \\ 1 & 1 \end{bmatrix} \quad (34)$$

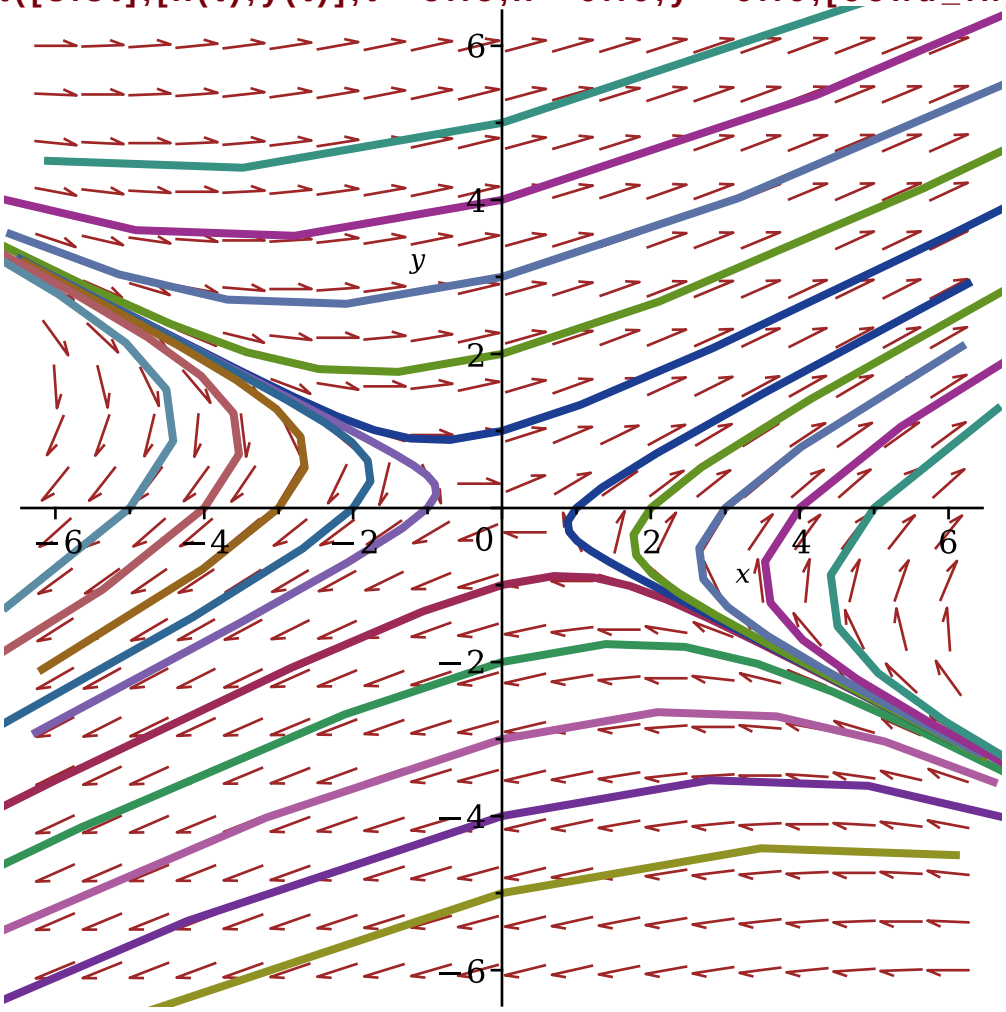
```
> eigenvals(A)
```

$$3, -1 \quad (35)$$

```
> # (0,0) e nod instabil e tip sa
> cond_in:=[x(0)=0,y(0)=i]$i=1..5,[x(0)=-i,y(0)=0]$i=1..5,[x(0)=0,y
  (0)=-i]$i=1..5,[x(0)=i,y(0)=0]$i=1..5
```

$$cond_in := [x(0) = 0, y(0) = 1], [x(0) = 0, y(0) = 2], [x(0) = 0, y(0) = 3], [x(0) = 0, y(0) = 4], [x(0) = 0, y(0) = 5], [x(0) = -1, y(0) = 0], [x(0) = -2, y(0) = 0], [x(0) = -3, y(0) = 0], [x(0) = -4, y(0) = 0], [x(0) = -5, y(0) = 0], [x(0) = 0, y(0) = -1], [x(0) = 0, y(0) = -2], [x(0) = 0, y(0) = -3], [x(0) = 0, y(0) = -4], [x(0) = 0, y(0) = -5], [x(0) = 1, y(0) = 0], [x(0) = 2, y(0) = 0], [x(0) = 3, y(0) = 0], [x(0) = 4, y(0) = 0], [x(0) = 5, y(0) = 0] \quad (36)$$

```
> DEplot([sist],[x(t),y(t)],t=-5..5,x=-6..6,y=-6..6,[cond_in])
```




```
> ec1:=diff(x(t),t)=2*x(t)-y(t)
```

$$ec1 := \frac{d}{dt} x(t) = 2\,x(t) - y(t) \quad (37)$$

```
> ec2:=diff(y(t),t)=x(t)+2*y(t)
```

$$ec2 := \frac{d}{dt} y(t) = x(t) + 2\,y(t) \tag{38}$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = 2\,x(t) - y(t),\ \frac{d}{dt} y(t) = x(t) + 2\,y(t) \tag{39}$$

```
> A:=matrix([[2,-1],[1,2]])
```

$$A := \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \tag{40}$$

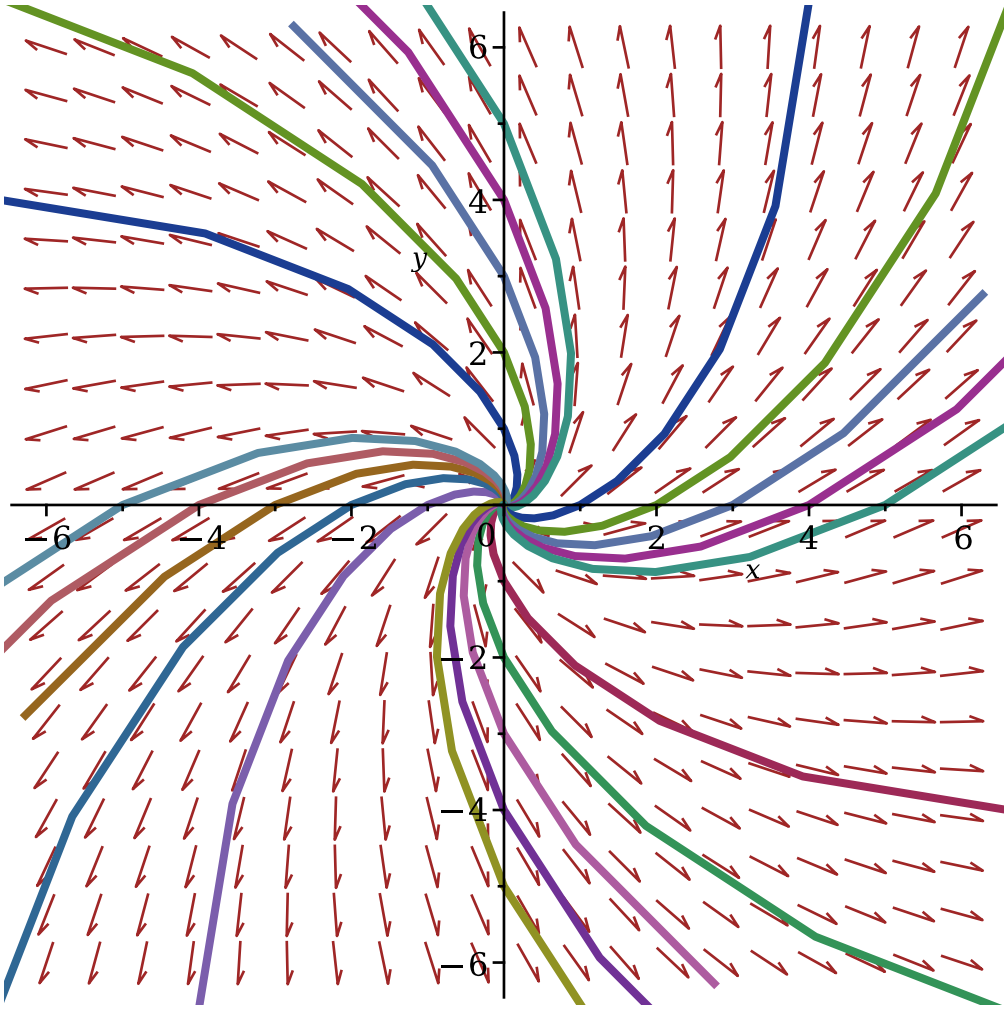
```
> eigenvals(A)
```

$$2 + I,\ 2 - I \tag{41}$$

```
> # (0,0) e nod instabil de tip focus
> cond_in:=[x(0)=0,y(0)=i]$i=1..5,[x(0)=-i,y(0)=0]$i=1..5,[x(0)=0,y
  (0)=-i]$i=1..5,[x(0)=i,y(0)=0]$i=1..5
```

$$cond_in := [x(0) = 0, y(0) = 1], [x(0) = 0, y(0) = 2], [x(0) = 0, y(0) = 3], [x(0) = 0, y(0) = 4], [x(0) = 0, y(0) = 5], [x(0) = -1, y(0) = 0], [x(0) = -2, y(0) = 0], [x(0) = -3, y(0) = 0], [x(0) = -4, y(0) = 0], [x(0) = -5, y(0) = 0], [x(0) = 0, y(0) = -1], [x(0) = 0, y(0) = -2], [x(0) = 0, y(0) = -3], [x(0) = 0, y(0) = -4], [x(0) = 0, y(0) = -5], [x(0) = 1, y(0) = 0], [x(0) = 2, y(0) = 0], [x(0) = 3, y(0) = 0], [x(0) = 4, y(0) = 0], [x(0) = 5, y(0) = 0] \tag{42}$$

```
> DEplot([sist],[x(t),y(t)],t=-5..5,x=-6..6,y=-6..6,[cond_in])
```




```
> ec1:=diff(x(t),t)=-x(t)-y(t)
```

$$ec1 := \frac{d}{dt}x(t) = -x(t) - y(t) \quad (43)$$

```
> ec2:=diff(y(t),t)=x(t)-3*y(t)
```

$$ec2 := \frac{d}{dt}y(t) = x(t) - 3\,y(t) \quad (44)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt}x(t) = -x(t) - y(t),\ \frac{d}{dt}y(t) = x(t) - 3\,y(t) \quad (45)$$

```
> A:=matrix([[-1,-1],[1,-3]])
```

$$A := \begin{bmatrix} -1 & -1 \\ 1 & -3 \end{bmatrix} \quad (46)$$

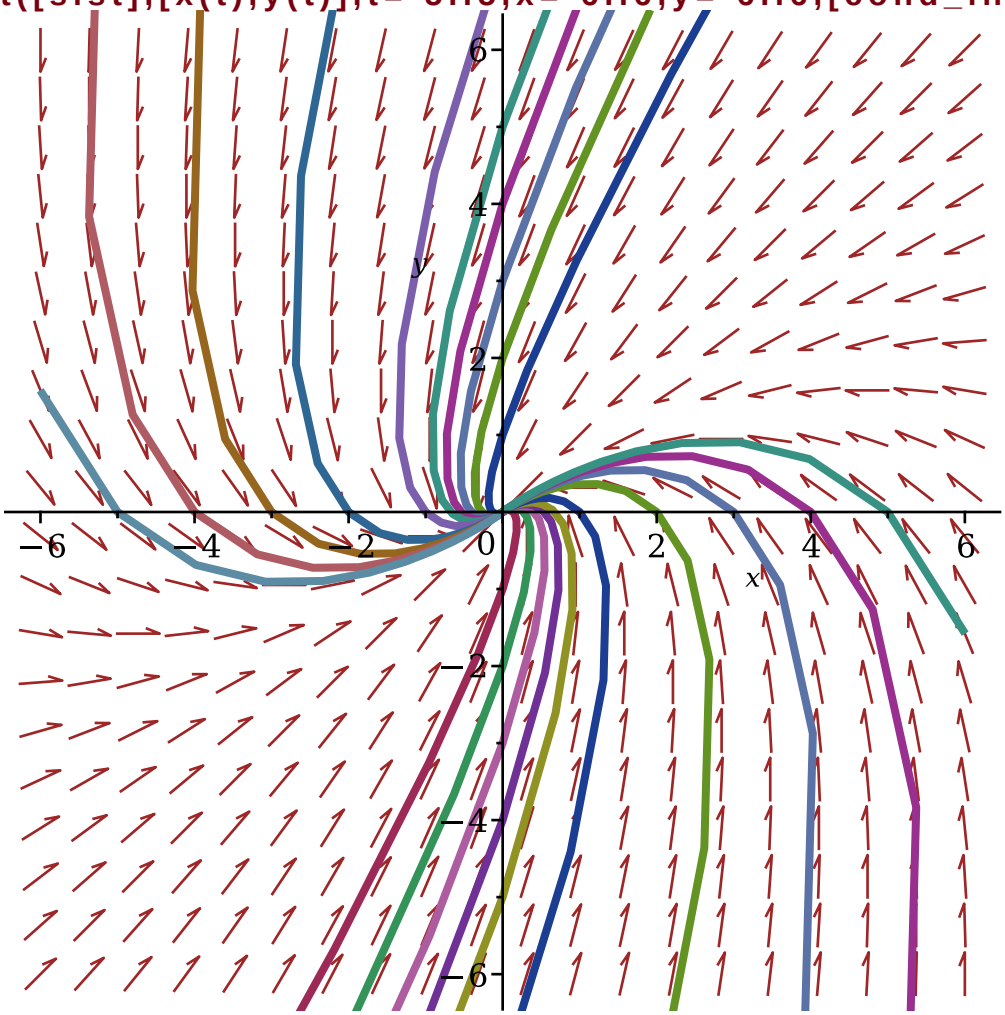
```
> eigenvals(A)
```

$$-2, -2 \quad (47)$$

```
> # (0,0) e nod stabil
> cond_in:=[x(0)=0,y(0)=i]$i=1..5,[x(0)=-i,y(0)=0]$i=1..5,[x(0)=0,y
  (0)=-i]$i=1..5,[x(0)=i,y(0)=0]$i=1..5
```

$$cond_in := [x(0) = 0, y(0) = 1], [x(0) = 0, y(0) = 2], [x(0) = 0, y(0) = 3], [x(0) = 0, y(0) = 4], [x(0) = 0, y(0) = 5], [x(0) = -1, y(0) = 0], [x(0) = -2, y(0) = 0], [x(0) = -3, y(0) = 0], [x(0) = -4, y(0) = 0], [x(0) = -5, y(0) = 0], [x(0) = 0, y(0) = -1], [x(0) = 0, y(0) = -2], [x(0) = 0, y(0) = -3], [x(0) = 0, y(0) = -4], [x(0) = 0, y(0) = -5], [x(0) = 1, y(0) = 0], [x(0) = 2, y(0) = 0], [x(0) = 3, y(0) = 0], [x(0) = 4, y(0) = 0], [x(0) = 5, y(0) = 0] \quad (48)$$

```
> DEplot([sist],[x(t),y(t)],t=-5..5,x=-6..6,y=-6..6,[cond_in])
```




```
> ec1:=diff(x(t),t)=-2*x(t)
```

$$ec1 := \frac{d}{dt} x(t) = -2\,x(t) \quad (49)$$

```
> ec2:=diff(y(t),t)=-4*x(t)-2*y(t)
```

$$ec2 := \frac{d}{dt} y(t) = -4\,x(t) - 2\,y(t) \quad (50)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = -2\,x(t),\ \frac{d}{dt} y(t) = -4\,x(t) - 2\,y(t) \quad (51)$$

```
> A:=matrix([[-2,0],[-4,-2]])
```

$$A := \begin{bmatrix} -2 & 0 \\ -4 & -2 \end{bmatrix} \quad (52)$$

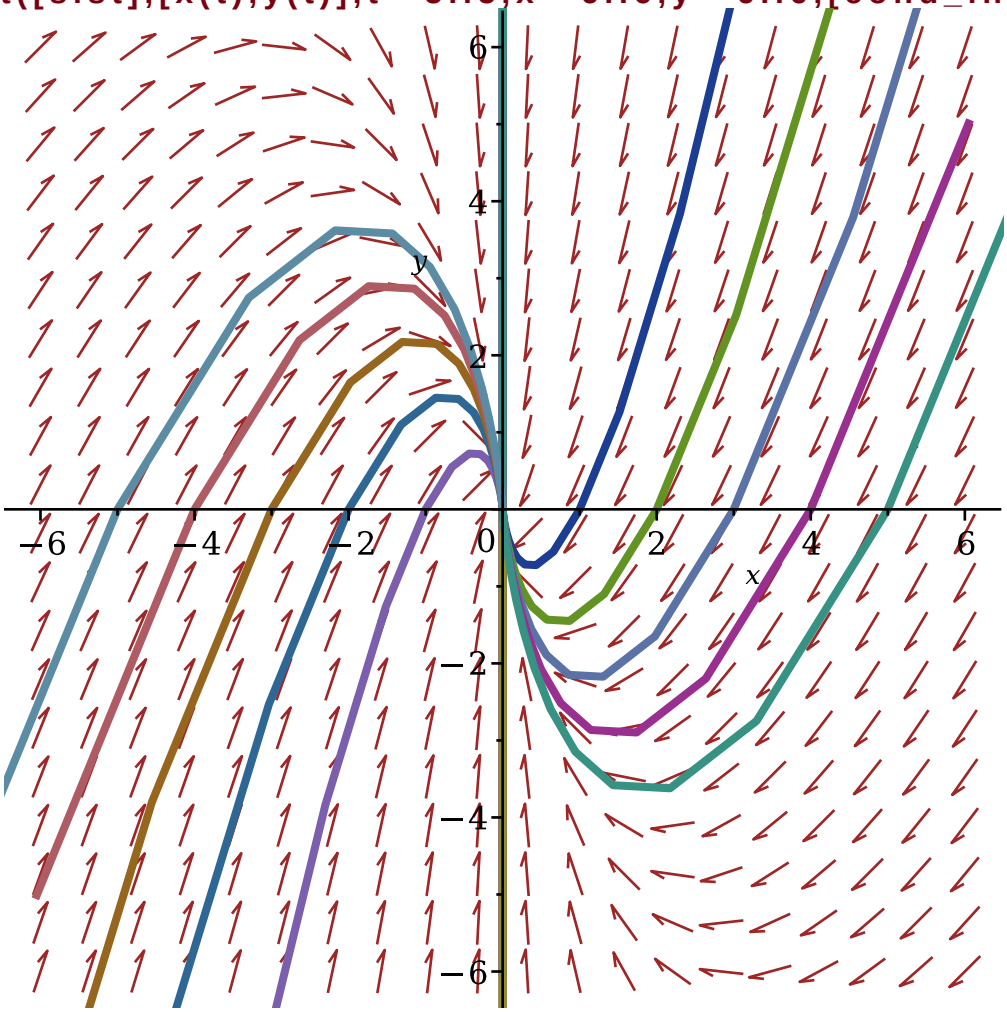
```
> eigenvals(A)
```

$$-2, -2 \quad (53)$$

```
> # (0,0) e nod stabil
> cond_in:=[x(0)=0,y(0)=i]$i=1..5,[x(0)=-i,y(0)=0]$i=1..5,[x(0)=0,y
  (0)=-i]$i=1..5,[x(0)=i,y(0)=0]$i=1..5
```

$$cond_in := [x(0) = 0, y(0) = 1], [x(0) = 0, y(0) = 2], [x(0) = 0, y(0) = 3], [x(0) = 0, y(0) = 4], [x(0) = 0, y(0) = 5], [x(0) = -1, y(0) = 0], [x(0) = -2, y(0) = 0], [x(0) = -3, y(0) = 0], [x(0) = -4, y(0) = 0], [x(0) = -5, y(0) = 0], [x(0) = 0, y(0) = -1], [x(0) = 0, y(0) = -2], [x(0) = 0, y(0) = -3], [x(0) = 0, y(0) = -4], [x(0) = 0, y(0) = -5], [x(0) = 1, y(0) = 0], [x(0) = 2, y(0) = 0], [x(0) = 3, y(0) = 0], [x(0) = 4, y(0) = 0], [x(0) = 5, y(0) = 0] \quad (54)$$

```
> DEplot([sist],[x(t),y(t)],t=-5..5,x=-6..6,y=-6..6,[cond_in])
```




```
> ec1:=diff(x(t),t)=-y(t)
```

$$ec1 := \frac{d}{dt} x(t) = -y(t) \quad (55)$$

```
> ec2:=diff(y(t),t)=x(t)
```

$$ec2 := \frac{d}{dt} y(t) = x(t) \tag{56}$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = -y(t), \frac{d}{dt} y(t) = x(t) \tag{57}$$

```
> A:=matrix([[0,-1],[1,0]])
```

$$A := \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \tag{58}$$

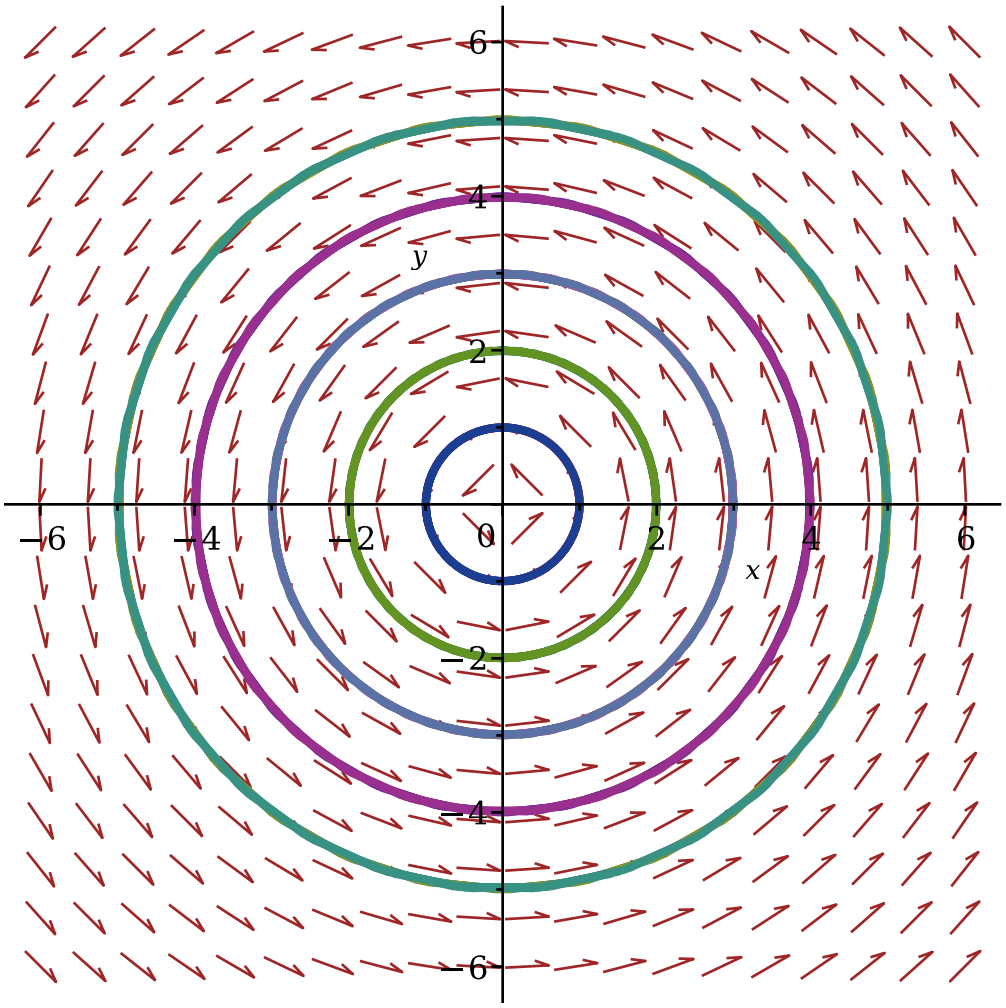
```
> eigenvals(A)
```

$$I, -I \tag{59}$$

```
> # (0,0) e nod stabil de tip centru
> cond_in:=[x(0)=0,y(0)=i]$i=1..5,[x(0)=-i,y(0)=0]$i=1..5,[x(0)=0,y
  (0)=-i]$i=1..5,[x(0)=i,y(0)=0]$i=1..5
```

$$cond_in := [x(0) = 0, y(0) = 1], [x(0) = 0, y(0) = 2], [x(0) = 0, y(0) = 3], [x(0) = 0, y(0) = 4], [x(0) = 0, y(0) = 5], [x(0) = -1, y(0) = 0], [x(0) = -2, y(0) = 0], [x(0) = -3, y(0) = 0], [x(0) = -4, y(0) = 0], [x(0) = -5, y(0) = 0], [x(0) = 0, y(0) = -1], [x(0) = 0, y(0) = -2], [x(0) = 0, y(0) = -3], [x(0) = 0, y(0) = -4], [x(0) = 0, y(0) = -5], [x(0) = 1, y(0) = 0], [x(0) = 2, y(0) = 0], [x(0) = 3, y(0) = 0], [x(0) = 4, y(0) = 0], [x(0) = 5, y(0) = 0] \tag{60}$$

```
> DEplot([sist],[x(t),y(t)],t=-5..5,x=-6..6,y=-6..6,[cond_in])
```




```
> ec1:=diff(x(t),t)=x(t)-4*y(t)
```

$$ec1 := \frac{d}{dt} x(t) = x(t) - 4\,y(t) \qquad (61)$$

```
> ec2:=diff(y(t),t)=5*x(t)-3*y(t)
```

$$ec2 := \frac{d}{dt} y(t) = 5\,x(t) - 3\,y(t) \qquad (62)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = x(t) - 4\,y(t),\ \frac{d}{dt} y(t) = 5\,x(t) - 3\,y(t) \qquad (63)$$

```
> A:=matrix([[1,-4],[5,-3]])
```

$$A := \begin{bmatrix} 1 & -4 \\ 5 & -3 \end{bmatrix} \qquad (64)$$

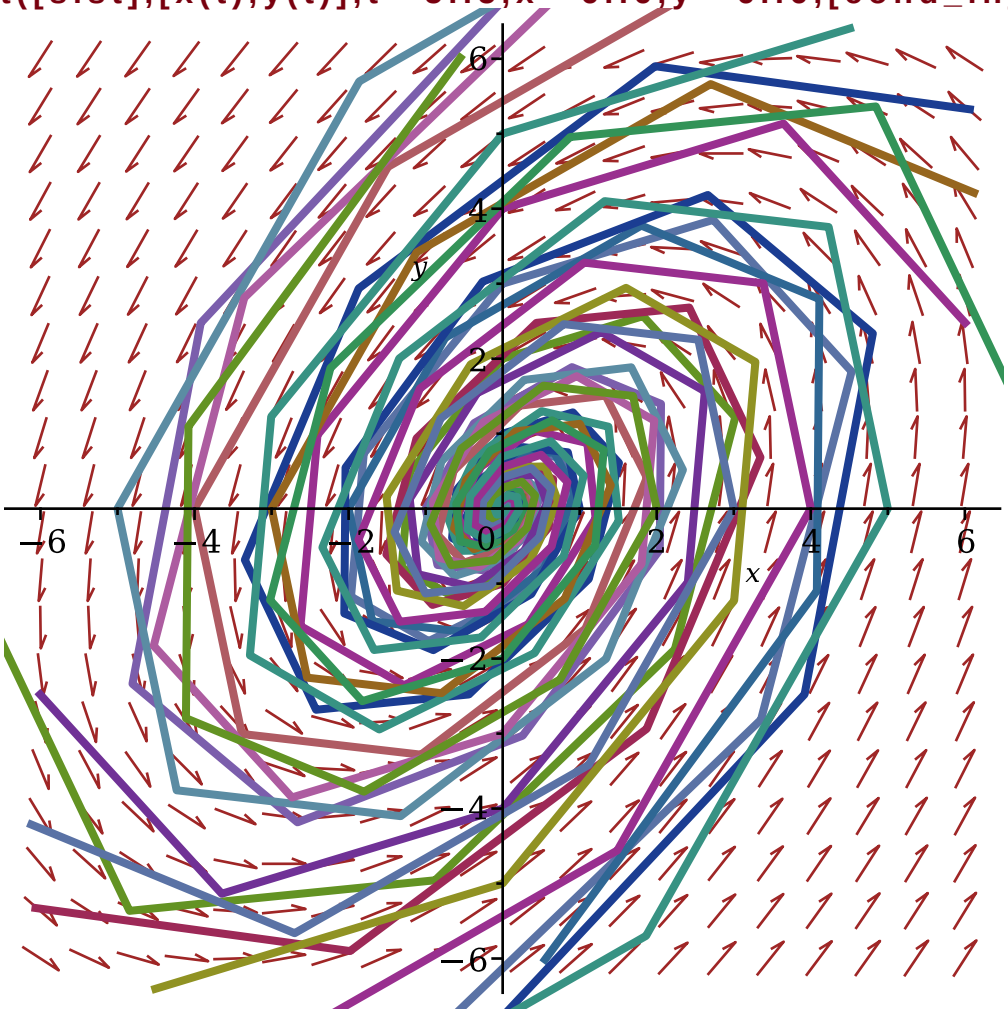
```
> eigenvals(A)
```

$$-1 + 4\,I,\ -1 - 4\,I \qquad (65)$$

```
> # (0,0) e nod stabil de tip focus
> cond_in:=[x(0)=0,y(0)=i]$i=1..5,[x(0)=-i,y(0)=0]$i=1..5,[x(0)=0,y
  (0)=-i]$i=1..5,[x(0)=i,y(0)=0]$i=1..5
```

$cond_in := [x(0) = 0, y(0) = 1], [x(0) = 0, y(0) = 2], [x(0) = 0, y(0) = 3], [x(0) = 0, y(0) = 4], [x(0) = 0, y(0) = 5], [x(0) = -1, y(0) = 0], [x(0) = -2, y(0) = 0], [x(0) = -3, y(0) = 0], [x(0) = -4, y(0) = 0], [x(0) = -5, y(0) = 0], [x(0) = 0, y(0) = -1], [x(0) = 0, y(0) = -2], [x(0) = 0, y(0) = -3], [x(0) = 0, y(0) = -4], [x(0) = 0, y(0) = -5], [x(0) = 1, y(0) = 0], [x(0) = 2, y(0) = 0], [x(0) = 3, y(0) = 0], [x(0) = 4, y(0) = 0], [x(0) = 5, y(0) = 0]$ **(66)**

```
> DEplot([sist],[x(t),y(t)],t=-5..5,x=-6..6,y=-6..6,[cond_in])
```




```
> restart
> with(DEtools), with(plots), with(linalg)
```

[*AreSimilar*, *Closure*, *DEnormal*, *DEplot*, *DEplot3d*, *DEplot_polygon*, *DFactor*, *DFactorLCLM*, *DFactorsols*, *Dchangevar*, *Desingularize*, *FindODE*, *FunctionDecomposition*, *GCRD*, *Gosper*, *Heunsols*, *Homomorphisms*, *IVPsol*, *IsHyperexponential*, *LCLM*, *MeijerGsols*, *MultiplicativeDecomposition*, *ODEInvariants*, *PDEchangecoords*, *PolynomialNormalForm*, *RationalCanonicalForm*, *ReduceHyperexp*, *RiemannPsols*, *Xchange*, *Xcommutator*, *Xgauge*, *Zeilberger*, *abelsol*, *adjoint*, *autonomous*, *bernoullisol*, *buildsol*, *buildsym*, *canoni*, *caseplot*, **(67)**

casesplit, *checkrank*, *chinisol*, *clairautsol*, *constcoeffsols*, *convertAlg*, *convertsys*, *dalembertsol*, *dcoeffs*, *de2diffop*, *dfieldplot*, *diff_table*, *diffop2de*, *dperiodic_sols*, *dpolyform*, *dsubs*, *eigenring*, *endomorphism_charpoly*, *equinv*, *eta_k*, *eulersols*, *exactsol*, *expsols*, *exterior_power*, *firint*, *firtest*, *formal_sol*, *gen_exp*, *generate_ic*, *genhomosol*, *gensys*, *hamilton_eqs*, *hypergeometricsols*, *hypergeomsols*, *hyperode*, *indicialeq*, *infgen*, *initialdata*, *integrate_sols*, *intfactor*, *invariants*, *kovacicsols*, *leftdivision*, *liesol*, *line_int*, *linearsol*, *matrixDE*, *matrix_riccati*, *maxdimsystems*, *moser_reduce*, *muchange*, *mult*, *mutest*, *newton_polygon*, *normalG2*, *ode_int_y*, *ode_y1*, *odeadvisor*, *odepde*, *parametricsol*, *particularsol*, *phaseportrait*, *poincare*, *polysols*, *power_equivalent*, *rational_equivalent*, *ratsols*, *redode*, *reduceOrder*, *reduce_order*, *regular_parts*, *regularsp*, *remove_RootOf*, *riccati_system*, *riccatisol*, *rifread*, *rifsimp*, *rightdivision*, *rtaylor*, *separablesol*, *singularities*, *solve_group*, *super_reduce*, *symgen*, *symmetric_power*, *symmetric_product*, *symtest*, *transinv*, *translate*, *untranslate*, *varparam*, *zoom*], [*animate*, *animate3d*, *animatecurve*, *arrow*, *changecoords*, *complexplot*, *complexplot3d*, *conformal*, *conformal3d*, *contourplot*, *contourplot3d*, *coordplot*, *coordplot3d*, *densityplot*, *display*, *dualaxisplot*, *fieldplot*, *fieldplot3d*, *gradplot*, *gradplot3d*, *implicitplot*, *implicitplot3d*, *inequal*, *interactive*, *interactiveparams*, *intersectplot*, *listcontplot*, *listcontplot3d*, *listdensityplot*, *listplot*, *listplot3d*, *loglogplot*, *logplot*, *matrixplot*, *multiple*, *odeplot*, *pareto*, *plotcompare*, *pointplot*, *pointplot3d*, *polarplot*, *polygonplot*, *polygonplot3d*, *polyhedra_supported*, *polyhedraplot*, *rootlocus*, *semilogplot*, *setcolors*, *setoptions*, *setoptions3d*, *shadebetween*, *spacecurve*, *sparsematrixplot*, *surfdata*, *textplot*, *textplot3d*, *tubeplot*], [*BlockDiagonal*, *GramSchmidt*, *JordanBlock*, *LUdecomp*, *QRdecomp*, *Wronskian*, *addcol*, *addrow*, *adj*, *adjoint*, *angle*, *augment*, *backsub*, *band*, *basis*, *bezout*, *blockmatrix*, *charmat*, *charpoly*, *cholesky*, *col*, *coldim*, *colspace*, *colspan*, *companion*, *concat*, *cond*, *copyinto*, *crossprod*, *curl*, *definite*, *delcols*, *delrows*, *det*, *diag*, *diverge*, *dotprod*, *eigenvals*, *eigenvalues*, *eigenvectors*, *eigenvects*, *entermatrix*, *equal*, *exponential*, *extend*, *ffgausselim*, *fibonacci*, *forwardsub*, *frobenius*, *gausselim*, *gaussjord*, *geneqns*, *genmatrix*, *grad*, *hadamard*, *hermite*, *hessian*, *hilbert*, *htranspose*, *ihermite*, *indexfunc*, *innerprod*, *intbasis*, *inverse*, *ismith*, *issimilar*, *iszero*, *jacobian*, *jordan*, *kernel*, *laplacian*, *leastsqrs*, *linsolve*, *matadd*, *matrix*, *minor*, *minpoly*, *mulcol*, *mulrow*, *multiply*, *norm*, *normalize*, *nullspace*, *orthog*, *permanent*, *pivot*, *potential*,

randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian]

```
> f1:=(x,y)->y
```

$$f1 := (x, y) \mapsto y \tag{68}$$

```
> f2:=(x,y)->x*(1-x^2)+y
```

$$f2 := (x, y) \mapsto x \cdot (1 - x^2) + y \tag{69}$$

```
> ec1:=diff(x(t),t)=f1(x(t),y(t))
```

$$ec1 := \frac{\mathrm{d}}{\mathrm{d}t}\, x(t) = y(t) \tag{70}$$

```
> ec2:=diff(y(t),t)=f2(x(t),y(t))
```

$$ec2 := \frac{\mathrm{d}}{\mathrm{d}t}\, y(t) = x(t)\ (1 - x(t)^2) + y(t) \tag{71}$$

```
> sist:=ec1,ec2
```

$$sist := \frac{\mathrm{d}}{\mathrm{d}t}\, x(t) = y(t),\ \frac{\mathrm{d}}{\mathrm{d}t}\, y(t) = x(t)\ (1 - x(t)^2) + y(t) \tag{72}$$

```
> sol:=solve({f1(x,y)=0,f2(x,y)=0},{x,y})
```

$$sol := \{x = 0, y = 0\},\ \{x = 1, y = 0\},\ \{x = -1, y = 0\} \tag{73}$$

```
> J:=jacobian([f1(x,y),f2(x,y)],[x,y])
```

$$J := \begin{bmatrix} 0 & 1 \\ -3x^2 + 1 & 1 \end{bmatrix} \tag{74}$$

```
> A1:=subs(sol[1,1],sol[1,2],eval(J))
```

$$A1 := \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix} \tag{75}$$

```
> eigenvals(A1)
```

$$\frac{1}{2} + \frac{\sqrt{5}}{2},\ \frac{1}{2} - \frac{\sqrt{5}}{2} \tag{76}$$

```
> # (0,0) e nod instabil de tip focus
> A2:=subs(sol[2,1],sol[2,2],eval(J))
```

$$A2 := \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix} \tag{77}$$

```
> eigenvals(A2)
```

$$\frac{1}{2} + \frac{\mathrm{I}\sqrt{7}}{2},\ \frac{1}{2} - \frac{\mathrm{I}\sqrt{7}}{2} \tag{78}$$

```
> # (1,0) e nod instabil de tip focus
> A3:=subs(sol[3,1],sol[3,2],eval(J))
```

$$A3 := \begin{bmatrix} 0 & 1 \\ -2 & 1 \end{bmatrix} \quad (79)$$

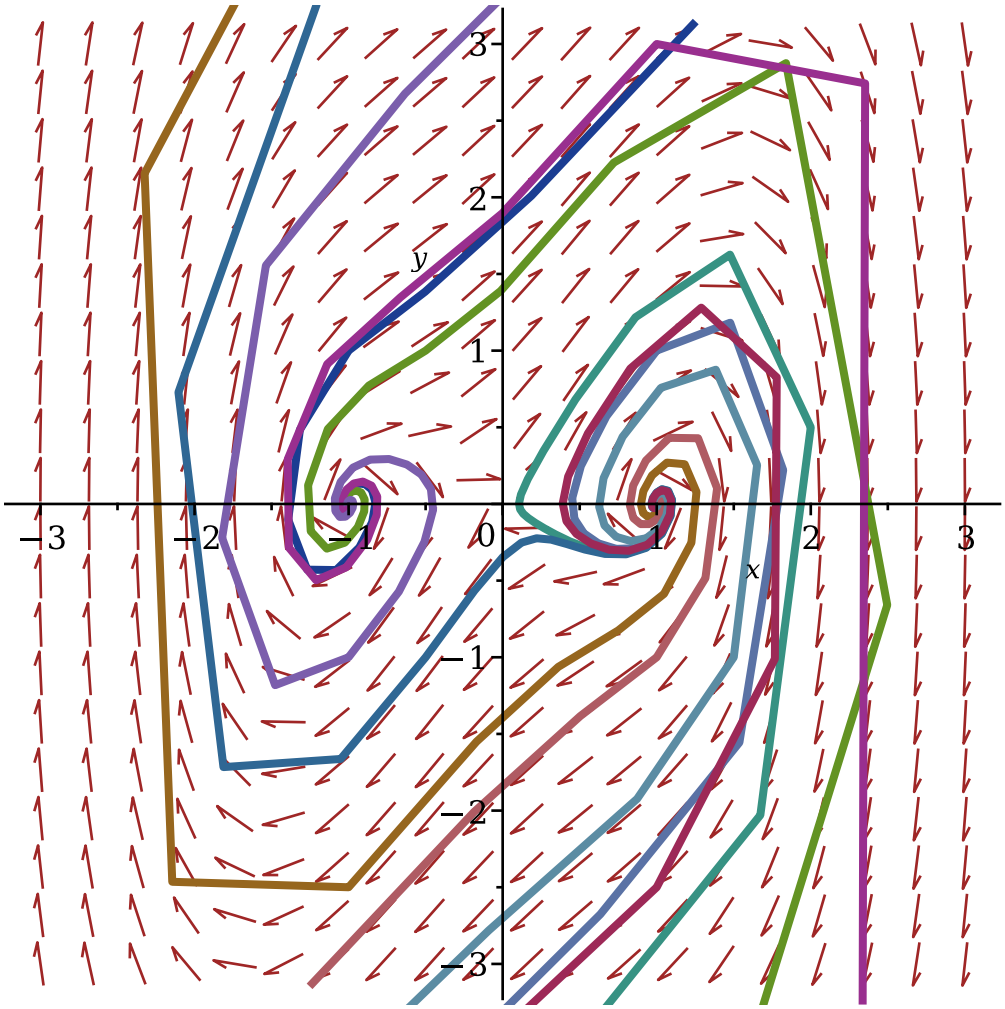
```
> eigenvals(A3)
```

$$\frac{1}{2} + \frac{I\sqrt{7}}{2}, \frac{1}{2} - \frac{I\sqrt{7}}{2} \quad (80)$$

```
> # (-1,0) e nod instabil de tip focus
> cond_in:=[x(0)=-1,y(0)=1],[x(0)=-0.5,y(0)=1],[x(0)=1,y(0)=1],[x
  (0)=1,y(0)=3],[x(0)=2,y(0)=0.5],[x(0)=-1,y(0)=-1],[x(0)=-0.5,y(0)
  =-1],[x(0)=-1,y(0)=-2.5],[x(0)=1,y(0)=-1],[x(0)=1.5,y(0)=-1],[x
  (0)=1,y(0)=-2.5]
```

$$cond_in := [x(0) = -1, y(0) = 1], [x(0) = -0.5, y(0) = 1], [x(0) = 1, y(0) = 1], [x(0) = 1, y(0) = 3], [x(0) = 2, y(0) = 0.5], [x(0) = -1, y(0) = -1], [x(0) = -0.5, y(0) = -1], [x(0) = -1, y(0) = -2.5], [x(0) = 1, y(0) = -1], [x(0) = 1.5, y(0) = -1], [x(0) = 1, y(0) = -2.5] \quad (81)$$

```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-3..3,y=-3..3,[cond_in])
```




```
> f1:=(x,y)->-2*x+y+2
```

$$f1 := (x, y) \mapsto -2 \cdot x + y + 2 \tag{82}$$

```
> f2:=(x,y)->x*y
```

$$f2 := (x, y) \mapsto y \cdot x \tag{83}$$

```
> ec1:=diff(x(t),t)=f1(x(t),y(t))
```

$$ec1 := \frac{\mathrm{d}}{\mathrm{d}t} x(t) = -2\,x(t) + y(t) + 2 \tag{84}$$

```
> ec2:=diff(y(t),t)=f2(x(t),y(t))
```

$$ec2 := \frac{\mathrm{d}}{\mathrm{d}t} y(t) = y(t)\,x(t) \tag{85}$$

```
> sist:=ec1,ec2
```

$$sist := \frac{\mathrm{d}}{\mathrm{d}t} x(t) = -2\,x(t) + y(t) + 2,\ \frac{\mathrm{d}}{\mathrm{d}t} y(t) = y(t)\,x(t) \tag{86}$$

```
> sol:=solve({f1(x,y)=0,f2(x,y)=0},{x,y})
```

$$sol := \{x = 1, y = 0\}, \{x = 0, y = -2\} \tag{87}$$

```
> J:=jacobian([f1(x,y),f2(x,y)],[x,y])
```

(88)

$$J := \begin{bmatrix} -2 & 1 \\ y & x \end{bmatrix} \quad (88)$$

```
> A1:=subs(sol[1,1],sol[1,2],eval(J))
```

$$A1 := \begin{bmatrix} -2 & 1 \\ 0 & 1 \end{bmatrix} \quad (89)$$

```
> eigenvals(A1)
```

$$-2, 1 \quad (90)$$

```
> # (1,0) e nod instabil de tip sa
> A2:=subs(sol[2,1],sol[2,2],eval(J))
```

$$A2 := \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix} \quad (91)$$

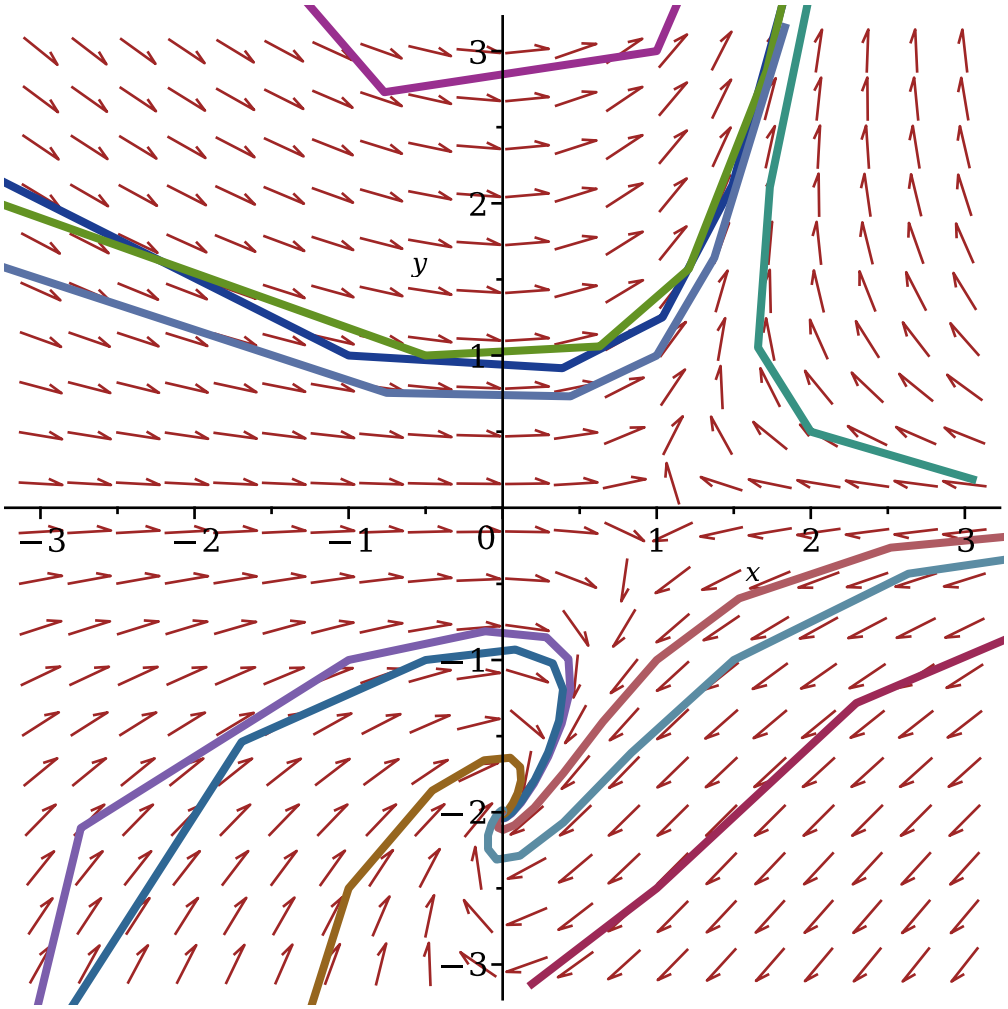
```
> eigenvals(A2)
```

$$-1 + I, -1 - I \quad (92)$$

```
> # (0,-2) e nod stabil de tip focus
> cond_in:=[x(0)=-1,y(0)=1],[x(0)=-0.5,y(0)=1],[x(0)=1,y(0)=1],[x
  (0)=1,y(0)=3],[x(0)=2,y(0)=0.5],[x(0)=-1,y(0)=-1],[x(0)=-0.5,y(0)
  =-1],[x(0)=-1,y(0)=-2.5],[x(0)=1,y(0)=-1],[x(0)=1.5,y(0)=-1],[x
  (0)=1,y(0)=-2.5]
```

$$cond_in := [x(0) = -1, y(0) = 1], [x(0) = -0.5, y(0) = 1], [x(0) = 1, y(0) = 1], [x(0) = 1, y(0) = 3], [x(0) = 2, y(0) = 0.5], [x(0) = -1, y(0) = -1], [x(0) = -0.5, y(0) = -1], [x(0) = -1, y(0) = -2.5], [x(0) = 1, y(0) = -1], [x(0) = 1.5, y(0) = -1], [x(0) = 1, y(0) = -2.5] \quad (93)$$

```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-3..3,y=-3..3,[cond_in])
```




```
> f1:=(x,y)->-y^2
```

$$f1 := (x, y) \mapsto -y^2 \quad (94)$$

```
> f2:=(x,y)->x
```

$$f2 := (x, y) \mapsto x \quad (95)$$

```
> ec1:=diff(x(t),t)=f1(x(t),y(t))
```

$$ec1 := \frac{\mathrm{d}}{\mathrm{d}t} x(t) = -y(t)^2 \quad (96)$$

```
> ec2:=diff(y(t),t)=f2(x(t),y(t))
```

$$ec2 := \frac{\mathrm{d}}{\mathrm{d}t} y(t) = x(t) \quad (97)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{\mathrm{d}}{\mathrm{d}t} x(t) = -y(t)^2, \frac{\mathrm{d}}{\mathrm{d}t} y(t) = x(t) \quad (98)$$

```
> sol:=solve({f1(x,y)=0,f2(x,y)=0},{x,y})
```

$$sol := \{x = 0, y = 0\} \quad (99)$$

```
> J:=jacobian([f1(x,y),f2(x,y)],[x,y])
```

(100)

$$J := \begin{bmatrix} 0 & -2y \\ 1 & 0 \end{bmatrix} \quad (100)$$

```
> A1:=subs(sol[1],sol[2],eval(J))
```

$$A1 := \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \quad (101)$$

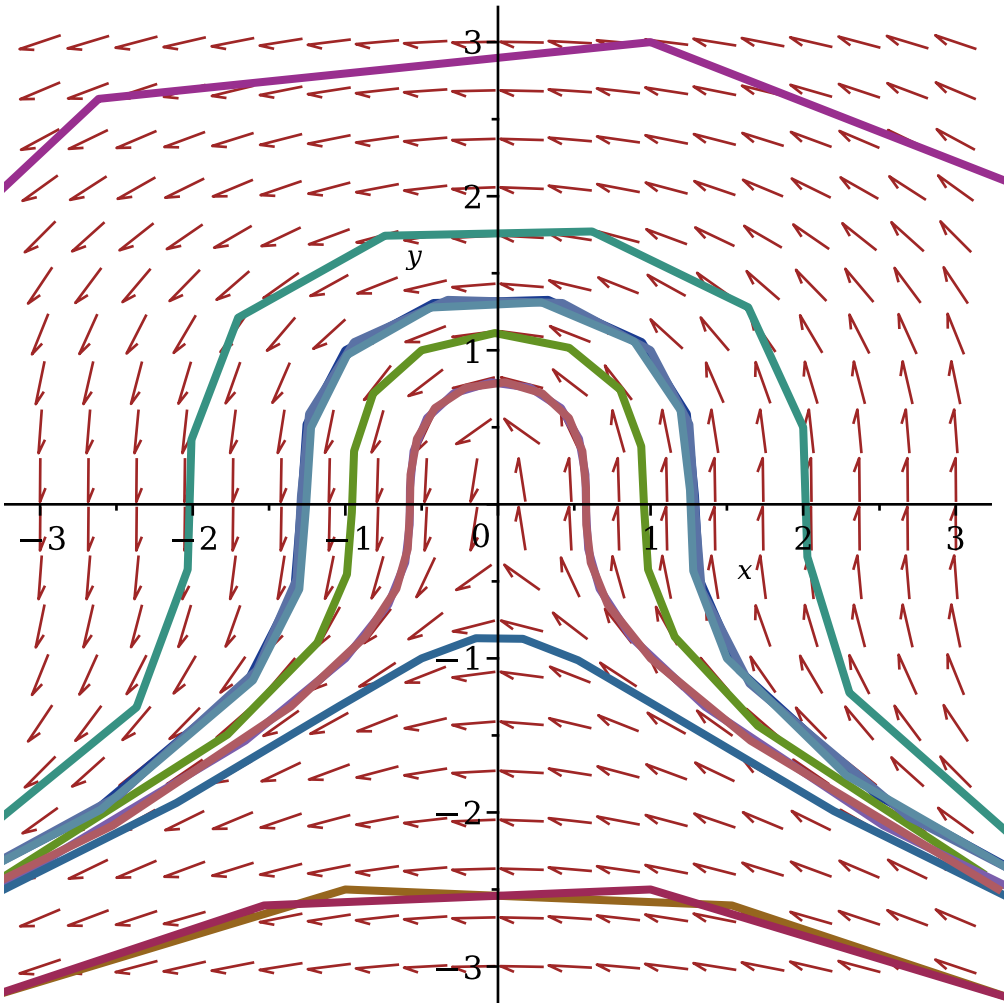
```
> eigenvals(A1)
```

$$0, 0 \quad (102)$$

```
> # (0,0) e nod stabil de tip centru
> cond_in:=[x(0)=-1,y(0)=1],[x(0)=-0.5,y(0)=1],[x(0)=1,y(0)=1],[x
  (0)=1,y(0)=3],[x(0)=2,y(0)=0.5],[x(0)=-1,y(0)=-1],[x(0)=-0.5,y(0)
  =-1],[x(0)=-1,y(0)=-2.5],[x(0)=1,y(0)=-1],[x(0)=1.5,y(0)=-1],[x
  (0)=1,y(0)=-2.5]
```

$$cond_in := [x(0) = -1, y(0) = 1], [x(0) = -0.5, y(0) = 1], [x(0) = 1, y(0) = 1], [x(0) = 1, y(0) = 3], [x(0) = 2, y(0) = 0.5], [x(0) = -1, y(0) = -1], [x(0) = -0.5, y(0) = -1], [x(0) = -1, y(0) = -2.5], [x(0) = 1, y(0) = -1], [x(0) = 1.5, y(0) = -1], [x(0) = 1, y(0) = -2.5] \quad (103)$$

```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-3..3,y=-3..3,[cond_in])
```




```
> f1:=(x,y)->-x^2-y^2
```

$$f1 := (x, y) \mapsto -x^2 - y^2 \quad (104)$$

```
> f2:=(x,y)->x*y-1
```

$$f2 := (x, y) \mapsto y \cdot x - 1 \quad (105)$$

```
> ec1:=diff(x(t),t)=f1(x(t),y(t))
```

$$ec1 := \frac{d}{dt} x(t) = -x(t)^2 - y(t)^2 \quad (106)$$

```
> ec2:=diff(y(t),t)=f2(x(t),y(t))
```

$$ec2 := \frac{d}{dt} y(t) = y(t)\, x(t) - 1 \quad (107)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = -x(t)^2 - y(t)^2,\ \frac{d}{dt} y(t) = y(t)\, x(t) - 1 \quad (108)$$

```
> sol:=solve({f1(x,y)=0,f2(x,y)=0},{x,y})
```

$$sol := \{x = -RootOf(_Z^4 + 1)^3,\ y = RootOf(_Z^4 + 1)\} \quad (109)$$

```
> J:=jacobian([f1(x,y),f2(x,y)],[x,y])
```

(110)

$$J := \begin{bmatrix} -2x & -2y \\ y & x \end{bmatrix} \quad (110)$$

```
> A1:=subs(sol[1],sol[2],eval(J))
```

$$A1 := \begin{bmatrix} 2\,RootOf(_Z^4+1)^3 & -2\,RootOf(_Z^4+1) \\ RootOf(_Z^4+1) & -RootOf(_Z^4+1)^3 \end{bmatrix} \quad (111)$$

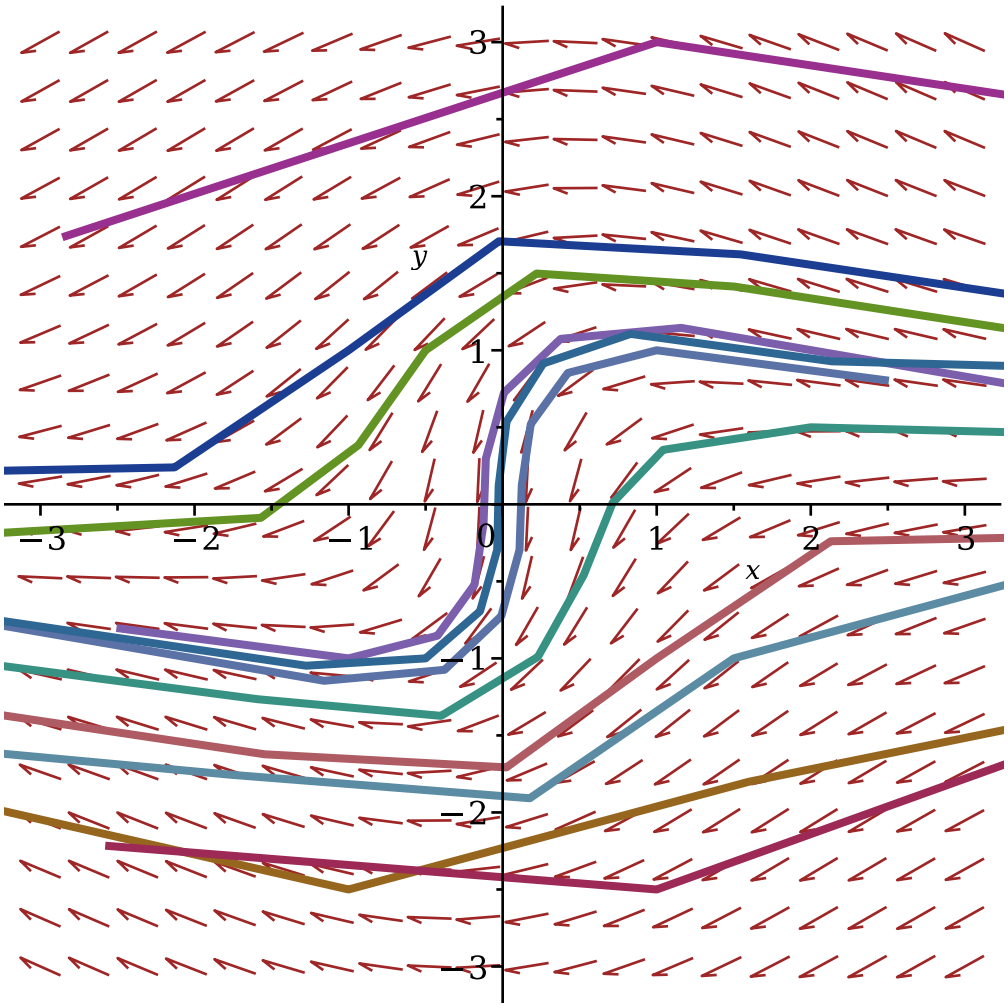
```
> evalf(eigenvals(A1))
```

$$1.104184583 - 1.104184583\,I \quad (112)$$

```
> # chestia ai e nod instabil de tip focus
> cond_in:=[x(0)=-1,y(0)=1],[x(0)=-0.5,y(0)=1],[x(0)=1,y(0)=1],[x
  (0)=1,y(0)=3],[x(0)=2,y(0)=0.5],[x(0)=-1,y(0)=-1],[x(0)=-0.5,y(0)
  =-1],[x(0)=-1,y(0)=-2.5],[x(0)=1,y(0)=-1],[x(0)=1.5,y(0)=-1],[x
  (0)=1,y(0)=-2.5]
```

$$cond_in := [x(0) = -1, y(0) = 1], [x(0) = -0.5, y(0) = 1], [x(0) = 1, y(0) = 1], [x(0) = 1, y(0) = 3], [x(0) = 2, y(0) = 0.5], [x(0) = -1, y(0) = -1], [x(0) = -0.5, y(0) = -1], [x(0) = -1, y(0) = -2.5], [x(0) = 1, y(0) = -1], [x(0) = 1.5, y(0) = -1], [x(0) = 1, y(0) = -2.5] \quad (113)$$

```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-3..3,y=-3..3,[cond_in])
Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further left of -.80599150, probably
a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further right of .74506202, probably
a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further right of .80599150, probably
a singularity
Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further right of .75566874, probably
a singularity
```




```
> restart
> with(DEtools); with(plots); with(linalg)
```

[*AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeometricsols, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor,*

invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom]

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*]

[*BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian*] **(114)**

```
> f1:=(x,y)->-2*x-1.2*x*y
```

$$f1 := (x, y) \mapsto -2 \cdot x - 1.2 \cdot y \cdot x \quad (115)$$

```
> f2:=(x,y)->-y+0.9*x*y
```

$$f2 := (x, y) \mapsto -y + 0.9 \cdot y \cdot x \quad (116)$$

```
> ec1:=diff(x(t),t)=f1(x(t),y(t))
```

$$ec1 := \frac{d}{dt} x(t) = -2\,x(t) - 1.2\,y(t)\,x(t) \quad (117)$$

```
> ec2:=diff(y(t),t)=f2(x(t),y(t))
```

$$ec2 := \frac{d}{dt} y(t) = -y(t) + 0.9\,y(t)\,x(t) \quad (118)$$

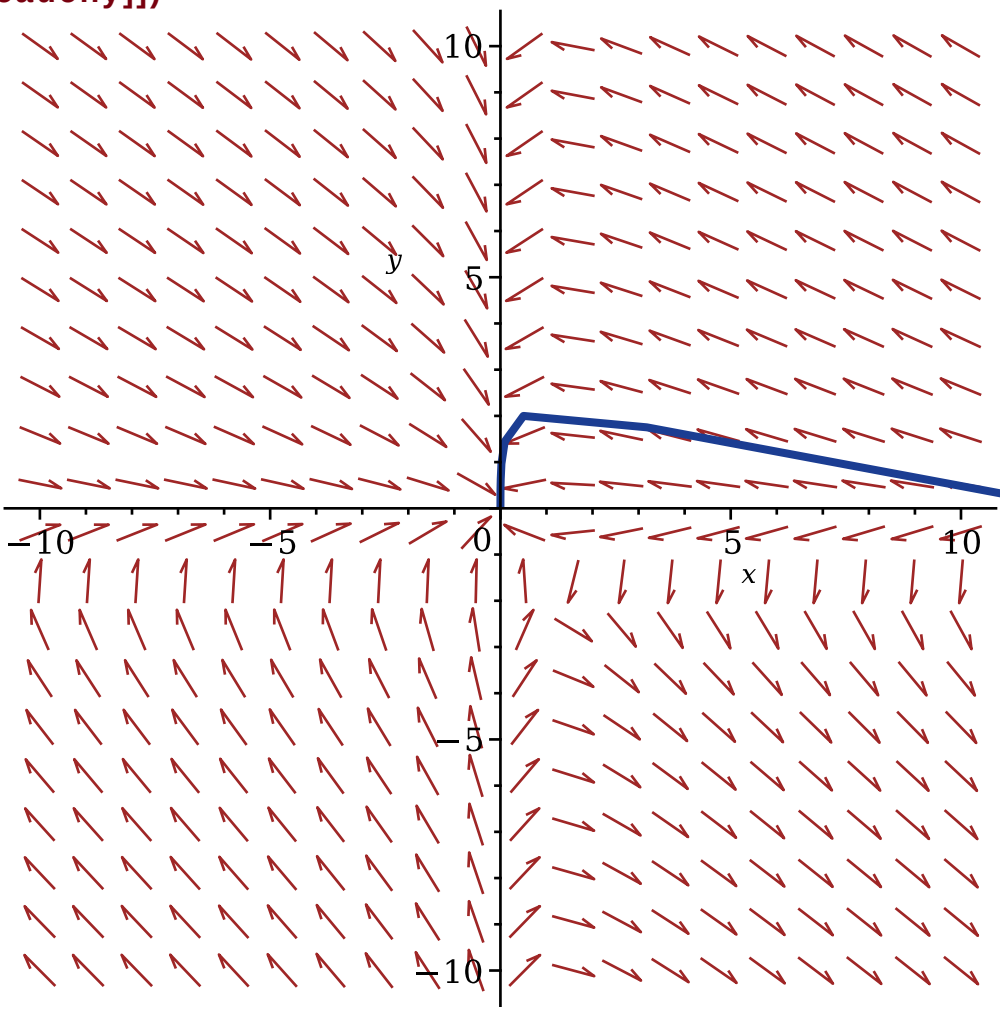
```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = -2\,x(t) - 1.2\,y(t)\,x(t),\ \frac{d}{dt} y(t) = -y(t) + 0.9\,y(t)\,x(t) \quad (119)$$

```
> cond_cauchy:=x(0)=0.5,y(0)=2
```

$$cond_cauchy := x(0) = 0.5,\ y(0) = 2 \quad (120)$$

```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10,[
  [cond_cauchy]])
```




```
> sol:=solve({f1(x,y)=0,f2(x,y)=0},{x,y})
```

$$sol := \{x = 0.,\ y = 0.\},\ \{x = 1.111111111,\ y = -1.666666667\} \quad (121)$$

```
> J:=jacobian([f1(x,y),f2(x,y)],[x,y])
```

$$J := \begin{bmatrix} -2 - 1.2\,y & -1.2\,x \\ 0.9\,y & -1 + 0.9\,x \end{bmatrix} \quad (122)$$

```
> A1:=subs(sol[1,1],sol[1,2],eval(J))
```

$$A1 := \begin{bmatrix} -2. & -0. \\ 0. & -1. \end{bmatrix} \quad (123)$$

```
> evalf(eigenvals(A1))
```

$$-2., -1. \quad (124)$$

```
> # (0,0) e punct de echilibru stabil de tip nod
> A2:=subs(sol[2,1],sol[2,2],eval(J))
```

$$A2 := \begin{bmatrix} 0. & -1.333333333 \\ -1.500000000 & -1. \times 10^{-10} \end{bmatrix} \quad (125)$$

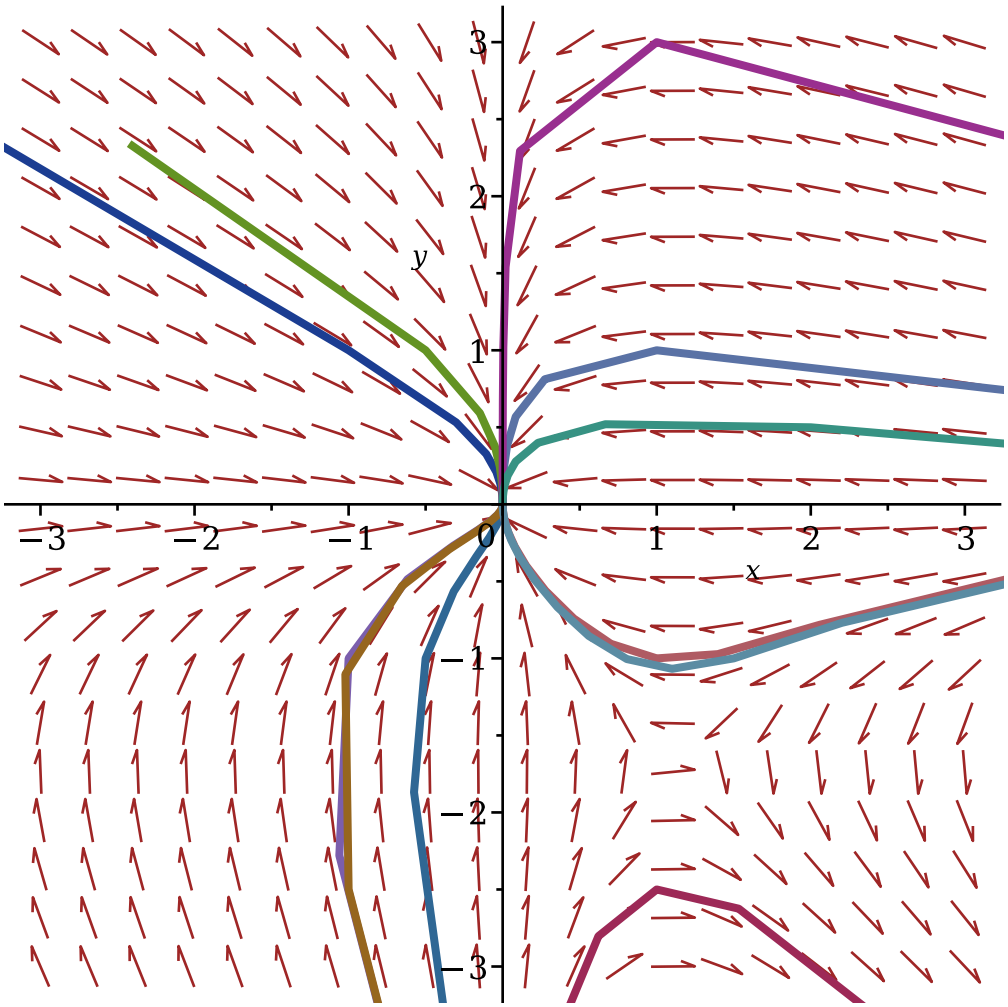
```
> evalf(eigenvals(A2))
```

$$1.41421356214632, -1.41421356224632 \quad (126)$$

```
> # (0,0) e punct de echilibru instabil de tip nod
> cond_in:=[x(0)=-1,y(0)=1],[x(0)=-0.5,y(0)=1],[x(0)=1,y(0)=1],[x
  (0)=1,y(0)=3],[x(0)=2,y(0)=0.5],[x(0)=-1,y(0)=-1],[x(0)=-0.5,y(0)
  =-1],[x(0)=-1,y(0)=-2.5],[x(0)=1,y(0)=-1],[x(0)=1.5,y(0)=-1],[x
  (0)=1,y(0)=-2.5]
```

$$cond_in := [x(0) = -1, y(0) = 1], [x(0) = -0.5, y(0) = 1], [x(0) = 1, y(0) = 1], [x(0) = 1, y(0) = 3], [x(0) = 2, y(0) = 0.5], [x(0) = -1, y(0) = -1], [x(0) = -0.5, y(0) = -1], [x(0) = -1, y(0) = -2.5], [x(0) = 1, y(0) = -1], [x(0) = 1.5, y(0) = -1], [x(0) = 1, y(0) = -2.5] \quad (127)$$

```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-3..3,y=-3..3,[cond_in])
Warning, plot may be incomplete, the following errors(s) were issued:
   cannot evaluate the solution further left of -.73211244, probably
a singularity
```




```
> restart
> with(DEtools); with(plots); with(linalg)
```

[*AreSimilar, Closure, DEnormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeometricsols, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor,*

invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom]

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*]

[*BlockDiagonal, GramSchmidt, JordanBlock, LUdecomp, QRdecomp, Wronskian, addcol, addrow, adj, adjoint, angle, augment, backsub, band, basis, bezout, blockmatrix, charmat, charpoly, cholesky, col, coldim, colspace, colspan, companion, concat, cond, copyinto, crossprod, curl, definite, delcols, delrows, det, diag, diverge, dotprod, eigenvals, eigenvalues, eigenvectors, eigenvects, entermatrix, equal, exponential, extend, ffgausselim, fibonacci, forwardsub, frobenius, gausselim, gaussjord, geneqns, genmatrix, grad, hadamard, hermite, hessian, hilbert, htranspose, ihermite, indexfunc, innerprod, intbasis, inverse, ismith, issimilar, iszero, jacobian, jordan, kernel, laplacian, leastsqrs, linsolve, matadd, matrix, minor, minpoly, mulcol, mulrow, multiply, norm, normalize, nullspace, orthog, permanent, pivot, potential, randmatrix, randvector, rank, ratform, row, rowdim, rowspace, rowspan, rref, scalarmul, singularvals, smith, stackmatrix, submatrix, subvector, sumbasis, swapcol, swaprow, sylvester, toeplitz, trace, transpose, vandermonde, vecpotent, vectdim, vector, wronskian*] **(128)**

```
> f1:=(x,y)->x-x*y-H*x
```

$$f1 := (x, y) \mapsto x - y \cdot x - H \cdot x \quad (129)$$

```
> f2:=(x,y)->-y+x*y-H*y
```

$$f2 := (x, y) \mapsto -y + y \cdot x - H \cdot y \quad (130)$$

```
> ec1:=diff(x(t),t)=f1(x(t),y(t))
```

$$ec1 := \frac{d}{dt} x(t) = x(t) - y(t)\, x(t) - H\, x(t) \quad (131)$$

```
> ec2:=diff(y(t),t)=f2(x(t),y(t))
```

$$ec2 := \frac{d}{dt} y(t) = -y(t) + y(t)\, x(t) - H\, y(t) \quad (132)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = x(t) - y(t)\, x(t) - H\, x(t),\ \frac{d}{dt} y(t) = -y(t) + y(t)\, x(t) - H\, y(t) \quad (133)$$

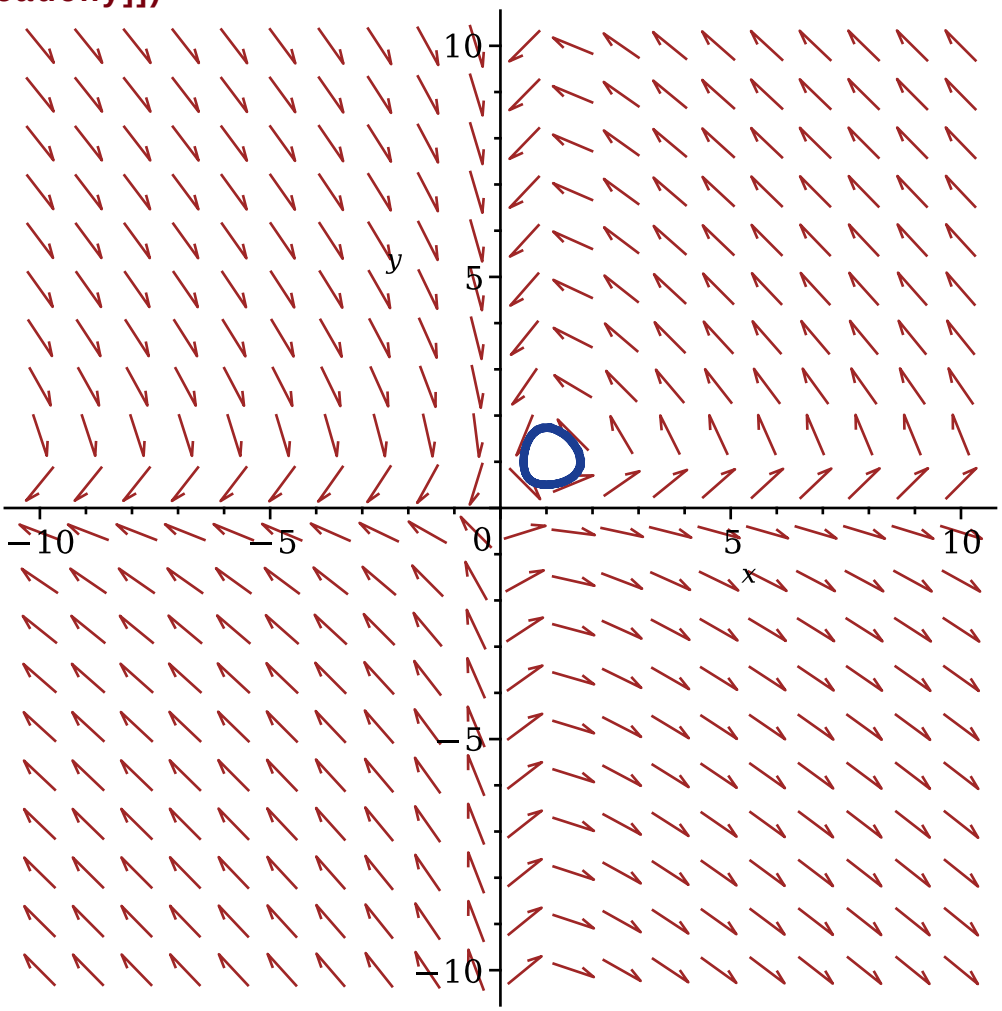
```
> cond_cauchy:=x(0)=1,y(0)=0.5
```

$$cond_cauchy := x(0) = 1, y(0) = 0.5 \quad (134)$$

```
> H:=0
```

$$H := 0 \quad (135)$$

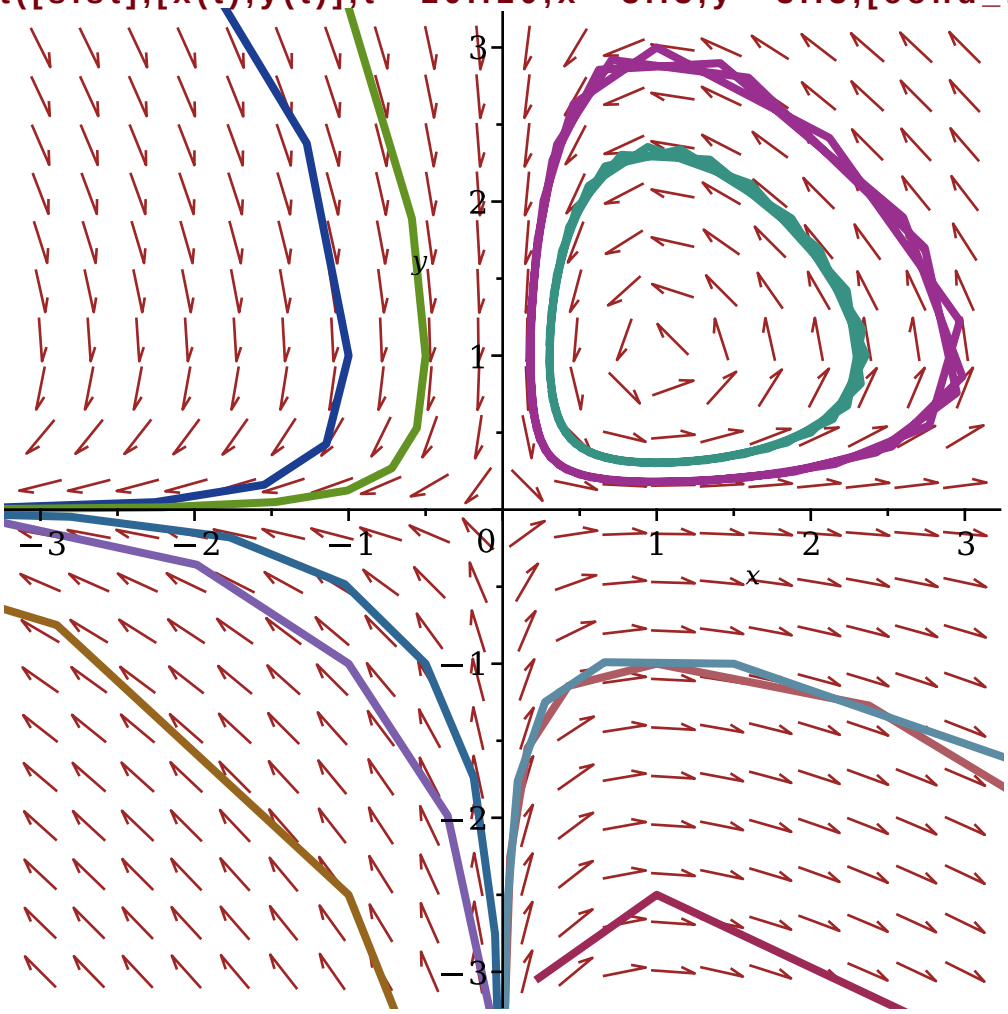
```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10,[
  [cond_cauchy]])
```



```
> cond_in:=[x(0)=-1,y(0)=1],[x(0)=-0.5,y(0)=1],[x(0)=1,y(0)=1],[x
  (0)=1,y(0)=3],[x(0)=2,y(0)=0.5],[x(0)=-1,y(0)=-1],[x(0)=-0.5,y(0)
  =-1],[x(0)=-1,y(0)=-2.5],[x(0)=1,y(0)=-1],[x(0)=1.5,y(0)=-1],[x
  (0)=1,y(0)=-2.5]
```

$$cond_in := [x(0) = -1, y(0) = 1], [x(0) = -0.5, y(0) = 1], [x(0) = 1, y(0) = 1], [x(0) = 1, y(0) = 3], [x(0) = 2, y(0) = 0.5], [x(0) = -1, y(0) = -1], [x(0) = -0.5, y(0) = -1], [x(0) = -1, y(0) = -2.5], [x(0) = 1, y(0) = -1], [x(0) = 1.5, y(0) = -1], [x(0) = 1, y(0) = -2.5] \quad (136)$$

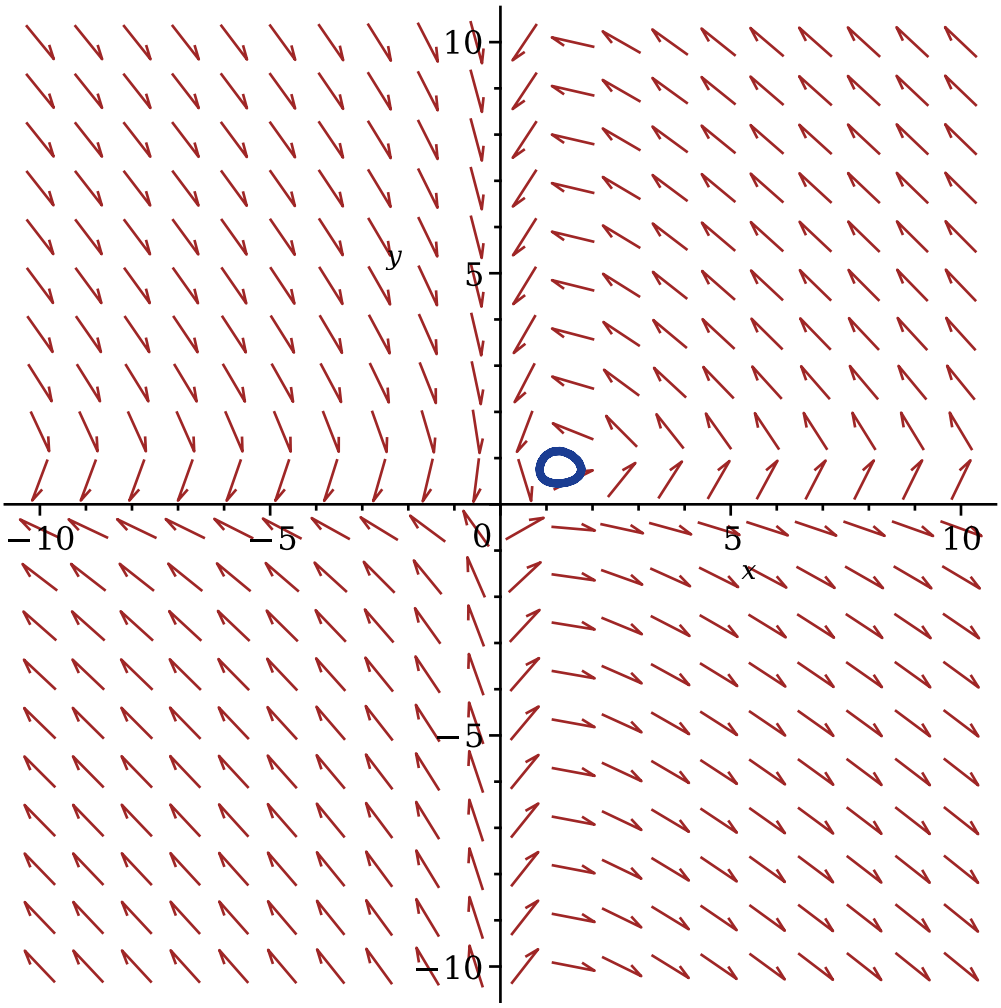
```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-3..3,y=-3..3,[cond_in])
```




```
> H:=0.25
```

$$H := 0.25 \quad (137)$$

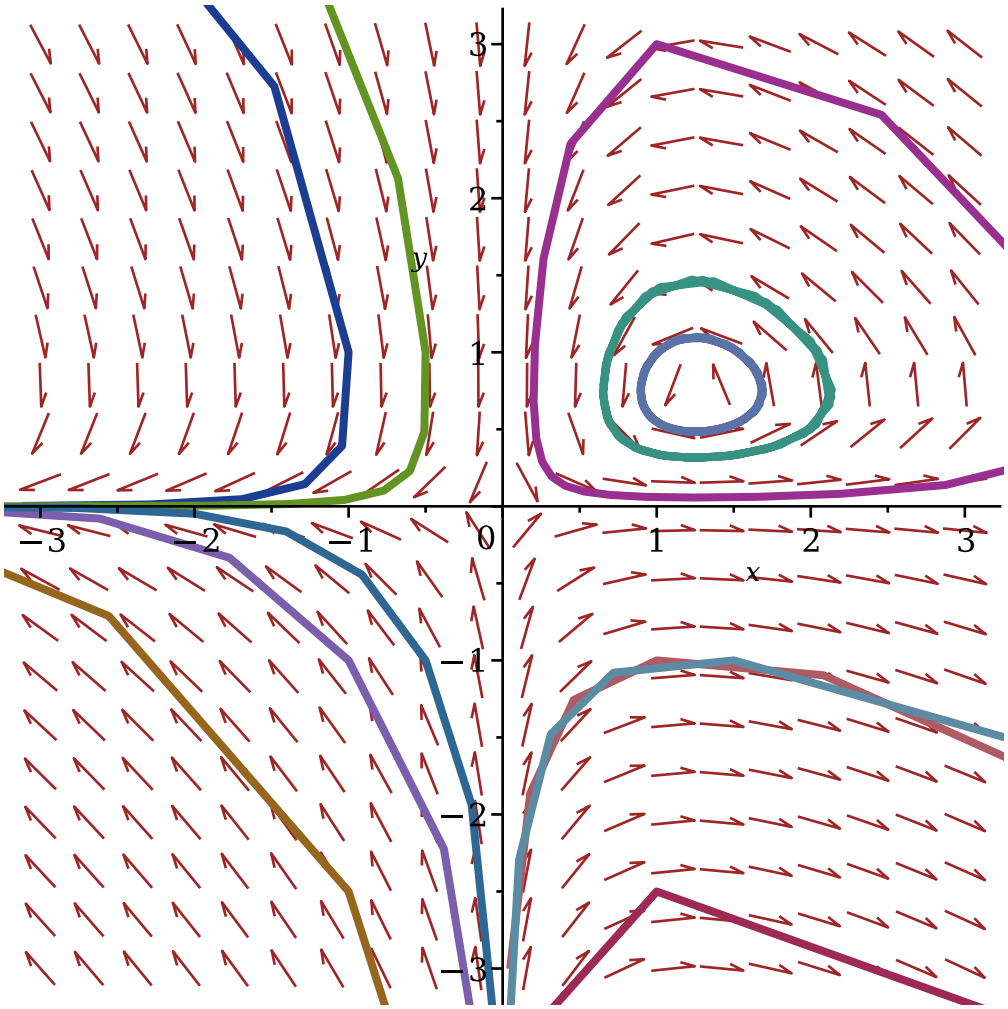
```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10,[
  [cond_cauchy]])
```




```
> cond_in:=[x(0)=-1,y(0)=1],[x(0)=-0.5,y(0)=1],[x(0)=1,y(0)=1],[x
  (0)=1,y(0)=3],[x(0)=2,y(0)=0.5],[x(0)=-1,y(0)=-1],[x(0)=-0.5,y(0)
  =-1],[x(0)=-1,y(0)=-2.5],[x(0)=1,y(0)=-1],[x(0)=1.5,y(0)=-1],[x
  (0)=1,y(0)=-2.5]
```

$$cond_in := [x(0) = -1, y(0) = 1], [x(0) = -0.5, y(0) = 1], [x(0) = 1, y(0) = 1], [x(0) = 1, y(0) = 3], [x(0) = 2, y(0) = 0.5], [x(0) = -1, y(0) = -1], [x(0) = -0.5, y(0) = -1], [x(0) = -1, y(0) = -2.5], [x(0) = 1, y(0) = -1], [x(0) = 1.5, y(0) = -1], [x(0) = 1, y(0) = -2.5] \quad (138)$$

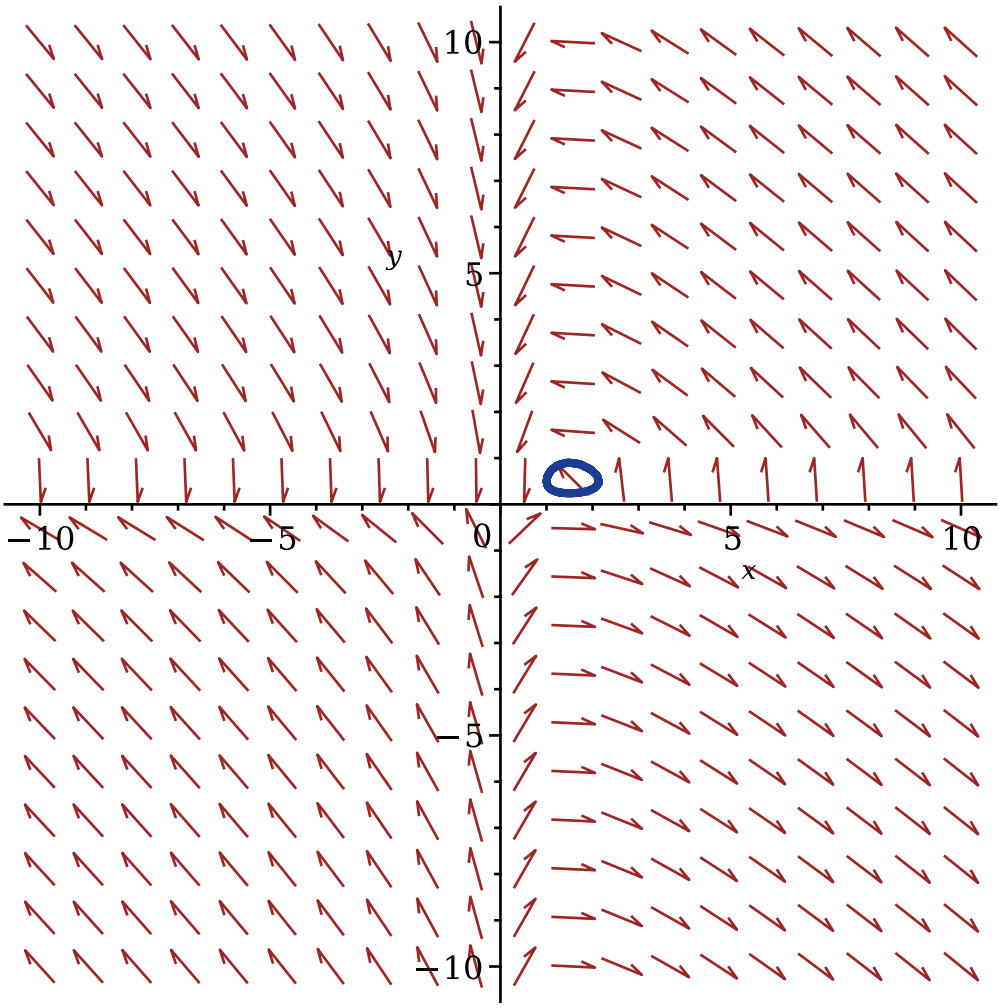
```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-3..3,y=-3..3,[cond_in])
```




```
> H:=0.5
```

$$H := 0.5 \qquad (139)$$

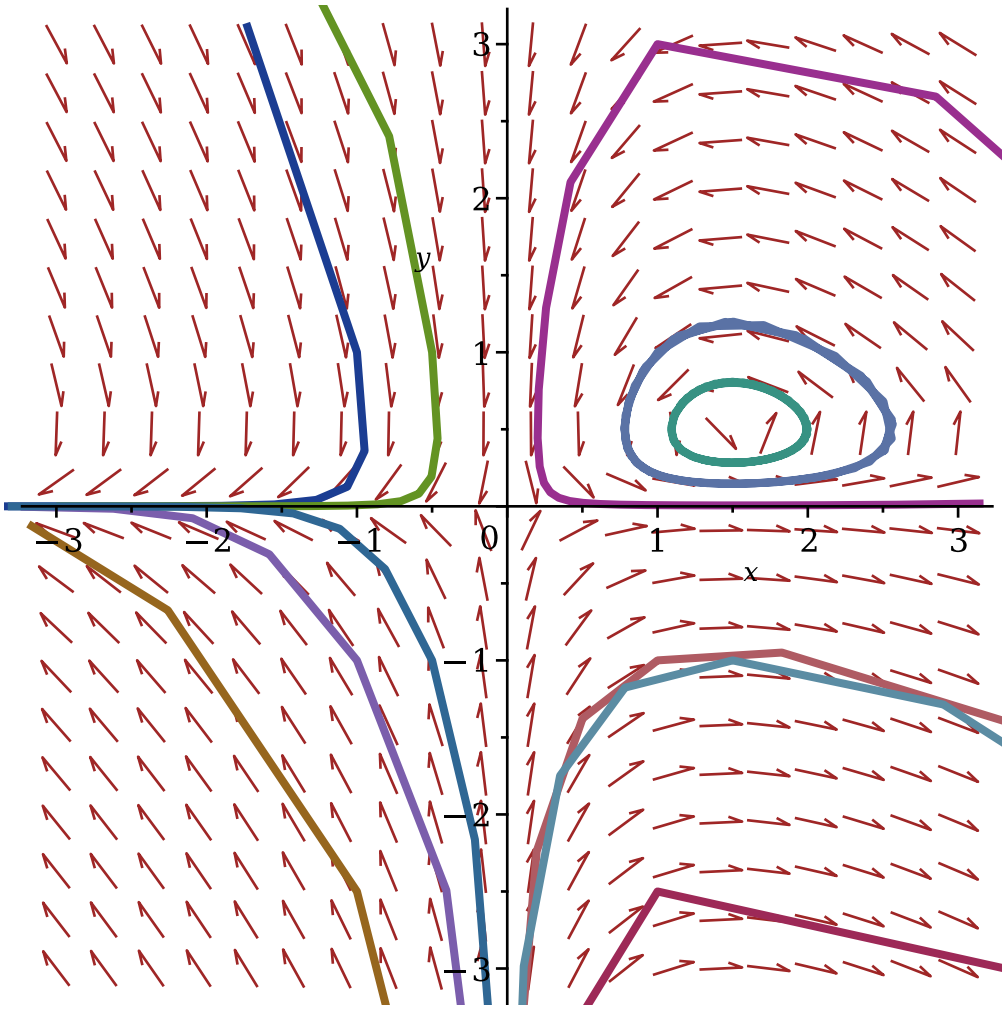
```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10,[
  [cond_cauchy]])
```




```
> cond_in:=[x(0)=-1,y(0)=1],[x(0)=-0.5,y(0)=1],[x(0)=1,y(0)=1],[x
  (0)=1,y(0)=3],[x(0)=2,y(0)=0.5],[x(0)=-1,y(0)=-1],[x(0)=-0.5,y(0)
  =-1],[x(0)=-1,y(0)=-2.5],[x(0)=1,y(0)=-1],[x(0)=1.5,y(0)=-1],[x
  (0)=1,y(0)=-2.5]
```

$$cond_in := [x(0) = -1, y(0) = 1], [x(0) = -0.5, y(0) = 1], [x(0) = 1, y(0) = 1], [x(0) = 1, y(0) = 3], [x(0) = 2, y(0) = 0.5], [x(0) = -1, y(0) = -1], [x(0) = -0.5, y(0) = -1], [x(0) = -1, y(0) = -2.5], [x(0) = 1, y(0) = -1], [x(0) = 1.5, y(0) = -1], [x(0) = 1, y(0) = -2.5] \quad (140)$$

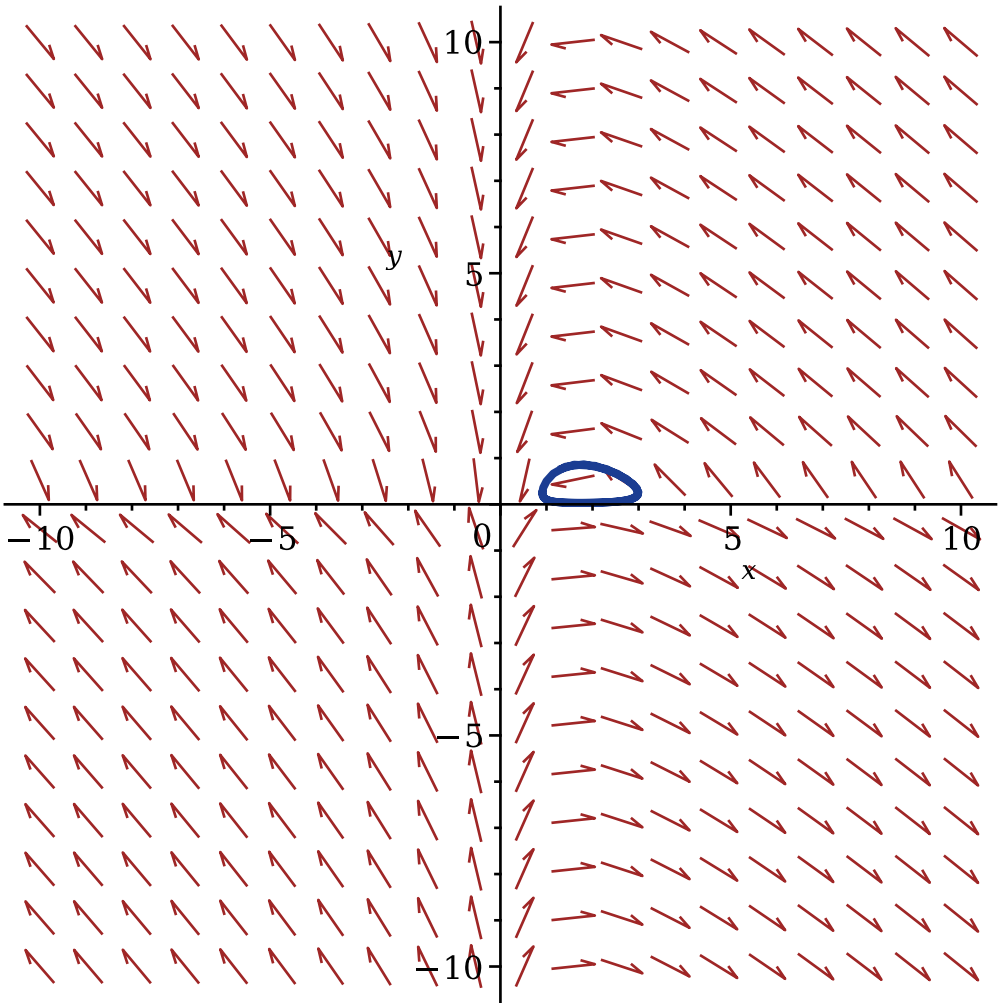
```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-3..3,y=-3..3,[cond_in])
```




```
> H:=0.75
```

$$H := 0.75 \qquad (141)$$

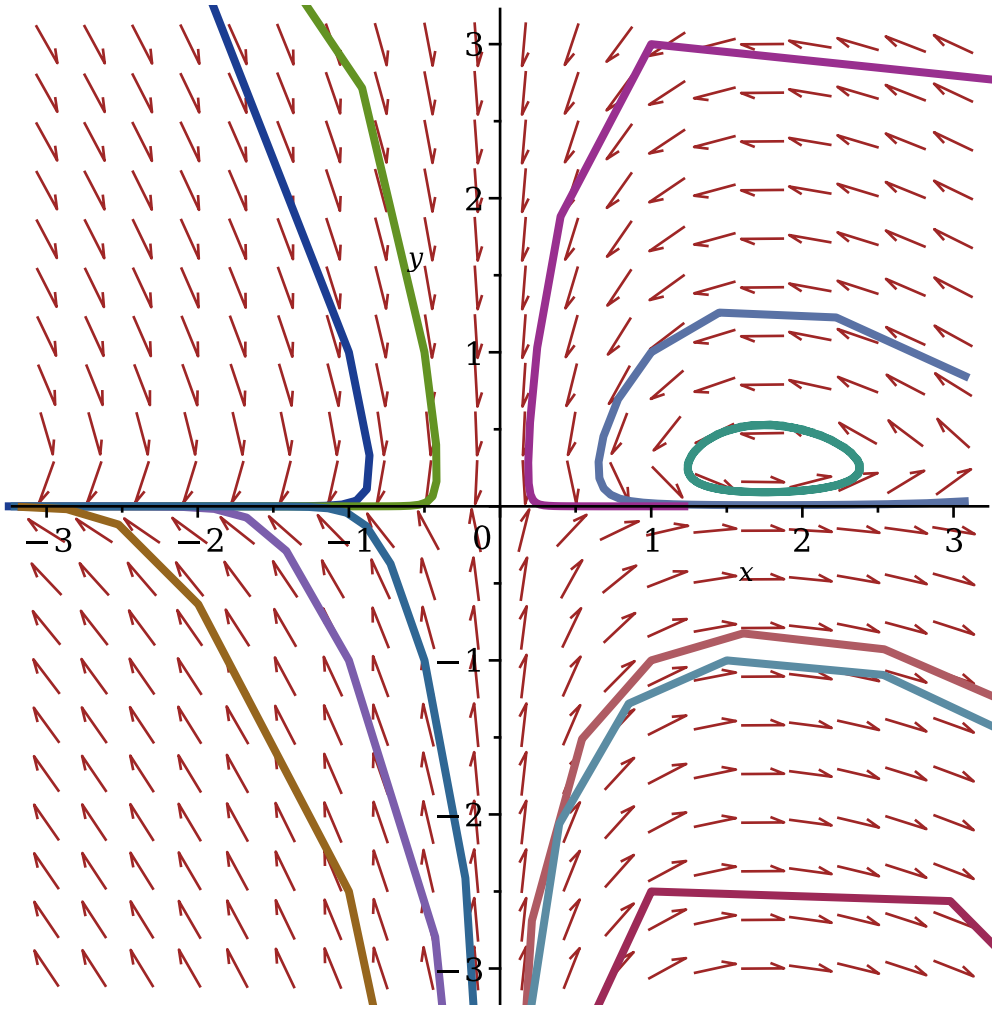
```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10,[
  [cond_cauchy]])
```




```
> cond_in:=[x(0)=-1,y(0)=1],[x(0)=-0.5,y(0)=1],[x(0)=1,y(0)=1],[x
  (0)=1,y(0)=3],[x(0)=2,y(0)=0.5],[x(0)=-1,y(0)=-1],[x(0)=-0.5,y(0)
  =-1],[x(0)=-1,y(0)=-2.5],[x(0)=1,y(0)=-1],[x(0)=1.5,y(0)=-1],[x
  (0)=1,y(0)=-2.5]
```

$$cond_in := [x(0) = -1, y(0) = 1], [x(0) = -0.5, y(0) = 1], [x(0) = 1, y(0) = 1], [x(0) = 1, y(0) = 3], [x(0) = 2, y(0) = 0.5], [x(0) = -1, y(0) = -1], [x(0) = -0.5, y(0) = -1], [x(0) = -1, y(0) = -2.5], [x(0) = 1, y(0) = -1], [x(0) = 1.5, y(0) = -1], [x(0) = 1, y(0) = -2.5] \quad (142)$$

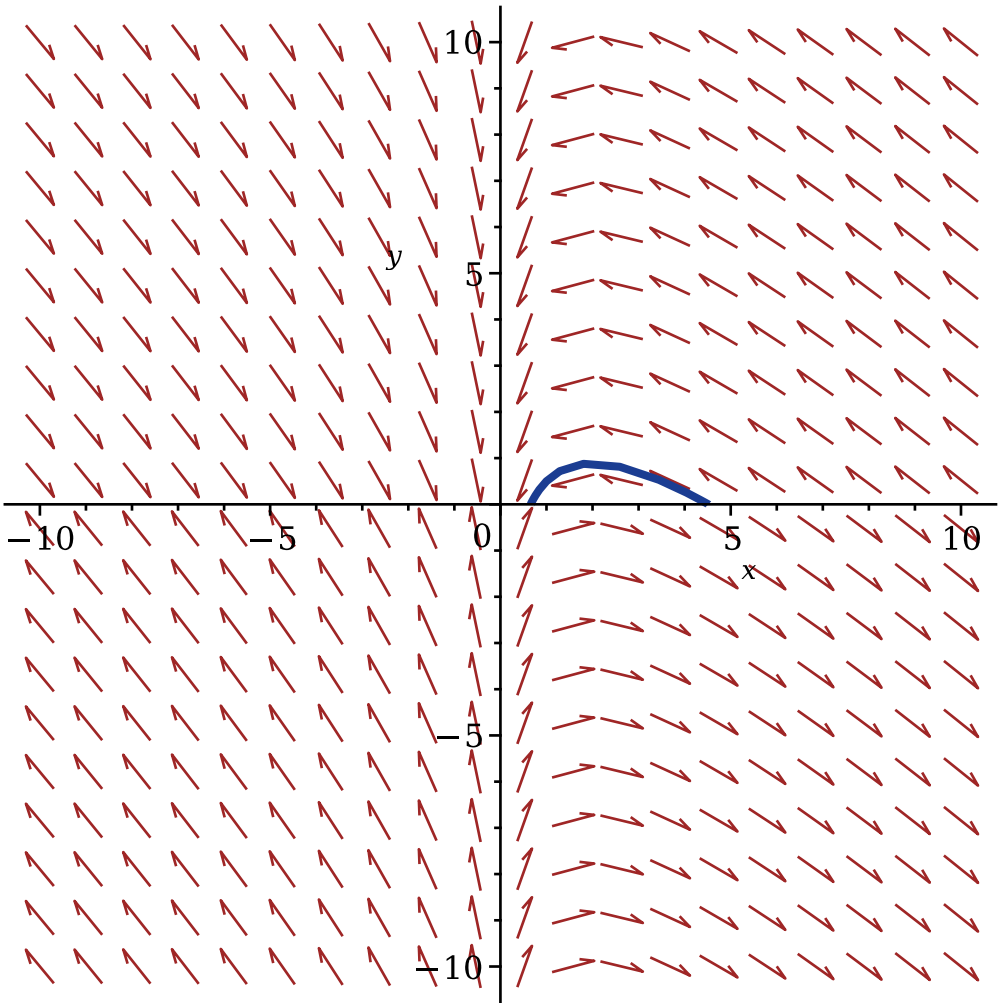
```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-3..3,y=-3..3,[cond_in])
```




```
> H:=1
```

$$H := 1 \qquad (143)$$

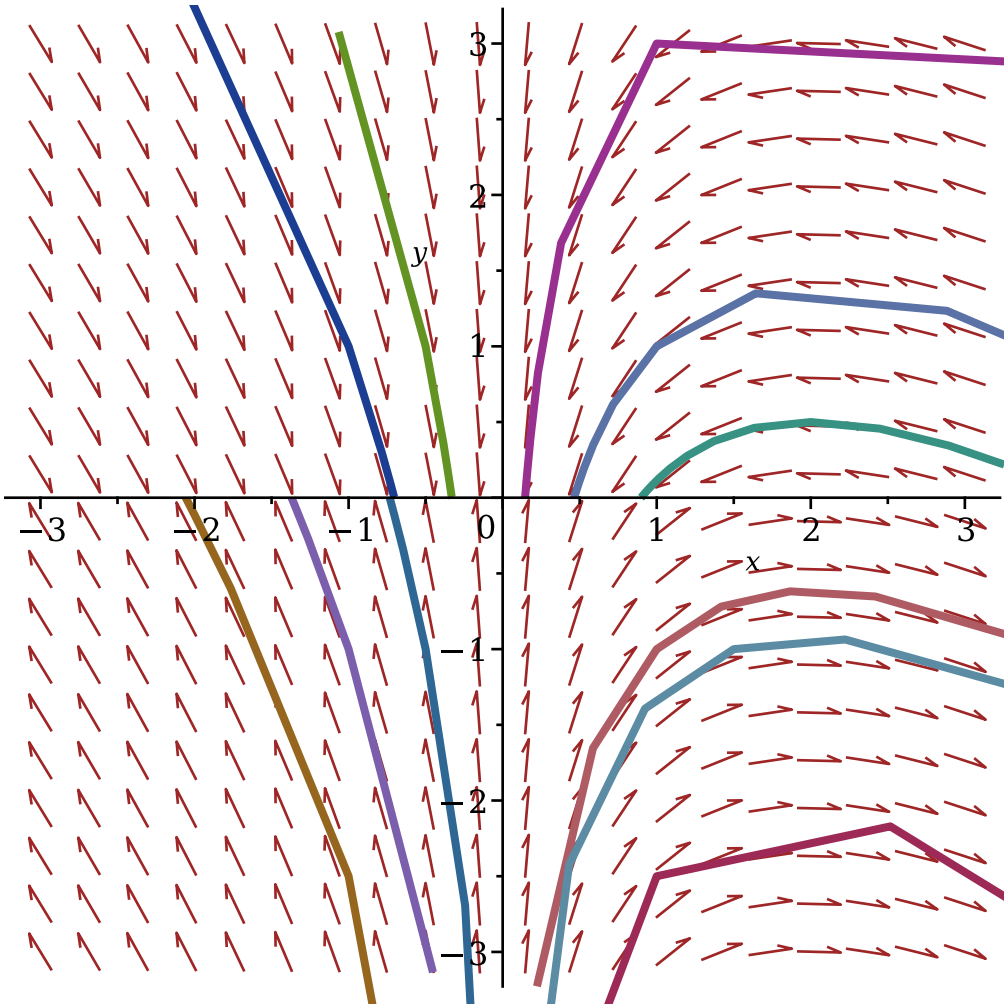
```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10,[
  [cond_cauchy]])
```




```
> cond_in:=[x(0)=-1,y(0)=1],[x(0)=-0.5,y(0)=1],[x(0)=1,y(0)=1],[x
  (0)=1,y(0)=3],[x(0)=2,y(0)=0.5],[x(0)=-1,y(0)=-1],[x(0)=-0.5,y(0)
  =-1],[x(0)=-1,y(0)=-2.5],[x(0)=1,y(0)=-1],[x(0)=1.5,y(0)=-1],[x
  (0)=1,y(0)=-2.5]
```

$$cond_in := [x(0) = -1, y(0) = 1], [x(0) = -0.5, y(0) = 1], [x(0) = 1, y(0) = 1], [x(0) = 1, y(0) = 3], [x(0) = 2, y(0) = 0.5], [x(0) = -1, y(0) = -1], [x(0) = -0.5, y(0) = -1], [x(0) = -1, y(0) = -2.5], [x(0) = 1, y(0) = -1], [x(0) = 1.5, y(0) = -1], [x(0) = 1, y(0) = -2.5] \quad (144)$$

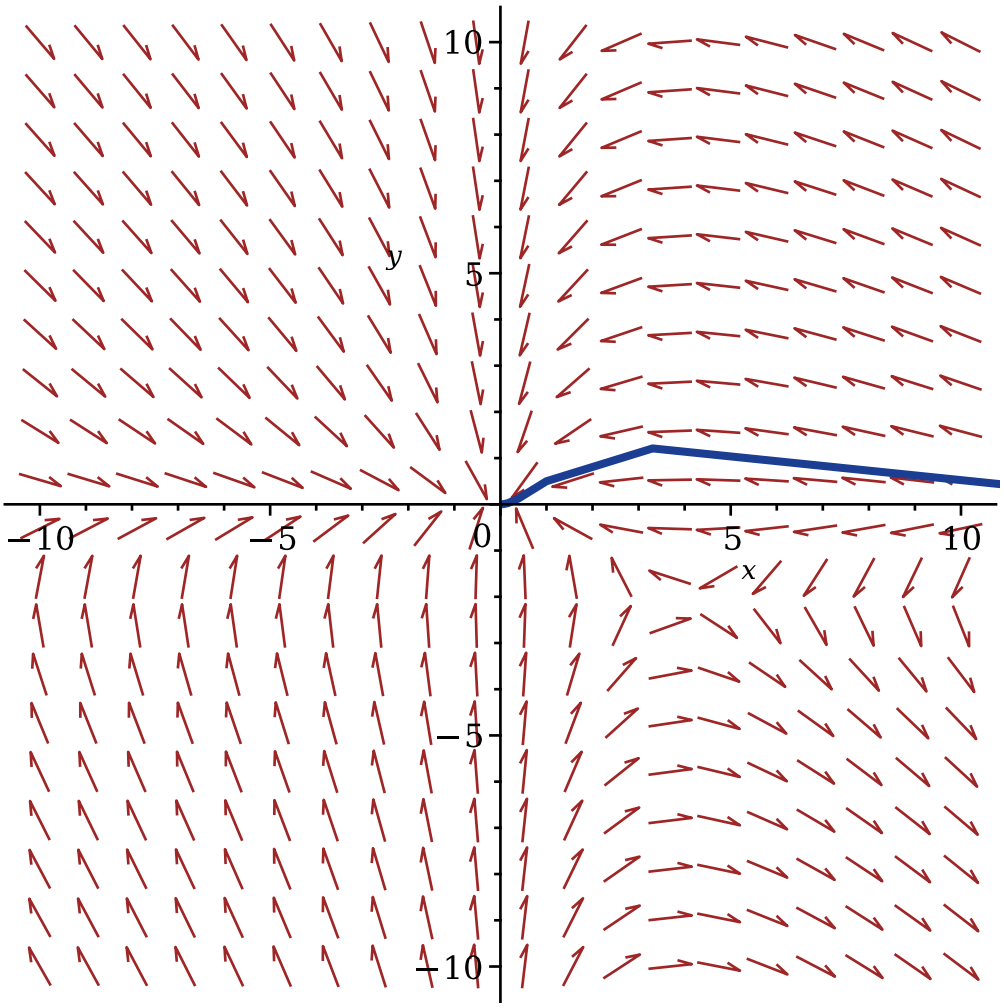
```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-3..3,y=-3..3,[cond_in])
```




```
> H:=3
```

$$H := 3 \qquad (145)$$

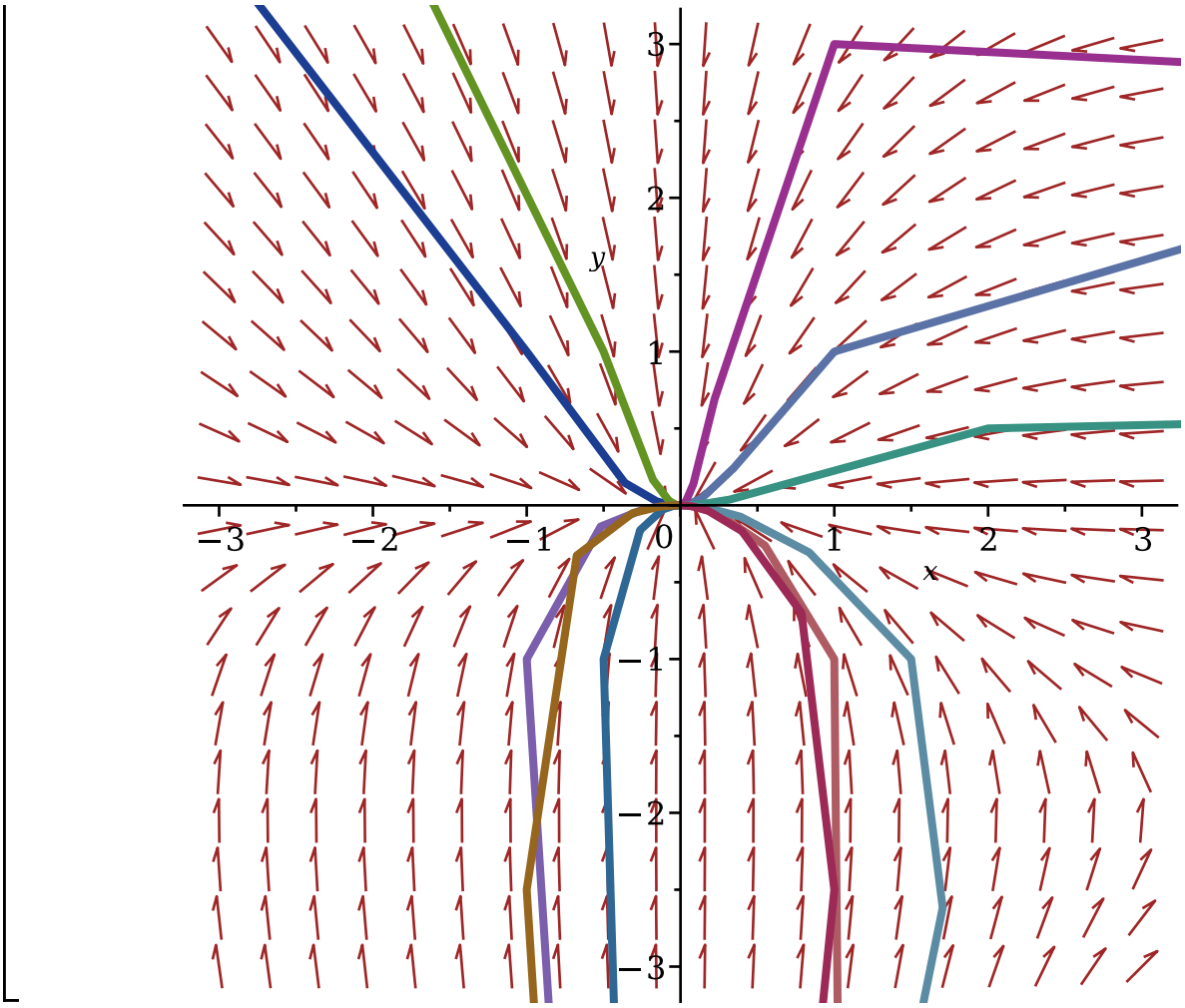
```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10,[
  [cond_cauchy]])
```




```
> cond_in:=[x(0)=-1,y(0)=1],[x(0)=-0.5,y(0)=1],[x(0)=1,y(0)=1],[x
  (0)=1,y(0)=3],[x(0)=2,y(0)=0.5],[x(0)=-1,y(0)=-1],[x(0)=-0.5,y(0)
  =-1],[x(0)=-1,y(0)=-2.5],[x(0)=1,y(0)=-1],[x(0)=1.5,y(0)=-1],[x
  (0)=1,y(0)=-2.5]
```

$$cond_in := [x(0) = -1, y(0) = 1], [x(0) = -0.5, y(0) = 1], [x(0) = 1, y(0) = 1], [x(0) = 1, y(0) = 3], [x(0) = 2, y(0) = 0.5], [x(0) = -1, y(0) = -1], [x(0) = -0.5, y(0) = -1], [x(0) = -1, y(0) = -2.5], [x(0) = 1, y(0) = -1], [x(0) = 1.5, y(0) = -1], [x(0) = 1, y(0) = -2.5] \quad (146)$$

```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-3..3,y=-3..3,[cond_in])
```


3
2
y
1
−3
−2
−1
0
1
2
3
x
−1
−2
−3