```
> with(plots)
```

[*animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot*] **(1)**

```
> ec:=(D@@2)(x)(t)+w0^2*x(t)=0
```

$$ec := \mathrm{D}^{(2)}(x)(t) + w0^2\, x(t) = 0 \quad \textbf{(2)}$$

```
> sol_a:=dsolve(ec,x(t))
```

$$sol\_a := x(t) = c_1 \sin(w0\, t) + c_2 \cos(w0\, t) \quad \textbf{(3)}$$

```
> sol_b:=subs({c__1=R*cos(delta), c__2=R*sin(delta)}, sol_a)
```

$$sol\_b := x(t) = R\cos(\delta)\sin(w0\, t) + R\sin(\delta)\cos(w0\, t) \quad \textbf{(4)}$$

```
> sol_b:=combine(sol_b)
```

$$sol\_b := x(t) = R\sin(w0\, t + \delta) \quad \textbf{(5)}$$

```
> cond:=x(0)=x0,D(x)(0)=v0
```

$$cond := x(0) = x0,\ \mathrm{D}(x)(0) = v0 \quad \textbf{(6)}$$

```
> sol:=dsolve({ec,cond}, x(t))
```

$$sol := x(t) = \frac{v0\sin(w0\, t)}{w0} + x0\cos(w0\, t) \quad \textbf{(7)}$$

```
> ec1:=R*cos(delta)=v0/w0
```

$$ec1 := R\cos(\delta) = \frac{v0}{w0} \quad \textbf{(8)}$$

```
> ec2:=R*sin(delta)=x0
```

$$ec2 := R\sin(\delta) = x0 \quad \textbf{(9)}$$

```
> sol_c:=solve({ec1,ec2},{R,delta})
```

$$sol\_c := \left\{ R = \frac{RootOf(-x0^2\, w0^2 + \_Z^2 - v0^2)}{w0},\ \delta = \arctan\left(\frac{x0\, w0}{RootOf(-x0^2\, w0^2 + \_Z^2 - v0^2)},\ \frac{v0}{RootOf(-x0^2\, w0^2 + \_Z^2 - v0^2)} \right) \right\} \quad \textbf{(10)}$$

```
> L:=39.24/100
```

$$L := 0.3924000000 \quad \textbf{(11)}$$

```
> g:=9.81
```

$$g := 9.81 \tag{12}$$

```
> w0:=sqrt(g/L)
```

$$w0 := 5.000000000 \tag{13}$$

```
> x0:=15/100
```

$$x0 := \frac{3}{20} \tag{14}$$

```
> v0:=0
```

$$v0 := 0 \tag{15}$$

```
> evalf(sol_c)
```

$$\{R = 0.1500000000, \delta = 1.570796327\} \tag{16}$$

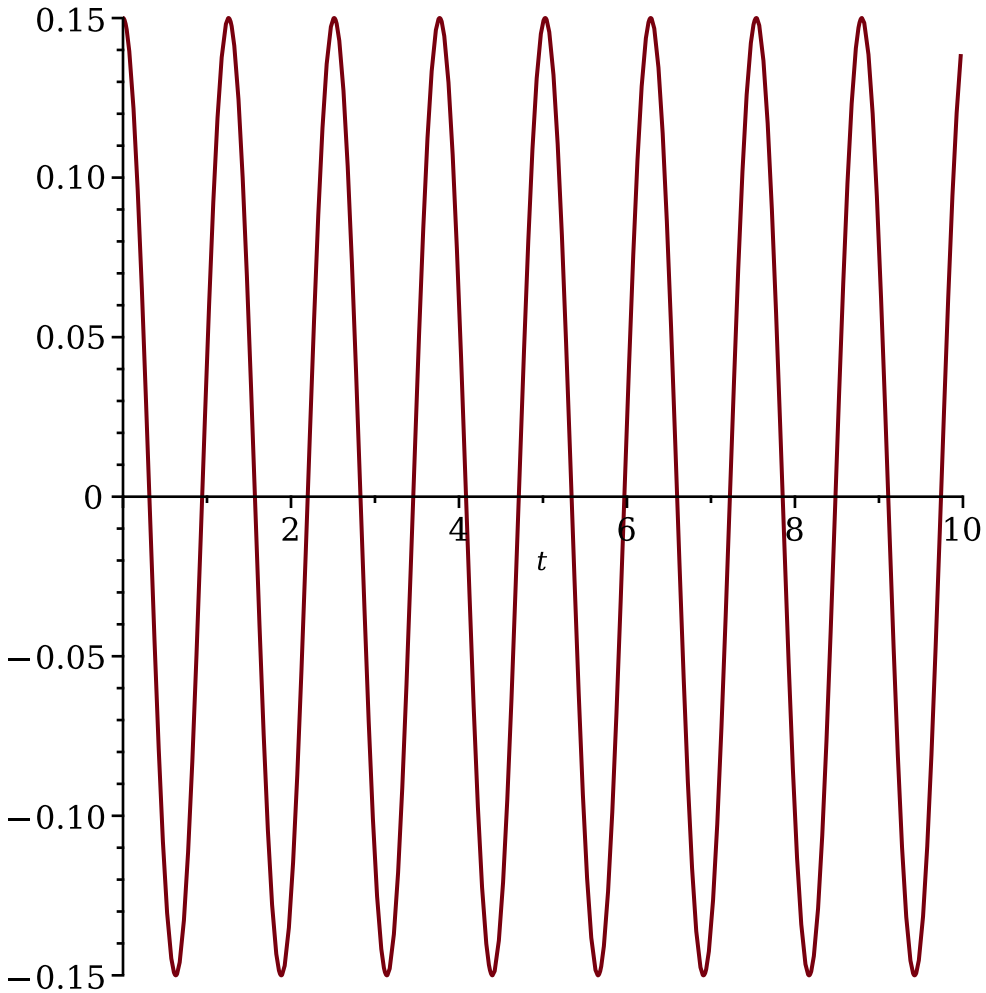
```
> sol_d:=evalf(sol)
```

$$sol\_d := x(t) = 0.1500000000 \cos(5.000000000\, t) \tag{17}$$

```
> y1:=unapply(rhs(sol_d),t)
```

$$y1 := t \mapsto 0.1500000000 \cdot \cos(5.000000000 \cdot t) \tag{18}$$

```
> plot(y1(t),t=0..10)
```




```
> restart
> with(plots)
```

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```
> ec:=(D@@2)(x)(t)+lambda*D(x)(t)+w0^2*x(t)=0
```

$$ec := \mathrm{D}^{(2)}(x)(t) + \lambda\,\mathrm{D}(x)(t) + w0^2\,x(t) = 0 \quad (20)$$

```
> sol_a:=dsolve({ec},x(t))
```

$$sol\_a := \left\{ x(t) = c_1\, \mathrm{e}^{\left(-\frac{\lambda}{2} + \frac{\sqrt{\lambda^2 - 4\,w0^2}}{2}\right) t} + c_2\, \mathrm{e}^{\left(-\frac{\lambda}{2} - \frac{\sqrt{\lambda^2 - 4\,w0^2}}{2}\right) t} \right\} \quad (21)$$

```
> lambda:=25
```

$$\lambda := 25 \quad (22)$$

```
> w0:=10
```

$$w0 := 10 \quad (23)$$

```
> cond:=x(0)=1,D(x)(0)=5
```

$$cond := x(0) = 1, \mathrm{D}(x)(0) = 5 \quad (24)$$

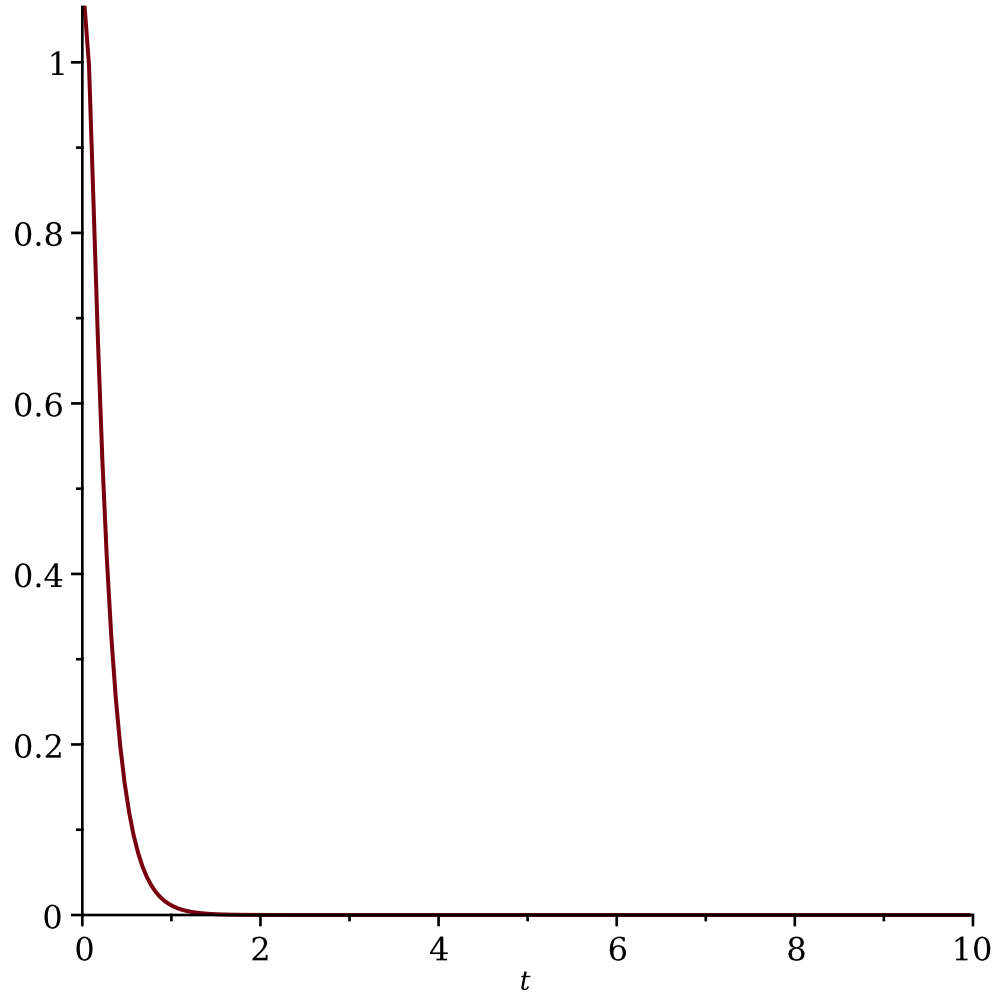
```
> sol_b:=dsolve({ec,cond},x(t))
```

$$sol\_b := x(t) = \frac{5\,\mathrm{e}^{-5\,t}}{3} - \frac{2\,\mathrm{e}^{-20\,t}}{3} \quad (25)$$

```
> y1:=unapply(rhs(sol_b) , t )
```

$$y1 := t \mapsto \frac{5 \cdot \mathrm{e}^{-5 \cdot t}}{3} - \frac{2 \cdot \mathrm{e}^{-20 \cdot t}}{3} \quad (26)$$

```
> plot(y1(t),t=0..10)
```




```
> lambda:='lambda'
```

$$\lambda := \lambda \tag{27}$$

```
> w0:='wo'
```

$$w0 := wo \tag{28}$$

```
> ec_c:=(D@@2)(x)(t)+lambda*D(x)(t)+lambda^2/4*x(t)=0
```

$$ec\_c := \mathrm{D}^{(2)}(x)(t) + \lambda\,\mathrm{D}(x)(t) + \frac{\lambda^2\,x(t)}{4} = 0 \tag{29}$$

```
> sol_c:=dsolve(ec_c,x(t))
```

$$sol\_c := x(t) = c_1\,\mathrm{e}^{-\frac{\lambda t}{2}} + c_2\,\mathrm{e}^{-\frac{\lambda t}{2}}\,t \tag{30}$$

```
> lambda:=20
```

$$\lambda := 20 \tag{31}$$

```
> w0:=10
```

$$w0 := 10 \tag{32}$$

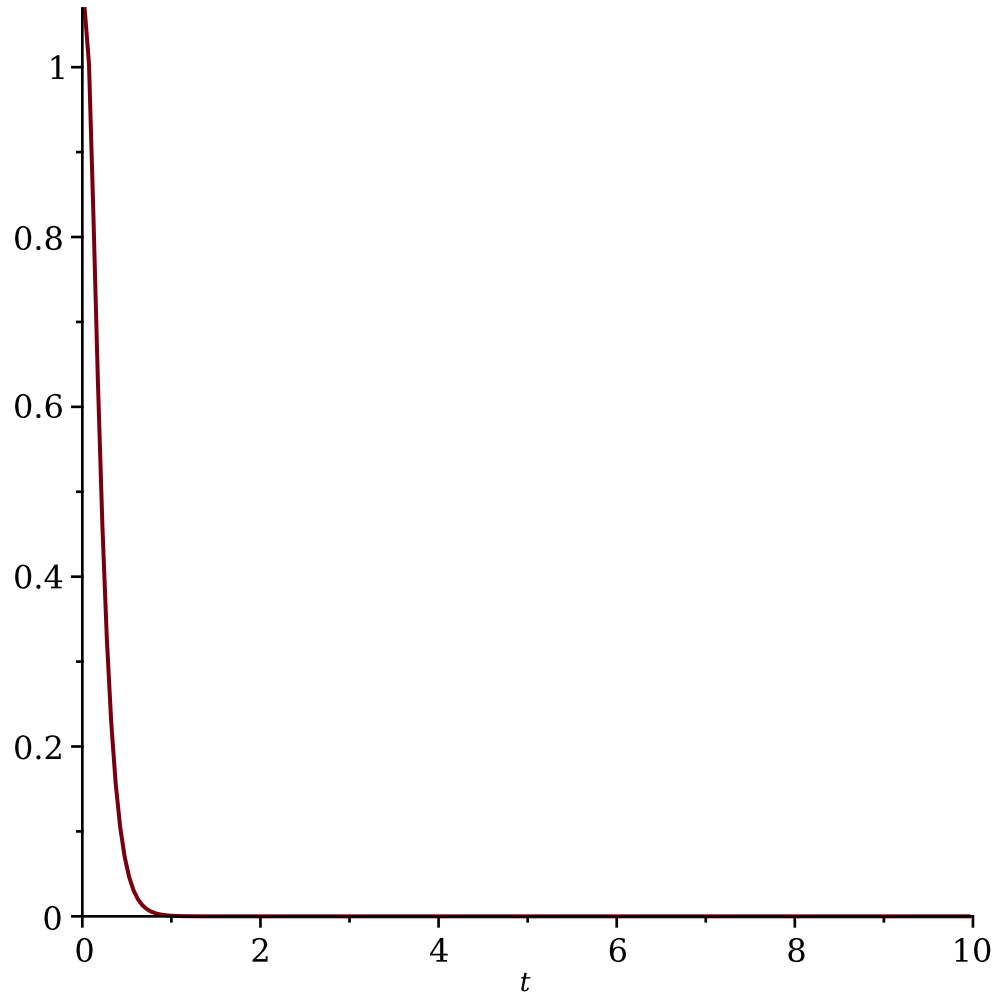
```
> sol_d:=dsolve({ec_c,cond},x(t))
```

$$sol\_d := x(t) = \mathrm{e}^{-10\,t}\,(1 + 15\,t) \tag{33}$$

```
> y2:=unapply(rhs(sol_d),t)
```

$$y2 := t \mapsto \mathrm{e}^{-10\cdot t}\cdot(1 + 15\cdot t) \tag{34}$$

```
> plot(y2(t),t=0..10)
```




```
> lambda:='lambda'
```

$$\lambda := \lambda \tag{35}$$

```
> w0:='wo'
```

$$w0 := wo \tag{36}$$

```
> assume (lambda^2<4*wo^2)
> sol_e:=dsolve({ec},x(t))
```

$$sol\_e := \left\{ x(t) = c_1 \, e^{-\frac{\lambda\sim t}{2}} \sin\left(\frac{\sqrt{-\lambda\sim^2 + 4\, wo\sim^2}\; t}{2} \right) + c_2 \, e^{-\frac{\lambda\sim t}{2}} \cos\left(\frac{\sqrt{-\lambda\sim^2 + 4\, wo\sim^2}\; t}{2} \right) \right\} \tag{37}$$

```
> lambda:=5
```

$$\lambda := 5 \tag{38}$$

```
> w0:=10
```

$$w0 := 10 \tag{39}$$

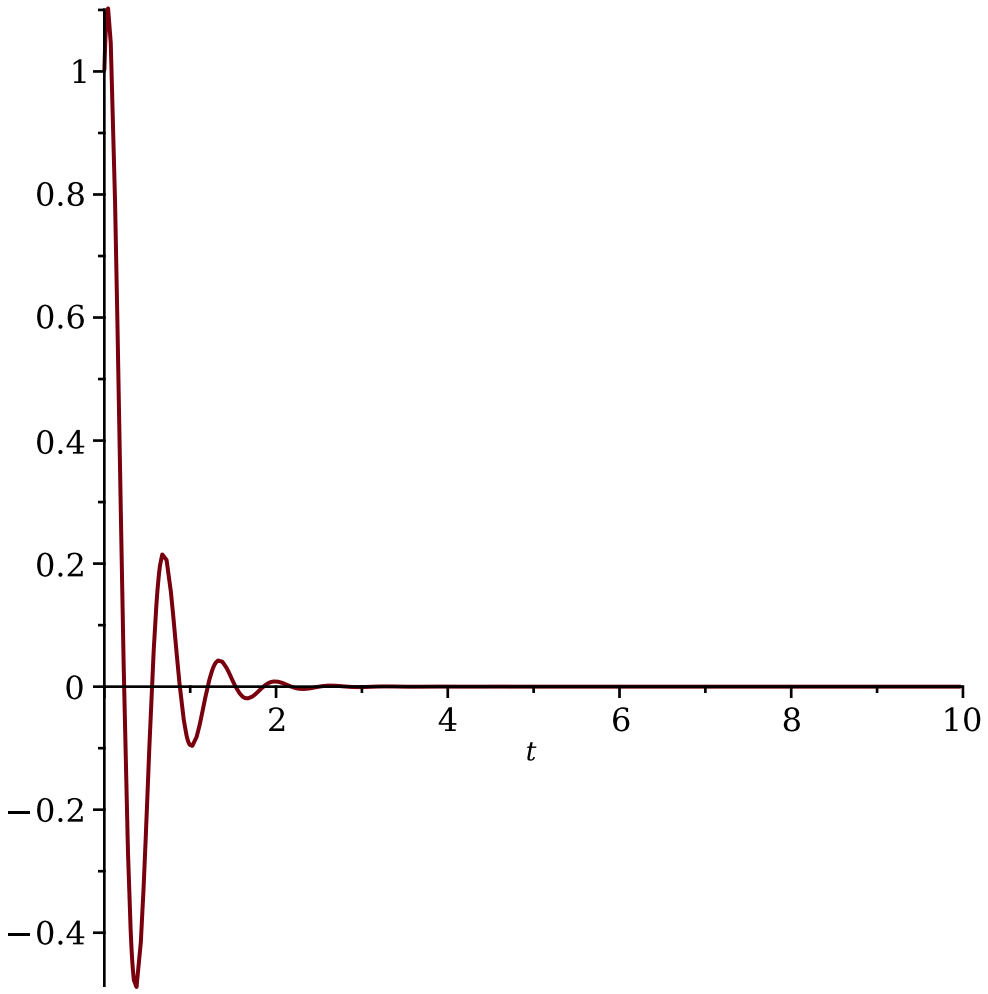
```
> sol_f:=dsolve({ec,cond},x(t))
```

$$sol\_f := x(t) = \frac{e^{-\frac{5t}{2}}\left(\sqrt{15}\sin\left(\frac{5\sqrt{15}\,t}{2}\right)+5\cos\left(\frac{5\sqrt{15}\,t}{2}\right)\right)}{5} \quad (40)$$

```
> y3:=unapply(rhs(sol_f),t)
```

$$y3 := t \mapsto \frac{e^{-\frac{5\cdot t}{2}}\cdot\left(\sqrt{15}\cdot\sin\left(\frac{5\cdot\sqrt{15}\cdot t}{2}\right)+5\cdot\cos\left(\frac{5\cdot\sqrt{15}\cdot t}{2}\right)\right)}{5} \quad (41)$$

```
> plot(y3(t),t=0..10)
```




```
> restart
> with(plots)
```

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polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]

```
> ec:=(D@@2)(x)(t)+w0^2*x(t)=F0*cos(w*t)
```

$$ec := \mathrm{D}^{(2)}(x)(t) + w0^2\, x(t) = F0 \cos(w\, t) \tag{43}$$

```
> sol_a:=dsolve(ec,x(t))
```

$$sol\_a := x(t) = \sin(w0\, t)\, c_2 + \cos(w0\, t)\, c_1 - \frac{F0 \cos(w\, t)}{w^2 - w0^2} \tag{44}$$

```
> cond:=x(0)=0,D(x)(0)=0
```

$$cond := x(0) = 0, \mathrm{D}(x)(0) = 0 \tag{45}$$

```
> w0:=5
```

$$w0 := 5 \tag{46}$$

```
> w:=5.5
```

$$w := 5.5 \tag{47}$$

```
> F0:=2
```

$$F0 := 2 \tag{48}$$

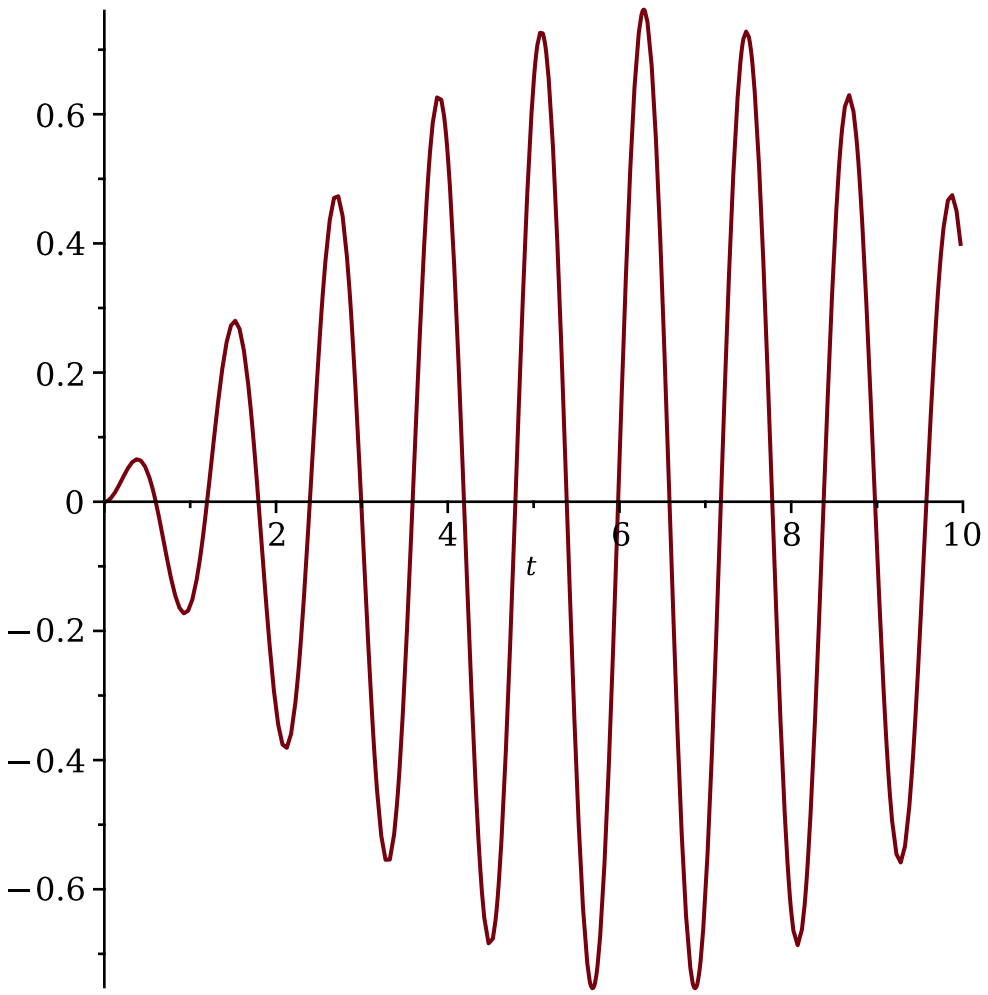
```
> sol_b:=dsolve({ec,cond},x(t))
```

$$sol\_b := x(t) = \frac{8 \cos(5\, t)}{21} - \frac{8 \cos\left(\frac{11\, t}{2}\right)}{21} \tag{49}$$

```
> y1:=unapply(rhs(sol_b),t)
```

$$y1 := t \mapsto \frac{8 \cdot \cos(5 \cdot t)}{21} - \frac{8 \cdot \cos\left(\frac{11 \cdot t}{2}\right)}{21} \tag{50}$$

```
> plot(y1(t),t=0..10)
```




```
> w:='w'
```

$$w := w \tag{51}$$

```
> w0:='w0';F0:='F0'
```

$$w0 := w0$$
$$F0 := F0 \tag{52}$$

```
> ec_c:=(D@@2)(x)(t)+w^2*x(t)=F0*cos(w*t)
```

$$ec\_c := \mathrm{D}^{(2)}(x)(t) + w^2\,x(t) = F0\cos(w\,t) \tag{53}$$

```
> sol_c:=dsolve(ec_c,x(t))
```

$$sol\_c := x(t) = \sin(w\,t)\,c_2 + \cos(w\,t)\,c_1 + \frac{F0\sin(w\,t)\,t}{2\,w} \tag{54}$$

```
> w:=5; F0:=2
```

$$w := 5$$
$$F0 := 2 \tag{55}$$

```
> sol_d:=dsolve({ec_c,cond},x(t))
```

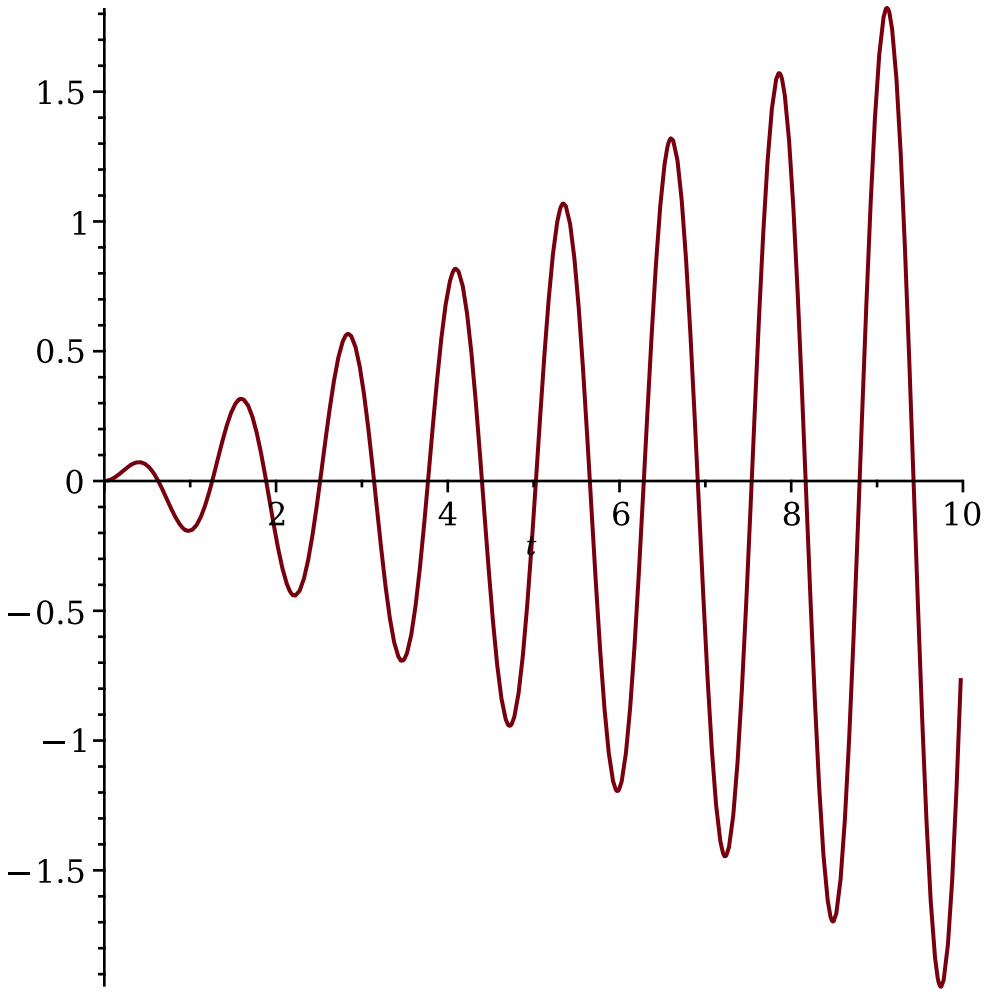
$$sol\_d := x(t) = \frac{\sin(5\,t)\,t}{5} \tag{56}$$

```
> y2:=unapply(rhs(sol_d),t)
```

(57)

$$y2 := t \mapsto \frac{\sin(5 \cdot t) \cdot t}{5} \quad (57)$$

```
> plot(y2(t),t=0..10)
```




```
> w:='w';F0:='F0'
```

$$w := w$$
$$F0 := F0 \quad (58)$$

```
> sol:=dsolve({ec,cond},x(t))
```

$$sol := x(t) = \frac{F0\,(\cos(w0\,t) - \cos(w\,t))}{w^2 - w0^2} \quad (59)$$

```
> y3:=unapply(rhs(sol),t,w)
```

$$y3 := (t, w) \mapsto \frac{F0 \cdot (\cos(w0 \cdot t) - \cos(w \cdot t))}{w^2 - w0^2} \quad (60)$$

```
> limit(y3(t,w),w=w0)
```

$$\frac{F0 \sin(w0\,t)\,t}{2\,w0} \quad (61)$$

```
> dsolve({ec_c,cond},x(t))
```

$$x(t) = \frac{F0 \sin(w\,t)\,t}{2\,w} \quad (62)$$