

$$\begin{aligned} &> \text{ec1} := \text{diff}(x(t), t) = x(t) + 4*y(t) \\ &\quad \text{ec1} := \frac{d}{dt} x(t) = x(t) + 4y(t) \end{aligned} \quad (1)$$

$$\begin{aligned} &> \text{ec2} := \text{diff}(y(t), t) = x(t) + y(t) \\ &\quad \text{ec2} := \frac{d}{dt} y(t) = x(t) + y(t) \end{aligned} \quad (2)$$

$$\begin{aligned} &> \text{sist} := \text{ec1}, \text{ec2} \\ &\quad \text{sist} := \frac{d}{dt} x(t) = x(t) + 4y(t), \frac{d}{dt} y(t) = x(t) + y(t) \end{aligned} \quad (3)$$

$$\begin{aligned} &> \text{infolevel}[\text{dsolve}] := 5 \\ &\quad \text{infolevel}_{\text{dsolve}} := 5 \end{aligned} \quad (4)$$

$$\begin{aligned} &> \text{dsolve}(\{\text{sist}\}, \{x(t), y(t)\}) \\ &\text{-> Solving each unknown as a function of the next ones using the} \\ &\text{order: } [y(t), x(t)] \\ &\text{-> Calling odsolve with the ODE } \text{diff}(\text{diff}(y(x) \ x) \ x) = 2*(\text{diff}(y \\ &\text{(x) x)}) + 3*y(x) \ y(x) \ \text{singsol} = \text{none} \\ &\text{Methods for second order ODEs:} \\ &\text{--- Trying classification methods ---} \\ &\text{trying a quadrature} \\ &\text{checking if the LODE has constant coefficients} \\ &\text{<- constant coefficients successful} \\ &\quad \left\{ x(t) = c_1 e^{3t} + c_2 e^{-t}, y(t) = \frac{c_1 e^{3t}}{2} - \frac{c_2 e^{-t}}{2} \right\} \end{aligned} \quad (5)$$

$$\begin{aligned} &> \text{sist} := \text{diff}(x(t), t) = 2*x(t) - y(t), \text{diff}(y(t), t) = x(t) + 2*y(t) \\ &\quad \text{sist} := \frac{d}{dt} x(t) = 2x(t) - y(t), \frac{d}{dt} y(t) = x(t) + 2y(t) \end{aligned} \quad (6)$$

$$\begin{aligned} &> \text{dsolve}(\{\text{sist}\}, \{x(t), y(t)\}) \\ &\text{-> Solving each unknown as a function of the next ones using the} \\ &\text{order: } [y(t), x(t)] \\ &\text{-> Calling odsolve with the ODE } \text{diff}(\text{diff}(y(x) \ x) \ x) = 4*(\text{diff}(y \\ &\text{(x) x)}) - 5*y(x) \ y(x) \ \text{singsol} = \text{none} \\ &\text{Methods for second order ODEs:} \\ &\text{--- Trying classification methods ---} \\ &\text{trying a quadrature} \\ &\text{checking if the LODE has constant coefficients} \\ &\text{<- constant coefficients successful} \\ &\quad \{x(t) = e^{2t} (c_2 \cos(t) + c_1 \sin(t)), y(t) = -e^{2t} (\cos(t) c_1 - \sin(t) c_2)\} \end{aligned} \quad (7)$$

$$\begin{aligned} &> \text{ec1} := \text{diff}(x(t), t) = x(t) - y(t) + z(t) \\ &\quad \text{ec1} := \frac{d}{dt} x(t) = x(t) - y(t) + z(t) \end{aligned} \quad (8)$$

$$\begin{aligned} &> \text{ec2} := \text{diff}(y(t), t) = x(t) + y(t) - z(t) \\ &\quad \text{ec2} := \frac{d}{dt} y(t) = x(t) + y(t) - z(t) \end{aligned} \quad (9)$$

$$\begin{aligned} &> \text{ec3} := \text{diff}(z(t), t) = -y(t) + 2z(t) \\ &\text{ec3} := \frac{d}{dt} z(t) = -y(t) + 2z(t) \end{aligned} \quad (10)$$

$$\begin{aligned} &> \text{sist} := \text{ec1}, \text{ec2}, \text{ec3} \\ \text{sist} := \frac{d}{dt} x(t) &= x(t) - y(t) + z(t), \frac{d}{dt} y(t) = x(t) + y(t) - z(t), \frac{d}{dt} z(t) = -y(t) \\ &+ 2z(t) \end{aligned} \quad (11)$$

$$\begin{aligned} &> \text{dsolve}(\{\text{sist}\}, \{x(t), y(t), z(t)\}) \\ &\rightarrow \text{Solving each unknown as a function of the next ones using the} \\ &\text{order: } [x(t), y(t), z(t)] \\ &\rightarrow \text{Calling odsolve with the ODE } \text{diff}(\text{diff}(\text{diff}(y(x) \ x) \ x) \ x) = \\ &4 * (\text{diff}(\text{diff}(y(x) \ x) \ x)) - 5 * (\text{diff}(y(x) \ x)) + 2 * y(x) \ y(x) \ \text{singsol} = \\ &\text{none} \\ &\text{Methods for third order ODEs:} \\ &\text{--- Trying classification methods ---} \\ &\text{trying a quadrature} \\ &\text{checking if the LODE has constant coefficients} \\ &<- \text{constant coefficients successful} \\ &\{x(t) = c_1 e^{2t} + c_2 e^t + c_3 e^t t + c_3 e^t, y(t) = e^t (c_3 t + c_2 - c_3), z(t) = c_1 e^{2t} + c_2 e^t \\ &+ c_3 e^t t\} \end{aligned} \quad (12)$$

$$\begin{aligned} &> \text{ec1} := \text{diff}(x(t), t) = 5x(t) + 3y(t) + 1 \\ &\text{ec1} := \frac{d}{dt} x(t) = 5x(t) + 3y(t) + 1 \end{aligned} \quad (13)$$

$$\begin{aligned} &> \text{ec2} := \text{diff}(y(t), t) = -6x(t) - 4y(t) + \exp(-t) \\ &\text{ec2} := \frac{d}{dt} y(t) = -6x(t) - 4y(t) + e^{-t} \end{aligned} \quad (14)$$

$$\begin{aligned} &> \text{sist} := \text{ec1}, \text{ec2} \\ \text{sist} := \frac{d}{dt} x(t) &= 5x(t) + 3y(t) + 1, \frac{d}{dt} y(t) = -6x(t) - 4y(t) + e^{-t} \end{aligned} \quad (15)$$

$$\begin{aligned} &> \text{dsolve}(\{\text{sist}\}, \{x(t), y(t)\}) \\ &\rightarrow \text{Solving each unknown as a function of the next ones using the} \\ &\text{order: } [y(t), x(t)] \\ &\rightarrow \text{Calling odsolve with the ODE } \text{diff}(\text{diff}(y(x) \ x) \ x) = \text{diff}(y(x) \\ &x) + 2 * y(x) + 4 + 3 * \exp(-x) \ y(x) \ \text{singsol} = \text{none} \\ &\text{Methods for second order ODEs:} \\ &\text{--- Trying classification methods ---} \\ &\text{trying a quadrature} \\ &\text{trying high order exact linear fully integrable} \\ &\text{trying differential order: 2; linear nonhomogeneous with} \\ &\text{symmetry [0,1]} \\ &\text{trying a double symmetry of the form [xi=0, eta=F(x)]} \\ &\rightarrow \text{Try solving first the homogeneous part of the ODE} \\ &\text{checking if the LODE has constant coefficients} \\ &<- \text{constant coefficients successful} \end{aligned}$$

-> Determining now a particular solution to the non-homogeneous ODE

building a particular solution using variation of parameters

<- solving first the homogeneous part of the ODE successful

$$\left\{ x(t) = e^{2t} c_2 + e^{-t} c_1 - 2 - t e^{-t} - \frac{e^{-t}}{3}, y(t) = -e^{2t} c_2 - 2 e^{-t} c_1 + \frac{e^{-t}}{3} + 2 t e^{-t} + 3 \right\} \quad (16)$$

> ec1:=diff(x(t),t)=x(t)+3*y(t)+cos(t)

$$ec1 := \frac{d}{dt} x(t) = x(t) + 3 y(t) + \cos(t) \quad (17)$$

> ec2:=diff(y(t),t)=x(t)-y(t)+2*t

$$ec2 := \frac{d}{dt} y(t) = x(t) - y(t) + 2 t \quad (18)$$

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = x(t) + 3 y(t) + \cos(t), \frac{d}{dt} y(t) = x(t) - y(t) + 2 t \quad (19)$$

> dsolve({sist},{x(t),y(t)})

-> Solving each unknown as a function of the next ones using the order: [x(t), y(t)]

-> Calling odsolve with the ODE diff(diff(y(x) x) x) = 2+4*y(x)

-2*x+cos(x) y(x) singsol = none

Methods for second order ODEs:

--- Trying classification methods ---

trying a quadrature

trying high order exact linear fully integrable

trying differential order: 2; linear nonhomogeneous with symmetry [0,1]

trying a double symmetry of the form [xi=0, eta=F(x)]

-> Try solving first the homogeneous part of the ODE

checking if the LODE has constant coefficients

<- constant coefficients successful

-> Determining now a particular solution to the non-homogeneous ODE

building a particular solution using variation of parameters

<- solving first the homogeneous part of the ODE successful

$$\left\{ x(t) = -e^{-2t} c_2 + 3 c_1 e^{2t} + \frac{\sin(t)}{5} - \frac{\cos(t)}{5} - \frac{3t}{2}, y(t) = e^{-2t} c_2 + c_1 e^{2t} - \frac{\cos(t)}{5} + \frac{t}{2} - \frac{1}{2} \right\} \quad (20)$$

> ec1:=diff(x(t),t)=x(t)-2*y(t)-2*z(t)+exp(-t)

(21)

$$ec1 := \frac{d}{dt} x(t) = x(t) - 2y(t) - 2z(t) + e^{-t} \quad (21)$$

> ec2:=diff(y(t),t)=-2*x(t)+y(t)+2*z(t)

$$ec2 := \frac{d}{dt} y(t) = -2x(t) + y(t) + 2z(t) \quad (22)$$

> ec3:=diff(z(t),t)=2*x(t)-y(t)-3*z(t)+exp(-t)

$$ec3 := \frac{d}{dt} z(t) = 2x(t) - y(t) - 3z(t) + e^{-t} \quad (23)$$

> sist:=ec1,ec2,ec3

$$sist := \frac{d}{dt} x(t) = x(t) - 2y(t) - 2z(t) + e^{-t}, \frac{d}{dt} y(t) = -2x(t) + y(t) + 2z(t), \quad (24)$$

$$\frac{d}{dt} z(t) = 2x(t) - y(t) - 3z(t) + e^{-t}$$

> dsolve({sist},{x(t),y(t),z(t)})

-> Solving each unknown as a function of the next ones using the order: [z(t), x(t), y(t)]

-> Calling odsolve with the ODE diff(y(x) x) = -y(x)-y(x)-(diff(y(x) x))+exp(-x) y(x) singsol = none

Methods for first order ODEs:

--- Trying classification methods ---

trying a quadrature

trying 1st order linear

<- 1st order linear successful

-> Calling odsolve with the ODE diff(diff(y(x) x) x) = 3*y(x) y(x) singsol = none

Methods for second order ODEs:

--- Trying classification methods ---

trying a quadrature

checking if the LODE has constant coefficients

<- constant coefficients successful

$$\left\{ x(t) = -c_2 e^{\sqrt{3}t} - c_3 e^{-\sqrt{3}t} + e^{-t} c_1 + t e^{-t}, y(t) = c_2 e^{\sqrt{3}t} + c_3 e^{-\sqrt{3}t}, z(t) \right. \quad (25)$$

$$\left. = \frac{c_2 \sqrt{3} e^{\sqrt{3}t}}{2} - \frac{c_3 \sqrt{3} e^{-\sqrt{3}t}}{2} - \frac{3c_2 e^{\sqrt{3}t}}{2} - \frac{3c_3 e^{-\sqrt{3}t}}{2} + e^{-t} c_1 + t e^{-t} \right\}$$

> restart

> with (DEtools), with (plots)

[AreSimilar, Closure, DENormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, (26)

RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeometricsols, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor, invariants, kovacicsols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom], [animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]

> ec1:=diff(x(t),t)=x(t)+4*y(t)

$$ec1 := \frac{d}{dt} x(t) = x(t) + 4y(t) \quad (27)$$

> ec2:=diff(y(t),t) = x(t)+y(t)

$$ec2 := \frac{d}{dt} y(t) = x(t) + y(t) \quad (28)$$

> cond:=x(0)=1,y(0)=2

$$cond := x(0) = 1, y(0) = 2 \quad (29)$$

> sist:=ec1,ec2

...

$$\text{sist} := \frac{d}{dt} x(t) = x(t) + 4y(t), \frac{d}{dt} y(t) = x(t) + y(t) \quad (30)$$

> infolevel[dsolve]:=5

$$\text{infolevel}_{\text{dsolve}} := 5 \quad (31)$$

> sol:=dsolve({sist,cond},{x(t),y(t)})

-> Solving each unknown as a function of the next ones using the order: [y(t), x(t)]

$$\text{sol} := \left\{ x(t) = \frac{5e^{3t}}{2} - \frac{3e^{-t}}{2}, y(t) = \frac{5e^{3t}}{4} + \frac{3e^{-t}}{4} \right\} \quad (32)$$

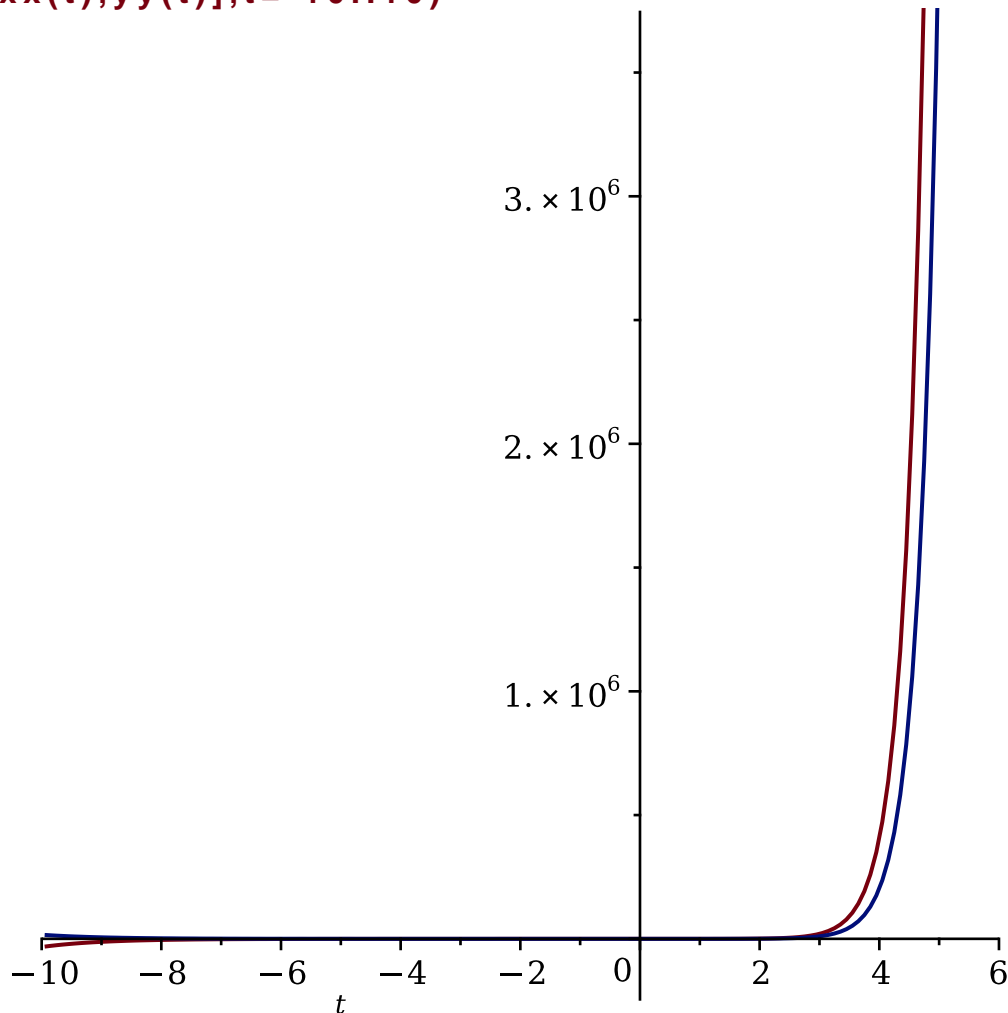
> xx:=unapply(rhs(sol[1]),t)

$$\text{xx} := t \mapsto \frac{5 \cdot e^{3 \cdot t}}{2} - \frac{3 \cdot e^{-t}}{2} \quad (33)$$

> yy:=unapply(rhs(sol[2]),t)

$$\text{yy} := t \mapsto \frac{5 \cdot e^{3 \cdot t}}{4} + \frac{3 \cdot e^{-t}}{4} \quad (34)$$

> plot([xx(t),yy(t)],t=-10..10)



> ec1:=diff(x(t),t)=x(t)-y(t)+t-1

(35)

$$ec1 := \frac{d}{dt} x(t) = x(t) - y(t) + t - 1 \quad (35)$$

> **ec2:=diff(y(t),t)=-2*x(t)+4*y(t)+cos(t)**

$$ec2 := \frac{d}{dt} y(t) = -2x(t) + 4y(t) + \cos(t) \quad (36)$$

> **sist:=ec1,ec2**

$$sist := \frac{d}{dt} x(t) = x(t) - y(t) + t - 1, \frac{d}{dt} y(t) = -2x(t) + 4y(t) + \cos(t) \quad (37)$$

> **cond:=x(0)=0,y(0)=1**

$$cond := x(0) = 0, y(0) = 1 \quad (38)$$

> **sol:=dsolve({sist,cond},{x(t),y(t)})**

-> Solving each unknown as a function of the next ones using the order: [y(t), x(t)]

-> Calling odsolve with the ODE diff(diff(y(x) x) x) = 5*(diff(y(x) x))+5-2*y(x)-4*x-cos(x) y(x) explicit singsol = none

Methods for second order ODEs:

--- Trying classification methods ---

trying a quadrature

trying high order exact linear fully integrable

trying differential order: 2; linear nonhomogeneous with symmetry [0,1]

trying a double symmetry of the form [xi=0, eta=F(x)]

-> Try solving first the homogeneous part of the ODE

checking if the LODE has constant coefficients

<- constant coefficients successful

-> Determining now a particular solution to the non-homogeneous ODE

building a particular solution using variation of parameters

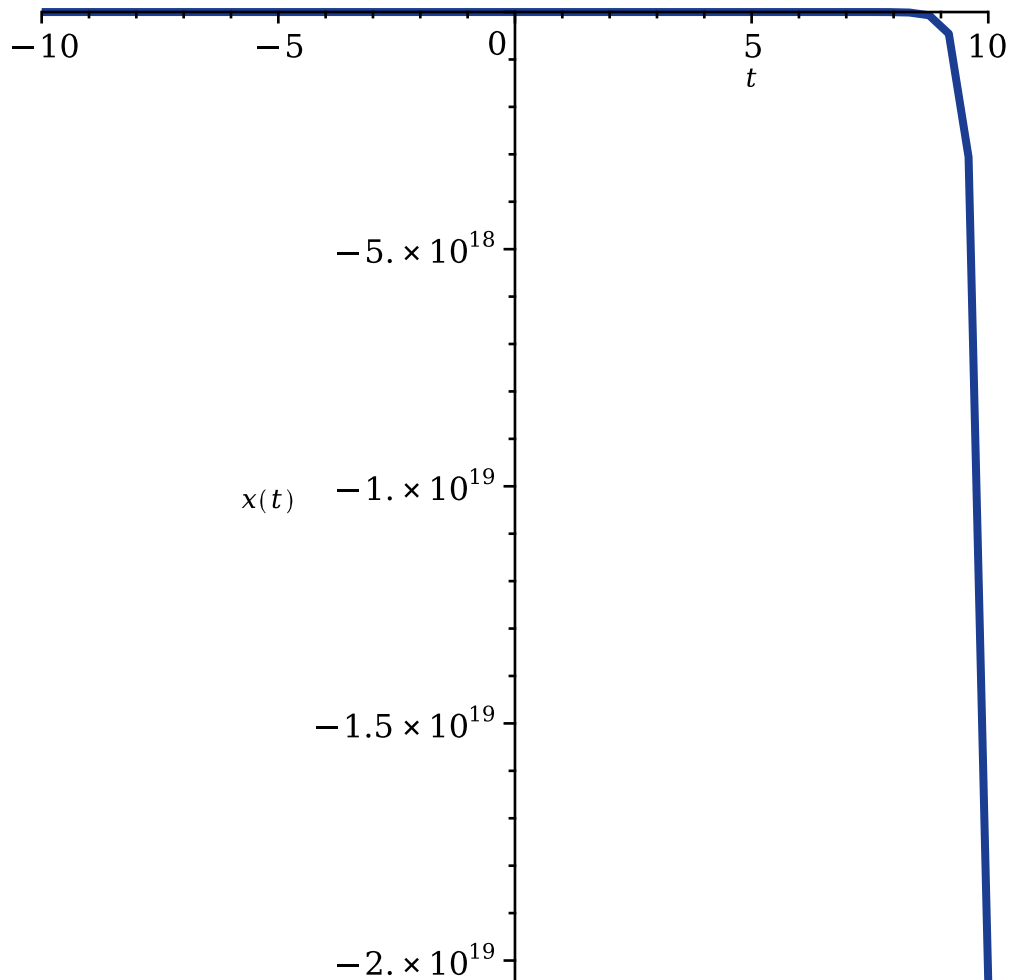
<- solving first the homogeneous part of the ODE successful

$$sol := \left\{ x(t) = e^{\frac{(5+\sqrt{17})t}{2}} \left(\frac{33}{26} - \frac{5\sqrt{17}}{13} \right) + e^{\frac{(-5+\sqrt{17})t}{2}} \left(\frac{33}{26} + \frac{5\sqrt{17}}{13} \right) - \frac{\cos(t)}{26} + \frac{5\sin(t)}{26} - 2t - \frac{5}{2}, y(t) = - \frac{e^{\frac{(5+\sqrt{17})t}{2}} \left(\frac{33}{26} - \frac{5\sqrt{17}}{13} \right) \sqrt{17}}{2} + \frac{e^{\frac{(-5+\sqrt{17})t}{2}} \left(\frac{33}{26} + \frac{5\sqrt{17}}{13} \right) \sqrt{17}}{2} - \frac{3e^{\frac{(5+\sqrt{17})t}{2}} \left(\frac{33}{26} - \frac{5\sqrt{17}}{13} \right)}{2} - \frac{3e^{\frac{(-5+\sqrt{17})t}{2}} \left(\frac{33}{26} + \frac{5\sqrt{17}}{13} \right)}{2} + \frac{2\sin(t)}{13} - \frac{3\cos(t)}{13} - t - \frac{3}{2} \right\} \quad (39)$$

```

> xx:=DEplot([sist],[x,y],t=-10..10,[[cond]],scene=[t,x(t)])
dsolve/numeric: entering dsolve/numeric
DEtools/convertsys: converted to first-order system  $Y'(x) = f(x, Y(x))$ 
namely (with  $Y'$  represented by  $YP$ )
 $[YP_1 = Y_1 - Y_2 + t - 1, YP_2 = -2Y_1 + 4Y_2 + \cos(t)]$ 
DEtools/convertsys: correspondence between  $Y[i]$  names and original
functions:
 $[Y_1 = x(t), Y_2 = y(t)]$ 
dsolve/numeric: the procedure  $F(x, Y, YP)$  for computing  $Y'(x)=f(x, Y(x))$ 
is: proc (N, X, Y, YP) option [Y[1] = x(t), Y[2] = y(t)]; YP[1] := Y
[1]-Y[2]+X-1; YP[2] := -2*Y[1]+4*Y[2]+cos(X); 0 end proc
dsolve/numeric: initial conditions:  $x_0=0.$ ,  $y_0=[0., 1.]$ 
dsolve/numeric/SC/firststep: Checking ODE for hardware floating point
computation
dsolve/numeric/SC/firststep: Initial point OK for hardware floating
point computation
dsolve/numeric/evalat_ext: loading hardware evalat

```



```

> yy:=DEplot([sist],[x,y],t=-10..10,[[cond]],linecolor=red,scene=
[t,y(t)])
dsolve/numeric: entering dsolve/numeric

```


DEtools/convertsys: converted to first-order system $Y'(x) = f(x, Y(x))$ namely (with Y' represented by YP)

$$[YP_1 = Y_1 - Y_2 + t - 1, YP_2 = -2Y_1 + 4Y_2 + \cos(t)]$$

DEtools/convertsys: correspondence between $Y[i]$ names and original functions:

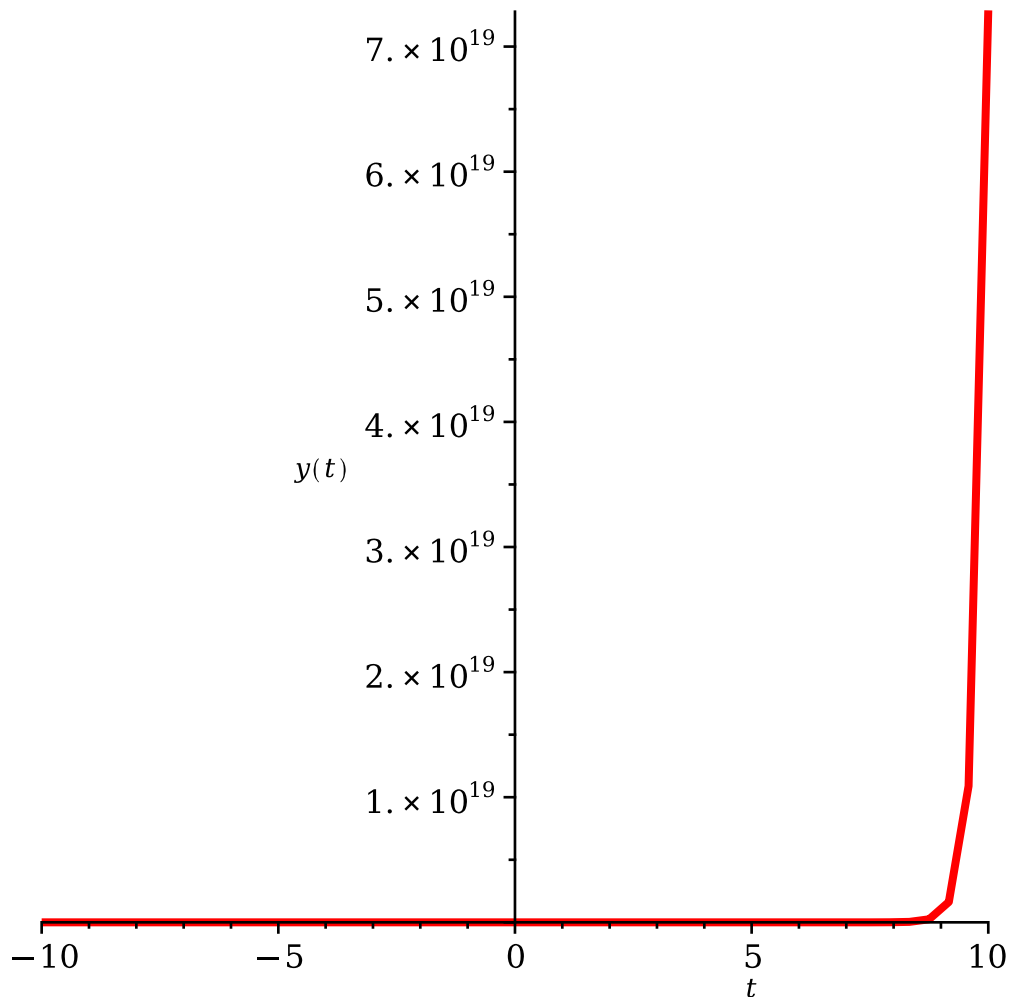
$$[Y_1 = x(t), Y_2 = y(t)]$$

dsolve/numeric: the procedure $F(x, Y, YP)$ for computing $Y'(x) = f(x, Y(x))$ is: `proc (N, X, Y, YP) option [Y[1] = x(t), Y[2] = y(t)]; YP[1] := Y[1] - Y[2] + X - 1; YP[2] := -2*Y[1] + 4*Y[2] + cos(X); 0 end proc`

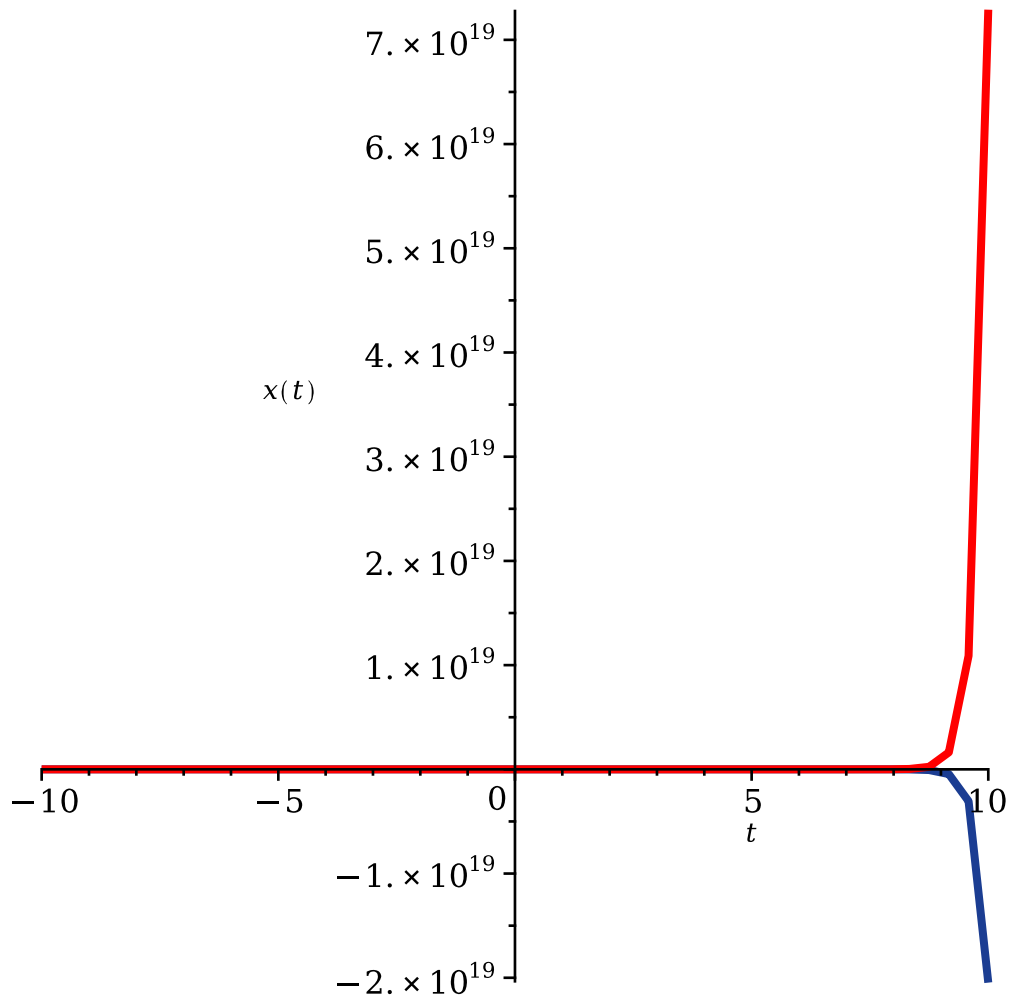
dsolve/numeric: initial conditions: $x_0 = 0.$, $y_0 = [0., 1.]$

dsolve/numeric/SC/firststep: Checking ODE for hardware floating point computation

dsolve/numeric/SC/firststep: Initial point OK for hardware floating point computation



> display([xx,yy])



```
> ec1:=diff(x(t),t)=x(t)+2*y(t)+exp(-t)
```

$$ec1 := \frac{d}{dt} x(t) = x(t) + 2y(t) + e^{-t} \quad (40)$$

```
> ec2:=diff(y(t),t)=-2*x(t)+y(t)+1
```

$$ec2 := \frac{d}{dt} y(t) = -2x(t) + y(t) + 1 \quad (41)$$

```
> cond:=x(0)=0,y(0)=1
```

$$cond := x(0) = 0, y(0) = 1 \quad (42)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = x(t) + 2y(t) + e^{-t}, \frac{d}{dt} y(t) = -2x(t) + y(t) + 1 \quad (43)$$

```
> sol:=dsolve({sist,cond},{x(t),y(t)})
```

```
-> Solving each unknown as a function of the next ones using the
order: [x(t), y(t)]
```

```
-> Calling odsolve with the ODE diff(diff(y(x) x) x) = 2*(diff(y
(x) x))-5*y(x)-1-2*exp(-x) y(x) explicit singsol = none
```

```
Methods for second order ODEs:
```

```
--- Trying classification methods ---
```

```
trying a quadrature
```

trying high order exact linear fully integrable
 trying differential order: 2; linear nonhomogeneous with
 symmetry [0,1]

trying a double symmetry of the form [xi=0, eta=F(x)]

-> Try solving first the homogeneous part of the ODE

checking if the LODE has constant coefficients

<- constant coefficients successful

-> Determining now a particular solution to the non-homogeneous ODE

building a particular solution using variation of parameters

<- solving first the homogeneous part of the ODE successful

$$\text{sol} := \left\{ x(t) = -\frac{3e^t \cos(2t)}{20} + \frac{29e^t \sin(2t)}{20} - \frac{e^{-t}}{4} + \frac{2}{5}, y(t) = \frac{3e^t \sin(2t)}{20} + \frac{29e^t \cos(2t)}{20} - \frac{1}{5} - \frac{e^{-t}}{4} \right\} \quad (44)$$

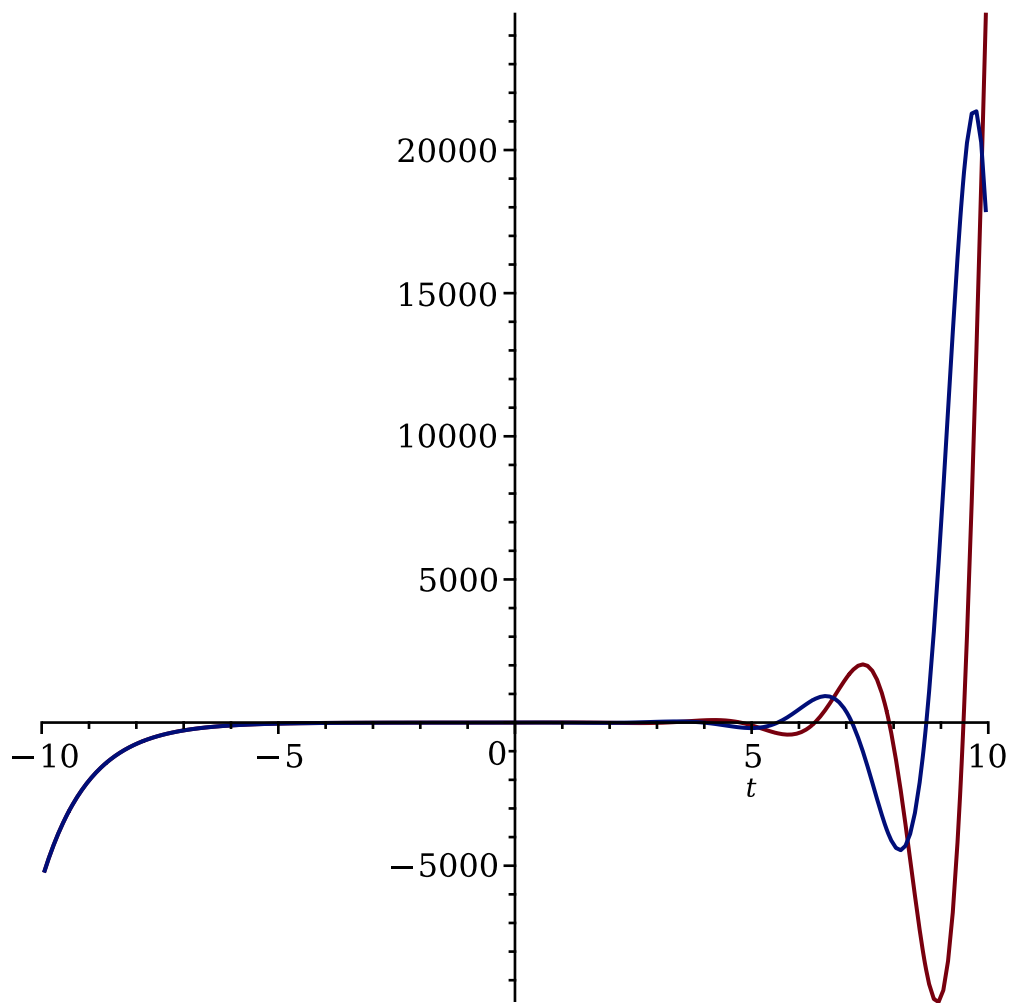
> **xx:=unapply(rhs(sol[1]),t)**

$$xx := t \mapsto -\frac{3 \cdot e^t \cdot \cos(2 \cdot t)}{20} + \frac{29 \cdot e^t \cdot \sin(2 \cdot t)}{20} - \frac{e^{-t}}{4} + \frac{2}{5} \quad (45)$$

> **yy:=unapply(rhs(sol[2]),t)**

$$yy := t \mapsto \frac{3 \cdot e^t \cdot \sin(2 \cdot t)}{20} + \frac{29 \cdot e^t \cdot \cos(2 \cdot t)}{20} - \frac{1}{5} - \frac{e^{-t}}{4} \quad (46)$$

> **plot([xx(t),yy(t)],t=-10..10)**



```
> ec1:=diff(x(t),t)=-x(t)+3*y(t)+3*z(t)+27*t^2
```

$$ec1 := \frac{d}{dt} x(t) = -x(t) + 3y(t) + 3z(t) + 27t^2 \quad (47)$$

```
> ec2:=diff(y(t),t)=2*x(t)-2*y(t)-5*z(t)+3*t
```

$$ec2 := \frac{d}{dt} y(t) = 2x(t) - 2y(t) - 5z(t) + 3t \quad (48)$$

```
> ec3:=diff(z(t),t)=-2*x(t)+3*y(t)+6*z(t)+3
```

$$ec3 := \frac{d}{dt} z(t) = -2x(t) + 3y(t) + 6z(t) + 3 \quad (49)$$

```
> cond:=x(0)=50,y(0)=-30,z(0)=26
```

$$cond := x(0) = 50, y(0) = -30, z(0) = 26 \quad (50)$$

```
> sist:=ec1,ec2,ec3
```

$$sist := \frac{d}{dt} x(t) = -x(t) + 3y(t) + 3z(t) + 27t^2, \frac{d}{dt} y(t) = 2x(t) - 2y(t) - 5z(t) + 3t, \frac{d}{dt} z(t) = -2x(t) + 3y(t) + 6z(t) + 3 \quad (51)$$

$$-5z(t) + 3t, \frac{d}{dt} z(t) = -2x(t) + 3y(t) + 6z(t) + 3$$

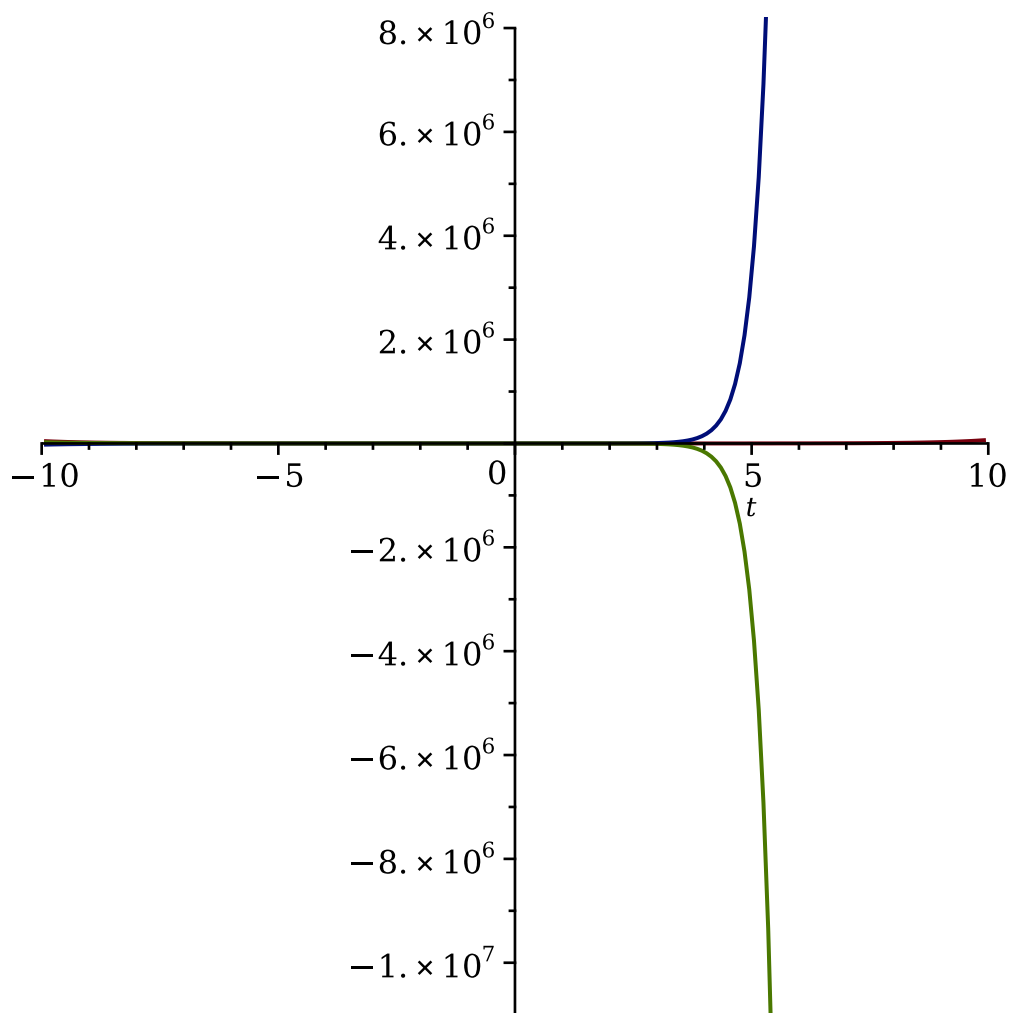
```
> sol:=dsolve({sist,cond},{x(t),y(t),z(t)})
```

```
-> Solving each unknown as a function of the next ones using the
```

```

order: [z(t), y(t), x(t)]
-> Calling odsolve with the ODE diff(y(x) x) = 45*x^2-(5/3)*
(diff(x(x) x))+(1/3)*x(x)+3*y(x)+3*x y(x) explicit singsol = none
Methods for first order ODEs:
--- Trying classification methods ---
trying a quadrature
trying 1st order linear
<- 1st order linear successful
-> Calling odsolve with the ODE diff(diff(y(x) x) x) = -27*x^2+y
(x)+63*x+9 y(x) explicit singsol = none
Methods for second order ODEs:
--- Trying classification methods ---
trying a quadrature
trying high order exact linear fully integrable
trying differential order: 2; linear nonhomogeneous with
symmetry [0,1]
trying a double symmetry of the form [xi=0, eta=F(x)]
-> Try solving first the homogeneous part of the ODE
    checking if the LODE has constant coefficients
    <- constant coefficients successful
    -> Determining now a particular solution to the non-
homogeneous ODE
        trying a rational particular solution
        <- rational particular solution successful
<- solving first the homogeneous part of the ODE successful
sol := {x(t) = 3et + 2e-t + 27t2 - 63t + 45, y(t) = e3t + 2et - 18t2 + 24t - 32 (52)
        - e-t, z(t) = -e3t - 27t + 26 + 18t2 + e-t}
=
> xx:=unapply(rhs(sol[1]),t)
      xx := t ↦ 3·et + 2·e-t + 27·t2 - 63·t + 45 (53)
=
> yy:=unapply(rhs(sol[2]),t)
      yy := t ↦ e3·t + 2·et - 18·t2 + 24·t - 32 - e-t (54)
=
> zz:=unapply(rhs(sol[3]),t)
      zz := t ↦ -e3·t - 27·t + 26 + 18·t2 + e-t (55)
=
> plot([xx(t),yy(t),zz(t)],t=-10..10)

```



> restart

> with (DEtools), with (plots)

[AreSimilar, Closure, DENormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeometricsols, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor,

(56)

invariants, kovaciccols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom], [animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]

> ec1:=diff(x(t),t)=x(t)+y(t)

$$ec1 := \frac{d}{dt} x(t) = x(t) + y(t) \quad (57)$$

> ec2:=diff(y(t),t)=-2*x(t)+4*y(t)

$$ec2 := \frac{d}{dt} y(t) = -2x(t) + 4y(t) \quad (58)$$

> cond1:=x(0)=3,y(0)=0

$$cond1 := x(0) = 3, y(0) = 0 \quad (59)$$

> cond2:=x(0)=0,y(0)=3

$$cond2 := x(0) = 0, y(0) = 3 \quad (60)$$

> cond3:=x(0)=-3,y(0)=0

$$cond3 := x(0) = -3, y(0) = 0 \quad (61)$$

> cond4:=x(0)=0,y(0)=-3

$$cond4 := x(0) = 0, y(0) = -3 \quad (62)$$

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = x(t) + y(t), \frac{d}{dt} y(t) = -2x(t) + 4y(t) \quad (63)$$

> infolevel[dsolve]:=3

(64)

$$\text{infolevel}_{\text{dsolve}} := 3 \quad (64)$$

```
> sol1:=dsolve({sist, cond1}, {x(t), y(t)})
```

-> Solving each unknown as a function of the next ones using the order: [y(t), x(t)]

$$\text{sol1} := \{x(t) = 6e^{2t} - 3e^{3t}, y(t) = 6e^{2t} - 6e^{3t}\} \quad (65)$$

```
> sol2:=dsolve({sist, cond2}, {x(t), y(t)})
```

-> Solving each unknown as a function of the next ones using the order: [y(t), x(t)]

$$\text{sol2} := \{x(t) = -3e^{2t} + 3e^{3t}, y(t) = -3e^{2t} + 6e^{3t}\} \quad (66)$$

```
> sol3:=dsolve({sist, cond3}, {x(t), y(t)})
```

-> Solving each unknown as a function of the next ones using the order: [y(t), x(t)]

$$\text{sol3} := \{x(t) = -6e^{2t} + 3e^{3t}, y(t) = -6e^{2t} + 6e^{3t}\} \quad (67)$$

```
> sol4:=dsolve({sist, cond4}, {x(t), y(t)})
```

-> Solving each unknown as a function of the next ones using the order: [y(t), x(t)]

$$\text{sol4} := \{x(t) = 3e^{2t} - 3e^{3t}, y(t) = 3e^{2t} - 6e^{3t}\} \quad (68)$$

```
> limit(sol1[1], t=infinity)
```

$$\lim_{t \rightarrow \infty} x(t) = -\infty \quad (69)$$

```
> limit(sol1[2], t=infinity)
```

$$\lim_{t \rightarrow \infty} y(t) = -\infty \quad (70)$$

```
> limit(sol2[1], t=infinity)
```

$$\lim_{t \rightarrow \infty} x(t) = \infty \quad (71)$$

```
> limit(sol2[2], t=infinity)
```

$$\lim_{t \rightarrow \infty} y(t) = \infty \quad (72)$$

```
> limit(sol3[1], t=infinity)
```

$$\lim_{t \rightarrow \infty} x(t) = \infty \quad (73)$$

```
> limit(sol3[2], t=infinity)
```

$$\lim_{t \rightarrow \infty} y(t) = \infty \quad (74)$$

```
> limit(sol4[1], t=infinity)
```

$$\lim_{t \rightarrow \infty} x(t) = -\infty \quad (75)$$

```
> limit(sol4[2], t=infinity)
```

$$\lim_{t \rightarrow \infty} y(t) = -\infty \quad (76)$$

```
> DEplot([sist], [x(t), y(t)], t=-10..10, x=-10..10, y=-10..10, [[cond1],
[cond2], [cond3], [cond4]])
```

dsolve/numeric: entering dsolve/numeric
DEtools/convertsys: converted to first-order system $Y'(x) = f(x, Y(x))$
namely (with Y' represented by YP)
 $[YP_1 = Y_1 + Y_2, YP_2 = -2Y_1 + 4Y_2]$

DEtools/convertsys: correspondence between $Y[i]$ names and original functions:

$$[Y_1 = x(t), Y_2 = y(t)]$$

dsolve/numeric: the procedure $F(x, Y, YP)$ for computing $Y'(x) = f(x, Y(x))$ is: proc (N, X, Y, YP) option [Y[1] = x(t), Y[2] = y(t)]; YP[1] := Y[1]+Y[2]; YP[2] := -2*Y[1]+4*Y[2]; 0 end proc

dsolve/numeric: initial conditions: $x_0=0.$, $y_0=[3., 0.]$

dsolve/numeric: entering dsolve/numeric

DEtools/convertsys: converted to first-order system $Y'(x) = f(x, Y(x))$ namely (with Y' represented by YP)

$$[YP_1 = Y_1 + Y_2, YP_2 = -2 Y_1 + 4 Y_2]$$

DEtools/convertsys: correspondence between $Y[i]$ names and original functions:

$$[Y_1 = x(t), Y_2 = y(t)]$$

dsolve/numeric: the procedure $F(x, Y, YP)$ for computing $Y'(x) = f(x, Y(x))$ is: proc (N, X, Y, YP) option [Y[1] = x(t), Y[2] = y(t)]; YP[1] := Y[1]+Y[2]; YP[2] := -2*Y[1]+4*Y[2]; 0 end proc

dsolve/numeric: initial conditions: $x_0=0.$, $y_0=[0., 3.]$

dsolve/numeric: entering dsolve/numeric

DEtools/convertsys: converted to first-order system $Y'(x) = f(x, Y(x))$ namely (with Y' represented by YP)

$$[YP_1 = Y_1 + Y_2, YP_2 = -2 Y_1 + 4 Y_2]$$

DEtools/convertsys: correspondence between $Y[i]$ names and original functions:

$$[Y_1 = x(t), Y_2 = y(t)]$$

dsolve/numeric: the procedure $F(x, Y, YP)$ for computing $Y'(x) = f(x, Y(x))$ is: proc (N, X, Y, YP) option [Y[1] = x(t), Y[2] = y(t)]; YP[1] := Y[1]+Y[2]; YP[2] := -2*Y[1]+4*Y[2]; 0 end proc

dsolve/numeric: initial conditions: $x_0=0.$, $y_0=[-3., 0.]$

dsolve/numeric: entering dsolve/numeric

DEtools/convertsys: converted to first-order system $Y'(x) = f(x, Y(x))$ namely (with Y' represented by YP)

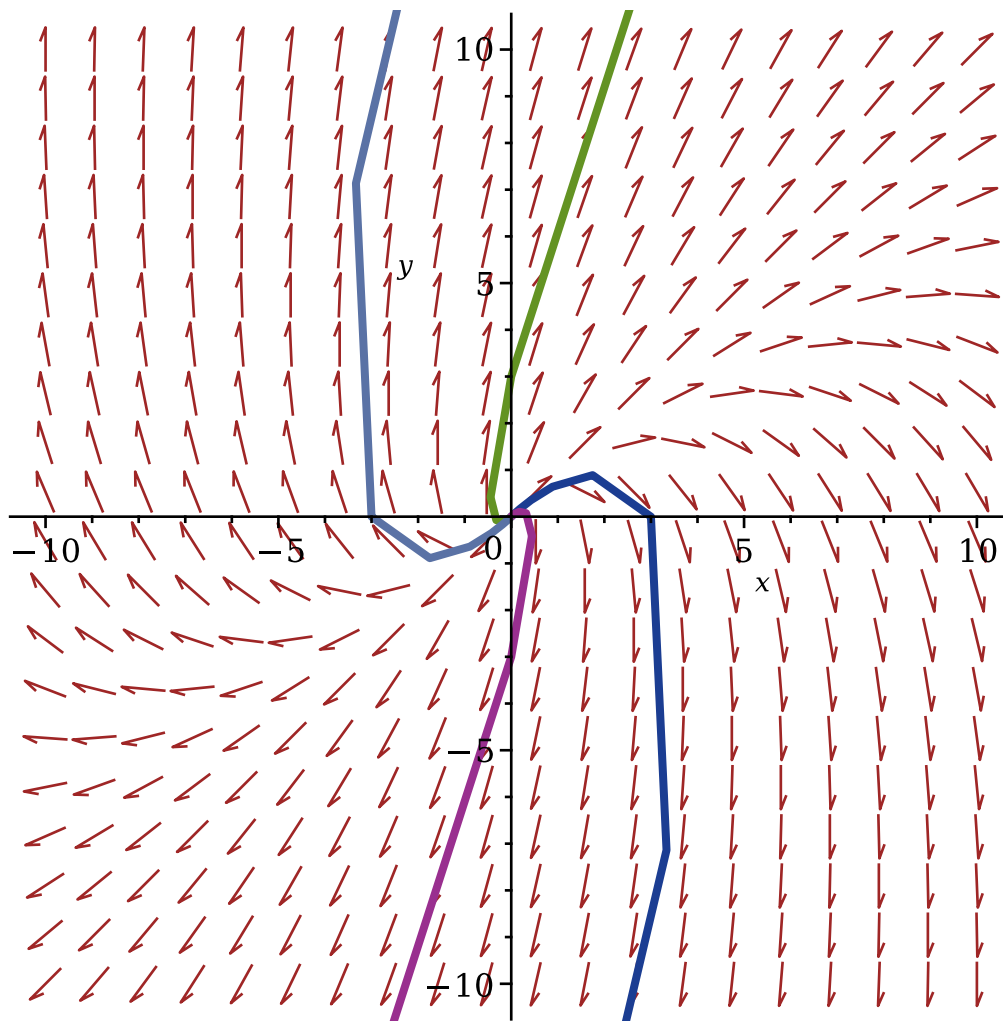
$$[YP_1 = Y_1 + Y_2, YP_2 = -2 Y_1 + 4 Y_2]$$

DEtools/convertsys: correspondence between $Y[i]$ names and original functions:

$$[Y_1 = x(t), Y_2 = y(t)]$$

dsolve/numeric: the procedure $F(x, Y, YP)$ for computing $Y'(x) = f(x, Y(x))$ is: proc (N, X, Y, YP) option [Y[1] = x(t), Y[2] = y(t)]; YP[1] := Y[1]+Y[2]; YP[2] := -2*Y[1]+4*Y[2]; 0 end proc

dsolve/numeric: initial conditions: $x_0=0.$, $y_0=[0., -3.]$



> restart

> with (DEtools), with (plots)

[AreSimilar, Closure, DENormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeometricsols, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor,

(77)

invariants, kovaciccols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom], [animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]

> ec1:=diff(x(t),t)=y(t)

$$ec1 := \frac{d}{dt} x(t) = y(t) \quad (78)$$

> ec2:=diff(y(t),t)=-x(t)-2*y(t)

$$ec2 := \frac{d}{dt} y(t) = -x(t) - 2y(t) \quad (79)$$

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = y(t), \frac{d}{dt} y(t) = -x(t) - 2y(t) \quad (80)$$

> sol:=dsolve({sist},{x(t),y(t)})

$$sol := \{x(t) = e^{-t} (c_2 t + c_1), y(t) = -e^{-t} (c_2 t + c_1 - c_2)\} \quad (81)$$

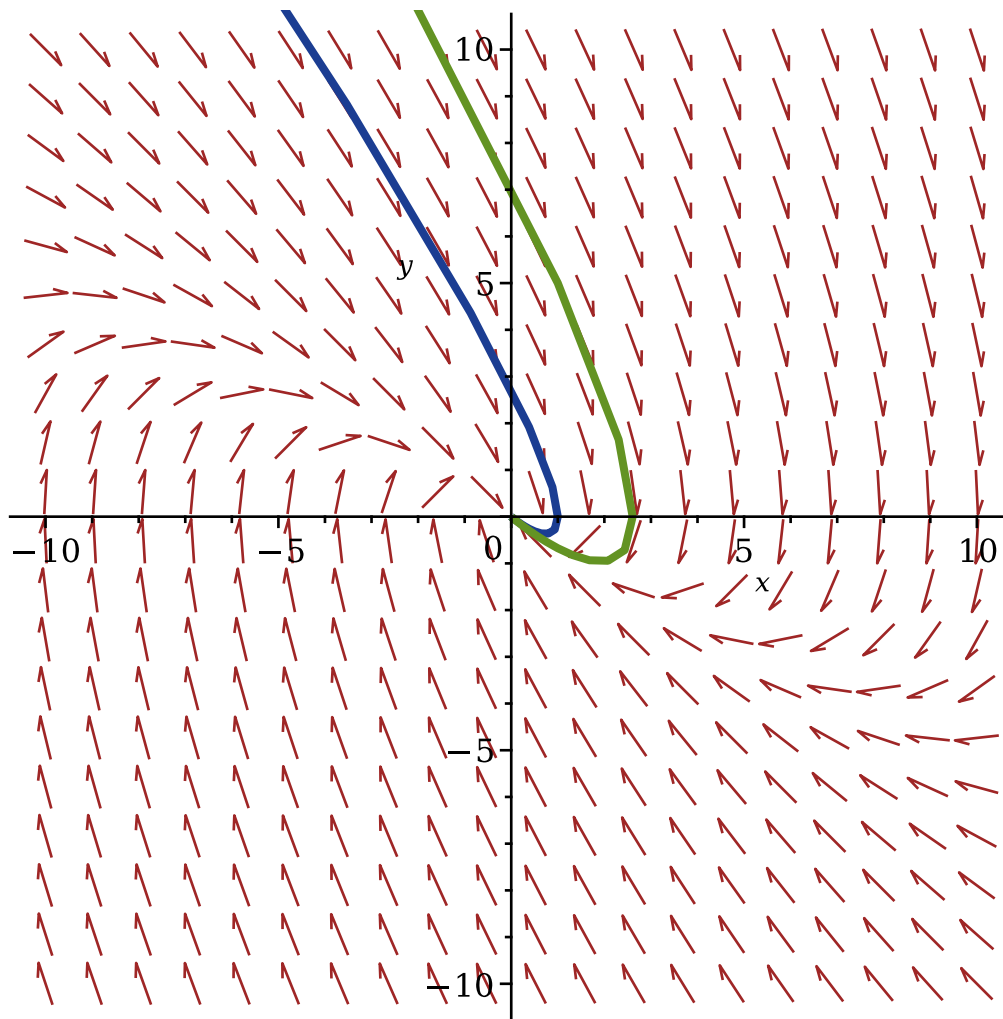
> limit(sol[1],t=infinity)

$$\lim_{t \rightarrow \infty} x(t) = 0 \quad (82)$$

> limit(sol[2],t=infinity)

$$\lim_{t \rightarrow \infty} y(t) = 0 \quad (83)$$

> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10,[[x(0)=1, y(0)=0], [x(0)=1,y(0)=5]])



> restart

> with (DEtools), with(plots)

[AreSimilar, Closure, DENormal, DEplot, DEplot3d, DEplot_polygon, DFactor, DFactorLCLM, DFactorsols, Dchangevar, Desingularize, FindODE, FunctionDecomposition, GCRD, Gosper, Heunsols, Homomorphisms, IVPsol, IsHyperexponential, LCLM, MeijerGsols, MultiplicativeDecomposition, ODEInvariants, PDEchangecoords, PolynomialNormalForm, RationalCanonicalForm, ReduceHyperexp, RiemannPsols, Xchange, Xcommutator, Xgauge, Zeilberger, abelsol, adjoint, autonomous, bernoullisol, buildsol, buildsym, canoni, caseplot, casesplit, checkrank, chinisol, clairautsol, constcoeffsols, convertAlg, convertsys, dalembertsol, dcoeffs, de2diffop, dfieldplot, diff_table, diffop2de, dperiodic_sols, dpolyform, dsubs, eigenring, endomorphism_charpoly, equinv, eta_k, eulersols, exactsol, expsols, exterior_power, firint, firtest, formal_sol, gen_exp, generate_ic, genhomosol, gensys, hamilton_eqs, hypergeometricsols, hypergeomsols, hyperode, indicialeq, infgen, initialdata, integrate_sols, intfactor,

(84)

invariants, kovaciccols, leftdivision, liesol, line_int, linearsol, matrixDE, matrix_riccati, maxdimsystems, moser_reduce, muchange, mult, mutest, newton_polygon, normalG2, ode_int_y, ode_y1, odeadvisor, odepde, parametricsol, particularsol, phaseportrait, poincare, polysols, power_equivalent, rational_equivalent, ratsols, redode, reduceOrder, reduce_order, regular_parts, regularsp, remove_RootOf, riccati_system, riccatisol, rifread, rifsimp, rightdivision, rtaylor, separablesol, singularities, solve_group, super_reduce, symgen, symmetric_power, symmetric_product, symtest, transinv, translate, untranslate, varparam, zoom], [animate, animate3d, animatecurve, arrow, changecoords, complexplot, complexplot3d, conformal, conformal3d, contourplot, contourplot3d, coordplot, coordplot3d, densityplot, display, dualaxisplot, fieldplot, fieldplot3d, gradplot, gradplot3d, implicitplot, implicitplot3d, inequal, interactive, interactiveparams, intersectplot, listcontplot, listcontplot3d, listdensityplot, listplot, listplot3d, loglogplot, logplot, matrixplot, multiple, odeplot, pareto, plotcompare, pointplot, pointplot3d, polarplot, polygonplot, polygonplot3d, polyhedra_supported, polyhedraplot, rootlocus, semilogplot, setcolors, setoptions, setoptions3d, shadebetween, spacecurve, sparsematrixplot, surfdata, textplot, textplot3d, tubeplot]

> ec1:=diff(x(t),t)=2*x(t)+y(t)

$$ec1 := \frac{d}{dt} x(t) = 2x(t) + y(t) \quad (85)$$

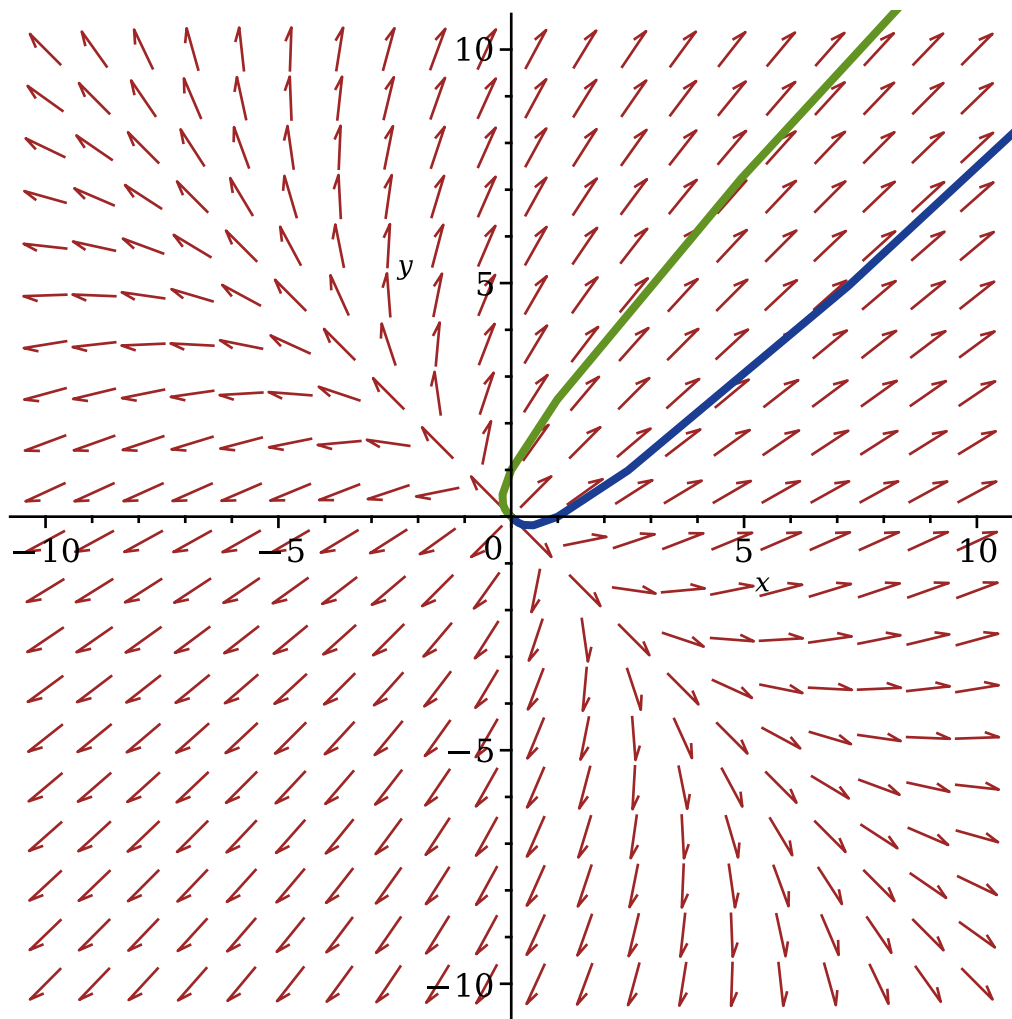
> ec2:=diff(y(t),t)=x(t)+2*y(t)

$$ec2 := \frac{d}{dt} y(t) = x(t) + 2y(t) \quad (86)$$

> sist:=ec1,ec2

$$sist := \frac{d}{dt} x(t) = 2x(t) + y(t), \frac{d}{dt} y(t) = x(t) + 2y(t) \quad (87)$$

> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10, [[x(0)=1,y(0)=0], [x(0)=0,y(0)=1]])



```
> #NU
```

```
> ec1:=diff(x(t),t)=-x(t)-y(t)
```

$$ec1 := \frac{d}{dt} x(t) = -x(t) - y(t) \quad (88)$$

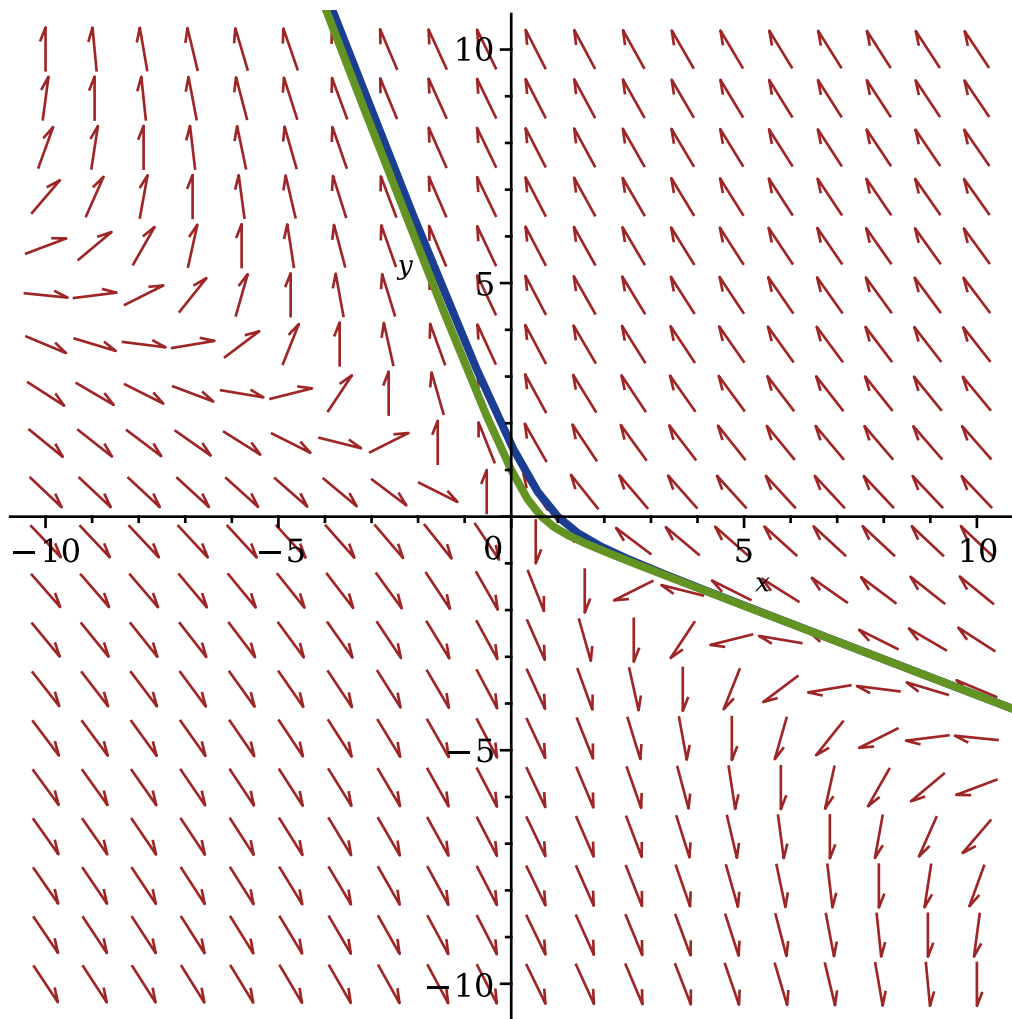
```
> ec2:=diff(y(t),t)=x(t)+2*y(t)
```

$$ec2 := \frac{d}{dt} y(t) = x(t) + 2y(t) \quad (89)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = -x(t) - y(t), \frac{d}{dt} y(t) = x(t) + 2y(t) \quad (90)$$

```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10, [[x(0)=
1,y(0)=0], [x(0)=0,y(0)=1]])
```



```
> #NU
```

```
> ec1:=diff(x(t),t)=y(t)
```

$$ec1 := \frac{d}{dt} x(t) = y(t) \quad (91)$$

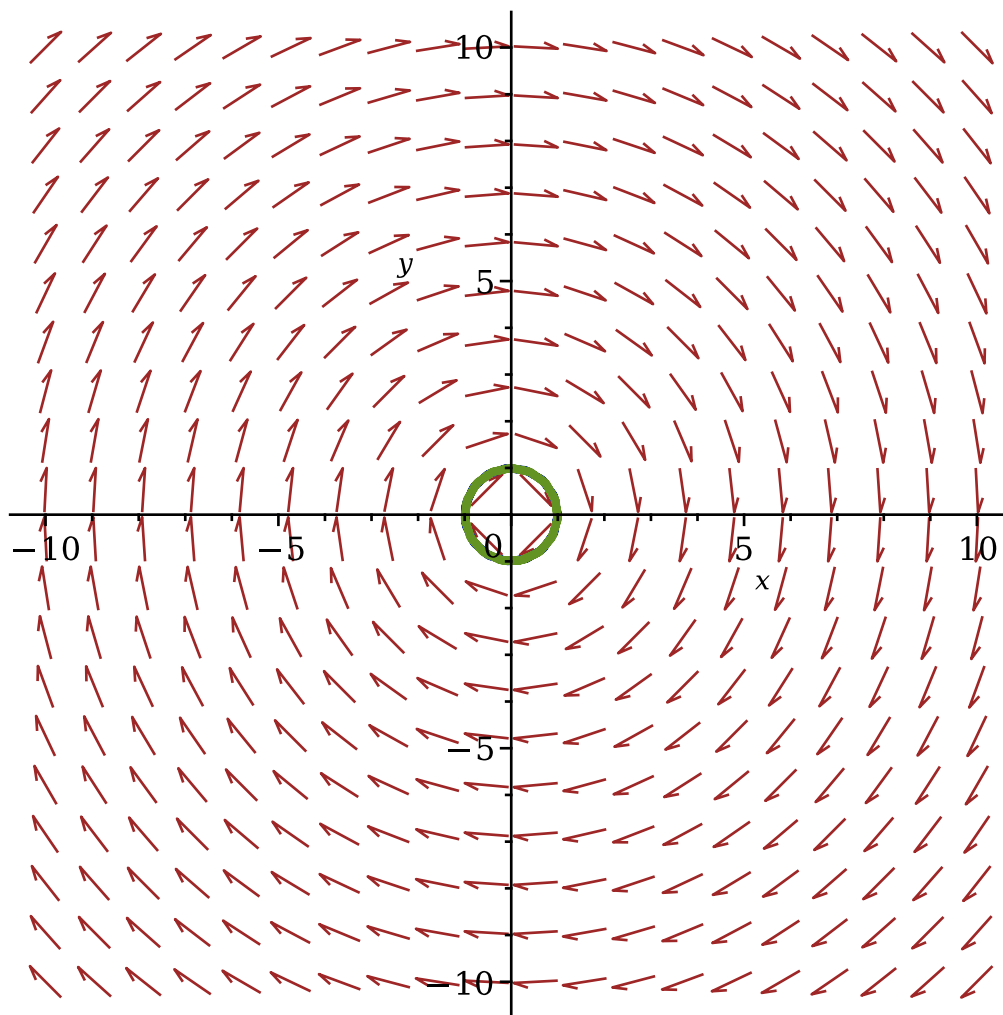
```
> ec2:=diff(y(t),t)=-x(t)
```

$$ec2 := \frac{d}{dt} y(t) = -x(t) \quad (92)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = y(t), \frac{d}{dt} y(t) = -x(t) \quad (93)$$

```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10, [[x(0)=
1,y(0)=0], [x(0)=0,y(0)=1]])
```



```
> #NU
```

```
> ec1:=diff(x(t),t)=-2*x(t)
```

$$ec1 := \frac{d}{dt} x(t) = -2x(t) \quad (94)$$

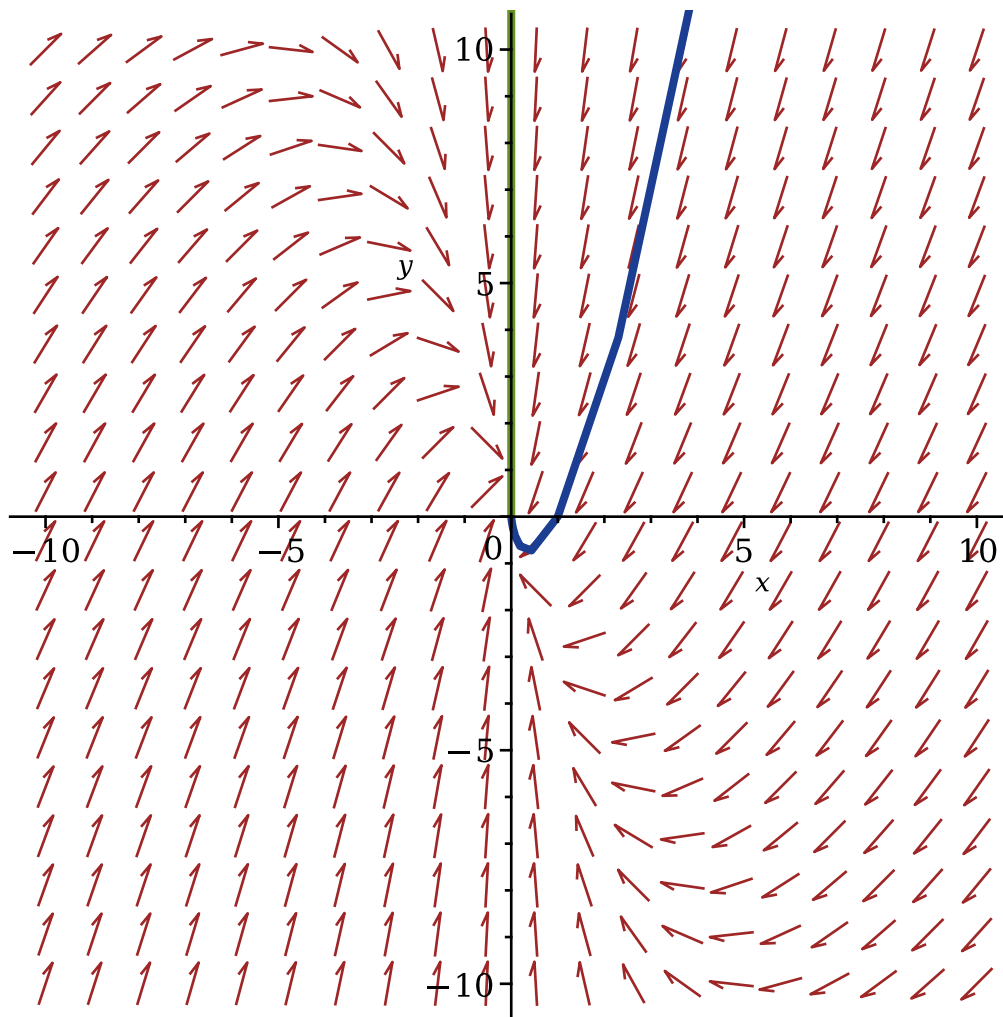
```
> ec2:=diff(y(t),t)=-4*x(t)-2*y(t)
```

$$ec2 := \frac{d}{dt} y(t) = -4x(t) - 2y(t) \quad (95)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = -2x(t), \frac{d}{dt} y(t) = -4x(t) - 2y(t) \quad (96)$$

```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10, [[x(0)=
1,y(0)=0], [x(0)=0,y(0)=1]])
```

```
> #DA
```

```
> ec1:=diff(x(t),t)=x(t)-4*y(t)
```

$$ec1 := \frac{d}{dt} x(t) = x(t) - 4y(t) \quad (97)$$

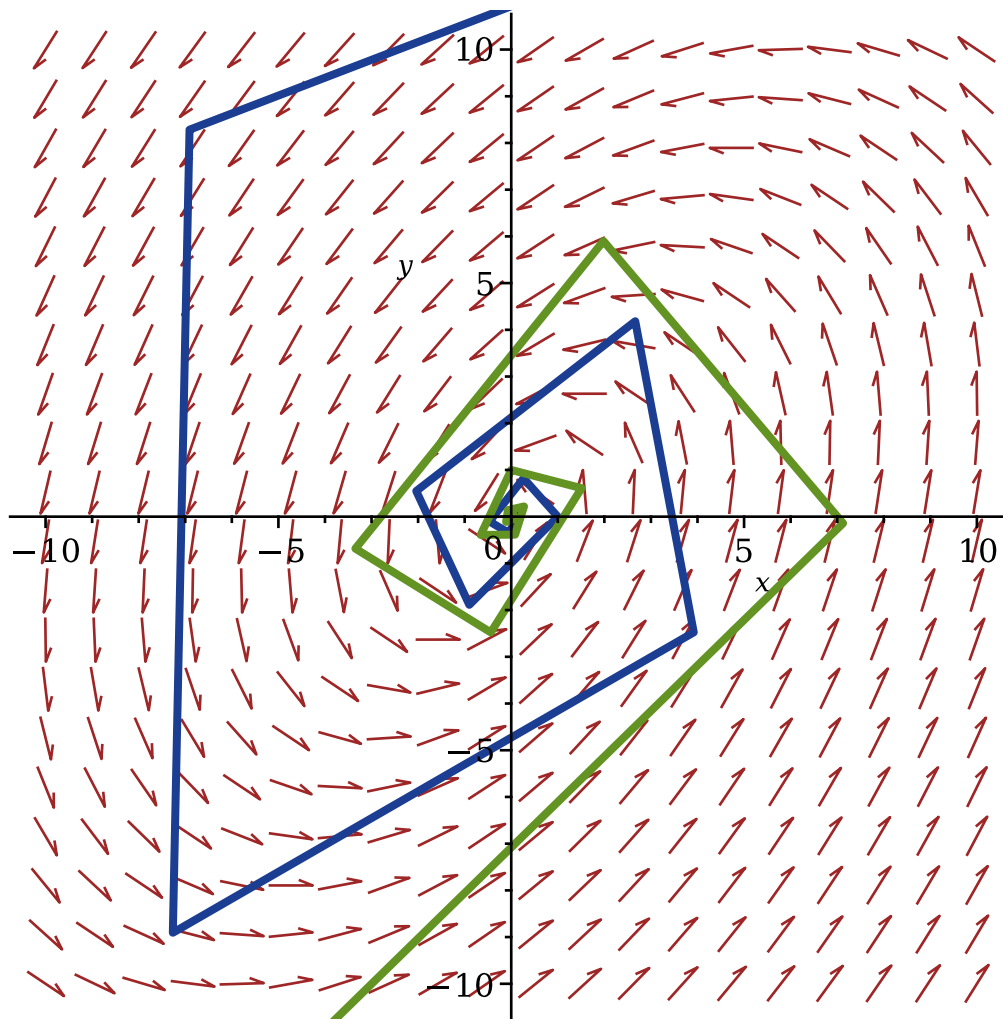
```
> ec2:=diff(y(t),t)=5*x(t)-3*y(t)
```

$$ec2 := \frac{d}{dt} y(t) = 5x(t) - 3y(t) \quad (98)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = x(t) - 4y(t), \frac{d}{dt} y(t) = 5x(t) - 3y(t) \quad (99)$$

```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10, [[x(0)=1,y(0)=0], [x(0)=0,y(0)=1]])
```



```
> #DA
```

```
> ec1:=diff(x(t),t)=3*x(t)-y(t)
```

$$ec1 := \frac{d}{dt} x(t) = 3x(t) - y(t) \quad (100)$$

```
> ec2:=diff(y(t),t)=y(t)
```

$$ec2 := \frac{d}{dt} y(t) = y(t) \quad (101)$$

```
> sist:=ec1,ec2
```

$$sist := \frac{d}{dt} x(t) = 3x(t) - y(t), \frac{d}{dt} y(t) = y(t) \quad (102)$$

```
> DEplot([sist],[x(t),y(t)],t=-10..10,x=-10..10,y=-10..10, [[x(0)=
1,y(0)=0], [x(0)=0,y(0)=1]])
```

