

$$Y = AK^\beta L^{1-\beta}$$

$$r = \beta A \left(\frac{K}{L} \right)^{\beta-1} - \kappa$$

$$w = (1 - \beta) A \left(\frac{K}{L} \right)^\beta$$

$$C = C^*(r, w) \quad \text{消费政策函数}$$

$$L = L^*(r, w)$$

$$a = a^*(C, L) \quad \text{个人财产分布}$$

$$\Phi = \Phi^*(C, L) \quad \text{个人医保账户分布}$$

$$TR = TR^*(C, L) \quad \text{税收收入}$$

$$LI = LI^*(C, L) \quad \text{统筹账户缺口}$$

$$(r - 1)D = TR - kY - LI \quad \text{政府负债}$$

$$(1 - k)AK^\beta L^{1-\beta} = C + \kappa K \quad \text{商品市场均衡}$$

$$K = a^* + \Phi^* + D$$

Proof

$$K = a^* + \Phi^* + D$$

$$K = a^* + \Phi^* + \frac{1}{r - 1} (TR - kY - LI)$$

$$K + \frac{k}{r - 1} Y = a^* + \Phi^* + \frac{1}{r - 1} (TR - LI)$$

$$\because Y = \frac{C^* + \kappa K}{1 - k}$$

$$\therefore K + \frac{k}{(r - 1)(1 - k)} (C^* + \kappa K) = a^* + \Phi^* + \frac{1}{r - 1} (TR - LI)$$

$$K = a^* + \Phi^* + \frac{1}{r - 1} \left[TR - LI - \frac{k}{1 - k} C^* - \frac{k\kappa}{1 - k} K \right]$$

最后整理得到 Gauss-Seidel 迭代式：

$$\begin{cases} L_{+1} = g_1 = L^*(r, w) \\ K_{+1} = g_2 = \frac{1}{1 - \frac{k}{r}} \left[a^*(C^*, L_{+1}) + \Phi^*(C^*, L_{+1}) + \frac{1}{r} \left[TR^*(C^*, L_{+1}) - LI^*(C^*, L_{+1}) + C^*(r, w) - AK^\beta L_{+1}^{1-\beta} \right] \right] \\ K = a^* + \Phi^* + \frac{1}{r - 1} \left[TR - LI - \frac{k}{1 - k} C^* - \frac{k\kappa}{1 - k} K \right] \end{cases}$$

使用该迭代式时先计算得到利率和工资然后再按该式逐个求解政策函数

其实是一个二元非线性方程组

使用该式时应当先固定一个利率 r ，然后用 r 和 L 倒算出 K 作为 the guess of capital factor