Denote $a + \Phi = A$

Retired:

$$\left(2 - \frac{1}{1 - F_s}\right) a_{s+1} = (1 + r_s) a_s + \Lambda_s - (1 + \mu) c_s + \frac{1}{1 + p_s} [p_s + (1 - c p_s^B)] q_s c_s$$

$$\left(2 - \frac{1}{1 - F_s}\right) \Phi_{s+1} = (1 + r_s) \Phi_s + \mathbb{P}_s - \frac{q_s p_s}{1 + p_s} c_s$$

Sum up;

$$\left(2 - \frac{1}{1 - F_s}\right) A_{s+1} = (1 + r_s) A_s + \Lambda_s + \mathbb{P}_s + \frac{1 - c p_s^B}{1 + p_s} q_s c_s - (1 + \mu) c_s$$

Get:

$$\left[(1 + \mu) - \frac{1 - c p_s^B}{1 + p_s} q_s\right] c_s = (1 + r_s) A_s - \left(2 - \frac{1}{1 - F_s}\right) A_{s+1} + \Lambda_s + \mathbb{P}_s$$

Working:

$$\left(2 - \frac{1}{1 - F_s}\right) a_{s+1} = (1 + r_s) a_s + (1 - \sigma - \pi_s - \pi_s^M) w_s (1 - l_s) - (1 + \mu) c_s + \frac{1}{1 + p_s} [p_s + (1 - c p_s^B)] q_s c_s$$

$$\left(2 - \frac{1}{1 - F_s}\right) \Phi_{s+1} = (1 + r_s) \Phi_s + \frac{\phi_s + \mathbb{a}_s \zeta_s}{1 + \eta_s + \zeta_s} w_s (1 - l_s) - \frac{q_s p_s}{1 + p_s} c_s$$

Sum up:

$$\left(2 - \frac{1}{1 - F_s}\right) A_{s+1} = (1 + r_s) A_s + \left(1 - \sigma - \pi_s - \pi_s^M + \frac{\phi_s + \mathbb{a}_s \zeta_s}{1 + \eta_s + \zeta_s}\right) w_s (1 - l_s) - (1 + \mu) c_s + \frac{1 - c p_s^B}{1 + p_s} q_s c_s$$

To solve:

$$\max L = \frac{1}{1-\gamma^{-1}}\left[\left((1-q)c\right)^{1-\varrho^{-1}} + \alpha l^{1-\varrho^{-1}}\right]^{\frac{1-\gamma^{-1}}{1-\varrho^{-1}}}$$

$$-\lambda\left\{(1+r_s)A_s + \left(1-\sigma-\pi_s-\pi_s^M+\frac{\phi_s+\mathbb{a}_s\zeta_s}{1+\eta_s+\zeta_s}\right)w_s(1-l_s) - (1+\mu)c_s + \frac{1-cp_s^B}{1+p_s}q_s c_s - \left(2-\frac{1}{1-F_s}\right)A_{s+1}\right\}$$

FOC:

1). $\frac{\partial L}{\partial c} = \left[\left((1-q)c\right)^{1-\varrho^{-1}} + \alpha l^{1-\varrho^{-1}}\right]^{\frac{1-\gamma^{-1}}{1-\varrho^{-1}}-1}(1-q)^{1-\varrho^{-1}}c^{-\varrho^{-1}} + \lambda\left((1+\mu)-\frac{1-cp_s^B}{1+p_s}q_s\right)$

2). $\frac{\partial L}{\partial l} = \left[\left((1-q)c\right)^{1-\varrho^{-1}} + \alpha l^{1-\varrho^{-1}}\right]^{\frac{1-\gamma^{-1}}{1-\varrho^{-1}}-1}\alpha l^{-\varrho^{-1}} + \lambda\left(1-\sigma-\pi_s-\pi_s^M+\frac{\phi_s+\mathbb{a}_s\zeta_s}{1+\eta_s+\zeta_s}\right)w_s$

3). $\frac{\partial L}{\partial \lambda} = (1+r_s)A_s + \left(1-\sigma-\pi_s-\pi_s^M+\frac{\phi_s+\mathbb{a}_s\zeta_s}{1+\eta_s+\zeta_s}\right)w_s(1-l_s) - (1+\mu)c_s + \frac{1-cp_s^B}{1+p_s}q_s c_s - \left(2-\frac{1}{1-F_s}\right)A_{s+1}$

Denote:

$$\mathcal{A} = (1+\mu) - \frac{1-cp_s^B}{1+p_s}q_s$$

$$\mathcal{B} = \left(1-\sigma-\pi_s-\pi_s^M+\frac{\phi_s+\mathbb{a}_s\zeta_s}{1+\eta_s+\zeta_s}\right)w_s$$

$$\mathcal{C} = \frac{(1-q)^{1-\varrho^{-1}}}{\alpha}$$

$$\mathcal{D} = (1+r_s)A_s - \left(2-\frac{1}{1-F_s}\right)A_{s+1}$$

Combine 1) 2) to get 4):

4). $\frac{(1-q)^{1-\varrho^{-1}}}{\alpha}\left[\frac{c}{l}\right]^{1-\varrho^{-1}} = \frac{(1+\mu)-\frac{1-cp_s^B}{1+p_s}q_s}{\left(1-\sigma-\pi_s-\pi_s^M+\frac{\phi_s+\mathbb{a}_s\zeta_s}{1+\eta_s+\zeta_s}\right)w_s}$

$$\frac{c_s}{l_s} = \left(\frac{\mathcal{A}}{\mathcal{B}\mathcal{C}}\right)^{\frac{1}{1-\varrho^{-1}}}$$

Rewrite 3):

$$\mathcal{D} + \mathcal{B} - \mathcal{B}l_s - \mathcal{A}c_s = 0$$

Substitute 4) to 3) then get 5):

$$\left[\mathcal{A}\left(\frac{\mathcal{A}}{\mathcal{BC}}\right)^{\frac{1}{1-\varrho^{-1}}}+\mathcal{B}\right]l_s=\mathcal{D}+\mathcal{B}$$

Finally use 5) & 4) to get c_s

Because we don't consider the constraint $l_s \in [0,1]$ and $c_s \geq 0$, there are checks & bounding to do after solution:

if $l_s < 0$

$l_s = 0$

$c_s = \frac{\mathcal{D}+\mathcal{B}}{\mathcal{A}}$

if $c_s < 0$

No feasible solution

elseif $l_s > 1$

$l_s = 1$

$c_s = \frac{\mathcal{D}}{\mathcal{A}}$

if $c_s < 0$

No feasible solution

Data used:

Retired:

$$A_s, A_{s+1}$$
$$r, F, q, p, cp^B, \mu$$
$$\Lambda, \mathbb{P}$$

Working:

$$A_s, A_{s+1}$$
$$r, F, q, p, cp^B, \mu$$
$$\sigma, \phi, \zeta, \eta, \theta, w, \alpha, \varrho, \mathbb{a}$$