

Sure! The Gauss-Seidel method is an iterative numerical method for solving systems of linear equations. It is an extension of the Jacobi method, where each equation in the system is solved for the unknown variable using the most recent values of the other variables.

To determine if the Gauss-Seidel method will converge for a given system of linear equations, we can use the diagonally dominant criteria. A matrix is said to be diagonally dominant if the absolute value of the diagonal element is greater than or equal to the sum of the absolute values of the other elements in the row:

$$|a(i,i)| \geq \sum_{j \neq i} |a(i,j)|$$

Here are the step-by-step instructions to determine if the Gauss-Seidel method will converge using the diagonally dominant criteria:

1. Write the system of linear equations in matrix form $Ax = b$, where A is the coefficient matrix, x is the vector of unknowns, and b is the vector of constants.
2. Check if the matrix A is diagonally dominant. For each row i , compare the absolute value of the diagonal element $a(i,i)$ to the sum of the absolute values of the other elements in the row. If the diagonal element is greater than or equal to the sum of the other elements, the matrix is diagonally dominant. If this holds for all rows, then the matrix is strictly diagonally dominant.
3. If the matrix A is strictly diagonally dominant, then the Gauss-Seidel method will converge for any initial guess. If the matrix is only diagonally dominant (i.e., some rows may not be strictly diagonally dominant), then the Gauss-Seidel method will converge if the initial guess is sufficiently close to the solution.

Now let's take an example to demonstrate the process. Consider the following system of linear equations:

$$3x + 2y + z = 1$$

$$2x + 5y + 2z = -4$$

$$x + y + 4z = 2$$

Writing this in matrix form, we get:

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 5 & 2 \\ 1 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 2 \end{bmatrix}$$

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To determine if the Gauss-Seidel method will converge, we need to check if the matrix A is diagonally dominant. Let's look at each row:

- Row 1: $|3| \geq |2| + |1|$, so this row is diagonally dominant.
- Row 2: $|5| \geq |2| + |2|$, so this row is diagonally dominant.
- Row 3: $|4| \geq |1| + |1|$, so this row is diagonally dominant.

Since all rows are diagonally dominant, the matrix A is strictly diagonally dominant, and the Gauss-Seidel method will converge for any initial guess.

I hope this helps! Let me know if you have any other questions.