

Assignment: Homework Twelve Name: Cody Strange

Disclaimer: This is my work, not that of others

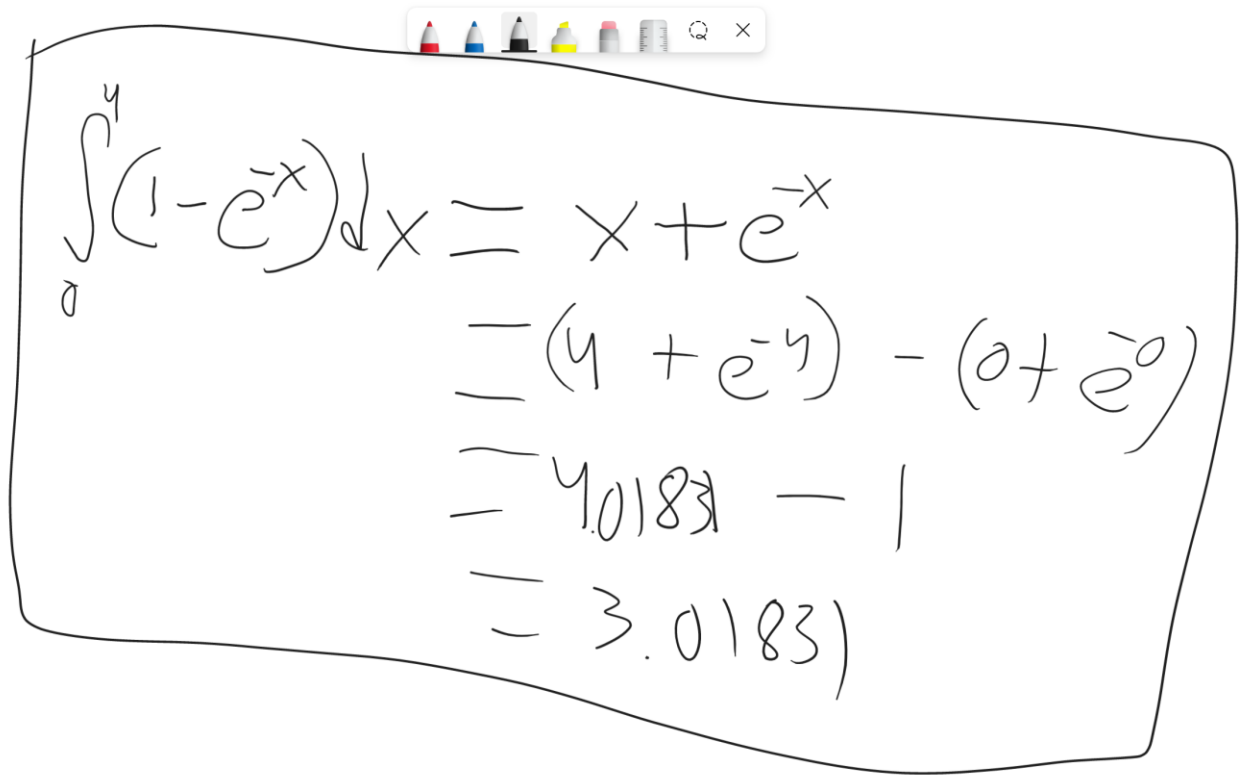
Total Score: 40 (in points, not percentage)

Problem 1 score: 20

Problem 2 score: 10

Problem 3 score: 10

1a.

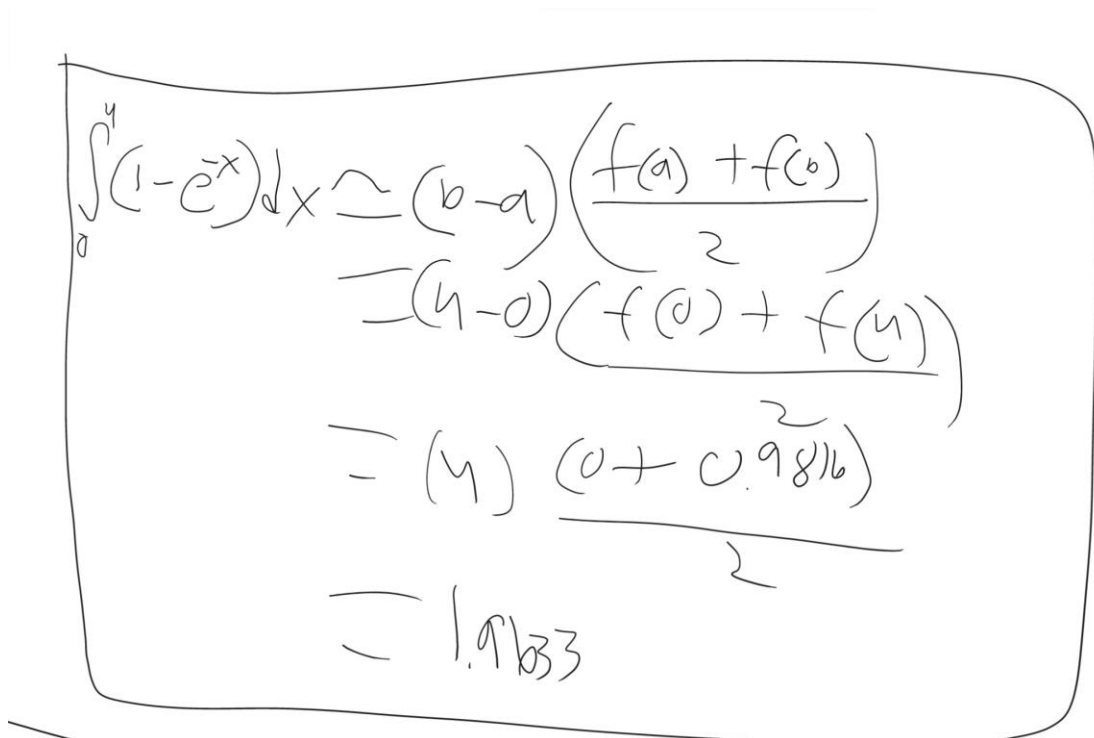


Handwritten solution for 1a using the Fundamental Theorem of Calculus. The work is enclosed in a hand-drawn box. At the top of the box, there is a digital toolbar with icons for various drawing tools (pen, highlighter, eraser, etc.) and a close button. The calculation proceeds as follows:

$$\begin{aligned}\int_0^4 (1 - e^{-x}) dx &= x + e^{-x} \\ &= (4 + e^{-4}) - (0 + e^{-0}) \\ &= 4.0183 - 1 \\ &= 3.0183\end{aligned}$$

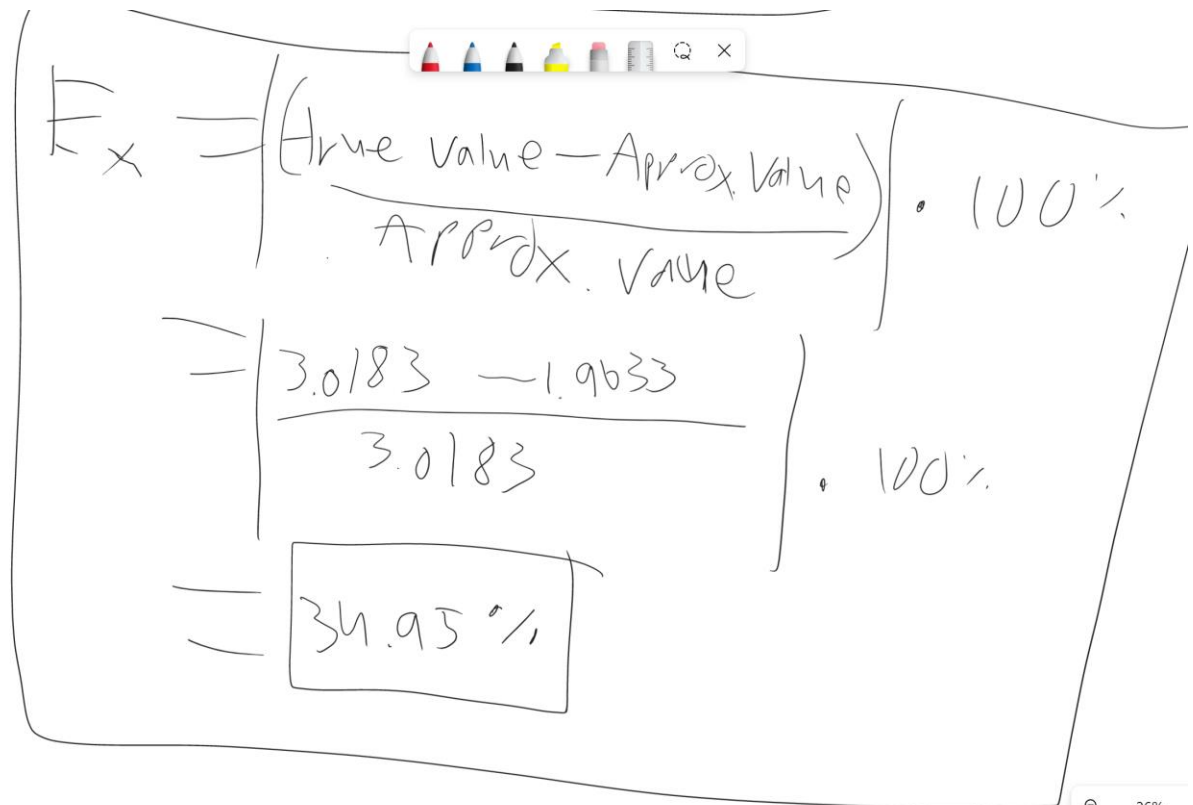
3.01831

1b.



Handwritten solution for 1b using the Trapezoidal Rule. The work is enclosed in a hand-drawn box. The calculation proceeds as follows:

$$\begin{aligned}\int_0^4 (1 - e^{-x}) dx &\approx (b-a) \left( \frac{f(a) + f(b)}{2} \right) \\ &= (4-0) \left( \frac{f(0) + f(4)}{2} \right) \\ &= (4) \frac{(0 + 0.9816)}{2} \\ &= 1.9633\end{aligned}$$



A handwritten calculation for the percentage error. The formula is written as  $E_x = \left( \frac{\text{True value} - \text{Approx. value}}{\text{Approx. value}} \right) \cdot 100\%$ . Below this, the values 3.0183 and 1.9633 are substituted into the formula. The result, 34.95%, is boxed. A toolbar with drawing tools is visible at the top of the handwriting area.

$$E_x = \left( \frac{\text{True value} - \text{Approx. value}}{\text{Approx. value}} \right) \cdot 100\%$$
$$= \left( \frac{3.0183 - 1.9633}{3.0183} \right) \cdot 100\%$$
$$= \boxed{34.95\%}$$

**Numerical Integration = 1.9633**

**Estimate Error = 34.95%**

1c.

$$n = 2$$

$$h = \frac{b-a}{n} = \frac{4-0}{2} = 2$$

$$\begin{aligned}\int_0^4 (1-e^x) dx &= \frac{b-a}{2n} \left[ f(a) + 2 \sum_{i=1}^{n-1} f(a+ih) + f(b) \right] \\ &= \frac{4-0}{2 \cdot 2} \left[ 0 + 2 \sum_{i=1}^1 f(2i) + 0.9817 \right] \\ &= \frac{4}{4} \left[ 2(0.8647) + 0.9817 \right] \\ &= 2.711\end{aligned}$$

$$n = 2$$

$$\begin{aligned}E_x &= \left( \frac{\text{True Value} - \text{Approx Value}}{\text{True Value}} \right) \cdot 100\% \\ &= \left( \frac{3.01831 - 2.7111}{3.01831} \right) \cdot 100\% \\ &= 0.1018 \cdot 100\% \\ &= 10.18\%\end{aligned}$$

$$h = 4$$

$$h = \frac{b-a}{2n} = \frac{4-0}{2 \cdot 4} = \frac{4}{8} = \frac{1}{2}$$

$$\begin{aligned} \int_0^4 (1-e^x) dx &= \frac{b-a}{2n} \left[ f(a) + 2 \sum_{i=1}^{n-1} f(a+ih) + f(b) \right] \\ &= \frac{1}{2} [0 + 2(0.6321) + 2(0.8647) + 2(0.9502) + 0.9817] \\ &= \frac{1}{2} [1.2642 + 1.7294 + 1.9004 + 0.9817] \\ &= 2.93785 \end{aligned}$$

$$h = 4$$

$$\begin{aligned} E_x &= \left( \frac{\text{True Value} - \text{Approx Value}}{\text{True Value}} \right) \cdot 100\% \\ &= \left( \frac{3.01831 - 2.93785}{3.01831} \right) \cdot 100\% \\ &= 0.0267 \cdot 100\% \\ &= 2.67\% \end{aligned}$$

**N=2:**

**Integral = 2.7111**

**Relative Error = 10.18%**

**N=4:**

**Integral = 2.93785**

**Relative Error = 2.67%**

1d.

$$h = \frac{5-1}{2} = \frac{4-0}{2} = 2$$

$$\begin{aligned}\int_0^4 (1-e^{-x}) dx &= \frac{h}{3} [f(x_0) + 4f(x_1) + f(x_2)] \\&= \frac{2}{3} [f(0) + 4f(2) + f(4)] \\&= \frac{2}{3} [0 + 3.4357 + 0.0817] \\&= \frac{2}{3} [4.5174] \\&= 2.9603\end{aligned}$$

$$\begin{aligned}E_x &= \left( \frac{\text{True Value} - \text{Approx Value}}{\text{True Value}} \right) \cdot 100\% \\&= \left( \frac{3.01831 - 2.9603}{3.01831} \right) \cdot 100\% \\&= 0.0192 \cdot 100\% \\&= 1.92\%\end{aligned}$$

**Integral = 2.9603**

**Relative Error = 1.92%**

1e.

$$h = \frac{5-0}{4} = \frac{5-0}{4} = 1$$

$$\begin{aligned}\int_0^5 (1-e^x) dx &= \frac{h}{3} [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)] \\&= \frac{1}{3} [f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)] \\&= \frac{1}{3} [1 + 4(0.6321) + 2(0.8647) + 4(0.9502) + 0.9801] \\&= \frac{1}{3} [9.0402] \\&= 3.0134\end{aligned}$$

$$\begin{aligned}E_x &= \left( \frac{\text{True Value} - \text{Approx Value}}{\text{True Value}} \right) \cdot 100\% \\&= \left( \frac{3.01831 - 3.0134}{3.01831} \right) \cdot 100\% \\&= 0.001627 \cdot 100\% \\&= 0.1627\%\end{aligned}$$

**Integral = 3.0134**

**Relative Error = 0.1627%**

2a.

$$T_n \approx \int_a^b f(x) dx$$

$$T_n = \frac{\Delta x}{2} [f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)]$$

$$= T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7$$

$$= \frac{1}{2}(5+6) + \frac{1.25}{2}(6+5.5) + \frac{1.25}{2}(5.5+7) + \frac{1.5}{2}(7+8.5)$$

$$+ \frac{1}{2}(8.5+2.8+6) + \frac{0.5}{2}(6+2.7+7) + \frac{1}{2}(7+5)$$

$$= 5.5 + 7.1875 + 7.8125 + 9.6875 + 15.25 + 6.75 + 6$$

$$= \frac{60.125}{4}$$

$$= 15.03125$$

Average = 6.6806 m/s



2b.

$$\begin{aligned}
 V(t) &= a_3 t^3 + a_2 t^2 + a_1 t + a_0 = y \\
 V(t) &= -0.0180x^3 + 0.1753x^2 + 0.0703x + 4.8507 \\
 \text{distance} &= \int_1^{10} (-0.0180x^3 + 0.1753x^2 + 0.0703x + 4.8507) \\
 &= \frac{-0.0180}{4} x^4 + \frac{0.1753}{3} x^3 + \frac{0.0703}{2} x^2 + 4.8507x \\
 &= 60.0359
 \end{aligned}$$

60.0359 m

3.

```

from numpy import array
from scipy import integrate
x=array([0, 4, 6, 8, 12, 16, 20])
rho=array([4.00, 3.95, 3.89, 3.80, 3.60, 3.41, 3.30])
area=array([100, 103, 106, 110, 120, 133, 150])
xCm=100*x
rhoAreaInKg = rho*area/1000
massTrap=integrate.trapezoid(rhoAreaInKg,xCm)
massSimp=integrate.simpson(rhoAreaInKg,xCm)
percentDiff = (abs(massTrap-massSimp)/massSimp)*100
print(f"Trapezoidal method: {massTrap}")
print(f"Simpson method: {massSimp}")
print(f"The percent difference is {percentDiff}")

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Trapezoidal method: 863.1350000000001
Simpson method: 861.4651666666668
The percent difference is 0.19383643099516984

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