Assignment: Homework Twelve   Name: Cody Strange
Disclaimer: This is my work, not that of others
Total Score: 40 (in points, not percentage)

Problem 1 score: 20
Problem 2 score: 10
Problem 3 score: 10

1a.

$$\int_0^4 (1-e^{-x})\,dx = x + e^{-x}$$
$$= (4 + e^{-4}) - (0 + e^{-0})$$
$$= 4.01831 - 1$$
$$= 3.01831$$

3.01831

1b.

$$\int_0^4 (1-e^{-x})\,dx \simeq (b-a)\left(\frac{f(a)+f(b)}{2}\right)$$
$$= (4-0)\left(\frac{f(0)+f(4)}{2}\right)$$
$$= (4)\,\frac{(0+0.9816)}{2}$$
$$= 1.9633$$

$$E_x = \left|\left(\frac{\text{true value} - \text{Aprox. Value}}{\text{Aprox. value}}\right)\right| \cdot 100\%$$

$$= \left|\frac{3.0183 - 1.9633}{3.0183}\right| \cdot 100\%$$

$$= \boxed{34.95\%}$$

Numerical Integration = 1.9633
Estimate Error = 34.95%

1c.

$n=2$

$h = \frac{b-a}{n} = \frac{4-0}{2} = 2$

$$\int_0^4 (1-e^{-x})dx = \frac{b-a}{2n}\left[f(a) + 2\sum_{i=1}^{n-1} f(a+ih) + f(b)\right]$$
$$= \frac{4-0}{2\cdot 2}\left[0 + 2\sum_{i=1}^{1} f(2i) + 0.9817\right]$$
$$= \frac{4}{4}\left[2(0.8647) + 0.9817\right]$$
$$= 2.7111$$

$n=2$

$$E_x = \left|\frac{(\text{True value} - \text{Aprox Value})}{\text{True Value}}\right| \cdot 100\%$$
$$= \left|\frac{(3.01831 - 2.7111)}{3.01831}\right| \cdot 100\%$$
$$= 0.1018 \cdot 100\%$$
$$= 10.18\%$$

$$n = 4$$

$$h = \frac{b-a}{2n} = \frac{4-0}{2 \cdot 4} = \frac{4}{8} = \frac{1}{2}$$

$$\int_0^4 (1-e^{-x})\,dx = \frac{b-a}{2n}\left[f(a) + 2\sum_{i=1}^{4-1} f(a+ih) + f(b)\right]$$

$$= \frac{1}{2}\left[0 + 2(0.6321) + 2(0.8647) + 2(0.9502) + 0.9817\right]$$

$$= \frac{1}{2}\left[1.2642 + 1.7294 + 1.9004 + 0.9817\right]$$

$$= 2.93785$$

$$n = 4$$

$$E_x = \left|\frac{(\text{True value} - \text{Aprox Value})}{\text{True Value}}\right| \cdot 100\%$$

$$= \left|\frac{(3.01831 - 2.93785)}{3.01831}\right| \cdot 100\%$$

$$= 0.0267 \cdot 100\%$$

$$= 2.67\%$$

N=2:

Integral = 2.7111

Relative Error = 10.18%

N=4:

Integral = 2.93785

Relative Error = 2.67%

1d.

$$h = \frac{b-a}{2} = \frac{4-0}{2} = 2$$

$$\int_0^4 (1-e^{-x})\,dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + f(x_2)\right]$$

$$= \frac{2}{3}\left[f(0) + 4f(2) + f(4)\right]$$

$$= \frac{2}{3}\left[0 + 3.45887 + 0.9817\right]$$

$$= \frac{2}{3}\left[4.4404\right]$$

$$= 2.9603$$

$$E_x = \left|\frac{(\text{True value} - \text{Aprox Value})}{\text{True Value}}\right| \cdot 100\%$$

$$= \left|\frac{(3.01831 - 2.9603)}{3.01831}\right| \cdot 100\%$$

$$= 0.0192 \cdot 100\%$$

$$= 1.92\%$$

Integral = 2.9603
Relative Error = 1.92%

1e.

$$h = \frac{b-a}{4} = \frac{4-0}{2} = 1$$

$$\int_0^4 (1-e^{-x})dx = \frac{h}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + f(x_4)\right]$$

$$= \frac{1}{3}\left[f(0) + 4f(1) + 2f(2) + 4f(3) + f(4)\right]$$

$$= \frac{1}{3}\left[0 + 4(0.6321) + 2(0.8647) + 4(0.9502) + 0.9817\right]$$

$$= \frac{1}{3}\left[9.0403\right]$$

$$= 3.0134$$

$$E_x = \left|\frac{(\text{True value} - \text{Aprox Value})}{\text{True Value}}\right| \cdot 100\%$$

$$= \left|\frac{(3.01831 - 3.0134)}{3.01831}\right| \cdot 100\%$$

$$= 0.001627 \cdot 100\%$$

$$= 0.1627\%$$

Integral = 3.0134
Relative Error = 0.1627%

2a.

$$T_n \approx \int_a^b f(x)\,dx$$

$$T_n = \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots 2f(x_{n-1}) + f(x_n)\right]$$

$$= T_1 + T_2 + T_3 + T_4 + T_5 + T_6 + T_7$$

$$= \frac{1}{2}(5+6) + \frac{1.25}{2}(6+5.5) + \frac{1.25}{2}(5.5+7) + \frac{1.5}{2}(7+8.5) + \frac{1}{2}(8.5+2.8+6) + \frac{0.5}{2}(6+2.7+7) + \frac{1}{2}(7+5)$$

$$= 5.5 + 7.1875 + 7.8125 + 9.6875 + 15.25 + 6.75 + 6$$

$$= \frac{60.125}{9}$$

$$= 6.6806$$

Average = 6.6806 m/s

2b.

$$V(t) = a_3 t^3 + a_2 t^2 + a_1 t^1 + a_0 = y$$

$$V(t) = -0.0180x^3 + 0.1753x^2 + 0.0703x + 4.8507$$

$$\text{distance} = \int_1^{10} \left(-0.0180x^3 + 0.1753x^2 + 0.0703x + 4.8507\right)$$

$$= \frac{-0.180}{4}x^4 + \frac{0.1753}{3}x^3 + \frac{0.0703}{2}x^2 + 4.8507x)$$

$$= 60.0359$$

60.0359 m

3.

```
from numpy import array
from scipy import integrate
x=array([0, 4, 6, 8, 12, 16, 20])
rho=array([4.00, 3.95, 3.89, 3.80, 3.60, 3.41, 3.30])
area=array([100, 103, 106, 110, 120, 133, 150])
xCm=100*x
rhoAreaInKg = rho*area/1000
massTrap=integrate.trapezoid(rhoAreaInKg,xCm)
massSimp=integrate.simpson(rhoAreaInKg,xCm)
percentDiff = (abs(massTrap-massSimp)/massSimp)*100
print(f"Trapezoidal method: {massTrap}")
print(f"Simpson method: {massSimp}")
print(f"The percent difference is {percentDiff}")
```

```
Trapezoidal method: 863.1350000000001
Simpson method: 861.4651666666668
The percent difference is 0.19383643099516984
```