CS 3320

Homework – Chapter 6

Roots of Equations – Open Methods

1. Use simple, fixed-point iteration to find a zero of the equation $x - \cos x = 0$. Use a calculator. Make sure your calculator is in *radian* mode, not degree mode. Describe the steps you used to find the root. Explain why your procedure converged to a solution. (10 pts.)

   After rewriting as $x_{i+1} = \cos x_i$, notice that no matter what value is used for x_0, the value of x_1 will be in the range [-1,1]. This means that x_2 and all subsequent values will be strictly *within* the range (-1,1), which means that $|g'(x)| = |-\sin x| < 1$ for all values after x_0, hence the process converges. With this in mind, using any starting value, you just keep pressing the cosine button on your calculator to converge to .7390851332.

2. Use Newton's method to solve the following:

   a. Form an equation whose root will yield the *square root* of the number a. Write the iteration formula to solve this equation using Newton's method. Use the formula with a calculator or a Python program to find the square root of 3. Report how many iterations the process took. (10 pts.)

   $$x^2 - a = 0 \Rightarrow x_{i+1} = x_1 - \frac{x_i^2 - a}{2x_1} = \frac{1}{2}\left(x_i + \frac{a}{x_i}\right)$$

   $x_0 = 1$

   $x_1 = 2$

   $x_2 = 1.75$

   $x_3 = 1.732142$

   $x_4 = 1.7320508$

   $x_5 = x_4$

   b. Repeat the previous part to find the *cube root* of 3. (10 pts.)

   $$x^3 - a = 0 \Rightarrow x_{i+1} = x_1 - \frac{x_i^3 - a}{3x_i^2} = \frac{1}{3}\left(2x_i + \frac{a}{x_i^2}\right)$$

   $x_0 = 1$

   $x_1 = \frac{5}{3}$

   $x_2 = 1.47111111$

   $x_3 = 1.442812$

   $x_4 = 1.44224979$

   $x_5 = 1.44224957$

   $x_6 = x_5$

3. When solving the equation $x^2 - 3x + 2 = 0$ by simple, fixed-point iteration, you can rearrange the evaluation as x = g(x) in different ways. First, solve for x = g(x) by isolating the middle term.

Second, solve for x = g(x) by adding x to both sides of the original equation. For each case:

a. In what interval can you choose an initial guess for the iteration that will guarantee that the iteration will converge to a root? (10 pts.)

If you rewrite the equation as $x = x^2 - 2x + 2$, then $g'(x) = 2x - 2$, which is less than 1 in the open interval $\left(\frac{1}{2}, \frac{3}{2}\right)$, which will converge to the root $x = 1$.

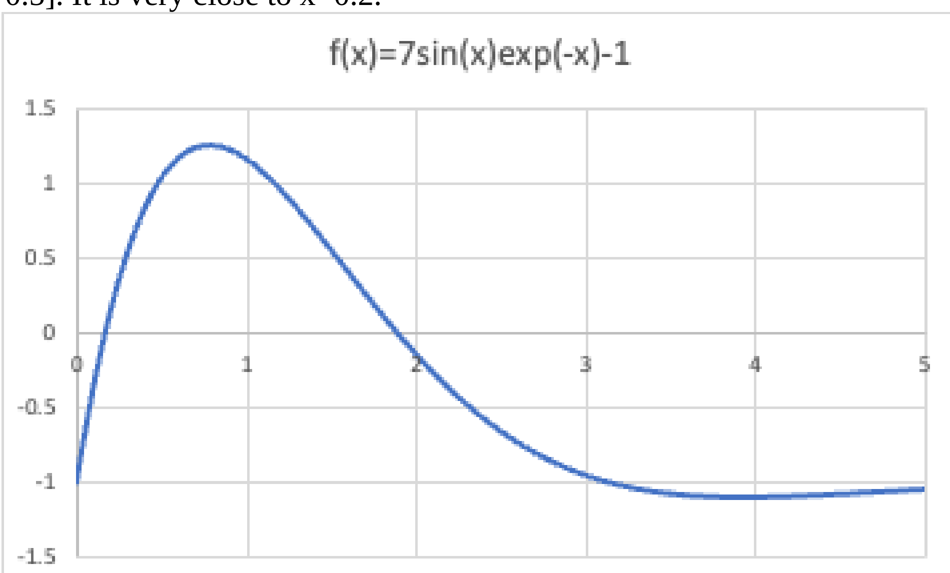
If you rewrite the equation as $x = \frac{x^2 + 2}{3}$, then $g'(x) = \frac{2}{3}x$, which is less than 1 on $\left(-\frac{3}{2}, \frac{3}{2}\right)$.

b. What is the order of convergence near the root where your formula converges in each case? (10 pts.)

If you choose $x = \sqrt{3x - 2}$, then $g'(x) = \frac{3}{2\sqrt{3x-2}}$. This will be less than 1 when $x > \frac{17}{12}$, and will converge linearly to the root 2, because $g'(2) = \frac{3}{4} \neq 0$.

4. Problem 6.4 parts (a) and (d) (10 pts.)

a. The following graph was done in Excel. Based on the plot the smallest positive root lies in [0, 0.5]. It is very close to x=0.2.



b. See table below:

Secant Iteration

iteration #	xi	δxi	xi+δxi	f(xi)	f(xi+δxi)	x(i+1)
1	0.3	0.0003	0.30030	0.53249	0.53351	0.14431
2	0.14431	0.00014431	0.14445	-0.12862	-0.12788	0.16941
3	0.16941	0.00016941	0.16958	-0.00372	-0.00290	0.17018
4	0.17018	0.00017018	0.17035	0.00000	0.00082	0.17018