

Assignment: Homework Four Name: Cody Strange

Disclaimer: This is my work, not that of others

Total Score: 30 (in points, not percentage)

Problem 1 score: 5

Problem 2 score: 5


Problem 3 score: 5

Problem 4 score: 5

Problem 5 score: 5

Problem 6 score: 5

1a.


$$\begin{aligned}f(x) &= \cos(5x) \\f'(x) &= -5\sin(5x) \\f''(x) &= -25\cos(5x) \\f(0) &= \cos(0) = 1 \\f'(0) &= -5\sin(0) = 0 \\f''(0) &= -25\cos(0) = -25\end{aligned}$$

$$\begin{aligned}f(x) &= f(0) + f'(0)x + \frac{1}{2!}f''(0)x^2 \\&= 1 + 0 \cdot x + \frac{-25x^2}{2 \cdot 1} \\&= 1 - \frac{25x^2}{2}\end{aligned}$$

$$1 - (25x/2)x^2$$

1b.

$$f(x) = \frac{1}{x+1}$$

$$f'(x) = \frac{d}{dx} \left(\frac{1}{x} \right) = -\frac{1}{x^2} = -\frac{1}{(x+1)^2}$$

$$f''(x) = \frac{2}{(x+1)^3}$$

$$f(0) = 1$$

$$f'(0) = -1$$

$$f''(0) = 2$$

$$\begin{aligned} f(x) &= f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 \\ &= 1 + -1x + \frac{2}{2!}x^2 \\ &= 1 - x + x^2 \end{aligned}$$

$$1 - x + x^2$$

2a.

$$f(x) = x^{-2} \quad -2x^{-3}$$

$$f'(x) = \frac{d}{dx}(x^{-2}) = -2x^{-2-1} = -2x^{-3}$$

$$f''(x) = \frac{d}{dx}(-2x^{-3}) = -2 \cdot \frac{d}{dx}(x^{-3}) = -2 \cdot \frac{d}{dx}(x^{-3-1}) = -2 \cdot \frac{d}{dx}(x^{-4}) = -2 \cdot (-4x^{-4-1}) = 8x^{-5}$$

$$f'''(x) = \frac{d}{dx}(8x^{-5}) = 8 \cdot \frac{d}{dx}(x^{-5}) = 8 \cdot \frac{d}{dx}(x^{-5-1}) = 8 \cdot \frac{d}{dx}(x^{-6}) = 8 \cdot (-6x^{-6-1}) = -48x^{-7}$$

$$f^{(4)}(x) = \frac{d}{dx}(-48x^{-7}) = -48 \cdot \frac{d}{dx}(x^{-7}) = -48 \cdot \frac{d}{dx}(x^{-7-1}) = -48 \cdot \frac{d}{dx}(x^{-8}) = -48 \cdot (-8x^{-8-1}) = 384x^{-9}$$

$$f^{(5)}(x) = \frac{d}{dx}(384x^{-9}) = 384 \cdot \frac{d}{dx}(x^{-9}) = 384 \cdot \frac{d}{dx}(x^{-9-1}) = 384 \cdot \frac{d}{dx}(x^{-10}) = 384 \cdot (-10x^{-10-1}) = -3840x^{-11}$$

$$f(1) = 1$$

$$f'(1) = -2$$

$$f''(1) = 8$$

$$f'''(1) = -48$$

$$f^{(4)}(1) = 384$$

$$f^{(5)}(1) = -3840$$

$$\begin{aligned} f(x) &= f(1) + f'(1)x + \frac{f''(1)}{2!}x^2 + \frac{f'''(1)}{3!}x^3 + \frac{f^{(4)}(1)}{4!}x^4 \\ &= 1 + -2x + \frac{8}{2}x^2 + \frac{-48}{3!}x^3 + \frac{384}{4!}x^4 \\ &= 1 + -2x + 4x^2 + \frac{-48}{6}x^3 + \frac{384}{24}x^4 \\ &= 1 + -2x + 4x^2 + \frac{-48}{6}x^3 + \frac{384}{24}x^4 \\ &= 1 + -2(x-1) + 3(x-1)^2 + -4(x-1)^3 + 5(x-1)^4 \end{aligned}$$

$$1 - 2(x-1) + 3(x-1)^2 - 4(x-1)^3 + 5(x-1)^4$$

2b.

2b.

$$f(x) = 1 - 2(x-1) + 3(x-1)^2 - 4(x-1)^3 + 5(x-1)^4 - 6(x-1)^5$$

$$f(0.9) = 1 - 2(0.9-1) + 3(0.9-1)^2 - 4(0.9-1)^3 + 5(0.9-1)^4 - 6(0.9-1)^5$$

$$= 1 + 0.2 + 0.03 + 0.004 + 0.0005 + 0.00006$$

$$= 1.2 + 0.034 + 0.00056$$

$$= 1.23456$$

$$f(0.9) = 1.2345$$

$$f(1.1) = 1 - 2(1.1-1) + 3(1.1-1)^2 - 4(1.1-1)^3 + 5(1.1-1)^4 - 6(1.1-1)^5$$

$$= 1 - 0.2 + 0.03 - 0.004 + 0.0005 - 0.00006$$

$$= 0.8265$$

$$f(1.1) = 0.8265$$

F(0.9) = 1.2345

F(1.1) = 0.8265

2c.

CC

$$\begin{aligned} R_n(x) &= \frac{f^{(n+1)}(c)}{(n+1)!} (x-a)^{n+1} \\ &= \frac{f^5(c)}{5!} (x-1)^5 \\ &= \frac{f^5(c)}{5!} (x-1)^5 \end{aligned}$$

$$(f^5(c)/5!)(x-1)^5$$

2d.

$$\frac{f^{(5)}(x)}{5!} (x-1)^5$$
$$f^{(5)}(0.9) = \frac{f^{(5)}(0.9)}{5!} (0.9-1)^5$$
$$= \frac{-720(0.9)^{-7}}{5!} (0.9-1)^5$$

$$-12.5445 (-0.0001) = 0.000125445$$

$$f^{(5)}(1.1) = \frac{f^{(5)}(1.1)}{5!} (1.1-1)^5$$
$$= \frac{-720(1.1)^{-7}}{5!} (1.1-1)^5$$
$$= -6 \cdot (1.1-1)^5$$
$$= -6 \cdot 0.00001$$

$$0.00006$$

$$F(0.9) = 0.000125445$$

$$F(1.1) = 0.00006$$