CS 3320

Module 5 – Chapter 5

Roots of Equations – Bracketing Methods

Use only bisection or false position for the first two problems. (You may use the code in the book or slides as the starting point. You need to modify it for the stopping condition.) Use both methods at least once. In each case, obtain maximum accuracy (withing one ulp) from your platform. Explain your work.

1. Find the two real roots of the equation $x^4 - 3x^2 + 75x - 10000 = 0$. Both roots are in the range [-20, 20]. (You may want to use incremental search method or graph method to narrow down the intervals for both roots.)

   Graphing the function reveals roots in the intervals (-11,-10) and (9, 10).

   The output for the first interval from method of false position is
   [-10.260964380932977, -10.260964380932977] (0 ulp) and the root is -10.260964380932977
   (Note: The values in the brackets are the approximated roots from the last two iterations.)

   The output for the second interval with bisection method is:
   [9.886002700947888, 9.886002700947891] (1 ulp) and the root is 9.88600270094789

2. Find all roots of the equation $e^x - 3x = 0$. Explain how you set up your procedure to find the solutions. (Graph the function first to find out the where the initial intervals should be.)

   Graphing (carefully) yields two roots in the intervals [0,1] and [1,2]. Using bisection on the first gives
   [0.6190612867359448, 0.6190612867359451] (1 ulp) and root is 0.619061286735945

   and method of false position on [1,2] gives
   [1.512134551657842, 1.512134551657842] (0 ulp) and the root is 1.512134551657842
   (Note: The values in the brackets are the approximated roots from the last two iterations.)

3. You plan for buy a Tesla Model S vehicle for $79,990 by paying 10% down and financing the balance over a 7-year term. You have budgeted a monthly payment of $1,000. So, now you need to shop for a loan at the required interest rate (or lower). The formula governs the loan calculation is

$$A = P\frac{i(1+i)^n}{(1+i)^n - 1}$$

   where A = the monthly payment, P = the loan amount, i = the monthly interest rate in a fraction, not a percentage (you need to divide the APR by 12), and n = the length of the loan in month. Use APR of 3% and 9% as the initial guesses.

The 10% down payment is $7,999.00. The loan amount is 79990 – 7999 = 71991.

The equation we want to solve is

$$f(i) = 71991 * \frac{i(1+i)^{84}}{(1+i)^{84}-1} - 1000 \leq 0$$

where *i* is APR/12. Finding the root of

$$f(i) = 71991 * \frac{i(1+i)^{84}}{(1+i)^{84}-1} - 1000 = 0$$

will give us the maximum allowable interest rate.

Using method of false position, we have a solution of 0.003732910498226829 which is equivalent to a maximum APR of 4.4795%.