Section 6.3:

Inference for a Difference in Proportions

One Proportion or Two?

- Two proportions: there are two separate categorical variables
- One proportion: there is only one categorical variable

You want to compare the proportion of students who use a Windows-based PC to the proportion who use a Mac. This is:

A. Inference for one proportion
B. Inference for two proportions

You want to compare the proportion of students who study abroad between those attending public universities and those at private universities. This is:

A. Inference for one proportion
B. Inference for two proportions

You want to compare the proportion of in-state students at a university to the proportion from outside the state. This is:

A. Inference for one proportion
B. Inference for two proportions

You want to compare the proportion of in-state students who get financial aid to the proportion of out-of-state students who get financial aid. This is:

A. Inference for one proportion

B. Inference for two proportions

Standard Error for $\hat{p}_1 - \hat{p}_2$

$$\boxed{SE = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}}$$

CLT for $\widehat{p}_1 - \widehat{p}_2$

Parameter: $p_1 - p_2$

Statistic: $\hat{p}_1 - \hat{p}_2$, based on sample sizes n_1 and n_2

If counts within each category (each cell of the two-way table) are at least 10

$$\hat{p}_1 - \hat{p}_2 \approx N\left(p_1 - p_2, \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}} \right)$$

Metal Tags and Penguins

Are metal tags detrimental to penguins? A study looked at the 10 year survival rate of penguins tagged either with a metal tag or an electronic tag. 20% of the 167 metal tagged penguins survived, compared to 36% of the 189 electronic tagged penguins.

Give a 90% confidence interval for the difference in proportions.

Source: Saraux, et. al. (2011). "Reliability of flipper-banded penguins as indicators of climate change," *Nature*, **469**, 203-206.

Metal Tags and Penguins

$$33 \geq 10, 134 \geq 10, 68 \geq 10, 121 \geq 10$$

$$\hat{p}_1 - \hat{p}_2 \pm z^* \times \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

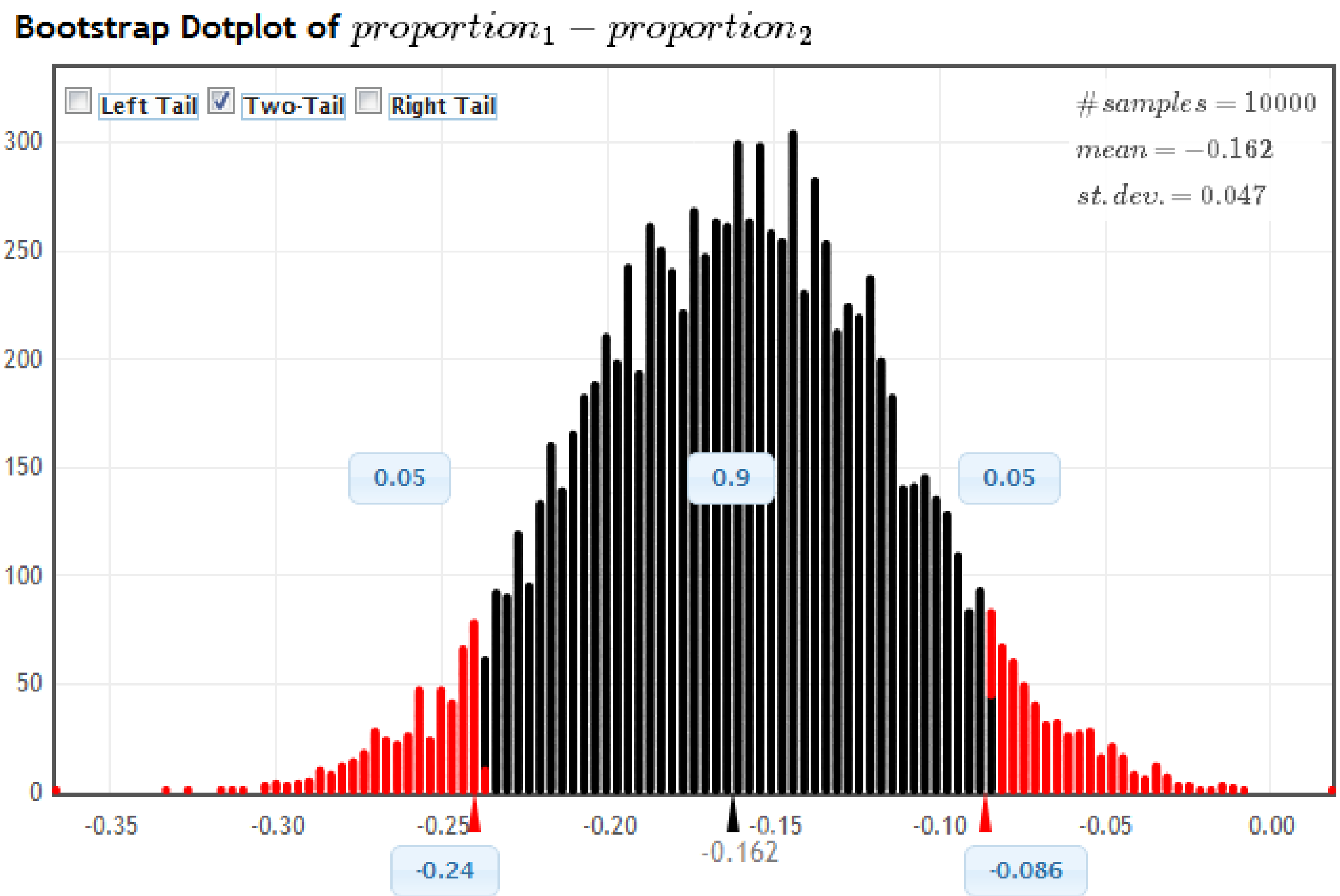
$$0.2 - 0.36 \pm 1.645 \times \sqrt{\frac{0.2(1-0.2)}{167} + \frac{0.36(1-0.36)}{189}}$$

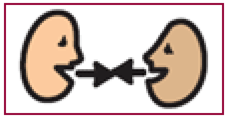
$$-0.16 \pm 1.645 \times 0.047$$

$$\boxed{(-0.237, -0.083)}$$

We are 90% confident that the survival rate is between 8.3% and 23.7% lower for metal tagged penguins, as opposed to electronically tagged.

Metal Tags and Penguins





Hypothesis Testing

$$SE = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

$$H_0 : p_1 = p_2$$

What should we use for p_1 and p_2 in the formula for SE for hypothesis testing?

Pooled Proportion

$$\boxed{\hat{p} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}}$$

Overall sample proportion across both groups. It will be in between the two observed sample proportions.

$$SE = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

$$\approx \sqrt{\frac{\hat{p}(1-\hat{p})}{n_1} + \frac{\hat{p}(1-\hat{p})}{n_2}} = \sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

Hypothesis Testing

$$z = \frac{\text{sample statistic} - \text{null value}}{\text{SE}}$$

$$z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

The p-value is the area in the tail(s) beyond *z* in a N(0,1)

Metal Tags and Penguins

20% of the 167 metal tagged penguins survived, compared to 36% of the 189 electronic tagged penguins.

Are metal tags detrimental to penguins?

$$H_0 : p_1 = p_2$$

$$H_a : p_1 < p_2$$

Metal Tags and Penguins

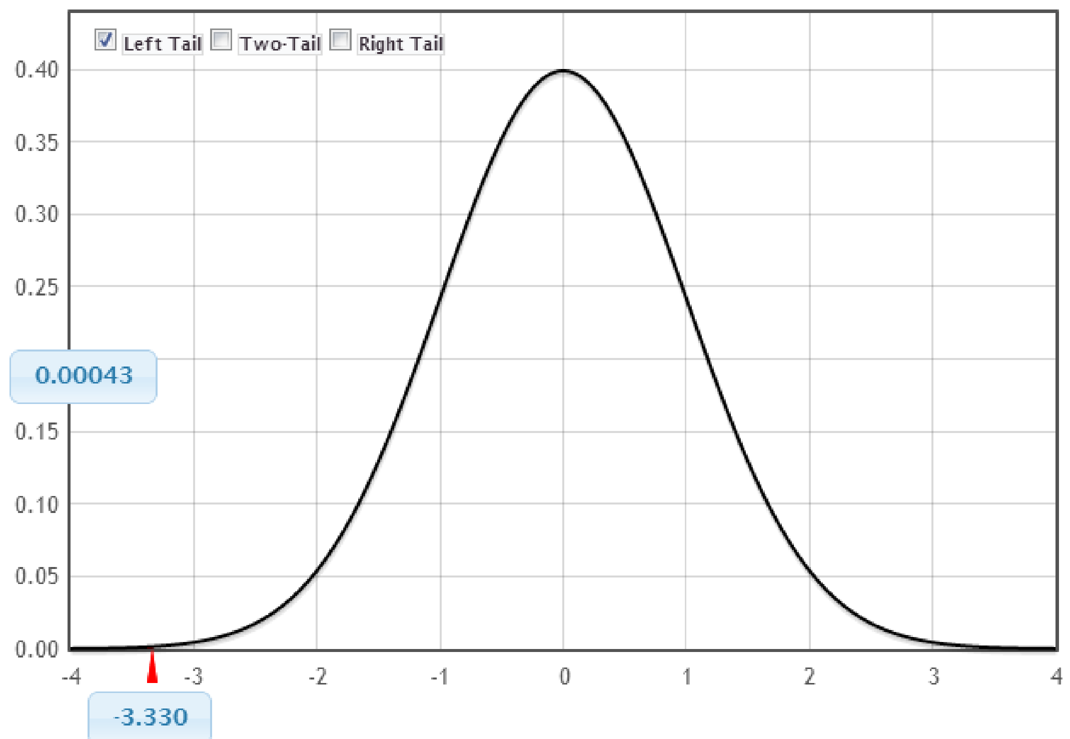
Are metal tags detrimental to penguins?

$$\boxed{z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}}$$

$$\hat{p} = \frac{33+68}{167+189} = 0.284$$

$$z = \frac{(0.2-0.36)-0}{\sqrt{0.284(1-0.284)\left(\frac{1}{167} + \frac{1}{189}\right)}} = \frac{-0.16}{0.048} = -3.33$$

Metal Tags and Penguins



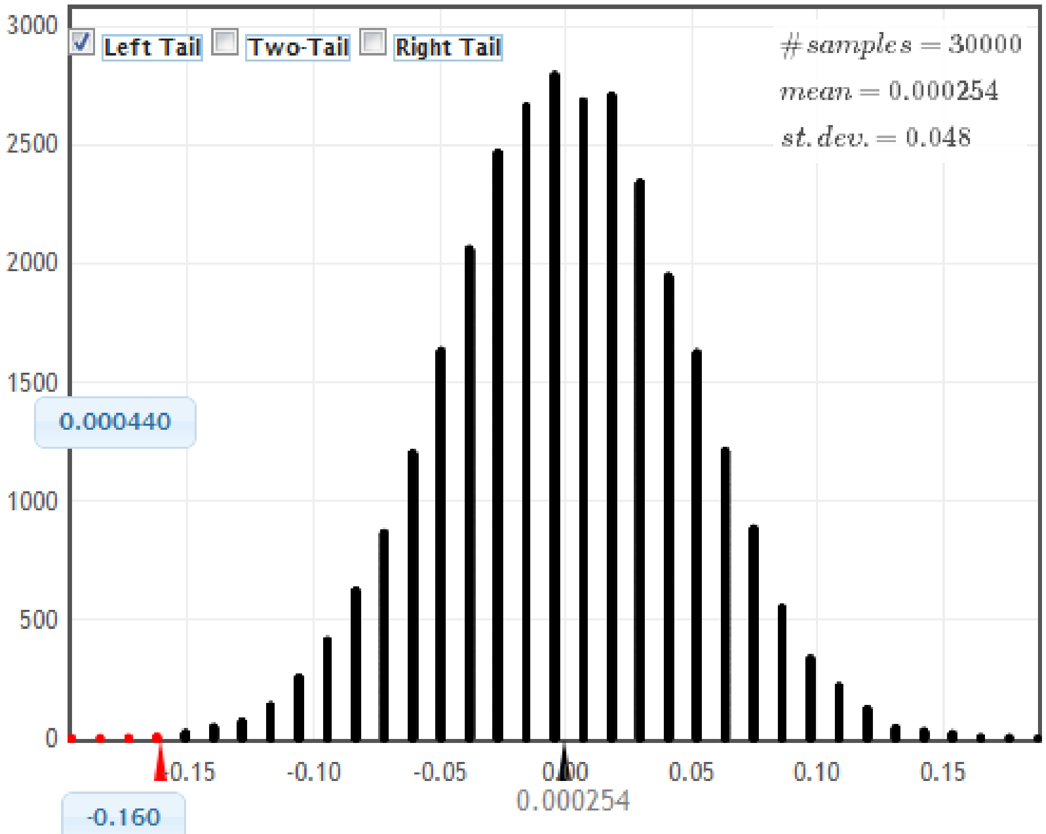
$$\boxed{p-value = 0.0004}$$

This is very strong evidence that metal tags are detrimental to penguins.

$$z = -3.33$$

Metal Tags and Penguins

Randomization Dotplot of $proportion_1 - proportion_2$ Null Hypothesis: $p1 = p2$



Original Sample

Group	Count	n	Proportion
Group 1	33	167	0.20
Group 2	68	189	0.36
Group 1-Group 2	-35	n/a	-0.16

Randomization Sample

Group	Count	n	Proportion
Group 1	41	167	0.25
Group 2	60	189	0.32
Group 1-Group 2	-19	n/a	-0.07

$$\boxed{p - value = 0.0004}$$

Accuracy

- The accuracy of intervals and p-values generated using simulation methods (bootstrapping and randomization) depends on the number of simulations (more simulations = more accurate)

- The accuracy of intervals and p-values generated using formulas and the normal distribution depends on the sample size (larger sample size = more accurate)

- If the distribution of the statistic is truly normal and you have generated many simulated randomizations, the p-values should be very close

In a randomized double-blind experiment, a sample of 150 patients were either given a treatment or a placebo. We want to test to see if the treatment is significantly better than a placebo at relieving pain. Using the data in the two-way table below, calculate the pooled proportion of patients that experienced pain relief.

$$\hat{p} = \frac{n_1 \hat{p}_1 + n_2 \hat{p}_2}{n_1 + n_2}$$

	Treatment	Placebo	Total
Relieved pain	36	21	57
Did not relieve pain	39	54	93
Total	75	75	150

A. 0.2

B. 0.212

C. 0.38

D. 0.5

In a randomized double-blind experiment, a sample of 150 patients were either given a treatment or a placebo. We want to test to see if the treatment is significantly better than a placebo at relieving pain. Calculate the test statistic. (Group 1 = Treatment, Group 2 = Placebo).

	Treatment	Placebo	Total
Relieved pain	36	21	57
Did not relieve pain	39	54	93
Total	75	75	150

$$z = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

A. 1.57

B. 1.65

C. 1.96

D. 2.52

In a randomized double-blind experiment, a sample of 150 patients were either given a treatment or a placebo. We want to test to see if the treatment is significantly better than a placebo at relieving pain. What is the p-value? (Group 1 = Treatment, Group 2 = Placebo). StatKey

	Treatment	Placebo	Total
Relieved pain	36	21	57
Did not relieve pain	39	54	93
Total	75	75	150

A. 0.0059

B. 0.059

C. 0.05

D. 0.842

Summary

- **Standard error** for a difference in proportions:

$$SE = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

- **Central Limit Theorem for a difference in proportions:** If counts within each category (each cell of the two-way table) are at least 10, then $\hat{p}_1 - \hat{p}_2 \approx N\left(p_1 - p_2, \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}\right)$.
  - **For a CI**, use $\hat{p}_1$ in place of p_1 *and* $\hat{p}_2$ in place of p_2:

$$\hat{p}_1 - \hat{p}_2 \pm z^* \times \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$$

  - **For a Hypothesis Test**, use the ***pooled proportion***, $\hat{p}$, in place of p_1 and p_2 when calculating the standardized statistic:

$$z = \frac{\hat{p}_1-\hat{p}_2-0}{\sqrt{\frac{\hat{p}(1-\hat{p})}{n_1}+\frac{\hat{p}(1-\hat{p})}{n_2}}} = \frac{\hat{p}_1-\hat{p}_2-0}{\sqrt{\hat{p}(1-\hat{p})\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}, \text{ where } \hat{p} = \frac{n_1\hat{p}_1+n_2\hat{p}_2}{n_1+n_2}$$