

Certainly! Here is the step-by-step process for constructing Newton's Interpolation Polynomials using divided differences:

Step 1: Gather the data points

First, you need to have a set of data points that you want to interpolate. Let's suppose you have the following set of data points:

...

x	y
---	---

1	3
---	---

2	5
---	---

4	7
---	---

5	8
---	---

...

Step 2: Calculate the divided differences

Next, you need to calculate the divided differences of the data points. The divided differences are defined as:

...

$$f[x_i] = y_i$$
$$f[x_i, x_{i+1}, \dots, x_j] = (f[x_{i+1}, x_{i+2}, \dots, x_j] - f[x_i, x_{i+1}, \dots, x_{j-1}]) / (x_j - x_i)$$

...

You start by calculating the first-order divided differences:

...

$$f[1,2] = (5 - 3) / (2 - 1) = 2$$

$$f[2,4] = (7 - 5) / (4 - 2) = 1$$

$$f[4,5] = (8 - 7) / (5 - 4) = 1$$

...

Then, you calculate the second-order divided differences:

...

$$f[1,2,4] = (1 - 2) / (4 - 1) = -1/3$$

$$f[2,4,5] = (1 - 1) / (5 - 2) = 0$$

...

Finally, you calculate the third-order divided difference:

...

$$f[1,2,4,5] = (0 - (-1/3)) / (5 - 1) = -1/36$$

...

Step 3: Write the interpolation polynomial

The interpolation polynomial is given by:

...

$$P_n(x) = f[x_0] + f[x_0, x_1](x - x_0) + f[x_0, x_1, x_2](x - x_0)(x - x_1) + \dots + f[x_0, x_1, \dots, x_n](x - x_0)(x - x_1) \dots (x - x_{n-1})$$

...

In our example, the interpolation polynomial is:

...

$$P_3(x) = 3 + 2(x - 1) - \frac{1}{3}(x - 1)(x - 2) - \frac{1}{36}(x - 1)(x - 2)(x - 4)$$

...

Step 4: Evaluate the interpolation polynomial

Finally, you can use the interpolation polynomial to estimate the value of the function at any point in the interval [1,5]. For example, if you want to estimate the value of the function at $x = 3$, you can evaluate the interpolation polynomial as follows:

...

$$P_3(3) = 3 + 2(3 - 1) - \frac{1}{3}(3 - 1)(3 - 2) - \frac{1}{36}(3 - 1)(3 - 2)(3 - 4) = 6$$

...

Therefore, the estimated value of the function at $x = 3$ is 6.