

Sure, here are the step-by-step instructions for finding LU factorization of a matrix with an example:

Step 1: Write down the matrix

Let's take the following matrix as an example:

...

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 20 & -7 & 12 \\ -8 & 13 & 17 \end{bmatrix}$$

...

Step 2: Decompose the matrix into an upper and lower triangular matrix

To do this, we first find the lower triangular matrix `L` and the upper triangular matrix `U`. We start by assuming that the diagonal elements of `L` are all equal to 1. Then, we apply Gaussian elimination to `A` to get `U`.

...

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 20 & -7 & 12 \\ -8 & 13 & 17 \end{bmatrix}$$

Step 3: Perform row operations on `A` to get `U`

We can use row operations to transform `A` into `U` by applying elementary matrices. The goal is to eliminate the entries below the diagonal of `A`. The elementary matrices are stored in `L`. We can start by subtracting 5 times the first row from the second row, and adding 2 times the first row to the third row:

...

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 20 & -7 & 12 \\ -8 & 13 & 17 \end{bmatrix}$$

$$L1 = \begin{bmatrix} 1 & 0 & 0 \\ -5 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$$

$$A1 = L1 @ A$$

$$A1 = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 3 & 7 \\ 0 & 9 & 19 \end{bmatrix}$$

Step 4: Continue with row operations until `A` is transformed to `U`

We can continue the process by subtracting 3 times the second row from the third row, and applying the corresponding elementary matrix `L2` to `A1`:

...

$$A1 = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 3 & 7 \\ 0 & 9 & 19 \end{bmatrix}$$

$$L2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -3 & 1 \end{bmatrix}$$

$$U = L2 @ A1$$

$$U = \begin{bmatrix} 4 & -2 & 1 \\ 0 & 3 & 7 \\ 0 & 0 & -4 \end{bmatrix}$$

Step 5: Write down the final `L` matrix

Now that we have `U`, we can write down `L` as the product of the elementary matrices `L1` and `L2`:

...

$$L = L2 @ L1$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ -2 & 3 & 1 \end{bmatrix}$$

...

Step 6: Check the result

To verify that we have correctly found the LU factorization, we can multiply `L` and `U` together and check that we get the original matrix `A`:

...

$$A = \begin{bmatrix} 4 & -2 & 1 \\ 20 & -7 & 12 \\ -8 & 13 & 17 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 5 & 1 & 0 \\ -2 & 3 & 1 \end{bmatrix}$$

$[5, 1, 0],$

$[-2, 3, 1]]$

$U = [[4, -2, 1],$

$[0, 3, 7],$

$[0, 0, -4]]$

$LU = L @ U$

$LU = [[4$