

The following is a step-by-step process for solving cubic spline interpolation:

1. Given a set of data points (x_i, y_i) , where $i = 0, 1, \dots, n$, the first step is to calculate the values of the second derivatives of the interpolating function at each data point, denoted by y''_i .
2. Next, divide the domain into n subintervals, each of which corresponds to a pair of adjacent data points. Let $[x_i, x_{i+1}]$ denote the i th subinterval.
3. Then, construct a cubic polynomial function $S_i(x)$ for each subinterval $[x_i, x_{i+1}]$, using the following formula:

$$S_i(x) = a_i + b_i(x - x_i) + c_i(x - x_i)^2 + d_i(x - x_i)^3$$

where $a_i = y_i$, $b_i = (y_{i+1} - y_i)/(x_{i+1} - x_i) - (2*y''_i + y''_{i+1})*(x_{i+1} - x_i)/6$, $c_i = y''_i/2$, and $d_i = (y''_{i+1} - y''_i)/(6*(x_{i+1} - x_i))$.

4. To obtain the values of the coefficients a_i , b_i , c_i , and d_i , we need to solve a system of linear equations. This can be done using various methods such as Gaussian elimination, LU decomposition, or tridiagonal matrix algorithm.

5. Once the coefficients have been calculated for all subintervals, the cubic spline function can be expressed as:

$$S(x) = S_i(x) \text{ for } x_i \leq x \leq x_{i+1}$$

6. Finally, to evaluate the value of the cubic spline function at a particular point x , we need to determine which subinterval $[x_i, x_{i+1}]$ x belongs to, and then evaluate $S_i(x)$ at that subinterval using the coefficients obtained in step 4.

That concludes the procedure for solving cubic spline interpolation.