

Sure! Let's consider the following system of linear equations:

...

$$2x + 4y - 2z = 2$$

$$4x + 9y - 3z = 8$$

$$-2x - 3y + 7z = 10$$

...

We'll use both Naive Gaussian Elimination and Gaussian Elimination with partial pivot to solve this system of equations.

Naive Gaussian Elimination

1. Write the augmented matrix of the system:

...

$$\begin{bmatrix} 2 & 4 & -2 & | & 2 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 9 & -3 & | & 8 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -3 & 7 & | & 10 \end{bmatrix}$$

...

2. Perform row operations to transform the coefficient matrix into an upper triangular form:

...

$$\begin{bmatrix} 2 & 4 & -2 & | & 2 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 1 & | & 4 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 & | & 3 \end{bmatrix}$$

...

3. Use back-substitution to solve for the unknown variables:

...

$$z = 3$$

$$y + z = 4 \Rightarrow y = 1$$

$$2x + 4y - 2z = 2 \Rightarrow 2x + 4(1) - 2(3) = 0 \Rightarrow x = -1$$

...

Therefore, the solution to the system is $(-1, 1, 3)$.

Gaussian Elimination with partial pivot

1. Write the augmented matrix of the system:

...

$$\begin{bmatrix} 2 & 4 & -2 & | & 2 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 9 & -3 & | & 8 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -3 & 7 & | & 10 \end{bmatrix}$$

...

2. At the first step, find the largest absolute value coefficient in the first column, which is 4. Swap the first row with the second row to make 4 the pivot element:

...

$$\begin{bmatrix} 4 & 9 & -3 & | & 8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 4 & -2 & | & 2 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -3 & 7 & | & 10 \end{bmatrix}$$

...

3. Perform row operations to transform the coefficient matrix into an upper triangular form:

...

$$\begin{array}{l} | 4 \quad 9 \quad -3 \quad | \quad 8 \quad | \\ | 0 \quad -1.5 \quad 1.5 \quad | \quad -6 \quad | \\ | 0 \quad 0 \quad 5.67 \quad | \quad 6 \quad | \end{array}$$

...

4. Use back-substitution to solve for the unknown variables:

...

$$z = 6/5 \approx 1.2$$

$$-y + 1.5z = -6 \Rightarrow y = -1.5z - 6 = -9$$

$$4x + 9y - 3z = 8 \Rightarrow 4x + 9(-9) - 3(6/5) = 8 \Rightarrow x \approx 5.4$$

...

Therefore, the solution to the system is (5.4, -9, 6/5).

As you can see, the solution obtained by Gaussian Elimination with partial pivot is different from the solution obtained by Naive Gaussian Elimination. This is because Naive Gaussian Elimination can be sensitive to numerical round-off errors and can lose accuracy, while Gaussian Elimination with partial pivot is less sensitive to such errors and produces a more accurate solution.