CS 3320 – Numerical Software

Module 12 Homework

1. Evaluate the following integral.

$$\int_0^4 (1 - e^{-x})dx$$

   a. (5 pts) Analytically.
   b. (5 pts) Single application of the trapezoidal rule.
   c. (5 pts) Composite trapezoidal rule with $n = 2$ and 4.
   d. (5 pts) Single application of Simpson's 1/3 rule.
   e. (5 pts) Composite Simpson's 1/3 rule with $n = 4$.

   For each of the numerical estimates (b) through (e), determine the true percent relative error based on (a).

2. Determine the distance traveled from the following velocity data:

t	1	2	3.25	4.5	6	7	8	8.5	9	10
v(t)	5	6	5.5	7	8.5	8	6	7	7	5

   a. (10 pts) Use the trapezoidal rule. In addition, determine the average velocity.
   b. (10 pts) Fit the data with a cubic equation using polynomial regression. Integrate the polynomial to determine the distance.

   (Hint: Distance traveled between t_i and t_f is $\int_{t_i}^{t_f} v(t)dt$.)

3. (10 pt) The total mass of a variable density rod of variable cross-section is given by:

$$m = \int_0^L \rho(x)A(x)dx$$

   where m = mass, ρ = density, A = cross-sectional area, x= the distance along the rod, and L = the total length of the rod. The following data have been measured for a 20-m rod. Determine the mass of the rod in grams using the Python functions `trapezoid` and `simpson` in `scipy.integrate`. What is the percentage difference between the two results? (Use the result from `simpson` as your base. Report your mass in kg.)

x, m	0	4	6	8	12	16	20
$\boldsymbol{\rho, g/cm^3}$	4.00	3.95	3.89	3.80	3.60	3.41	3.30
$\boldsymbol{A, cm^2}$	100	103	106	110	120	133	150