2.1.2(b): $4n + 3$

1. $4n + 3 = 2k + 1$
2. $4n + 2 = 2k$
3. $2n + 1 = k$
4. $k = 2n + 1$
5. $4n + 3 = 2(2n + 1) + 1$
6. $4n + 3 = 4n + 2 + 1$
7. $4n + 3 = 4n + 3$
8. Since $2k+ 1$ = odd and $4n + 3 = 2k + 1$; **Therefore $4n + 3$ = odd**

2.1.2(c): $10n^3 + 8n – 4$

1. $10n^3 + 8n – 4 = 2k$
2. $k = 5n^3 +4n - 2$
3. $10n^3 + 8n – 4 = 2(5n^3 +4n - 2)$
4. $10n^3 + 8n – 4 = 10n^3 + 8n – 4$
5. Since $2k$ = even and $10n^3 + 8n – 4 = 2k$; **Therefore $10n^3 + 8n – 4$ = even**

2.1.6(a): It is not true that $x < 7$

1. **$x \geq 7$**

2.1.6(d): it is not true that $x \geq 7$

1. **$x < 7$**
2. **sdfsdfsdf**
3. **sdfsdf**

2.2.2(b): For every integer n such that $0 \leq n \leq 4$, $2^{(n+2)} > 3^n$

1. $n = 0$, $2^{(n + 2)} > 3^n$
    a. $2^{(0 + 2)} > 3^0$
    b. $2^{(2)} > 1$
    c. $4 > 1$
    d. true
2. $n = 1$, $2^{(n + 2)} > 3^n$
    a. $2^{(1 + 2)} > 3^1$
    b. $2^{(3)} > 3$
    c. $8 > 3$
    d. true
3. $n = 2$, $2^{(n + 2)} > 3^n$
    a. $2^{(2 + 2)} > 3^2$
    b. $2^{(4)} > 9$

   c. $16 > 9$
   d. true
4. $n = 3$, $2^{(n+2)} > 3^n$
   a. $2^{(3+2)} > 3^3$
   b. $2^{(5)} > 27$
   c. $32 > 27$
5. $n = 4$, $2^{(n+2)} > 3^n$
   a. $2^{(4+2)} > 3^4$
   b. $2^{(6)} > 81$
   c. $64 > 81$
   d. false
6. For every integer n such that $0 \leq n \leq 4$, $2^{(n+2)} > 3^n$
   a. **False**

2.2.3(c): For every positive integer x, $x^3 < 2^x$

1. $x = 1$
2. $1^3 < 2^1$
3. $1 < 2$
4. counter example, $x = 1$

2.2.4(b): There is no largest integer

Let x be an integer, x will always be smaller than x + 1, Therefore x cannot be the largest integer

2.2.5(e) There are three positive integers, x, y, and z, that satisfy $x^2 + y^2 = z^2$

1. $\exists x \exists y \exists z(x^2 + y^2 = z^2)$
2. $x = 1$, $y = 1$, $z = 2$
3. $1^2 + 1^2 = 2^2$
4. $1 + 1 = 2$
5. $2 = 2$

2.3.2(a)

Proof.

Let w, x, y, z be integers such that w divides x and y divides z.
Since, by assumption, w divides x, then x = kw for some integer k and w ≠ 0. Since, by assumption, y divides z, then z = ky for some integer k and y ≠ 0. Plug in the expression kw for x and ky for z in the expression xz to get xz=(kw)(ky)=(k2)(wy)
Since k is an integer, then k^2 is also an integer.
Since w ≠ 0 and y ≠ 0, then wy ≠ 0.
Since xz equals wy times an integer and wy ≠ 0, then wy divides xz. ■

The calculation xz=(kw)(ky)=(k2)(wy) is incorrect. xz=(kw)(ky) = k^2wy

2.3.3(a) **Theorem:** If n and m are odd integers, then $n^2 + m^2$ is even

Proof.
m = 7 is odd because 7 = 2·3+1. n = 9 is odd because 9 = 2·4 + 1.

72+92=49+81=130=2·65

Since $7^2 + 9^2$ is equal to 2 times an integer, $7^2 + 9^2$ is even. Therefore the theorem is true. ■
The proof is generalizing from examples, just because the theorem holds true for one set of integers does not mean that it holds true for all sets of integers

2.4.1(c)
Theorem: The square of an odd integer is an odd integer.

Proof:

Suppose x is an odd integer

As x is odd, ∃k so that x = 2k + 1

Since x = 2k + 1 then $x^2 = (2k + 1)^2$

$(2k + 1)(2k + 1) = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$

Let j be some integer such that $j = 2k^2 + 2k$

Thus $x^2 = 2j + 1$

Therefore x^2 is odd ▪

2.4.3(b)

Theorem: If x is a real number and $x \leq 3$, then $12 - 7x + x^2 \geq 0$.

Proof:

Suppose x is a real number and $x \leq 3$

If we factor $12 - 7x + x^2 \geq 0$ we get $(3 - x)(4 - x) \geq 0$

Since x is a real number, any number added/subtracted from x is also a real number

Therefore (4-x) is a real number, 0 divided by any real number is 0

Therefore $3 - x \geq 0$

Using algebra, $3 \geq x$ or $x \leq 3$

Therefore $12 - 7x + x^2 \geq 0$ ▪

2.4.4(e)

Theorem: If x and y are positive real numbers and x<y, then x2<y2.

Proof:

Suppose x and y are positive real numbers

Since positive numbers are not negative numbers

Thus $x \geq 0$ and $y \geq 0$

Since $x < y$ are positive real numbers you can multiply both sides by 2

Therefore $x2 < y2$ ▪

2.5.1(d)

Theorem: For every integer n, if $n^2 - 2n+7$ is even, then n is odd

Proof:

if n is even, then $n^2 - 2n + 7$ is odd

The integer n is even, by definition there is some integer k that $n = 2k$

Therefore $(2k)^2 - 2(2k) + 7$, simplified this equals, $4k^2 - 4k + 7$

Since $n^2 - 2n + 7$ is odd, therefore $4k^2 - 4k + 7$ is also odd, by definition there is some integer j that $4k^2 - 4k + 7 = 2j + 1$

$j = 2k^2 - 2k + 3$

Therefore $4k^2 - 4k + 7 = 2(2k^2 - 2k + 3) + 1$

Since $4k^2 - 4k + 7 = 4k^2 - 4k + 7$ then we can conclude that if n is even then $n^2 - 2n + 7$ is odd ▪

2.5.4(c)

Theorem: For every pair of positive real numbers x and y, if xy>400, then x>20 or y>20
Proof:
For every pair of positive real numbers x and y, if $x < 20$ and $y < 20$, then $xy < 400$
if $x < 20$ and $y < 20$ then by assumption $x * y < 20 * 20$
Therefore $xy < 400$ ▪

2.6.1(a)
Theorem: $\sqrt{2}/2$ is irrational.
Proof:
Suppose $\sqrt{2}/2$ is rational
By definition, a number is rational if there exists integers x an y such that $y \neq 0$ and $r = x/y$
Let $r = \sqrt{2}/2$, therefore $\sqrt{2}/2 = x/y$
Square both sides, $2/4 = x^2/y^2$
multiply both sides by y^2, $y^2 2/4 = x^2$
multiply both sides by 4, $2y^2 = 4x^2$ ▪

Since x is equal to two times another number then by assumption, it is even, by definition $x = 2k$
Since y is equal to an even number, then it is also an even number, by definition $y = 2j$
Therefore $2(2j)^2 = 4(2k)^2$, simplified, $8j^2 = 16k^2$
By assumption, using algebra, $2/4 = k^2/j^2$, then take the square root of both sides
By assumption, using algebra $\sqrt{2}/2 = k/j$, this is back where proof started
$\sqrt{2}/2$ creates an infinite loops, therefore it is not rational ▪

2.6.4(d)
Theorem: Among any group of 1000 people, at least three of the people have the same birthday.
Proof:
Suppose in a group of 1000 no more than two people two have the same birthday
By assumption, a year has 365 days
Therefore after 745 days for three people to not share a birthday two people would have to share a birthday on each day of the year.
Thus on the 746-day three people would have to share a birthday at a minimum
Since 1000 > 746 then at least three people would have to have same birthday ▪

2.7.1(b)
Theorem: for every integer n, $n^2 \geq n$
Case 1: n is 1 or 0
Since n is 1, then $1^2 \geq 1$
Since n is 0, then $0^2 \geq 0$

Case 2: n is not 1 or 0
For every integer that is not 1 or 0, by definition, multiplication, if an integer is multiplied by an integer of the same sign, the integer will get larger.
Therefore $n^2 \geq n$ ▪

2.7.3(c)
Theorem:
Proof:
For all real numbers x and y, $|x-y|=|y-x|$.
By assumption, when you subtract two numbers the order in which you subtract the two numbers only affects the sign that will be attached to said number
Therefore the absolute value of two numbers will always be the same no matter which order they are subtracted in
In other words $|x - y| = |y - x|$▪