Section P.1

Probability Rules

Outline

- Probabilities for equally likely outcomes
- Complement rule: P(not A)
- Additive rule: P(A or B)
- Conditional probability: P(B if A)
- Multiplicative rule: P(A and B)
- Special cases:
    - Independent
    - Mutually exclusive

Event

- An ***event*** is something that either happens or doesn’t happen, or something that either is true or is not true

- Examples:
    - A randomly selected card is a Heart
    - The response variable $Y > 90$
    - A randomly selected person is male
    - It rains today

Probability

The ***probability*** of event A, **P(A)**, is the long run frequency or proportion of times the event occurs.

- Probability is always between 0 and 1
- Probability always refers to an event
- P(A) = 1 means A will definitely happen
- P(A) = 0 means A will definitely not happen

Probability Examples

- Y = number of siblings

  $P(Y = 1) = 0.481$

  (based on survey data)

- Y= final grade in STAT 101

  $P(Y > 90) = 0.338$

  (based on previous classes)

- For U.S, college students in 2010

  P(Gender = female) = 0.585

  (World DataBank)

- P(it rains today) = 0.3 (www.weather.com)

Equally Likely Outcomes

If all possible outcomes are equally likely, the probability of an event is

$$\mathrm{P}(A) = \frac{Number\ of\ outcomes\ in\ A}{Total\ number\ of\ outcomes}$$

Example: Draw a card from a standard 52 card deck

$$P(Jack) = \frac{4\ jacks}{52\ cards} = \frac{1}{13}$$

Sexual Orientation

- What are the sexual orientation demographics of American adults?
- We need data!
- Data collected in 2009 on a random sample of American adults (National Survey of Sexual Health and Behavior)

Sexual Orientation Data

	Male	Female	Total
Heterosexual	2325	2348	4673
Homosexual	105	23	128
Bisexual	66	92	158
Other	25	58	83
Total	2521	2521	5042

Herbenick D, Reece M, Schick V, Sanders SA, Dodge B, and Fortenberry JD (2010). *Sexual behavior in the United States: Results from a national probability sample of men and women ages 14–94*. Journal of Sexual Medicine;7(suppl 5):255–265.

Sexual Orientation Probability

	Male	Female	Total
Heterosexual	2325	2348	4673
Homosexual	105	23	128
Bisexual	66	92	158
Other	25	58	83
Total	2521	2521	5042

What is the probability that an American adult is homosexual?

a) 128/5042 = 0.025

b) 128/4673 = 0.027

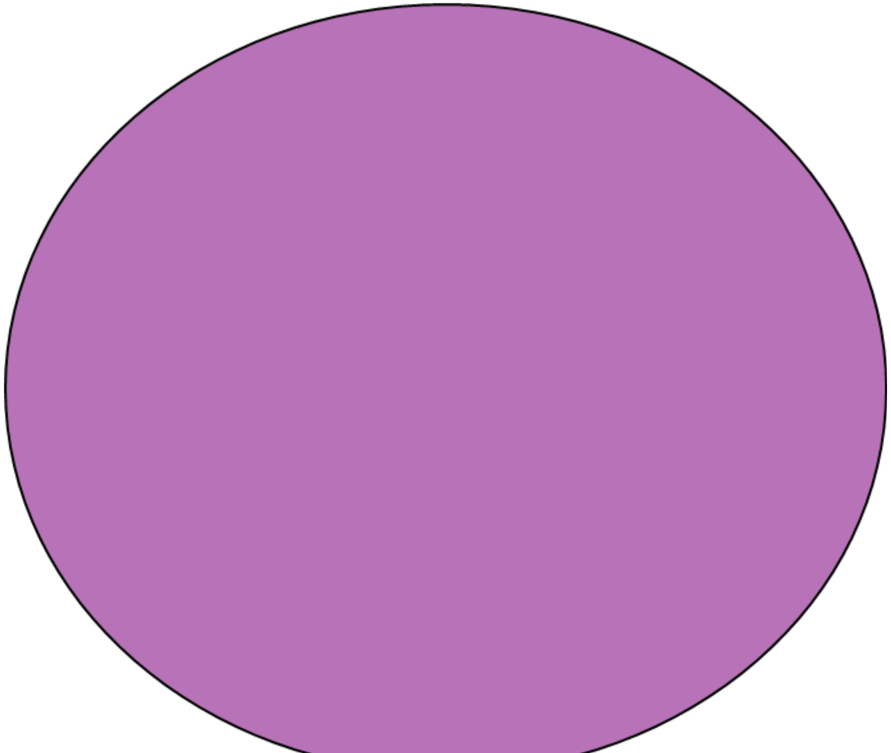
c) 105/2521 = 0.04

d) I got a different answer

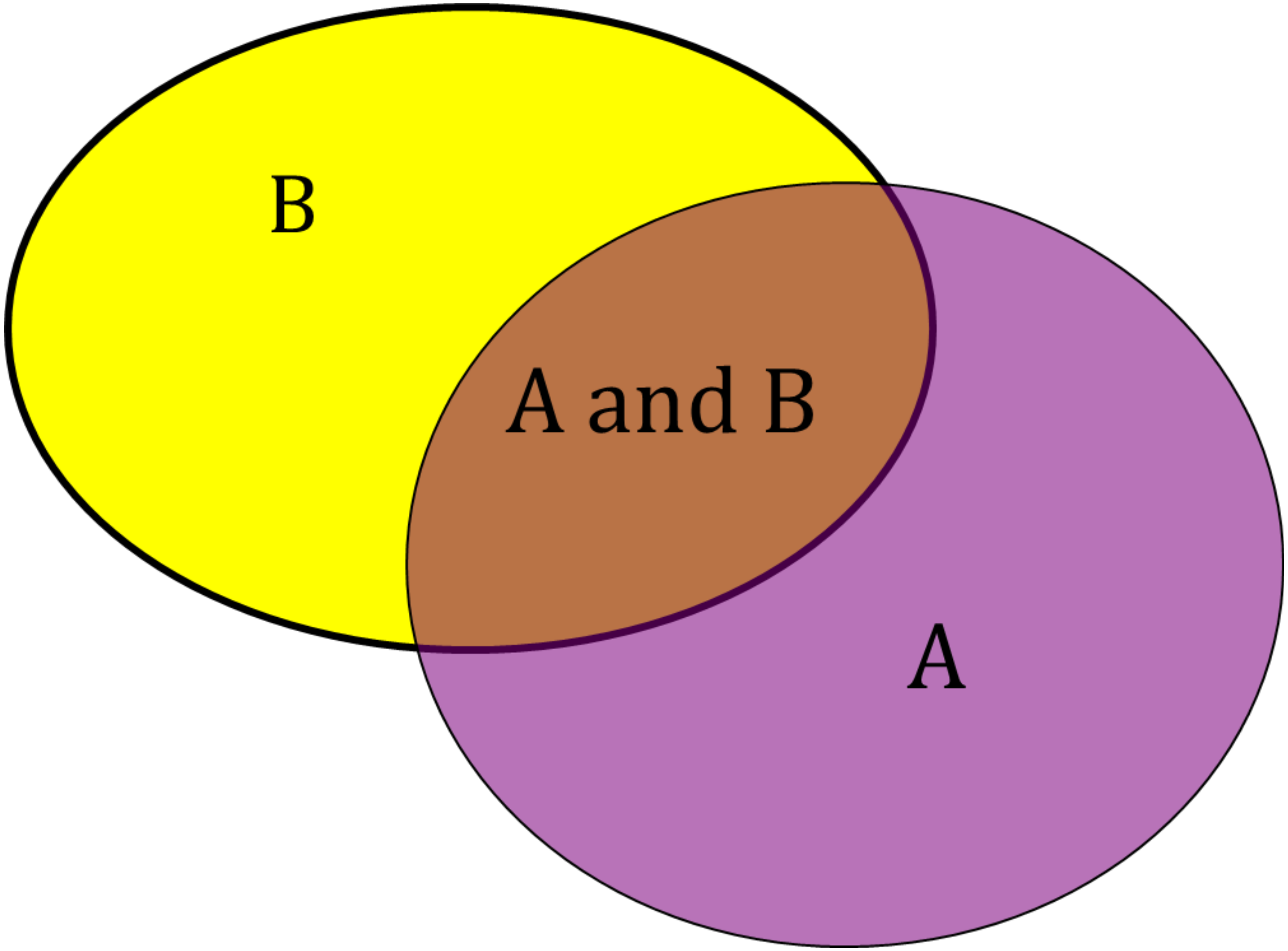
Combining Events

- ***P(A and B)*** is the probability that both events A *and* B will happen
- ***P(A or B)*** is the probability that either event A *or* event B will happen (or both)

Combining Events: Start With One Event

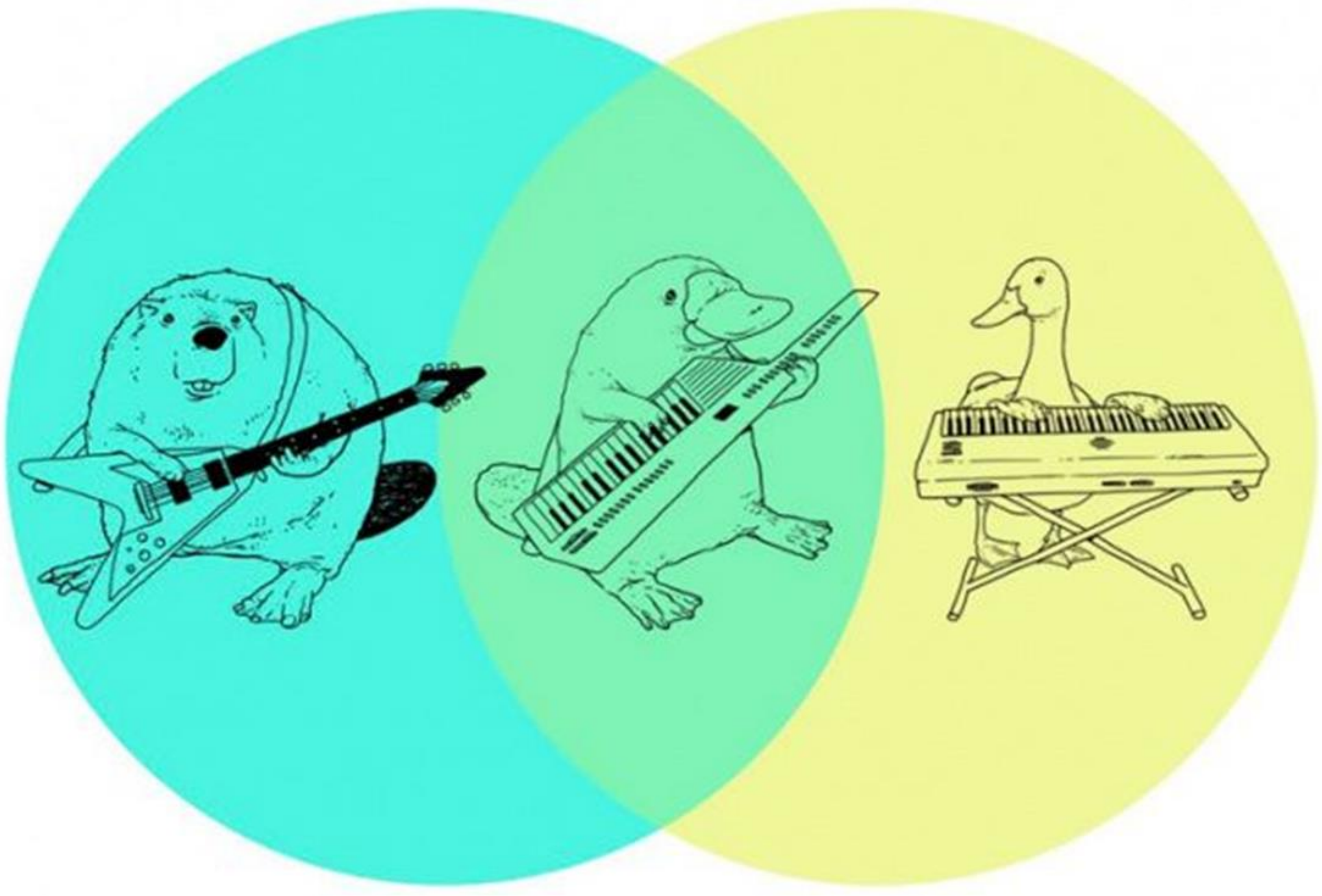


Combining Events: Add Another Event



A or B:
all color

Venn Diagram



Sexual Orientation: And Probability

	Male	Female	Total
Heterosexual	2325	2348	4673
Homosexual	105	23	128
Bisexual	66	92	158
Other	25	58	83
Total	2521	2521	5042

What is the probability that an American adult is male and homosexual?

a) 105/128 = 0.82
b) 105/2521 = 0.04
c) 105/5042 = 0.021
d) I got a different answer

Sexual Orientation: Or Probability

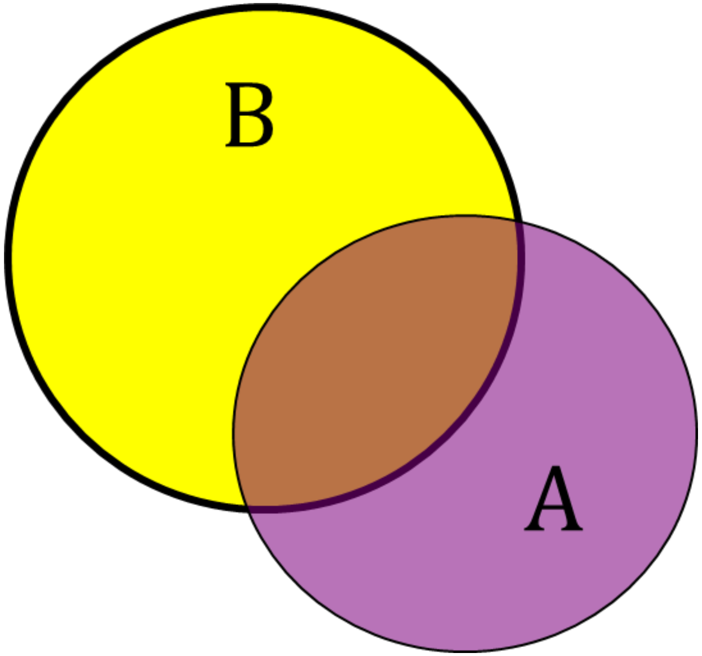
	Male	Female	Total
Heterosexual	2325	2348	4673
Homosexual	105	23	128
Bisexual	66	92	158
Other	25	58	83
Total	2521	2521	5042

What is the probability that an American adult is female or bisexual?

a) 2679/5042 = 0.531
b) 2587/5042 = 0.513
c) 92/2521 = 0.036
d) I got a different answer

Additive Rule: P(A or B)

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$



Sexual Orientation: Additive Rule

	Male	Female	Total
Heterosexual	2325	2348	4673
Homosexual	105	23	128
Bisexual	66	92	158
Other	25	58	83
Total	2521	2521	5042

What is the probability that an American adult is female or bisexual?

$$P(F \text{ or } B) = P(F) + P(B) - P(F \text{ and } B)$$

$$= \frac{2521}{5042} + \frac{158}{5042} - \frac{92}{6042} = \frac{2587}{5042} = 0.513$$

Sexual Orientation: Complement Rule

	Male	Female	Total
Heterosexual	2325	2348	4673
Homosexual	105	23	128
Bisexual	66	92	158
Other	25	58	83
Total	2521	2521	5042

What is the probability that an American adult is not heterosexual?

a) 369/5042 = 0.073
b) 2587/5042 = 0.513
c) 92/2521 = 0.036
d) I got a different answer

Complement rule: P(not A)

$$P(not\ A) = 1 - P(A)$$

$$P(\text{not Heterosexual}) = 1 - P(\text{Heterosexual})$$
$$= 1 - \frac{4673}{5042}$$
$$= \frac{369}{5042} = 0.073$$

Caffeine

Based on recent survey data, 52% of students drink caffeine in the morning, 48% of students drink caffeine in the afternoon, and 37% drink caffeine in the morning and the afternoon. What percent of students do not drink caffeine in the morning or the afternoon?

a) 63%
b) 37%
c) 100%
d) 50%

$P(\text{ not}(\ M \text{ or } A))$
$= 1 - P(\ M \text{ or } A\ )$
$= 1 - [P(M)+P(A) - P(M \text{ and } A)]$
$= 1 - [0.52 + 0.48 - 0.37]$
$= 1 - 0.63$
$= 0.37$

Conditional Probability

- ***P(A if B)*** is the probability of A, if we know B has happened

- This is read in multiple ways:

  - "probability of A if B"
  - "probability of A given B"
  - "probability of A conditional on B"

- You may also see this written as P(A | B)

Sexual Orientation: Condition is Male

	Male	Female	Total
Heterosexual	2325	2348	4673
Homosexual	105	23	128
Bisexual	66	92	158
Other	25	58	83
Total	2521	2521	5042

What is the probability that an American adult male is homosexual?

a) 105/128 = 0.82
b) 105/2521 = 0.04
c) 105/5042 = 0.021
d) I got a different answer

P(Homosexual if Male)

Sexual Orientation: Condition is Male/Homosexual

	Male	Female	Total
Heterosexual	2325	2348	4673
Homosexual	105	23	128
Bisexual	66	92	158
Other	25	58	83
Total	2521	2521	5042

What is the probability that an American adult homosexual is male?

a) 105/128 = 0.82

b) 105/2521 = 0.04

c) 105/5042 = 0.021

d) I got a different answer

P(Male if Homosexual)

Conditional Probability: Is Order Important?

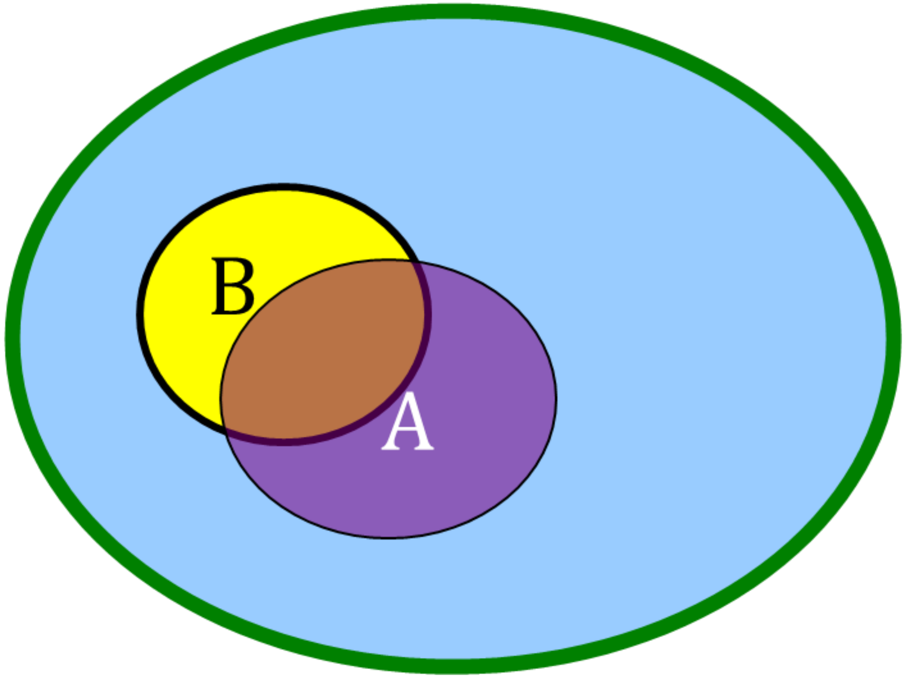
Note: $P(A \text{ if } B) \neq P(B \text{ if } A)$

P(Homosexual if Male) = 0.04

P(Male if Homosexual) = 0.82

Conditional Probability Rule

$$P\left(A \, if \;\; B\right) = \frac{P\left(A \, and \;\; B\right)}{P\left(B\right)}$$



Caffeine: Conditional Probability

Based on recent survey data, 52% of students drink caffeine in the morning, 48% of students drink caffeine in the afternoon, and 37% drink caffeine in the morning and the afternoon. What percent of students who drink caffeine in the morning also drink caffeine in the afternoon?

a) 77%
b) 37%
c) 71%
d) I got a different answer

P(A if M)
= P(A and M)/P(M)
= 0.37/0.52
= 0.71

Helpful Tip

If the table problems are easier for you than the sentence problems, try to first convert what you know into a table.

52% of students drink caffeine in the morning, 48% of students drink caffeine in the afternoon, and 37% drink caffeine in the morning and the afternoon

	Caffeine Afternoon	No Caffeine Afternoon	Total
Caffeine Morning	37	15	52
No Caffeine Morning	11	37	48
Total	48	52	100

P(afternoon if morning) = 37/52 = 0.71

P(not(morning or afternoon))= 37/100=0.37

Multiplicative Rule Again

$$P(A\ if\ B) = \frac{P(A\ and\ B)}{P(B)}$$

Multiplicative Rule: P(A and B)

$$P(A\ if\ B) = \frac{P(A\ and\ B)}{P(B)}$$

$$\boxed{P(A\ and\ B) = P(A\ if\ B)P(B)}$$

Equivalent form:

$$\boxed{P\ (A\ and\ B) = P(A)\ P(B\ if\ A)}$$

College Rank and Experience

60% of STAT 101 students rank their college experience as "Excellent," and this was the first choice school for 59% of those who ranked their experience as excellent. What percentage of STAT 101 students had this college as a first choice and rank their experience as excellent?

a) 60%
b) 59%
c) 35%
d) 41%

P(first choice and excellent)
= P(excellent) P(first choice if excellent)
= 0.60 × 0.59
= 0.354

Summary: Probability Rules

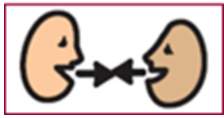
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A)\, P(B \text{ if } A)$$

$$P(\text{not } A) = 1 - P(A)$$

$$P(A \text{ if } B) = \frac{P(A \text{ and } B)}{P(B)}$$

Disjoint Events



- Events A and B are ***disjoint*** or ***mutually exclusive*** if only one of the two events can happen

- Think of two events that *are* disjoint, and two events that *are not* disjoint.

Disjoint Events and Probability

Pick the best choice:

If A and B are disjoint, then

a) $P(A \text{ or } B) = P(A) + P(B)$

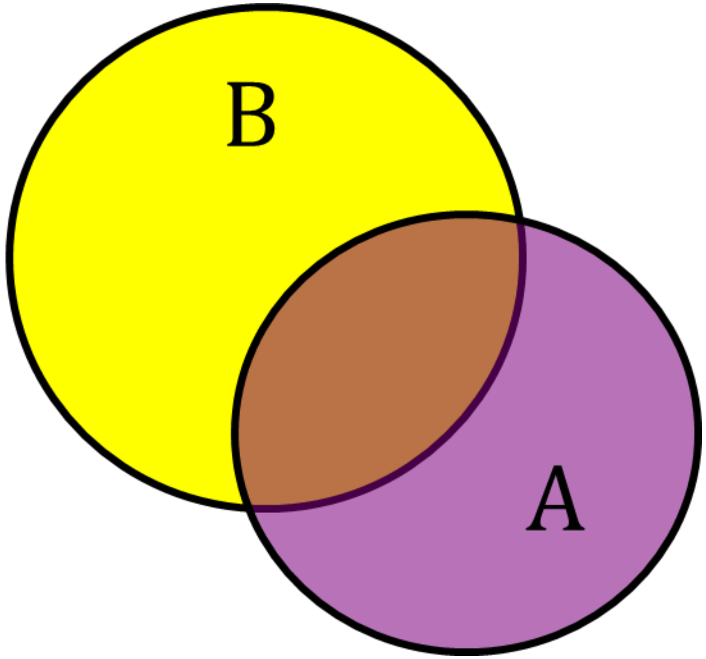
b) $P(A \text{ and } B) = P(A)P(B)$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

If A and B are disjoint, then both cannot happen, so $P(A \text{ and } B) = 0$.

P(A or B)

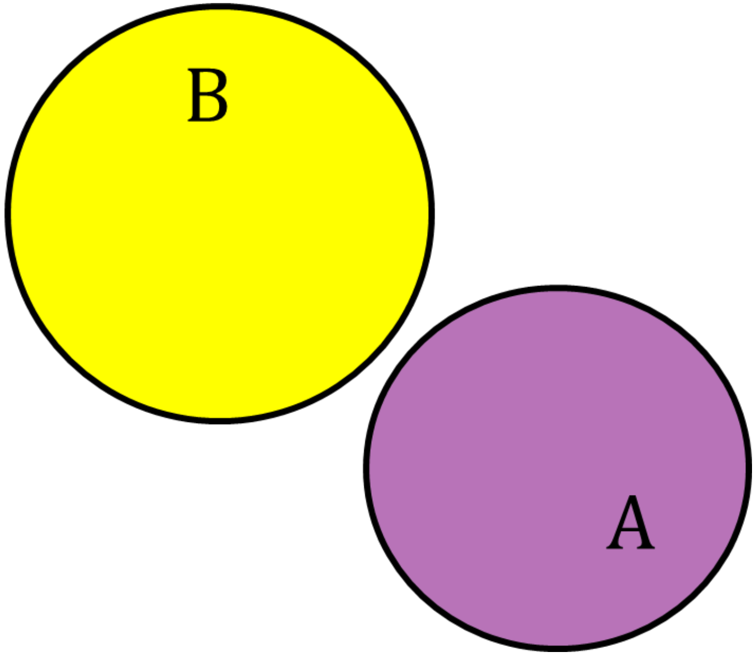
P(A or B) =
P(A)+P(B)–P(A and B)



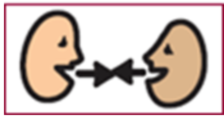
Special Case:

If A and B are ***disjoint***

P(A or B)=P(A)+P(B)



Independence



- Events A and B are ***independent*** if P(A if B) = P(A).
- Intuitively, knowing that event B happened does not change the probability that event A happened.
- Think of two events that *are* independent, and two events that *are not* independent.

Independent Events

Pick the best choice:

If A and B are independent, then

a) $P(A \text{ or } B) = P(A) + P(B)$

b) $P(A \text{ and } B) = P(A)P(B)$

$P(\ A \text{ and } B) = P(A \text{ if } B)P(B)$

If A and B are independent, then $P(A \text{ if } B) = P(A)$,
so $P(A \text{ and } B) = P(A)P(B)$

P(A and B)

$$P(A \text{ and } B) = P(A)\, P(B \text{ if } A)$$

If A and B are independent, then
$P(B \text{ if } A) = P(A)$, so

Special Case:

If A and B are ***independent***

$$P(A \text{ and } B) = P(A)\, P(B)$$

Disjoint and Independent

Assuming that $P(A) > 0$ and $P(B) > 0$, then disjoint events are:

a) Always independent
b) Never independent
c) Sometimes independent – we need more information to judge

If A and B are disjoint, then A cannot happen if B has happened, so P(A if B) = 0.

If P(A) > 0, then P(A if B) ≠ P(A) so A and B are not independent.

Summary: Special Cases

- If events A and B are **disjoint** (mutually exclusive):
  - P(A or B) = P(A) + P(B)
  - P(A and B) = 0
- If events A and B are **independent**:
  - P(A if B) = P(A)
  - P(A and B) = P(A) P(B)