Section 6.4:

Inference for a Difference in Means

SE for Difference in Means

$$\boxed{SE = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

df = smaller of $n_1 - 1$ and $n_2 - 1$

SE for Difference in Means

$$SE = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

- We don't know the population standard deviations σ_1 or σ_2, so we estimate them with the sample standard deviations, s_1 *and* s_2

$$SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Confidence Interval: Difference in Means

If $n_1 \geq 30$ and $n_2 \geq 30$,*

$$\boxed{\left(\bar{X}_1 - \bar{X}_2\right) \pm t^* \times \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

For a difference in means, the degrees of freedom for the *t*-distribution is the smaller of **$n_1 - 1$ and $n_2 - 1$**

*Smaller sample sizes may be sufficient for symmetric distributions, and 30 may not be sufficient for skewed distributions

Video Games and GPA

- 210 first-year college students were *randomly assigned* roommates
- For the 78 students assigned to roommates who brought a videogame system to college: average GPA after the first semester was 2.84, with a sd of 0.669.
- For the 132 students assigned to roommates who did not bring a videogame system to college, average GPA after the first semester was 3.105, with a sd of 0.625.

How much does getting assigned a roommate who brings a videogame system to college affect your first semester GPA? Give a 90% confidence interval.

78 students whose roommates brought a videogame system to college: average GPA was 2.84, with a sd of 0.669. 132 students whose roommates did not bring a videogame system to college: average GPA was 3.105, with a sd of 0.625. For a 90% confidence interval, what are the degrees of freedom for t*?

A. 77
B. 78
C. 131
D. 132
E. 209

78 students whose roommates brought a videogame system to college: average GPA was 2.84, with a sd of 0.669. 132 students whose roommates did not bring a videogame system to college: average GPA was 3.105, with a sd of 0.625. For a 90% confidence interval, what is t*?

A. 1.645
B. 1.960
C. 2.576
D. 1.665
E. 1.678

78 students whose roommates brought a videogame system to college: average GPA was 2.84, with a sd of 0.669. 132 students whose roommates did not bring a videogame system to college: average GPA was 3.105, with a sd of 0.625. For a 90% CI, what is the relevant sample statistic?
(Group 1 = Videogame System, Group 2 = No Video Games)

A. 2.84
B. 3.105
C. -0.265
D. -1.005
E. 1.678

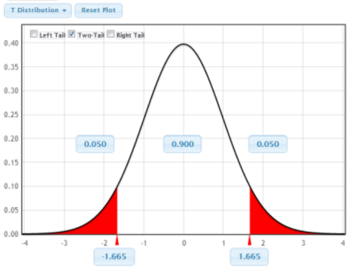
78 students whose roommates brought a videogame system to college: average GPA was 2.84, with a sd of 0.669. 132 students whose roommates did not bring a videogame system to college: average GPA was 3.105, with a sd of 0.625. For a 90% CI, what is the standard error?

A. 1.960
B. 0.625
C. -0.265
D. 0.044
E. 0.093

$$\boxed{SE = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Videogames and GPA

1. Check conditions: $78 \geq 30, 132 \geq 30$



2. Find t^*:

3. Calculate statistic:

4. Calculate standard error:

5. Calculate CI: $\bar{x}_1 - \bar{x}_2 \pm t^* \cdot SE$

6. Interpret in context:

T-Test for a Difference in Means

$H_0: \mu_1 = \mu_2$

$\Leftrightarrow H_0: \mu_1 - \mu_2 = 0$

$$\frac{statistic \ - null}{SE}$$

$$t = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\frac{{s_1}^2}{n_1} + \frac{{s_2}^2}{n_2}}}$$

- If the population is approximately normal or if sample sizes are large ($n_1 \geq 30$, $n_2 \geq 30$), the p-value can be computed as the area in the tail(s) beyond t of a t-distribution with degrees of freedom equal to the smaller of $n_1 - 1$ and $n_2 - 1$

The Pygmalion Effect

Teachers were told that certain children (chosen randomly) were expected to be "growth spurters," based on the Harvard Test of Inflected Acquisition (a test that didn't actually exist). These children were selected randomly.

The response variable is change in IQ over the course of one year.

Source: Rosenthal, R. and Jacobsen, L. (1968). "Pygmalion in the Classroom: Teacher Expectation and Pupils' Intellectual Development." Holt, Rinehart and Winston, Inc.

The Pygmalion Effect

	n	$\bar{x}$	s
Control Students	255	8.42	12.0
"Growth Spurters"	65	12.22	13.3

Research Question: Does this provide evidence that merely expecting a child to do well actually *causes* the child to do better?

If so, how much better?

*s_1 and s_2 were not given in the article, so they were selected to ensure the p-value specified in the article.

Pygmalion Effect

1. State hypotheses:

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_a : \mu_1 - \mu_2 > 0$$

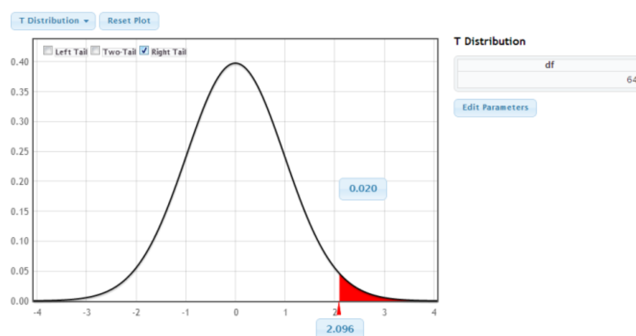
where μ_1 : Mean IQ change for "growth spurters" and μ_2 : Mean IQ change for control students

2. Check conditions: $255 \geq 30, 65 \geq 30$

3. Calculate t statistic:

$$t = \frac{\bar{X}_1 - \bar{X}_2 - 0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{12.22 - 8.42}{\sqrt{\frac{13.3^2}{65} + \frac{12^2}{255}}} = \frac{3.8}{1.813} = 2.096$$

4. Compute p-value:



$$p - value = .02$$

5. Interpret in context:

Parameter	Distribution	Conditions	Standard Error
Proportion	Normal	All counts at least 10: $np \geq 10$, $n(1-p) \geq 10$	$\sqrt{\frac{p(1-p)}{n}}$
Difference in Proportions	Normal	All counts at least 10: $n_1p_1 \geq 10$, $n_1(1-p_1) \geq 10$, $n_2p_2 \geq 10$, $n_2(1-p_2) \geq 10$	$\sqrt{\frac{p_1(1-p_1)}{n_1}+\frac{p_2(1-p_2)}{n_2}}$
Mean	t, df $= n-1$	$n \geq 30$ or data normal	$\sqrt{\frac{s^2}{n}}$
Difference in Means	t, df = smaller of n_1-1, n_2-1	$n_1 \geq 30$ or data normal, $n_2 \geq 30$ or data normal	$\sqrt{\frac{s_1^{\ 2}}{n_1}+\frac{s_2^{\ 2}}{n_2}}$