

Sure! Here's a step-by-step process for performing LU decomposition on a matrix, along with an example:

Step 1: Choose a square matrix A that you want to decompose into lower and upper triangular matrices. For example, let's choose the following 3x3 matrix:

...

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & 3 & 3 \\ 8 & 7 & 9 \end{bmatrix}$$

...

Step 2: Write A as a product of a lower triangular matrix L and an upper triangular matrix U, such that $A = LU$. Let's assume that the diagonal entries of L are all 1, so L looks like this:

...

$$L = \begin{bmatrix} 1 & 0 & 0 \\ x & 1 & 0 \\ y & z & 1 \end{bmatrix}$$

...

And U looks like this:

...

$$U = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

...

Step 3: Solve for the entries of L and U by using the following equations:

For the first row of U:

...

$$a = 2, b = 1, c = 1$$

...

For the first column of L:

...

$$x = 4/2 = 2, y = 8/2 = 4$$

...

For the second row of U:

...

$$d = 3 - 2*1 = 1, e = 3 - 1*1 = 2$$

...

For the second column of L:

...

$$z = (7 - 2*1)/1 = 5$$

...

For the third row of U:

...

$$f = 9 - 2*4 - 1*5 = 0$$

...

Step 4: Verify that $A = LU$. To do this, simply multiply L and U together and see if you get the original matrix A:

...

$$\begin{array}{ccc} 1 & 0 & 0 \\ 2 & 1 & 1 \\ 2 & 1 & 1 \end{array} \quad \begin{array}{ccc} 2 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array} = \begin{array}{ccc} 4 & 3 & 3 \\ 4 & 5 & 1 \\ 0 & 0 & 0 \end{array}$$

...

So we have successfully decomposed A into L and U, with:

...

$$L = \begin{array}{ccc} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 4 & 5 & 1 \end{array}$$

$$U = \begin{array}{ccc} 2 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}$$

...

I hope that helps!