

Assignment: Homework Eight Name: Cody Strange

Disclaimer: This is my work, not that of others

Total Score: 50 (in points, not percentage)

Problem 1 score: 10

Problem 2 score: 30

Problem 3 score: 10

The image shows a dual-screen setup. On the left is a Microsoft Whiteboard with handwritten mathematical work. On the right is a VS Code editor with a Python script and its terminal output.

Whiteboard Content:

Initial equations:

$$50 = 5x_3 - 7x_2$$
$$4x_2 + 7x_3 + 30 = 0$$
$$x_1 - 7x_3 = 40 - 3x_2 + 5x_3$$

Red arrows point to the rearranged equations:

$$0x_1 - 7x_2 + 5x_3 = 50$$
$$0x_1 + 4x_2 + 7x_3 = -30$$
$$4x_1 - 3x_2 + 7x_3 = -40$$

A purple arrow points to the matrix equation:

$$\begin{bmatrix} 0 & -7 & 5 \\ 0 & 4 & 7 \\ 4 & -3 & 7 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} 50 \\ -30 \\ -40 \end{Bmatrix}$$

VS Code Content:

matrix.py

```
1 from numpy import linalg, matrix
2 a = matrix('0,-7,5; 0, 4, 7; 4, -3, 7')
3 b = matrix('50;-30;-40')
4
5 solved = linalg.solve(a,b)
6
7 print(solved)
```

Terminal Output:

```
PS D:\School\CS3320\HW\HW-8> & C:/Users/cody1/AppData/Local/Programs/Python/Python310/python.exe d:/School/CS3320/HW/HW-8/matrix.py
[[-15.18115942]
 [-7.24637681]
 [-0.14492754]]
PS D:\School\CS3320\HW\HW-8>
```

1.

x1 = -1518115942, x2 = -7.24637681, x3 = -0.14492754

2. A. etc

The image shows a dual-screen setup. On the left is a Microsoft Whiteboard with handwritten mathematical equations and a matrix. On the right is a Visual Studio Code editor with a Python script and its terminal output.

Whiteboard Content:

$$\begin{aligned} -3x_2 + 7x_3 &= 4 \\ x_1 + 2x_2 - x_3 &= 0 \\ 5x_1 - 2x_2 &= 3 \end{aligned}$$

Arrows indicate the equations are being rearranged into a standard form:

$$\begin{aligned} 0x_1 - 3x_2 + 7x_3 &= 4 \\ x_1 + 2x_2 - x_3 &= 0 \\ 5x_1 - 2x_2 + 0x_3 &= 3 \end{aligned}$$

Below these, a matrix equation is written:

$$\begin{bmatrix} 0 & -3 & 7 \\ 1 & 2 & -1 \\ 5 & -2 & 0 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$$

Visual Studio Code Content:

The Explorer pane shows a file named `problem_two.py`. The code in the editor is:

```
1 import numpy
2
3 a = numpy.matrix('0,-3,7; 1,2,-1; 5,-2,0')
4 b = numpy.matrix('4;0;3')
5
6 determinant = numpy.linalg.det(a)
7 print(determinant)
```

The TERMINAL pane shows the command prompt output:

```
PS D:\School\CS3320\HW\HW-8> & C:\Users\codyl\AppData\Local\Programs\Python\Python310\python.exe d:\School\CS3320\HW\HW-8\problem_two.py
-68.999999999999996
PS D:\School\CS3320\HW\HW-8>
```

Determinant = -68.999999999999996

B.

Gauss Elimination

$$\begin{array}{c}
 \begin{bmatrix} 0 & -3 & 7 & 4 \\ 1 & 2 & -1 & 0 \\ 5 & -2 & 0 & 3 \end{bmatrix} \xrightarrow{\text{Pivot } R_1 \text{ and } R_3} \begin{bmatrix} 5 & -2 & 0 & 3 \\ 1 & 2 & -1 & 0 \\ 0 & -3 & 7 & 4 \end{bmatrix} \xrightarrow{R_2 - \frac{1}{5} R_1} \begin{bmatrix} 5 & -2 & 0 & 3 \\ 0 & 2.4 & -1 & -0.6 \\ 0 & -3 & 7 & 4 \end{bmatrix} \\
 \downarrow \text{Pivot } R_2 \text{ and } R_3 \\
 \begin{bmatrix} 5 & -2 & 0 & 3 \\ 0 & -3 & 7 & 4 \\ 0 & 0 & 4.6 & 2.6 \end{bmatrix} \xleftarrow{R_3 - \frac{2.4}{-3} R_2} \begin{bmatrix} 5 & -2 & 0 & 3 \\ 0 & -3 & 7 & 4 \\ 0 & 2.4 & -1 & -0.6 \end{bmatrix}
 \end{array}$$

$x_{24} = 0 - \frac{3}{-3} = 1$
 $x_{14} = 2 - \frac{-2}{5} = 2.4$
 $x_{34} = -1 - \frac{-2}{5} = -1$

$x_{32} = 2.4 - \frac{2.4}{-3} \cdot -3 = 0$
 $x_{33} = -1 - \frac{2.4}{-3} \cdot 7 = 4.6$
 $x_{34} = (-0.6) - \frac{2.4}{-3} \cdot 4 = 2.6$

Backwards Substitution

$$\begin{aligned}
 x_3 &= \frac{2.6}{4.6} = 0.5652173913 \\
 x_2 &= -\frac{1}{3}(4 - 7x_3) = -0.0144927536 \\
 x_1 &= \frac{1}{5}(3 - 2x_2) = 0.5942028986
 \end{aligned}$$

$$x_3 = 0.5652173913$$

$$x_2 = -0.0144927536$$

$$x_1 = 0.5942028986$$

C.

$$-3x_2 + 7x_3 = 4$$

$$x_1 + 2x_2 - x_3 = 0$$

$$5x_1 - 2x_2 = 3$$

$$x_1 = 0.5942028986$$

$$x_2 = -0.0144927536$$

$$x_3 = 0.5652173913$$

$$-3(-0.0144927536) + 7(0.5652173913) = 4$$

↓

$$0.0434782608 + 3.9565217391 = 4$$

↓

$$3.9999999991 = 4$$

$$0.5942028986 + 2(-0.0144927536) - 0.5652173913 = 0$$

$$-0.0289855072$$

$$0.0000000001 = 0$$

$$5(0.5942028986) - 2(-0.0144927536) = 3$$

↓

$$2.971014493 - 0.289855072 = 3$$

$$2.681159421 = 3$$

3.

pivot P_1 and P_2

$$A = \begin{bmatrix} 4 & -1 & 2 \\ 2 & 1 & -2 \\ 2 & -1 & -1 \end{bmatrix} \quad L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{2} & -\frac{1}{3} & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4 & -1 & 2 \\ 0 & \frac{3}{2} & -3 \\ 2 & -1 & -1 \end{bmatrix} \quad -2R_1 + R_2$$

$$\begin{bmatrix} 4 & -1 & 2 \\ 0 & \frac{3}{2} & -3 \\ 0 & -\frac{1}{2} & -2 \end{bmatrix} \quad -\frac{2}{3}R_1 + R_3$$

$$\begin{bmatrix} 4 & -1 & 2 \\ 0 & \frac{3}{2} & -3 \\ 0 & 0 & -1 \end{bmatrix} \quad -\left(\frac{-\frac{1}{2}}{\frac{3}{2}}\right)R_2 + R_3$$

$$-\left(-\frac{1}{2} \div \frac{3}{2} \rightarrow -\frac{1}{2} \cdot \frac{2}{3} = -\frac{1}{3}\right) = \frac{1}{3}$$

$$B = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{2} & -\frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}$$

$$y_1 = 5$$

$$\frac{1}{2}y_1 + y_2 = 1$$

$$\frac{1}{2}y_1 - \frac{1}{3}y_2 + y_3 = 2$$

$$\frac{1}{2}(5) + y_2 = 1$$

$$\frac{5}{2} + y_2 = 1$$

$$y_2 = 1 - \frac{5}{2} = -\frac{3}{2}$$

$$y_2 = -\frac{3}{2}$$

$$y_2 = -\frac{3}{2}$$

$$y_2 = -\frac{3}{2}$$

$$\frac{1}{2}(5) - \frac{1}{3}(-\frac{3}{2}) + y_3 = 2$$

$$\frac{5}{2} + \frac{1}{2} + y_3 = 2$$

$$\frac{1}{2} + \frac{1}{2} + y_3 = 0$$

$$1 + y_3 = 0$$

$$y_3 = -1$$

$$y_1 = 5$$

$$y_2 = -\frac{3}{2}$$

$$y_3 = -1$$

$$U = \begin{bmatrix} 4 & -1 & 2 \\ 0 & \frac{3}{2} & -3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -\frac{3}{2} \\ -1 \end{bmatrix}$$

$$4x_1 - x_2 + 2x_3 = 5$$

$$\frac{3}{2}x_2 - 3x_3 = -\frac{3}{2}$$

$$-x_3 = -1$$

$$x_3 = 1$$

$$x_2 = 1$$

$$x_1 = 1$$

$$\frac{3}{2}x_2 - 3 = -\frac{3}{2}$$

$$\frac{3}{2}x_2 = 3 - \frac{3}{2} = \frac{3}{2}$$

$$x_2 = \frac{3}{2} \cdot \frac{2}{3} = 1$$

$$x_2 = 1$$

$$x_2 = 1$$

$$x_2 = 1$$

$$4x_1 - 1 + 2 = 5$$

$$4x_1 + 1 = 5$$

$$4x_1 = 4$$

$$x_1 = 1$$

$$x_1 = 1$$

$$x_1 = 1$$

$$B = \begin{bmatrix} 6 \\ 0 \\ 3 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{2} & -\frac{1}{3} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 0 \\ 3 \end{bmatrix}$$

$$y_1 = 6$$

$$\frac{1}{2}y_1 + y_2 = 0$$

$$\frac{1}{2}y_1 - \frac{1}{3}y_2 + y_3 = 3$$

$$y_1 = 6$$

$$y_2 = -3$$

$$y_3 = -1$$

$$\frac{1}{2}(6) + y_2 = 0$$

$$3 + y_2 = 0$$

$$y_2 = -3$$

$$\frac{1}{2}(6) - \frac{1}{3}(-3) + y_3 = 3$$

$$3 + 1 + y_3 = 3$$

$$y_3 = -1$$

$$U = \begin{bmatrix} 4 & -1 & 2 \\ 0 & \frac{3}{2} & -3 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ -1 \end{bmatrix}$$

$$4x_1 - x_2 + 2x_3 = 6$$

$$\frac{3}{2}x_2 - 3x_3 = -3$$

$$-x_3 = -1$$

$$x_3 = 1$$

$$x_2 = 0$$

$$x_1 = 1$$

$$\frac{3}{2}(x_2) - 3(1) = -3$$

$$\frac{3}{2}(x_2) = 0$$

$$x_2 = 0$$

$$4(1) - 1(-2) + 2(0) = 6$$

$$4(1) + 2 = 6$$

$$4(1) = 4$$

$$x_1 = 1$$