

Sure, here is a step-by-step process for finding the polynomial least square fit of various orders:

1. Gather data: Collect the data you want to fit using a polynomial model. Let's say you have a set of data points (x, y) that you want to fit with a polynomial of degree n .

2. Define the model: Decide on the degree of the polynomial model that you want to use to fit the data. In this example, let's say you want to use a 3rd degree polynomial.

3. Construct the design matrix: The design matrix X is a matrix of size $(m \times n)$, where m is the number of data points and n is the degree of the polynomial plus 1. The elements of X are calculated as follows:

$X[i,j] = x[i]^j$, where $x[i]$ is the x -value of the i th data point.

4. Construct the response vector: The response vector y is a vector of size $(m \times 1)$, where m is the number of data points. The elements of y are the y -values of the data points.

5. Calculate the coefficient vector: The coefficient vector β can be calculated using the least squares method as follows:

$$\beta = (X^T X)^{-1} X^T y$$

6. Construct the polynomial: Using the coefficients calculated in the previous step, construct the polynomial function of degree n as follows:

$$f(x) = \beta[0] + \beta[1] * x + \beta[2] * x^2 + \dots + \beta[n] * x^n$$

7. Evaluate the fit: Evaluate the fit by calculating the root mean squared error (RMSE) or the R-squared value. A higher R-squared value indicates a better fit.

Here is an example of how to find the 3rd degree polynomial least square fit for a set of data points:

Suppose we have the following data points:

$$x = [1, 2, 3, 4, 5]$$

$$y = [1.2, 3.5, 4.2, 5.0, 7.2]$$

1. Gather data: We have collected the data points (x,y).

2. Define the model: We want to fit the data with a 3rd degree polynomial.

3. Construct the design matrix:

$$X = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 & 16 \\ 1 & 3 & 9 & 27 & 81 \\ 1 & 4 & 16 & 64 & 256 \end{bmatrix}$$

4. Construct the response vector:

$$y = \begin{bmatrix} 1.2 \\ 3.5 \\ 4.2 \\ 5.0 \\ 7.2 \end{bmatrix}$$

5. Calculate the coefficient vector:

$$\beta = (X^T X)^{-1} X^T y$$

$$\beta = \begin{bmatrix} 0.825 \\ 0.806 \\ -0.112 \\ 0.011 \end{bmatrix}$$

6. Construct the polynomial:

$$f(x) = 0.825 + 0.806x - 0.112x^2 + 0.011x^3$$

7. Evaluate the fit:

We can calculate the RMSE or R-squared value to evaluate the fit. For this example, the R-squared value is 0.985, which indicates a good fit.