Assignment: Module 9 Homework Name: Cody Strange
Disclaimer: This is my work, not that of others:
Total Score: 55.0
Problem 1.a score: 5.0
Problem 1.b score: 10.0
Problem 1.c score: 10.0
Problem 2 score: 10.0
Problem 3 score: 10.0
Problem 4 score: 10.0

1. The following system of equations is designed to determine the concentrations (cs in g/m^3) in a series of coupled, well-mixed tanks as a function of mass input to each tank. The right-hand side of the equations below represent these inputs in g/day.

$$15c_1 - 3c_2 - c_3 = 4000$$
$$-3c_1 + 18c_2 - 6c_3 = 1200$$
$$-4c_1 - c_2 + 12c_3 = 2350$$

   a. (5 pt) Determine the inverse of the coefficient matrix. (You can use Python to solve the inverse.)

question_one.py > ...

```python
import numpy as np

A = [[15, -3, -1], [-3, 18, -6], [-4,-1, 12]]
b = [4000,1200,2350]
inverse_A = np.linalg.inv(A)
print(inverse_A)
```

```
[[0.07253886 0.01278066 0.01243523]
 [0.02072539 0.06079447 0.03212435]
 [0.02590674 0.00932642 0.09015544]]
PS D:\School\CS3320\HW\HW-9>
```

   b. (10 pt) Use the inverse to determine the solution. (Do this by hand.)

$$\begin{bmatrix} 0.07253886x_1, 0.01278066x_2, 0.01243523x_3 \\ 0.02072539x_1, 0.06079447x_2, 0.03212435x_3 \\ 0.02590674x_1, 0.00932642x_2, 0.09015544x_3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \begin{bmatrix} 4000 \\ 1200 \\ 2350 \end{bmatrix}$$

$0.07253886(4000) + 0.01278066(1200) + 0.01243523(2350)$

$290.15544 + 15.336792 + 29.2227905 = 334.7150225$

$0.02072539(4000) + 0.06079447(1200) + 0.03212435(2350)$

$82.90156 + 72.953364 + 75.4922225 = 231.3471465$

$0.02590674(4000) + 0.00932642(1200) + 0.09015544(2350)$

$103.62696 + 11.19170 + 211.865284 = 327.409284$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 334.7150225 \\ 231.3471465 \\ 327.409284 \end{bmatrix}$$

   c. (10 pt) Determine how much the rate of mass input to tank 3 must increase to induce a 10 g/m3 rise in the concentration in tank 1.

$$A'[1,3] = A[3,1] = 0.01243523$$

$$\frac{0.01243523 \cdot \Delta I_3}{0.01243523} = \frac{10}{0.01243523}$$

$$\Delta I_3 = 804.166871059$$

804.166871059

2. (10 pt) Determine $\|A\|_f$, $\|A\|_1$, and $\|A\|_\infty$ for

$$A = \begin{bmatrix} 8 & 2 & -10 \\ -9 & 1 & 3 \\ 15 & -1 & 6 \end{bmatrix}.$$

$$A = \begin{bmatrix} 8 & 2 & -10 \\ -9 & 1 & 3 \\ 15 & -1 & 6 \end{bmatrix} \qquad n=3$$

$$\|A\|_F = \sqrt{\sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij}^2}$$

$$= \sqrt{\sum_{i=1}^{3} \sum_{j=1}^{3} a_{ij}^2}$$

question_two.py > ...

```python
from math import sqrt


A = [[8,2,-10,],[-9,1,3],[15,-1,6]]
n = 3
sumation = 0
for i in range(n):
    for j in range(n):
        sumation += A[i][j]**2

A2 = sqrt(sumation)
print(A2)
```

```
PS D:\School\CS3320\HW\HW-9> & C:
22.825424421026653
PS D:\School\CS3320\HW\HW-9>
```

$\|A\|_F$ = 22.825424421026653

$$A = \begin{bmatrix} 8 & 2 & -10 \\ -9 & 1 & 3 \\ 15 & -1 & 6 \end{bmatrix} \quad n=3$$

$$\|A\|_1 = \sum_{i=1}^{3} |A_i|$$

$8+9+15=32$

$32 > 2+1+1=4$

$32 > 10+3+6=19$

max $= 32$

$\|A\|_1 = 32$

$$n=3 \quad A = \begin{bmatrix} 8 & 2 & -10 \\ -9 & 1 & 3 \\ 15 & -1 & 6 \end{bmatrix}$$

$$\|A\|_\infty = \max_{1 \le i \le n} |a_i|$$

$15+1+6=22$

$9+1+3=13$

$10+8+2=20$

$22 > 20 > 13$

max $= 22$

$\|A\|_{inf} = 22$

3. (10 pt) Solve the following system using three iterations of the Gauss-Seidel method. If necessary, rearrange the equations. Show all the steps in your solution. At the end of your computation, compute the true error of your final results. (Do this by hand.)

$3x_1 + 8x_2 = 11$

$7x_1 - x_2 = 5$

$$\begin{bmatrix} 3x_1 & 8x_2 \\ 7x_1 & -x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 11 \\ 5 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 7x_1 & -x_2 \\ 3x_1 & 8x_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 11 \end{bmatrix}$$

$x_1^0 = 0$

$x_2^0 = 0$

$x_1^1 = \frac{5 - -1(0)}{7} = \frac{5}{7}$

$x_2^1 = \frac{11 - 3(\frac{5}{7})}{8} = 1.10714$

$x_1^2 = \frac{5 + 1(1.10714)}{7} = 0.87244$

$x_2^2 = \frac{11 - 3(0.87244)}{8} = 1.047835$

$x_1^3 = \frac{5 + 1(1.047835)}{7} = .86397$

$x_2^3 = \frac{11 - 3(0.86397)}{8} = 1.0510$

$x_1 = 0.86397$

$x_2 = 1.0510$

$\varepsilon_1 = \left| \frac{0.86397 - 0.87244}{0.87244} \right| = 0.97\%$

$\varepsilon_2 = \left| \frac{1.0510 - 1.047835}{1.047835} \right| = 0.3\%$

4. (10 pt) Use the Gauss-Seidel method (a) without relaxation and (b) with relaxation ($\lambda = 1.2$) to solve the following set of linear equation to meet an error tolerance of $\varepsilon_s = 5\%$. If necessary, rearrange the equations to achieve convergence.

$2x_1 - 6x_2 - x_3 = -38$

$-3x_1 - x_2 + 7x_3 = -34$

$-8x_1 + x_2 - 2x_3 = -20$

Without Relax

$x^0 = 0 \quad z^0 = 0$
$y^0 = 0$

$$\begin{bmatrix} -8 & x & -2 \\ 2 & -6 & -1 \\ -3 & -1 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} -20 \\ -38 \\ -34 \end{bmatrix}$$

$x^1 = 2.500$

$y^1 = 7.167$

$z^1 = -2.762$

$x^2 = 4.086375 \rightarrow 38.82\%$

$y^2 = 8.1557757365 \rightarrow 12.12\%$

$z^2 = -1.94072843805 \rightarrow 42.32\%$

$x^3 = 4.0045408872 \rightarrow 2.04\%$

$y^3 = 7.99167277258 \rightarrow 2.05\%$

$z^3 = -1.99919499447 \rightarrow 2.92\%$

with Relax

$x^0 = 9 \quad z^0 = 0$
$y^0 = 0$

$$\begin{bmatrix} -8 & x & -2 \\ 2 & -6 & -1 \\ -3 & -1 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} -20 \\ -38 \\ -34 \end{bmatrix}$$

$x^1 = 3$
$y^1 = 8.8$
$z^1 = -2.777142857$

$x^2 = 4.55314285542 \rightarrow 34.11\%$
$y^2 = 8.2166857136 \rightarrow 7.10\%$
$z^2 = -1.52295183689 \rightarrow 82.35\%$
$x^3 = 3.77573983662 \rightarrow 20.49\%$
$y^3 = 7.90606001697 \rightarrow 3.93\%$
$z^3 = -2.22329428425 \rightarrow 31.56\%$
$x^4 = 4.0977453203 \rightarrow 7.78\%$
$y^4 = 8.10294491458 \rightarrow 2.43\%$
$z^4 = -1.88702491 04 \rightarrow 17.93\%$
$x^5 = 3.96200000622 \rightarrow 3.43\%$
$y^5 = 7.9416158882 \rightarrow 2.03\%$
$z^3 = -2.0521466768 \rightarrow 8.05\%$
$x^6 = 4.01448638503 \rightarrow 1.31\%$
$y^6 = 8.02790071173 \rightarrow 1.07\%$
$z^6 = -19773375461 \rightarrow 3.78\%$

$$f(x) = 1.2\left(\frac{(-20-(7.9416158882)+2(-2.0521466768))}{-8}\right)+(1-1.2)3.96200000622$$

$$g(x) = 1.2\left(\frac{(-38-(2(4.01448638503))+(-2.0521466768))}{-6}\right)+(1-1.2)7.9416158882$$

$$h(x) = 1.2\frac{((-34+3(4.01448638503)+8.02790071173))}{7}+(1-1.2)(-2.0521466768)$$

$f(x)$

$= 4.01448638503$

$g(x)$

$= 8.02790071173$

$h(x)$

$= -1.97733754461$

$$\frac{(4.01448638503-3.96200000622)}{4.01448638503}\cdot 100$$

$= 1.30742450655$

$$\frac{(8.02790071173-7.9416158882)}{8.02790071173}\cdot 100$$

$= 1.07481179238$

$$\frac{(-1.97733754461--2.0521466768)}{-1.97733754461}\cdot 100$$

$= -3.78332634172$