

Sure, here's an example of how to use LU factorization to solve a system of linear equations:

Suppose we have the following system of linear equations:

$$x + 2y + 3z = 6$$

$$4x + 5y + 6z = 15$$

$$7x + 8y + 10z = 28$$

We can rewrite this system in matrix form as $Ax = b$, where:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$$

$$x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$b = \begin{bmatrix} 6 \\ 15 \\ 28 \end{bmatrix}$$

To solve this system using LU factorization, we first factorize the matrix A into its lower triangular matrix L and upper triangular matrix U such that $A = LU$. To do this, we can use Gaussian elimination with partial pivoting.

After applying Gaussian elimination with partial pivoting, we get:

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 1 & 0; \\ 0.57 & 0.32 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 7 & 8 & 10; \\ 0 & -1.14 & -2.43; \\ 0 & 0 & 0.18 \end{bmatrix}$$

Now we can solve the system by first solving $Ly = b$ for y using forward substitution, and then solving $Ux = y$ for x using backward substitution.

Solving $Ly = b$ for y using forward substitution:

$$\begin{aligned} y_1 &= b_1 = 6 \\ y_2 &= b_2 - L(2,1)y_1 = 15 - 4(6) = -9 \\ y_3 &= b_3 - L(3,1)y_1 - L(3,2)y_2 = 28 - 7(6) - 8(-9) = 2 \end{aligned}$$

So we have $y = [6; -9; 2]$.

Now we can solve $Ux = y$ for x using backward substitution:

$$\begin{aligned} z &= y_3 / U(3,3) = 2 / 0.18 = 11.11 \\ y_2 &= (y_2 - U(2,3)z) / U(2,2) = (-9 + 2.43(11.11)) / (-1.14) = 7.05 \\ x &= (y_1 - U(1,2)y_2 - U(1,3)z) / U(1,1) = (6 - 8(7.05) - 10(11.11)) / 7 = -2.33 \end{aligned}$$

So we have $x = -2.33$, $y = 7.05$, and $z = 11.11$ as the solution to the system of linear equations.