

Certainly! The Gauss-Seidel iteration is a numerical method for solving a system of linear equations. Here's the step-by-step process for performing Gauss-Seidel iteration manually, using an example:

Suppose we have the following system of equations:

...

$$3x + y + z = 7$$

$$x - 4y + 2z = -11$$

$$-2x + y - 5z = 15$$

...

Step 1: Rearrange the equations so that each variable is on the left-hand side of one equation and all other variables are on the right-hand side:

...

$$x = (7 - y - z)/3$$

$$y = (1 + x + 2z)/4$$

$$z = (-15 + 2x - y)/5$$

...

Step 2: Choose an initial guess for each variable. Let's start with $x = 0$, $y = 0$, $z = 0$.

Step 3: Substitute the initial guesses into the rearranged equations to obtain new estimates for each variable.

...

$$x = (7 - 0 - 0)/3 = 7/3$$

$$y = (1 + 0 + 2(0))/4 = 1/4$$

$$z = (-15 + 2(0) - 0)/5 = -3$$

...

Step 4: Use the new estimates to update the original guesses for each variable.

...

$$x_{\text{new}} = 7/3$$

$$y_{\text{new}} = 1/4$$

$$z_{\text{new}} = -3$$

...

Step 5: Repeat steps 3 and 4 until the estimates for each variable converge to a solution. Let's perform one more iteration:

...

$$x = (7 - 1/4 - (-3))/3 = 10/3$$

$$y = (1 + 10/3 + 2(-3))/4 = -7/6$$

$$z = (-15 + 2(10/3) - (-7/6))/5 = -4/5$$

...

...

$$x_{\text{new}} = 10/3$$

$$y_{\text{new}} = -7/6$$

$$z_{\text{new}} = -4/5$$

...

We can continue iterating until the estimates converge to a solution that is accurate enough for our needs. In this case, we can check that the estimates are converging by computing the absolute difference between the current and previous estimates for each variable:

...

$$|x_{\text{new}} - x| = |10/3 - 7/3| = 1/3$$

$$|y_{\text{new}} - y| = |-7/6 - 1/4| = 5/6$$

$$|z_{\text{new}} - z| = |-4/5 - (-3)| = 1/5$$

...

Since all of these absolute differences are decreasing with each iteration, we can be confident that the estimates are converging. The final solution is:

...

$$x = 10/3$$

$$y = -7/6$$

$$z = -4/5$$

...

This solution satisfies all three of the original equations.