

The truncation error of a numerical method is the difference between the exact solution of a problem and the approximate solution obtained using the method, when the method is applied with a finite number of iterations or steps. Here are the truncation errors of some common numerical methods:

#### 1. Trapezoidal Rule:

The truncation error of the Trapezoidal Rule for approximating the definite integral of a function  $f(x)$  over the interval  $[a,b]$  is proportional to  $(b-a)^3/n^2$ , where  $n$  is the number of subintervals used. In other words, the error decreases as  $n$  increases and as the interval  $[a,b]$  becomes smaller.

#### 2. Simpson's Rule:

The truncation error of Simpson's Rule for approximating the definite integral of a function  $f(x)$  over the interval  $[a,b]$  is proportional to  $(b-a)^5/n^4$ , where  $n$  is the number of subintervals used. In other words, the error decreases more rapidly than the Trapezoidal Rule as  $n$  increases and as the interval  $[a,b]$  becomes smaller.

#### 3. Euler's Method:

The truncation error of Euler's Method for approximating the solution of a first-order ordinary differential equation  $y' = f(x,y)$  with initial condition  $y(x_0) = y_0$  is proportional to  $h^2$ , where  $h$  is the step size used in the method. In other words, the error decreases as the step size decreases, but the method is generally not very accurate for large values of  $h$ .

#### 4. Runge-Kutta Methods:

The truncation error of a Runge-Kutta Method for approximating the solution of a first-order ordinary differential equation is typically of order  $O(h^4)$  or higher, where  $h$  is the step size used in the method. In other words, the error decreases more rapidly than Euler's Method as the step size decreases, and the method can be very accurate for small values of  $h$ .

It's important to note that the truncation error is only one source of error in numerical methods, and there may be other sources of error such as rounding errors or errors in function evaluations. However, understanding the truncation error can give insight into how to choose appropriate values of parameters such as the number of subintervals or the step size to obtain accurate solutions.