Yes, I'd be happy to explain the step-by-step process for the Simpson's 1/3 rule.

The Simpson's 1/3 rule is a numerical integration method that approximates the definite integral of a function over an interval. It works by approximating the area under the curve of a function using quadratic polynomial interpolation.

Here are the steps involved in the Simpson's 1/3 rule:

1. Choose a function to integrate over a specified interval [a,b].
2. Divide the interval [a,b] into an even number of sub-intervals of equal width, denoted by h. This means that the number of sub-intervals is given by n = (b-a)/h, where n must be an even integer.
3. Evaluate the function at each of the endpoints of the sub-intervals and at the midpoint of each sub-interval. This will give us a set of n+1 function values.
4. Use the set of function values to construct quadratic polynomials that approximate the function over each sub-interval. To do this, we can use the Lagrange polynomial interpolation formula, which involves using three function values to construct a quadratic polynomial.
5. Integrate each quadratic polynomial over its corresponding sub-interval. This can be done analytically using integration rules for quadratics, or by approximating the integral using a numerical integration method such as the trapezoidal rule.
6. Add up the sub-interval integrals to get an approximation of the definite integral of the function over the interval [a,b].

The formula for the Simpson's 1/3 rule is:

$I \approx (h/3)[f(a) + 4f(a+h) + 2f(a+2h) + 4f(a+3h) + ... + 2f(b-h) + 4f(b-h) + f(b)]$,

where I is the approximation of the definite integral of the function over the interval [a,b], f(x) is the value of the function at x, h is the width of each sub-interval, and the coefficients 4 and 2 alternate for each sub-interval.

It's worth noting that the Simpson's 1/3 rule is only accurate when the function being integrated is smooth and has a continuous second derivative over the interval [a,b]. If the function is not smooth or has discontinuities, the Simpson's 1/3 rule may give inaccurate results.

Sure! Here's an example of how to use the Simpson's 1/3 rule to approximate the definite integral of a function over an interval:

Suppose we want to approximate the definite integral of the function f(x) = x^3 over the interval [0,2].

1. Choose the function f(x) = x^3 over the interval [0,2].

2. Divide the interval [0,2] into sub-intervals of equal width. Let's choose to use four sub-intervals, so h = (2-0)/4 = 0.5. Therefore, the sub-intervals are [0,0.5], [0.5,1], [1,1.5], and [1.5,2].

3. Evaluate the function at each endpoint and midpoint of the sub-intervals:

f(0) = 0

f(0.5) = (0.5)^3 = 0.125

f(1) = 1

f(1.5) = (1.5)^3 = 3.375

f(2) = 8

4. Use the set of function values to construct quadratic polynomials that approximate the function over each sub-interval. For example, the quadratic polynomial that approximates f(x) over the interval [0,0.5] is:

p(x) = f(0)[(x-0.5)(x-1.5)]/[(0-0.5)(0-1.5)] + f(0.5)[(x-0)(x-1.5)]/[(0.5-0)(0.5-1.5)] + f(1)[(x-0)(x-0.5)]/[(1-0)(1-0.5)]

Evaluating this polynomial at x = 0 and x = 0.5 gives:

p(0) = 0

p(0.5) = 0.125

Similarly, we can construct quadratic polynomials for the other sub-intervals.

5. Integrate each quadratic polynomial over its corresponding sub-interval. For example, the integral of the quadratic polynomial that approximates f(x) over the interval [0,0.5] is:

$\int(0 \text{ to } 0.5)\ p(x)\ dx = \int(0 \text{ to } 0.5)\ [0.25x^2 - 0.375x + 0.125]\ dx = 0.015625$

We can repeat this process for each sub-interval to get the integrals of the corresponding quadratic polynomials.

6. Add up the sub-interval integrals to get an approximation of the definite integral of the function over the interval [0,2].

Using the Simpson's 1/3 rule formula:

$I \approx (h/3)[f(0) + 4f(0.5) + 2f(1) + 4f(1.5) + f(2)]$

$I \approx (0.5/3)[0 + 4(0.125) + 2(1) + 4(3.375) + 8] = 8.5$

Therefore, the Simpson's 1/3 rule gives an approximation of the definite integral of f(x) over [0,2] as 8.5. The actual value of the definite integral is 8, so the approximation is quite accurate.