

CS 3320 – Numerical Software
Module 9 Homework

1. The following system of equations is designed to determine the concentrations (c 's in g/m^3) in a series of coupled, well-mixed tanks as a function of mass input to each tank. The right-hand side of the equations below represent these inputs in g/day .

$$15c_1 - 3c_2 - c_3 = 4000$$

$$-3c_1 + 18c_2 - 6c_3 = 1200$$

$$-4c_1 - c_2 + 12c_3 = 2350$$

- a. (5 pt) Determine the inverse of the coefficient matrix. (You can use Python to solve the inverse.)
- b. (10 pt) Use the inverse to determine the solution. (Do this by hand.)
- c. (10 pt) Determine how much the rate of mass input to tank 3 must increase to induce a 10 g/m^3 rise in the concentration in tank 1.

2. (10 pt) Determine $\|A\|_f$, $\|A\|_1$, and $\|A\|_\infty$ for

$$A = \begin{bmatrix} 8 & 2 & -10 \\ -9 & 1 & 3 \\ 15 & -1 & 6 \end{bmatrix}.$$

3. (10 pt) Solve the following system using three iterations of the Gauss-Seidel method. If necessary, rearrange the equations. Show all the steps in your solution. At the end of your computation, compute the true error of your final results. (Do this by hand.)

$$3x_1 + 8x_2 = 11$$

$$7x_1 - x_2 = 5$$

4. (10 pt) Use the Gauss-Seidel method (a) without relaxation and (b) with relaxation ($\lambda = 1.2$) to solve the following set of linear equation to meet an error tolerance of $\varepsilon_s = 5\%$. If necessary, rearrange the equations to achieve convergence.

$$2x_1 - 6x_2 - x_3 = -38$$

$$-3x_1 - x_2 + 7x_3 = -34$$

$$-8x_1 + x_2 - 2x_3 = -20$$