Sure, I can provide you with the step-by-step process to formulate the Normal Equation for the least square curve fit along with an example.

Step 1: Gather Data

- Collect the data points that you want to fit a curve to. Let's say we have the following data points:

$(x1, y1), (x2, y2), (x3, y3), \ldots, (xn, yn)$

Step 2: Define the Model

- Choose a model that you believe will fit the data. In this example, we will use a linear model of the form:

$y = mx + b$

- We want to find the values of m and b that best fit the data.

Step 3: Formulate the Objective Function

- We want to minimize the sum of the squared errors between the model predictions and the actual data. The squared error for each point is:

$(y - mx - b)^2$

- The objective function is the sum of the squared errors over all the data points:

$J(m,b) = \Sigma(y - mx - b)^2$

Step 4: Minimize the Objective Function

- To minimize the objective function, we take the partial derivatives of J with respect to m and b and set them equal to zero:

$\partial J/\partial m = -2\Sigma x(y - mx - b) = 0$

$\partial J/\partial b = -2\Sigma(y - mx - b) = 0$

- Solving for m and b gives us the normal equations:

$m = (\Sigma xy - \Sigma x\Sigma y/n) / (\Sigma x^2 - \Sigma x^2/n)$

$b = (\Sigma y - m\Sigma x) / n$

Step 5: Apply the Model to New Data

- Once we have found the values of m and b that minimize the objective function, we can use the model to make predictions for new values of x.

Example:

Suppose we have the following data points:

(1, 3), (2, 5), (3, 7), (4, 9)

We want to fit a linear model to this data.

Step 1: Gather Data

- We have the data points: (1, 3), (2, 5), (3, 7), (4, 9)

Step 2: Define the Model

- We will use a linear model of the form: $y = mx + b$

Step 3: Formulate the Objective Function

- The objective function is: $J(m,b) = \Sigma(y - mx - b)^2$

- Substituting in the data points gives:

$J(m,b) = (3 - m - b)^2 + (5 - 2m - b)^2 + (7 - 3m - b)^2 + (9 - 4m - b)^2$

Step 4: Minimize the Objective Function

- Taking the partial derivatives of J with respect to m and b and setting them equal to zero gives us the normal equations:

$m = 2$

$b = 1$

Step 5: Apply the Model to New Data

- The model we found is: $y = 2x + 1$

- If we plug in $x = 5$, we get $y = 11$.

Therefore, the linear model that best fits the given data points is $y = 2x + 1$, and if we want to predict the value of y for a new value of x, we can use this equation.