

Certainly! The midpoint rule is a numerical integration technique used to approximate the definite integral of a function. Here are the step-by-step instructions for using the midpoint rule:

1. Choose the function you want to integrate and the interval $[a, b]$ over which you want to integrate it.
2. Determine the number of subintervals, n , into which you want to divide the interval $[a, b]$. This will depend on the accuracy you want to achieve and the computational resources available.
3. Calculate the width of each subinterval, h , using the formula $h = (b - a) / n$.
4. Choose a set of points within each subinterval to evaluate the function. For the midpoint rule, you will choose the midpoint of each subinterval. The midpoint of the i th subinterval is given by $x_i = a + (i - 1/2) * h$.
5. Evaluate the function at each of the chosen points. That is, calculate $f(x_i)$ for $i = 1, 2, \dots, n$.
6. Approximate the definite integral of the function over the interval $[a, b]$ using the formula:

$$\int_{\text{from } a \text{ to } b} f(x) \, dx \approx h * (f(x_1) + f(x_2) + \dots + f(x_n))$$

That is, you multiply the width of each subinterval by the average value of the function over that subinterval, which is approximated by the function value at the midpoint of the subinterval.

As an example, suppose you want to approximate the definite integral of the function $f(x) = x^2$ over the interval $[0, 2]$ using the midpoint rule with 4 subintervals.

1. Function: $f(x) = x^2$, interval: $[0, 2]$
2. Number of subintervals: $n = 4$
3. Width of each subinterval: $h = (2 - 0) / 4 = 0.5$
4. Midpoints of the subintervals:
 - $x_1 = 0.25$
 - $x_2 = 0.75$
 - $x_3 = 1.25$
 - $x_4 = 1.75$
5. Function values at the midpoints:
 - $f(x_1) = 0.0625$

$$- f(x_2) = 0.5625$$

$$- f(x_3) = 1.5625$$

$$- f(x_4) = 3.0625$$

6. Approximation of the definite integral:

$$\int_{\text{from } 0 \text{ to } 2} x^2 \, dx \approx 0.5 * (0.0625 + 0.5625 + 1.5625 + 3.0625) = 2.25$$