In numerical software, composite methods are numerical integration techniques that use a combination of multiple smaller subintervals to approximate the integral of a function over a larger interval. Composite methods are used when the interval to be integrated is too large to use a single quadrature rule, or when a higher degree of accuracy is desired.

The basic idea of composite methods is to divide the interval of integration into multiple smaller subintervals, and then use a quadrature rule to approximate the integral over each subinterval. The approximations for each subinterval are then combined in a certain way to give an overall approximation for the entire interval.

There are several types of composite methods, including composite trapezoidal rule, composite Simpson's rule, and composite midpoint rule. In each case, the subintervals are divided into equally sized pieces, and a quadrature rule is used to approximate the integral over each subinterval. The approximations are then combined using a weighted sum to give an overall approximation for the entire interval.

Composite methods are often used in numerical software because they can provide accurate approximations for integrals over large intervals without requiring excessive computation time or memory. They are also useful in cases where the integrand is highly oscillatory or has sharp peaks, which can make it difficult to accurately integrate using a single quadrature rule.

1. Composite Trapezoidal Rule: In this method, the interval of integration is divided into multiple equally sized subintervals, and the trapezoidal rule is used to approximate the integral over each subinterval. The approximations are then combined using a weighted sum to give an overall approximation for the entire interval.

2. Composite Simpson's Rule: This method is similar to the composite trapezoidal rule, but instead of using the trapezoidal rule, Simpson's rule is used to approximate the integral over each subinterval.

3. Composite Midpoint Rule: In this method, the interval of integration is divided into multiple equally sized subintervals, and the midpoint rule is used to approximate the integral over each subinterval. The approximations are then combined using a weighted sum to give an overall approximation for the entire interval.

4. Composite Gauss-Legendre Rule: This method is a more advanced composite method that uses a higher degree of accuracy than the previous methods. In this method, the interval of integration is divided into multiple equally sized subintervals, and the Gauss-Legendre quadrature rule is used to

approximate the integral over each subinterval. The approximations are then combined using a weighted sum to give an overall approximation for the entire interval.

All of these methods are commonly used in numerical software for approximating integrals of functions over large intervals. The choice of method depends on the desired level of accuracy, the complexity of the function being integrated, and other factors such as computational resources and memory limitations.

Sure, here are some examples of using Simpson's rule to approximate definite integrals:

Example 1:

Approximate the definite integral of $f(x) = x^2$ over the interval $[0,2]$ using Simpson's rule with 4 subintervals.

Solution:

We need to divide the interval $[0,2]$ into 4 subintervals, which requires 5 equally spaced points:

$x_0 = 0$, $x_1 = 0.5$, $x_2 = 1$, $x_3 = 1.5$, $x_4 = 2$.

Then, for each adjacent pair of subintervals, we apply Simpson's rule using the formula:

$\int(x_{j-1}, x_j) f(x)\, dx \approx (\Delta x/3) [f(x_{j-1}) + 4f(x_{j-1/2}) + f(x_j)]$

Using this formula for each pair of subintervals and summing the results, we get:

$\int(0,2) x^2\, dx \approx (0.5/3) [0^2 + 4(0.5)^2 + 1^2 + 4(1.5)^2 + 2^2]$

≈ 2.6667

Example 2:

Approximate the definite integral of $f(x) = \sin(x)$ over the interval $[0,\pi/2]$ using Simpson's rule with 6 subintervals.

Solution:

We need to divide the interval $[0,\pi/2]$ into 6 subintervals, which requires 7 equally spaced points:

$x0 = 0$, $x1 = \pi/12$, $x2 = \pi/6$, $x3 = \pi/4$, $x4 = \pi/3$, $x5 = 5\pi/12$, $x6 = \pi/2$.

Then, applying Simpson's rule to each pair of adjacent subintervals and summing the results, we get:

$\int(0,\pi/2) \sin(x)\, dx \approx (\pi/12) [\sin(0) + 4\sin(\pi/12) + 2\sin(\pi/6) + 4\sin(\pi/4) + 2\sin(\pi/3) + 4\sin(5\pi/12) + \sin(\pi/2)]$

≈ 0.9999

Note that the exact value of the integral is 1, so Simpson's rule with 6 subintervals gives a very accurate approximation.