CS 3320 – Numerical Software

Module 1 Homework

Key

Do the following problems in the textbook.

1. (10 pt) Problem 1.6

   The solution to the free-fall bungee jumper with linear drag coefficient is:

$$v(t) = \frac{mg}{c'}\left(1 - e^{-\frac{c'}{m}t}\right)$$

Solution:

Jumper 1: $m = 70$ kg and $c' = 12$ kg/s. Velocity at $t = 9$:

$$v(9) = \frac{70\cdot 9.81}{12}\left(1 - e^{-\frac{12}{70}\cdot 9}\right) = 57.225(1 - 0.2138) = 44.9903 \text{ m/s}$$

Jumper 2: $m = 80$ kg and $c' = 15$ kg/s. Time to reach velocity 44.9903 m/s

$$44.9903 = \frac{80\cdot 9.81}{15}\left(1 - e^{-\frac{15}{80}t}\right)$$

$$\frac{44.9903\cdot 15}{80\cdot 9.81} = 1 - e^{-\frac{15}{80}t}$$

$$e^{-\frac{15}{80}t} = 1 - 0.8599 = 0.1401$$

Take natural log on both sides:

$$-\frac{15}{80}t = \ln(0.1401) = -1.9654$$

$$t = 1.9654\cdot\frac{80}{15} = \underline{10.4824}\,sec$$

2. (10 pt) Problem 1.15

   Instead of from t=0 to t=10 min, solve the problem from t=0 to t=1 min. (Solve the problem by hand.)

Solution:

Given $\frac{dV}{dt} = -kA$, with $k = 0.08$ mm/min, the Euler's method for this equation is

$$V(t + \Delta t) = V(t) + (-kA)\cdot\Delta t$$

For a sphere: Volume (V) and the surface area (A) can be calculated as

$$V = \frac{4}{3}\pi r^3; A = 4\pi r^2$$

The iteration process is tabulated below:

Time	$r(t)$	$V(t)$	$A(t)$	$-kA$	$V(t+\Delta t)$
0.0	2.5	65.4498	78.5398	–6.2832	63.8791
0.25	2.4798	63.8791	77.2781	–6.1822	62.3335
0.50	2.4597	62.3335	76.0265	–6.0821	60.8130
0.75	2.4395	60.8130	74.7851	–5.9828	59.3173
1.00	2.4193	59.3173	73.5538	–5.8843	57.8462

Average evaporate rate $k_{avg} = \frac{2.5 - 2.4193}{1} = 0.0807\ mm/min$

Alternative approach:

Since $V = \frac{4}{3}\pi r^3$ and $A = 4\pi r^2$, $A = 4\pi(\frac{3V}{4\pi})^{\frac{2}{3}}$. Therefore,

$$\frac{dV}{dt} = -4k\pi(\frac{3V}{4\pi})^{\frac{2}{3}}$$

$V(0) = \frac{4}{3}\pi(2.5)^3 = 65.4498$.

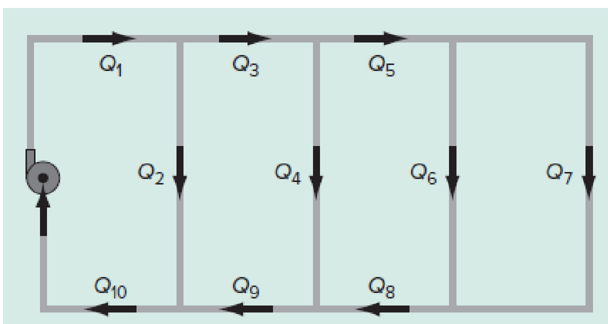
Time	$V(t)$	$\frac{dV}{dt}$	$V(t + \Delta t)$
0.0	65.4498	–6.2832	63.8791
0.25	63.8791	–6.1822	62.3335
0.50	62.3335	–6.0821	60.8130
0.75	60.8130	–5.9828	59.3173
1.00	59.3173	–5.8843	57.8462

Validation step is the same as the previous approach.

3. (10 pt) Problem 1.16

Solution:

FIGURE P1.16



From continuity at each node:

$Q_1 = Q_2 + Q_3 = 0.7 + 0.5 = 1.2\ m^3/s$

$Q_{10} = Q_1 = 1.2\ m^3/s$

$Q_9 = Q_{10} - Q_2 = 1.2 - 0.7 = 0.5\ m^3/s$

$Q_4 = Q_9 - Q_8 = 0.5 - 0.3 = 0.2\ m^3/s$

$Q_5 = Q_3 - Q_4 = 0.5 - 0.2 = 0.3\ m^3/s$

$Q_6 = Q_5 - Q_7 = 0.3 - 0.1 = 0.2\ m^3/s$

4. (10 pt) Problem 1.17

Solution:

You can use a spreadsheet application or a python program to solve this problem. However, your answer should include the coffer temperature at each time step.

Given $\frac{dT}{dt} = -k(T - T_a)$, with $k = 0.019/min$, $T(0) = 70$°C, and $T_a = 20$°C.

Euler's method: $T(t + \Delta t) = T(t) + \frac{dT}{dt}\Delta t$.

Time	$T(t)$	$T(t) - T_a$	$k * (T(t) - T_a)$	$T(t + \Delta t)$
0	70.0000	50.0000	1.9000	68.1000
2	68.1000	48.1000	1.8278	66.2722
4	66.2722	46.2722	1.7583	64.5139
6	64.5139	44.5139	1.6915	62.8223
8	62.8223	42.8223	1.6272	61.1951
10	61.1951	41.1951	1.5654	59.6297
12	59.6297	39.6297	1.5059	58.1237
14	58.1237	38.1237	1.4487	56.6750
16	56.6750	36.6750	1.3937	55.2814
18	55.2814	35.2814	1.3407	53.9407
20	53.9407	33.9407	1.2897	52.6509

5. (10 pt) Problem 1.24
   Hint: The weight of the ice sphere is balanced with the buoyancy force. The buoyancy force is equal to the weight of the seawater displaced by the ice sphere.

   Solution:

   Weight of the ice sphere: $\rho_s g(\frac{4}{3}\pi r^3)$

   Submerged volume: (volume of the sphere) – (volume above the water)

   Volume above the water: $V_a = \int_0^h A(h)dh = \int_0^h \pi(r^2 - (r-h)^2)dh$

$$V_a = \pi \int_0^h (2rh - h^2)dh = \pi\left(rh^2 - \frac{1}{3}h^3\right) = \frac{\pi h^2}{3}(3r - h)$$

   Buoyancy Force: $\rho_f g\left(\frac{4}{3}\pi r^3 - \frac{\pi h^2}{3}(3r - h)\right)$.

   Force balance:

$$\rho_s g\left(\frac{4}{3}\pi r^3\right) - \rho_f g\left(\frac{4}{3}\pi r^3 - \frac{\pi h^2}{3}(3r - h)\right) = 0$$

   Since $\pi \neq 0$ and $g \neq 0$,

$$\rho_s\left(\frac{4}{3}r^3\right) - \rho_f\left(\frac{4}{3}r^3 - \frac{h^2}{3}(3r - h)\right) = 0$$

$$\frac{4}{3}r^3(\rho_s - \rho_f) + \rho_f\left(\frac{h^2}{3}(3r - h)\right) = 0$$

$$\frac{4}{3}r^3(\rho_s - \rho_f) + \rho_f r h^2 - \frac{\rho_f}{3}h^3 = 0$$