CS 3320
Final
Review Sheet and Practice Problem

Basic Info:
Closed-book, open note (1 full page of 8 ½ x 11 single sided note), timed (210 minutes), no proctorio. Covers modules 8 – 13.
Three sections:

- Computer (auto) graded problems: 13 problems with 4 points each.
- Manually graded problems: 7 problems total 48 points. You can work on your solution on a piece of paper then upload the image to Canvas. However, make sure your handwriting is legible and easy to follow.
- Bonus problems: optional, 1 or 2 problems with 5 points each.

Topics:
Module 8 Systems of Equations I – Gaussian Elimination, LU factorization

- Perform Gaussian elimination manually
- Difference between Naïve Gaussian Elimination and Gaussian Elimination with partial pivot
- Computational resources (time/operation wise) required for Gaussian Elimination
- Procedure of finding LU factorization
- Solution procedure using LU factorization
- Definition and purpose of permutation matrices

Module 9 Systems of Equations II – Matrix/Vector Norms, Gauss-Seidel Iteration with/without relaxation

- Find the various norms of a given matrix/vector
- Find the inverse of a matrix using Gaussian elimination/LU Decomposition
- Identify if Gauss-Seidel with converge using the diagonally dominant criteria
- Perform Gauss-Seidel iteration manually

Module 10 Least Squares Curve Fit

- Formulate Normal Equation for least square curve fit
- Find the polynomial least square fit of various order
- Find the SS_E of a polynomial fit
- Linearize exponential, power law, and other nonlinear models to use first order polynomial least square fit

Module 11 Interpolation

- Use the following methods for to find interpolating polynomials: straight forward polynomial interpolation, Newton Interpolation polynomials (Divided Differences), Lagrange Interpolation polynomials.
- Discuss the advantages/disadvantages using polynomial interpolation versus spline interpolations
- Procedure of solving quadratic/cubic spline interpolation

- Meaning of various end conditions for quadratic/cubic splines

Module 12 Numerical Integration

- Understand and able to use the following methods: Midpoint rule, trapezoidal rule, Simpson's 1/3 rule, Simpson's 3/8 rule, and adaptive quadrature (with trapezoidal and Simpson's 1/3 rule
- Composite methods
- Number of intervals required for Simpson's rules
- Truncation Errors of various methods

Module 13 Random Numbers

- Basic Statistics: Mean (average), median, mode, etc.
- Use Linear Congruential Generator to create uniform random numbers
- Use Revised Box-Mueller Method to transform uniform random numbers to normal random numbers
- Apply Monte-Carlo simulation
- Use Python random number functions

Sample questions for manually graded portion.

1. Use Lagrange interpolation to find a polynomial that passes through the following points: (1, 4), (2, 7), (3, 20).

2. Use Newton Divided Differences to find the Newton Interpolation Polynomial passes through (0, 1), (1, -5), (2, -9), and (3, -5)

3. Given four data points (0, 2), (1, 3), (2, 1), and (3, 12), the least squares second order polynomial fit is $y = 2.5x^2 - 4.7x + 2.8$. Find the SS_E of the solution.

4. Find the first three random numbers (to the fifth digits after the decimal point) using the Linear Congruent Generator with $a = 3, m = 17, b = 0$, and 11 as the seed.

5. Fit the data in the following table to the exponential model, $y = e^{(x-b)/a}$. In other words, find a and b to the fourth place after the decimal point.

x	1	2	3	4	5
y	0.5	2	2.9	3.5	4

6. Find the area under the function $f(x) = xe^{-x/2}$ from $a = 1$ to $b = 4$ using Simpson's 3/8 rule with $h = 0.5$. (Report your answer to the fourth place after the decimal point.)

7. Given the following system of equations, what is the approximated solution after two iterations using Gauss-Seidel method with $x = 1, y = 0, and\ z = 2$ as the initial guess. (Make sure the algorithm with converge to the solution before you start the iterative process. Report your answer to the fourth place after the decimal point or in fraction.)
   $x - 2y + 4z = -4$
   $3x - y - z = 6$
   $2x + 5y - 2z = 11$