2.1.2(b): 4n + 3

1. 4n + 3 = 2k + 1
2. 4n + 2 = 2k
3. 2n + 1 = k
4. k = 2n + 1
5. 4n + 3 = 2(2n + 1) + 1
6. 4n + 3 = 4n + 2 + 1
7. 4n + 3 = 4n + 3
8. Since 2k+ 1 = odd and 4n + 3 = 2k + 1; **Therefore 4n + 3 = odd**

2.1.2(c): 10n3 + 8n – 4

1. 10n3 + 8n – 4 = 2k
2. k = 5n3 +4n - 2
3. 10n3 + 8n – 4 = 2(5n3 +4n - 2)
4. 10n3 + 8n – 4 = 10n3 + 8n – 4
5. Since 2k = even and 10n3 + 8n – 4 = 2k; **Therefore 10n3 + 8n – 4 = even**

2.1.6(a): It is not true that x < 7

1. **x ≥ 7**

2.1.6(d): it is not true that x ≥ 7

1. **x < 7**
2. **sdfsdfsdf**
3. **sdfsdf**

2.2.2(b): For every integer n such that 0 ≤ n ≤ 4, 2(n+2) > 3n

1. n = 0, 2(n + 2) > 3n
   1. 2(0 + 2) > 30
   2. 2(2) > 1
   3. 4> 1
   4. true
2. n = 1, 2(n + 2) > 3n
   1. 2(1 + 2) > 31
   2. 2(3) > 3
   3. 8> 3
   4. true
3. n = 2, 2(n + 2) > 3n
   1. 2(2 + 2) > 32
   2. 2(4) > 9
   3. 16 > 9
   4. true
4. n = 3, 2(n + 2) > 3n
   1. 2(3 + 2) > 33
   2. 2(5) > 27
   3. 32 > 27
5. n = 4, 2(n + 2) > 3n
   1. 2(4 + 2) > 34
   2. 2(6) > 81
   3. 64 > 81
   4. false
6. For every integer n such that 0 ≤ n ≤ 4, 2(n+2) > 3n 
   1. **False**

2.2.3(c): For every positive integer x, x3 < 2x

1. x = 1
2. 13 < 21
3. 1 < 2
4. counter example, x = 1

2.2.4(b): There is no largest integer

**Let x be an integer, x will always be smaller than x + 1, Therefore x cannot be the largest integer**

2.2.5(e) There are three positive integers, x, y, and z, that satisfy x2 + y2 = z2

1. ⱻxⱻyⱻz(x2 + y2 = z2)
2. x = 1, y = 1, z = 2
3. 12 + 12 = 22
4. 1 + 1 = 2
5. 2 = 2

2.3.2(a) **Proof.**

Let w, x, y, z be integers such that w divides x and y divides z.

Since, by assumption, w divides x, then x = kw for some integer k and w ≠ 0. Since, by assumption, y divides z, then z = ky for some integer k and y ≠ 0. Plug in the expression kw for x and ky for z in the expression xz to get xz=(kw)(ky)=(k2)(wy)

Since k is an integer, then k2 is also an integer.

Since w ≠ 0 and y ≠ 0, then wy ≠ 0.

Since xz equals wy times an integer and wy ≠ 0, then wy divides xz. ■

**The calculation xz=(kw)(ky)=(k2)(wy) is incorrect. xz=(kw)(ky) = k2wy**

2.3.3(a) **Theorem:** If n and m are odd integers, then n2 + m2 is even

**Proof.**

m = 7 is odd because 7 = 2·3+1. n = 9 is odd because 9 = 2·4 + 1.

72+92=49+81=130=2⋅65

Since 72 + 92 is equal to 2 times an integer, 72 + 92 is even. Therefore the theorem is true. ■

**The proof is generalizing from examples, just because the theorem holds true for one set of integers does not mean that it holds true for all sets of integers**

2.4.1(c)   
Theorem: The square of an odd integer is an odd integer.

Proof:

**Suppose x is an odd integer**

**As x is odd, ⱻk so that x = 2k + 1**

**Since x = 2k + 1 then x2 = (2k + 1)2**

**(2k + 1)(2k + 1) = 4k2 + 4k + 1 = 2(2k2 + 2k) + 1**

**Let j be some integer such that j = 2k2 + 2k**

**Thus x2 = 2j + 1**

**Therefore x2 is odd ▪**

2.4.3(b)

Theorem: If x is a real number and x≤3, then 12−7x+x2≥0.

Proof:

**Suppose x is a real number and x ≤ 3**

**If we factor 12 – 7x + x2 ≥ 0 we get (3 – x)(4 – x) ≥ 0**

**Since x is a real number, any number added/subtracted from x is also a real number**

**Therefore (4-x) is a real number, 0 divided by any real number is 0**

**Therefore 3 – x ≥ 0**

**Using algebra, 3 ≥ x or x ≤ 3**

**Therefore 12 – 7x + x2 ≥ 0 ▪**

2.4.4(e)

Theorem: If x and y are positive real numbers and x<y, then x2<y2.

Proof:

**Suppose x and y are positive real numbers**

**Since positive numbers are not negative numbers**

**Thus x ≥ 0 and y≥ 0**

**Since x < y are positive real numbers you can multiply both sides by 2**

**Therefore x2 < y2 ▪**

2.5.1(d)

Theorem: For every integer n, if n2−2n+7 is even, then n is odd

Proof:

**if n is even, then n2 – 2n + 7 is odd**

**The integer n is even, by definition there is some integer k that n = 2k**

**Therefore (2k)2 – 2(2k) + 7, simplified this equals, 4k2 – 4k + 7**

**Since n2 – 2n + 7 is odd, therefore 4k2 – 4k + 7 is also odd, by definition there is some integer j that 4k2 – 4k + 7 = 2j + 1**

**j = 2k2  - 2k + 3**

**Therefore 4k2 – 4k + 7 = 2 (2k2- 2k + 3) + 1**

**Since 4k2 – 4k + 7 = 4k2 – 4k + 7 then we can conclude that if n is even then n2 – 2n + 7 is odd ▪**

2.5.4(c)

Theorem: For every pair of positive real numbers x and y, if xy>400, then x>20 or y>20

Proof:

**For every pair of positive real numbers x and y, if x < 20 and y < 20, then xy < 400**

**if x < 20 and y < 20 then by assumption x \* y < 20 \* 20**

**Therefore xy < 400 ▪**

2.6.1(a)

Theorem: √2/2 is irrational.

Proof:

**Suppose √2/2 is rational**

**By definition, a number is rational if there exists integers x an y such that y ≠ 0 and r = x/y**

**Let r = √2/2, therefore √2/2 = x/y**

**Square both sides, 2/4 = x2/y2**

**multiply both sides by y2, y22/4 = x2**

**multiply both sides by 4, 2y2 = 4x2 ▪**

**Since x is equal to two times another number then by assumption, it is even, by definition x = 2k**

**Since y is equal to an even number, then it is also an even number, by definition y = 2j**

**Therefore 2(2j)2 = 4(2k)2, simplified, 8j2 = 16k2**

**By assumption, using algebra, 2/4 = k2/j2, then take the square root of both sides**

**By assumption, using algebra √2/2 = k/j, this is back where proof started**

**√2/2 creates an infinite loops, therefore it is not rational ▪**

2.6.4(d)

Theorem: Among any group of 1000 people, at least three of the people have the same birthday.

Proof:

**Suppose in a group of 1000 no more than two people two have the same birthday**

**By assumption, a year has 365 days**

**Therefore after 745 days for three people to not share a birthday two people would have to share a birthday on each day of the year.**

**Thus on the 746-day three people would have to share a birthday at a minimum**

**Since 1000 > 746 then at least three people would have to have same birthday ▪**

2.7.1(b)

Theorem: for every integer n, n2 ≥ n

**Case 1: n is 1 or 0**

**Since n is 1, then 12 ≥ 1**

**Since n is 0, then 02 ≥ 0**

**Case 2: n is not 1 or 0**

**For every integer that is not 1 or 0, by definition, multiplication, if an integer is multiplied by an integer of the same sign, the integer will get larger.**

**Therefore n2 ≥ n ▪**

2.7.3(c)

Theorem:

Proof:  
**For all real numbers x and y, |x−y|=|y−x|.**

**By assumption, when you subtract two numbers the order in which you subtract the two numbers only affects the sign that will be attached to said number**

**Therefore the absolute value of two numbers will always be the same no matter which order they are subtracted in**

**In other words |x – y| = |y – x|▪**