7.1.1(e)

1, 4, 7, 10, 13, 16, 19, 22, 25, 28

increasing, non-decreasing

7.1.2(b)

non-decreasing

7.2.1(a)

1, 2, 2, 4, 8, 32

7.3.1(e)

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7.3.2(a)

7.4.2(b)

**Theorem**: = (n – 1)2n+1 + 2, for any positive integer n

**Proof.**

By induction on n.

**Base Case:** n = 1

When n = 1, the left side of the equation is

When n = 1, the right side of the equation is (1-1)21+1 + 2 = 2

Therefore = (1-1)21+1 + 2

**Inductive step:** Suppose that for positive integer k, (k – 1)2k+1 + 2,

then we will show that = k2k+2 + 2

starting with the left side of the equation to be proven:

= by separating out the last term

= (k-1)2k+1 + 2 +(k + 1) by the inductive hypothesis

= k2k+2 + 2 by algebra

Therefore, = k2k + 2 + 2 ∎

7.5.1(b)

**Theorem:** For every positive integer n, 6 evenly divides 7n – 1.

**Proof:** By induction on n.

**Base case:** n = 1

71 – 1 =6. Since 6 evenly divides 6, the theorem holds for the case n = 1

**Inductive step:** Suppose that for positive integer k, 6 evenly divides 7n – 1.

Then we will show that 6 evenly divides 7k+1 – 1.

By the inductive hypothesis, 6 evenly divides 7k – 1, which means that 7k – 1 = 6m for some integer m.

By adding 1 to both sides of the equation 6m =7k – 1, we get 7k = 6m + 1 which is an equivalent statement of the inductive hypothesis.

We must show that 7k+1 – 1 can be expressed as 6 times an integer.

7k+1 – 1 = (6m+1) -1 = 6m

Since m is an integer, (6m) is also an integer.

Therefore 7k+1 – 1 is equal to 3 times an integer which means that 7k+1 – 1 is divisible by 6 ∎

7.5.3(b)

**Theorem:** for n ≥ 0, bn = 2n+1 – 1.

**Proof:** By induction on n.

**Base case:** n = 1

b1 = 2(b1-1) + 1 = 2(1) + 1 = 3

b1 = 21+1 – 1 = 3

21+1 – 1 = 2(b1-1) + 1

**Inductive step:** Suppose that for k ≥ 0, bk = 2k+1 – 1.

Then we will show that bk+1 = 2(k+1)+1 – 1

By the inductive hypothesis, bk = 2k+1 - 1, which means that 2(k+1)+1 – 1 = 2(bk) + 1

21+1+1 – 1 = 7

2(b1) + 1 = 7

therefore 2(k+1)+1 – 1 = 2(bk) + 1∎

7.6.1(a)

**Theorem:** any amount of postage worth 8 cents or more can be made from 3-cent or 5-cent stamps

**Proof. By strong induction**

**Base case: Prove P(8), P(9), P(10) directly**

**Inductive step: for** k ≥ 10, assume that P(j) is true for any j in range 8 through k

P(k-2) falls under range of 8 through k because k ≥ 10 therefore k-2 ≥ 8∎

7.8.2(d)

Base case:

λ ∈ bCount(x)

Recursive rule:

if x ∈ bCount(x)

x1 ∈ bCount(x)

7.8.4(c)

Base case: |λ| = 0

Recursive rule:

same number of 0’s and 1‘s

7.9.2(b)

a is the only base case, therefore every x is going to be an s. There is one x in all of the rules, therefore every string in S will contain exactly one a

7.10.1(a)

g(n):

g0 = 0

gn = gn-1 + n3

7.11.1(a)

g1 = 1

gn = n3 + gn-1

assume gk = k3 + gk – 1, then gk+1 = (k+1)3 + gk

g2+1 = (2+1)3 + g2 = 36

g3 = 33 + g2 = 36

gk+1 = (k+1)3 + gk = gn = n3 + gn-1

7.11.10(a)

if x and y are ≥0

FastMultx,0 = 0

FastMultx,y = FastMult(x,⌊y/2⌋)

if y is an even return 2p

else return 2p + x

7.12.5(a)

T(n) = T(n-1) + T(n-2) + O(1)

7.13.4(b)

Best run time: 15

Worst run time: 35