1.6.2(b): ⱻx (x + 2 = 1), D{All Int}

**True, x = -1**

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1.6.2(c): Ɐx (x2 – x ≠ 1), D{All Int}

**True**

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1.6.3(d): Every number is less than or equal to its square, D{All real numbers}

**Ɐx(x ≤ x2)**

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1.6.4(b): ⱻx P(x), D{a,b,c,d}

|  |  |  |  |
| --- | --- | --- | --- |
|  | P | Q | R |
| a | T | T | F |
| b | T | F | F |
| c | T | F | F |
| d | T | F | F |

**True, P(a) = T**

---------------------------------------------------------------------------------------------------------------------

1.6.4(c): Ɐx Q(x)

|  |  |  |  |
| --- | --- | --- | --- |
|  | P | Q | R |
| a | T | T | F |
| b | T | F | F |
| c | T | F | F |
| d | T | F | F |

**False, Q(b) = F**

---------------------------------------------------------------------------------------------------------------------

1.7.6(a): Ɐx(A(x) → E(x)), D{employees of company}

A(x): x is on the board of directors. (Note: members of the board of directors are also employees.)

E(x): x earns more than $100,000

W(x): x works more than 60 hours per week

**Every employee that is on the board of directors earns more than $100,000**

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1.7.6(f): ∃x (A(x) ∧ ¬E(x) ∧ W(x)), D{employees of company}

A(x): x is on the board of directors. (Note: members of the board of directors are also employees.)

E(x): x earns more than $100,000

W(x): x works more than 60 hours per week

**There is an employee that is on the board of directors and an employee that doesn’t make more than $100,000 and an employee that works more than 60 hours per week**

1.7.10(a): ∀x ¬(O(x) ↔ D(x)), D{All club members}

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Name | M(x) | O(x) | D(x) |  |
| Hillary | T | F | T |  |
| Bernie | F | T | F |  |
| Donald | F | T | F |  |
| Jeb | F | T | T |  |
| Carly | F | T | F |  |

M(x): person x came to meeting on time

O(x): person x is an officer of the club

D(x): person x has paid his/her club dues  
**False, ¬(O(Jeb) ↔ D(Jeb)) = F**

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1.7.10(d): ∃x (M(x) ∧ D(x)), D{All club members}

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Name | M(x) | O(x) | D(x) |  |
| Hillary | T | F | T |  |
| Bernie | F | T | F |  |
| Donald | F | T | F |  |
| Jeb | F | T | T |  |
| Carly | F | T | F |  |

M(x): person x came to meeting on time

O(x): person x is an officer of the club

D(x): person x has paid his/her club dues

**True, (M(Hillary) ∧ D(Hillary)) = T**

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1.8.2(a): Every patient was given the medication, D{All patients}

P(x): x was given the placebo

D(x): x was given the medication

M(x): x had migraines

**- Ɐx(D(x))**

**-** Negation: **⌐ Ɐx(D(x))**

**-** Applying De Morgan’s law: **ⱻx(⌐D(x))**

-English: **This is a patient who was not given the medication**

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1.8.2(c): There is a patient who took the medication and had migraines, D{All patients}

P(x): x was given the placebo

D(x): x was given the medication

M(x): x had migraines

**- ⱻx(D(x) ꓥ M(x)**

- Negation: ⌐ **ⱻx(D(x) ꓥ M(x)**

**-**Applying De Morgan’s law: **Ɐx(⌐D(x) V ⌐M(x)**

-English: **All patients either didn’t take the medication or didn’t have migraines**

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1.9.3(b): ∃x∀y (xy = 0), D{All real numbers}

**True**

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1.9.3(d): ∀x∃y∀z (z = (x - y)/3) , D{All real numbers}

**False**

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1.9.3(f): ∃x∃y∃z (x2 + y2 = z2) , D{All real numbers}

**True**

----------------------------------------------------------- ---------------------------------------------------------

1.9.4(c): ∃x ∀y (P(x, y) → Q(x, y)) , D{All real numbers}

**Ɐxⱻy(P(x,y) ꓥ ⌐Q(x,y))**

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1.9.4(e): ∃x ∃y P(x, y) ∧ ∀x ∀y Q(x, y) , D{All real numbers}

**Ɐx Ɐy ⌐P(x,y) V ⱻx ⱻy ⌐Q(x,y)**

---------------------------------------------------------------------------------------------------------------------

1.10.2(a): ∀x ∃y (x + y = 0), D{All real numbers}

**True, (x + (y = -x) = 0)**

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1.10.2(b): ∃x ∀y (x + y = 0), D{All real numbers}

**False, ((x = y + 1) + y ≠ 0)**

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1.10.4(a): There are two numbers whose ratio is less than 1, D{All real numbers}

**ⱻx ⱻy((x / y) < 1)**

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1.10.4(e): The reciprocal of every positive number less than one is greater than one, D{All real numbers}

**Ɐx((0 > 1/x < 1) → (1/x > 1))**

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1.10.7(d): Sam knows the phone number of everyone who missed the deadline, D{group working on a project at a company }, One of the members of the group is named Sam

P(x, y): x knows y's phone number. (A person may or may not know their own phone number.)

D(x): x missed the deadline.

N(x): x is a new employee.

**D(y) → P(Sam, y)**

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1.10.7(f): Exactly one new employee missed the deadline, D{group working on a project at a company }, One of the members of the group is named Sam

P(x, y): x knows y's phone number. (A person may or may not know their own phone number.)

D(x): x missed the deadline.

N(x): x is a new employee.

**ⱻx(D(x) ꓥ Ɐy((y ≠ x) → ⌐D(y))**

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1.10.10(e): Everyone other than Sam has taken at least two different math classes

The domain for the first input variable to predicate T is a set of students at a university

The domain for the second input variable to predicate T is the set of Math classes offered at that university

The predicate T(x, y) indicates that student x has taken class y

Sam is a student at the university and Math 101 is one of the courses offered at the university

**Ɐx ⱻy T(x ≠ Sam, y) ꓥ T(x ≠ Sam, z ≠y)**