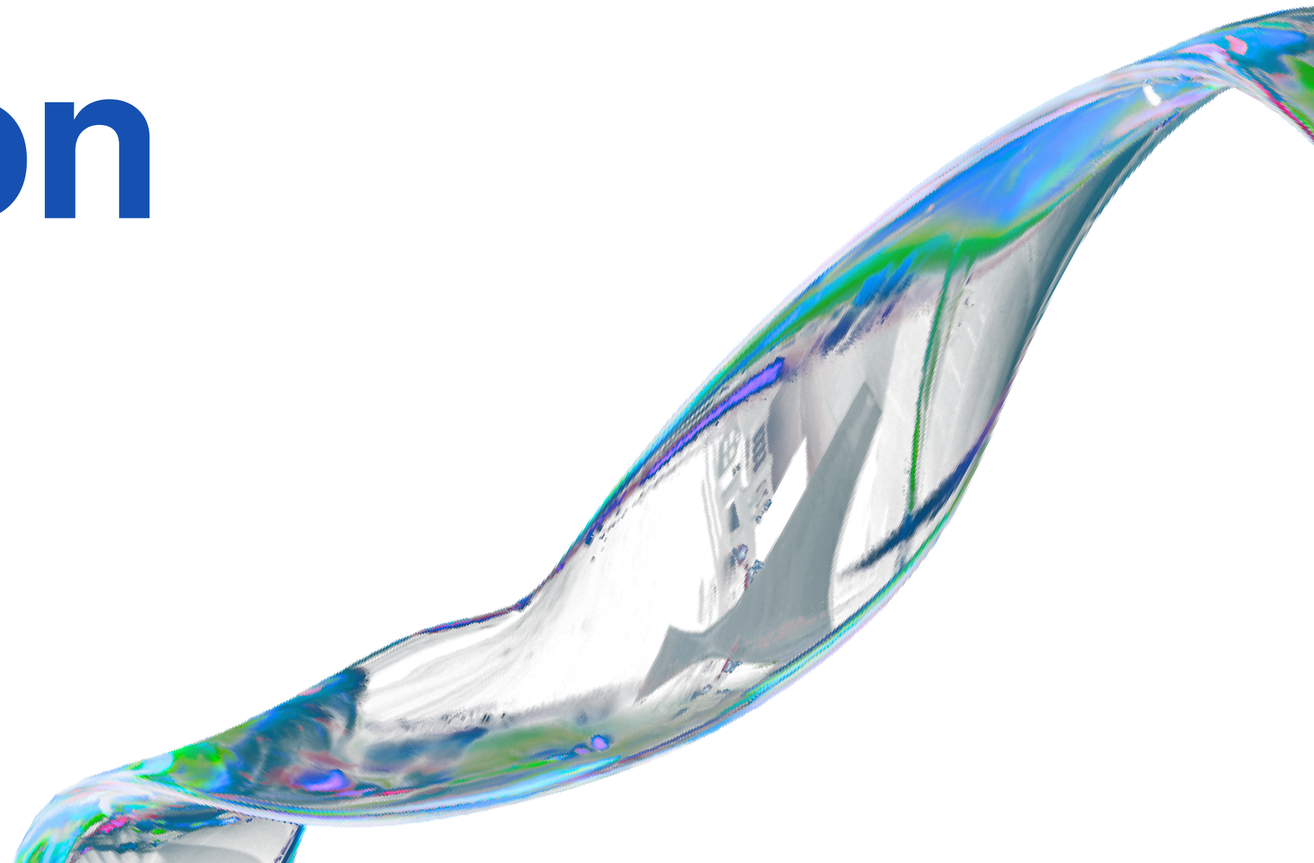
Team
Super(position)

Github: @quantum_for_portfolio_optimization

Portfolio Optimization

Using QAOA

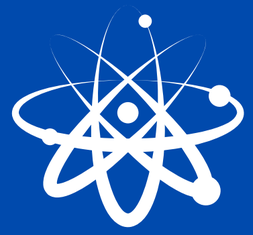


portfolio problem QUBO → **quantum Hamiltonian** → **ground state = optimal portfolio**

Challenge: Complex portfolio optimization with multiple constraints

- Portfolio size constraints (target N bonds from n available)
- Cash flow bounds (yield requirements)
- Risk characteristic limits (duration/credit targets)

Exponential complexity, local optima traps



QAOA for combinatorial optimization

Deliverables

1) Review the mathematical formulation provided below, focusing on binary decision variables, linear constraints, and the quadratic objective.

2) Convert the binary optimization problem to a formulation that is compatible with a quantum optimization algorithm. For example, convert the constrained problem to an unconstrained problem.

3) Write a quantum optimization program for handling problems of the type in (2). An example of such an optimization routine which is used in portfolio optimization is the Variational Quantum Eigensolver (see resources below), however you may pursue what you judge to be the best solution.

4) Solve the optimization problem in (1) using your quantum formulation.

5) Validate your solution in (4) using a classical optimization routine. Compare the solution quality against the benchmark classical solution in terms of the cost function, and include relevant performance metrics(e.g., convergence of the optimization routine, and scaling properties with problem size).

The Equations, Parameters, Constraints, and Objective Function

Input parameters

A set of securities C with:
- p_c market price
- m_c min trade
- M_c max trade
- i_c basket inventory
- δ_c minimum increment

A set L of risk buckets
- $\mathbb{K}_\ell$ are the bonds in bucket ℓ (not mutually exclusive)

A set J of characteristics with:
- $K_{\ell,j}^{\text{target}}$ target of characteristic j in risk bucket ℓ
- $b_{\ell,j}^{\text{up}}$ and $b_{\ell,j}^{\text{low}}$ guardrails for characteristic j in risk bucket ℓ
- $\beta_{c,j}$ contribution of a unit of bond c to the target of characteristic j
- $K_{\ell,j}^{\text{up}}$ and $K_{\ell,j}^{\text{low}}$ guardrails for characteristic j in risk bucket ℓ (binary version)

Global parameters:
- N max number of bonds in portfolio
- Min/max residual cash flow of portfolio

Decision variables

- y_c whether bond c is included in the portfolio

Here x_c (how much of bond c is included in the basket) is not a variable, but is fixed to the average value it is allowed to have if c is included at all in the portfolio:

$$x_c = \frac{m_c + \min\{M_c, i_c\}}{2\,\delta_c} y_c$$

Constraints

- Maximum number of bonds in basket

$$\sum_{c \in C} y_c \le N$$

- Residual cash flow of portfolio

$$\frac{minRC}{MV^b} \le \sum_{c \in C} \frac{p_c}{100} \frac{\delta_c}{MV^b} x_c \le \frac{maxRC}{MV^b}$$

- Min/max value of each characteristic j in each risk group ℓ

$$\sum_{c \in \mathbb{K}_\ell} \frac{p_c}{100} \frac{\delta_c}{MV^b} \beta_{c,j} x_c \le b_j^{\text{up}}, \quad \forall j \in J, \ell \in L,$$

$$\sum_{c \in \mathbb{K}_\ell} \frac{p_c}{100} \frac{\delta_c}{MV^b} \beta_{c,j} x_c \ge b_j^{\text{low}}, \quad \forall j \in J, \ell \in L.$$

- Similar constraint on the binary variable y_c

$$\sum_{c \in \mathbb{K}_\ell} \beta_{c,j} y_c \le K_j^{\text{up}}, \quad \forall j \in J, \ell \in L,$$

$$\sum_{c \in \mathbb{K}_\ell} \beta_{c,j} y_c \ge K_j^{\text{low}}, \quad \forall j \in J, \ell \in L.$$

Constraints that guarantee consistency between x_c and y_c are no longer needed since x_c is not a variable in this model

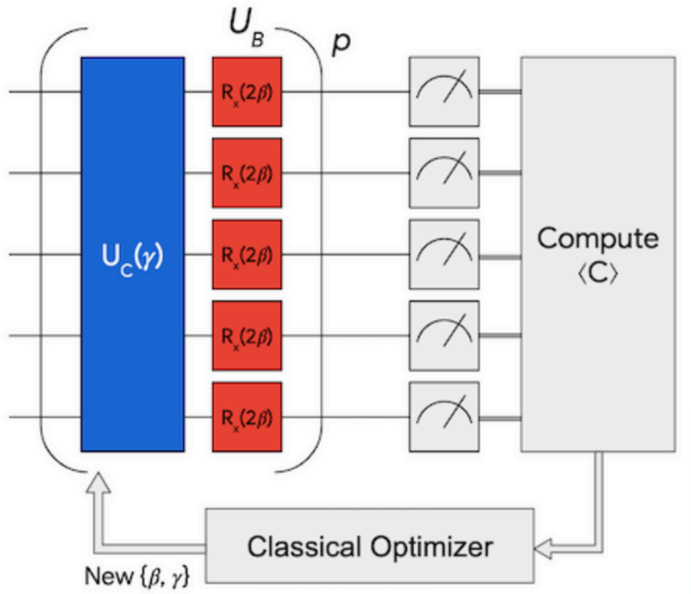
Objective function

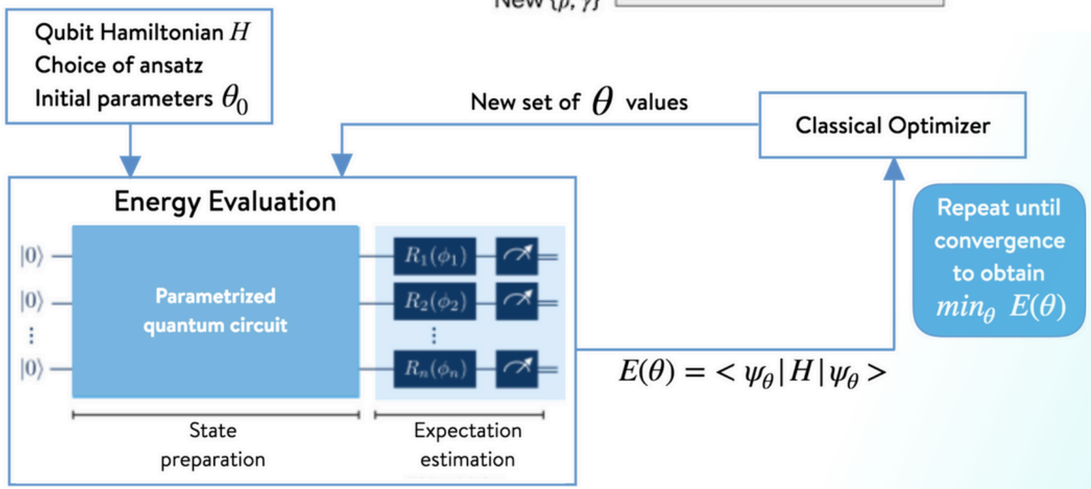
Match the value of each characteristic j in each risk group ℓ to its target (based on the quantity x_c in the basket)

$$\min \sum_{\ell \in L} \sum_{j \in J} \rho_j \left(\sum_{c \in \mathbb{K}_\ell} \beta_{c,j} x_c - K_{\ell,j}^{\text{target}} \right)^2$$

Overall summary of process

Overview diagrams of quantum circuits





Loading in Data + QUBO Formulation

Building+Solving QAOA

Backend + memory

Post-processing + selecting best solution

Step 2: Loading in the 2600+ Bonds Data (Part I)

To Load the Data:

- Filter for corporate bonds (main focus for Main Fund)
- Select top holdings by market value for quantum optimization:
  - Ensures we work best for the most significant positions
- Extract expected returns from bond characteristics:
  - 1.) Credit spread (OAS) - Compensation for credit risk
  - 2.) Duration - Interest Rate sensitivity
  - 3.) Current Yield Environment
  - Make sure risk-free rate is 4.5% to approximate the current 10-year treasury
  - Extract OAS in basis points → Convert to decimal (divide by 10000)
  - Expected Return = Risk-free rate + Credit Spread
- Extract Risk measures from bond duration and speed characteristics:
  - Interest Rate Risk and Credit Risk (measured by duration and spread volatility, respectively)
  - Higher Spreads → Higher credit risk.
  - Combine Risk Measure and ensure reasonable risk bounds to 0.5% to 15% annual volatility

Step 2: Loading in the 2600+ Bonds Data (Part II)

To Load the Data:

- Estimate correlation matrix based on sector and credit quality clustering, duration buckets, and general market factor
  - Loop through the number of assets in $O(N^2)$ Time
    - Make 0.05 be the base market correlation
      - If same sector correlation boost, add 0.25 to correlation matrix
      - If same credit quality correlation boost, add 0.15 to correlation matrix
      - Otherwise (similar duration), if within 2 years duration, add 0.10
    - Ensure to cap max correlation between under 0.85
    - Ensure Positive Semi-definite matrix and normalize diagonal to 1
- Of course, calculate current weights, get asset identities, and extract risk factors and sector classifications

- And then, you're done! You now can load the data into your workflow!

Step 2.5 : Converting the QUBO problem into the right formation to be used by our VQA with QAOA Algorithms

Our QUBO problem consists of expected returns, risk measures, asset names, current weights, and the portfolio information, as well as the correlation matrix for sector and credits

- Target basket size should be minimum 8 bonds for optimization OR it depends on the bonds (qubits) you're simulating on
- Min Portfolio Yield (3%) & Max Portfolio yield (6%) due to the dataset
- Market value base is normalized (1.0) to avoid any division by 0
- K_target is based on the portfolio average
- Portfolio size is 1000, cash flow upper and lower bound penalty is 300, and risk characteristic penalty is 200
- Scaled current wrights to optimization range (current weights * bonds * 2)

Step 2.5 (cont): Converting the QUBO problem into the right formation to be used by our VQA with QAOA Algorithms

To build the QUBO Formulation Part 1:

- Prepare a single bond "bucket" indexed by ‘I,j’ and pulls out its weight and target return
- Prepare 2 structures:
  - Q: an n x n matrix capturing all quadratic (pairwise) couplings between decision bits
  - q: A length-n vector capturing all linear biases on each bit
- Prepare Risk-Adjusted Return Term:
  - Penalized or rewards selecting pairs (i,k) in proportion to their joint exposure (β[i,j]*β[k,j]) scaled by the bond weight and any client-specific sizes x_c
  - Adds a real world risk penalty via the covariance between assets i and k

Step 2.5 (cont): Converting the QUBO problem into the right formation to be used by our VQA with QAOA Algorithms

To build the QUBO Formulation Part 2:

Apply a linear reward (negative bias) for selecting each asset i, scaled by how it helps hit the target return + add additional incentive proportional to the asset's standalone expected return

basket-size constraint:

- Implement a soft constraint on the total # of bonds selected
  - Uniform positive couplings between all pairs push the solver to pick exactly N ones
- This ensures a proper penalty

risk-contribution bounds:

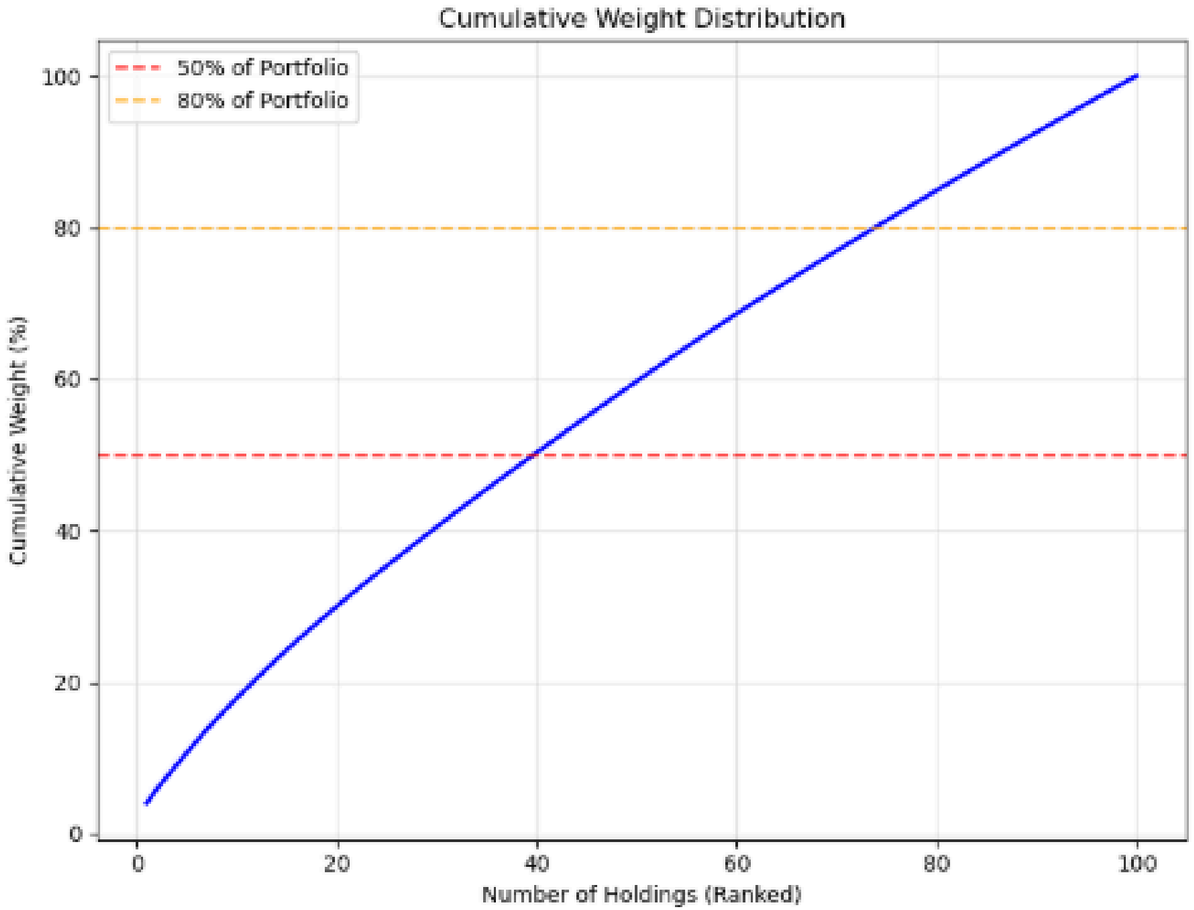
- Computes each asset's fractional contribution to total risk (a_cf) (see github code)
- Adds 2 squared-error penalties to ensure each individual contribution stays between rc min and rc max

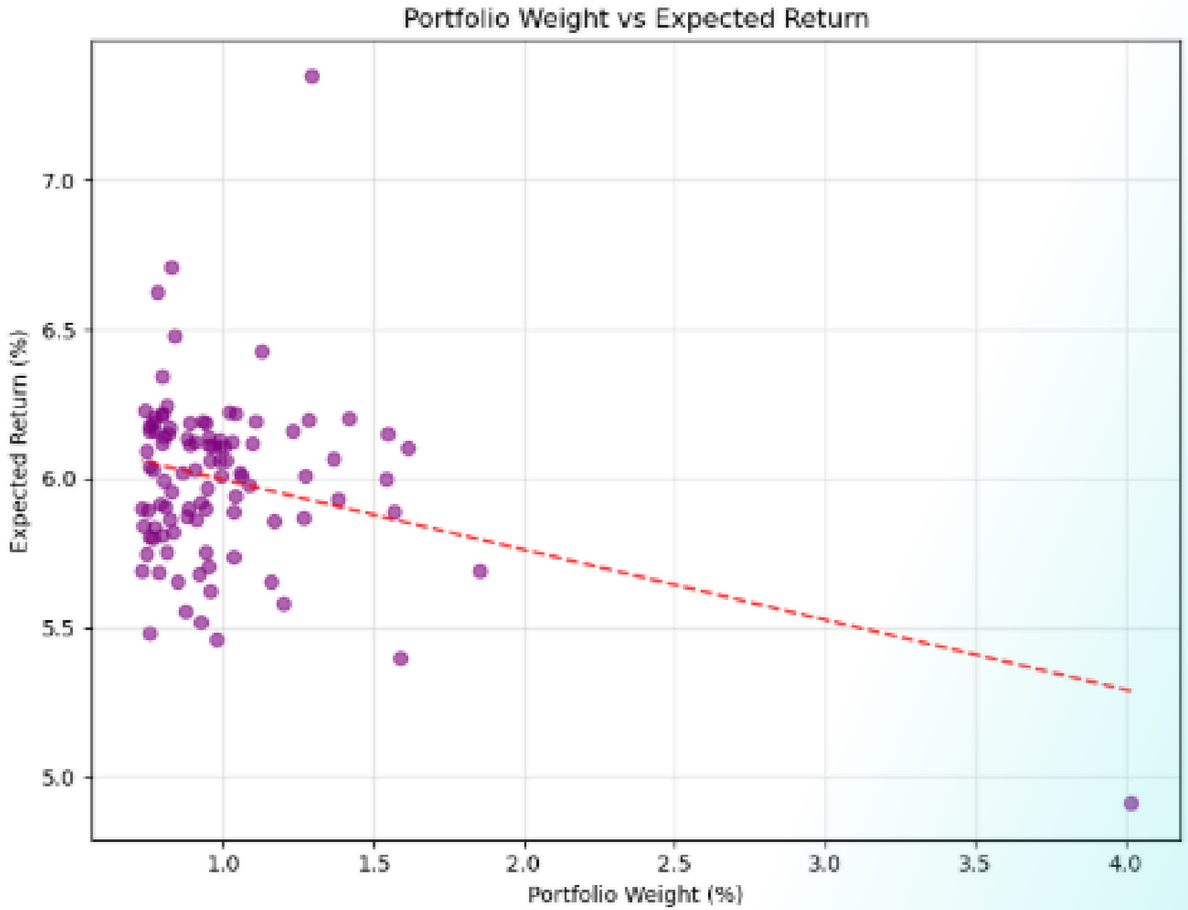
characteristic-level bounds:

- Builds a penalty forcing the portfolio's overall exposure, e.g. duration, credit quality, to lie between a lower bound and upper bound (b_lo and b_up)

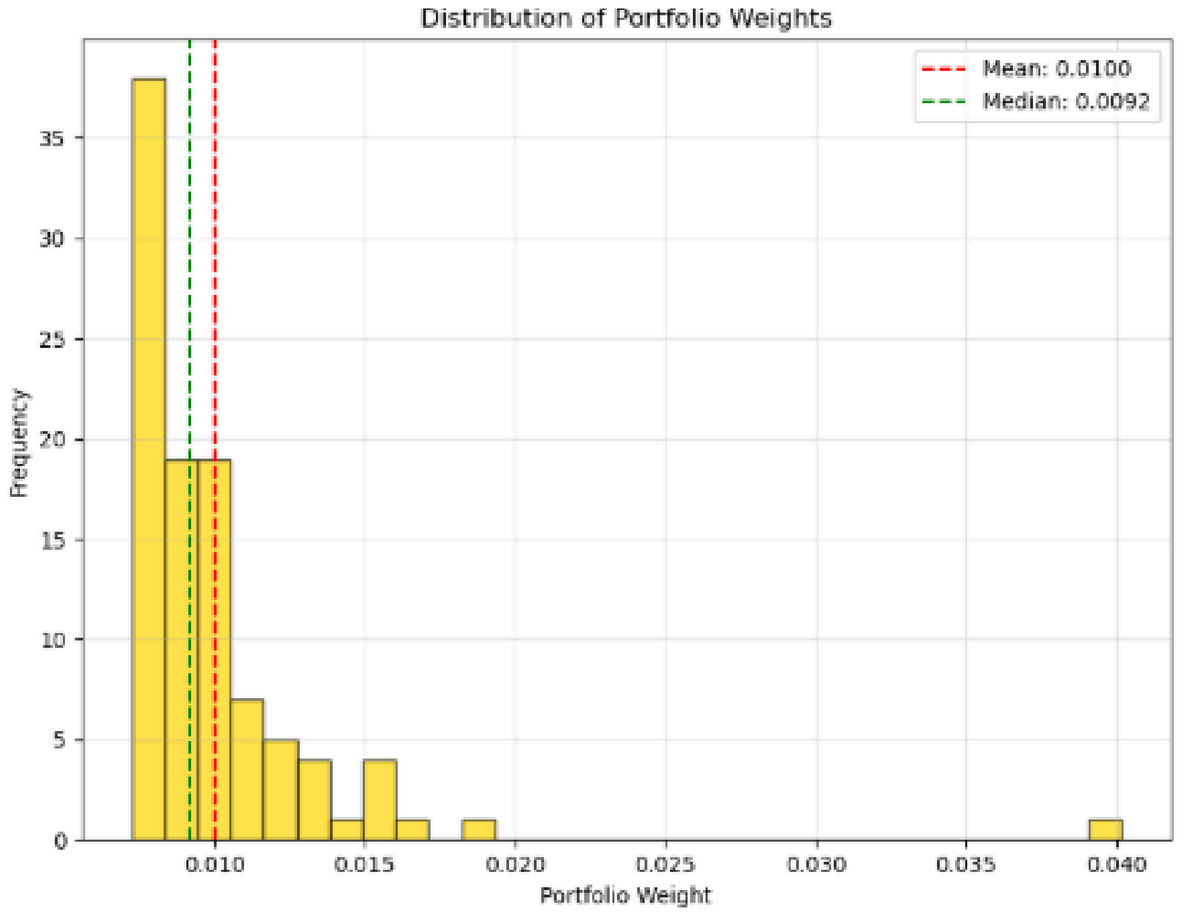
You MUST ensure Q is perfectly symmetric (REQUIRED by most QUBO solvers)

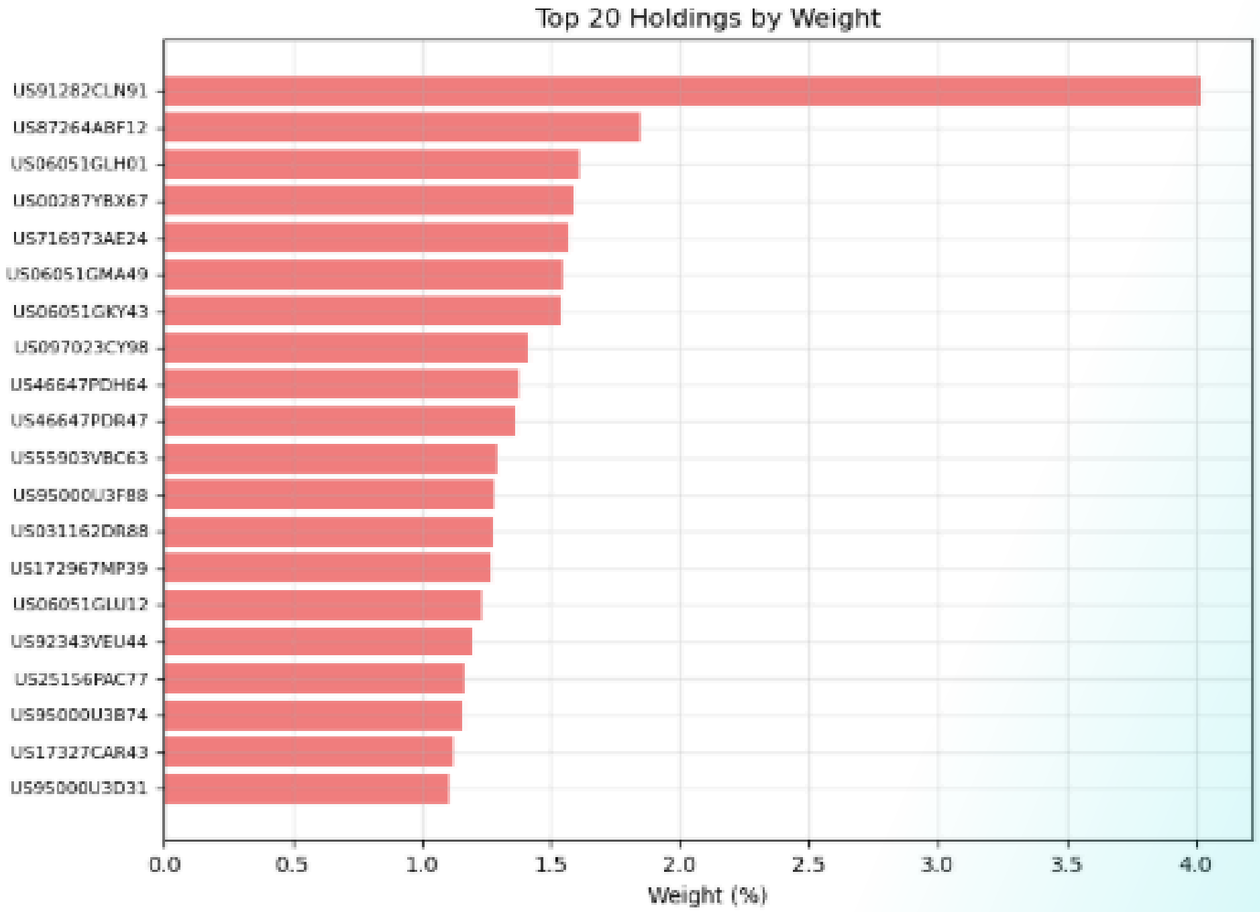
Overview of the Data (Quick Classical Run: No Quantum Computing yet)





Overview of the Data Part II

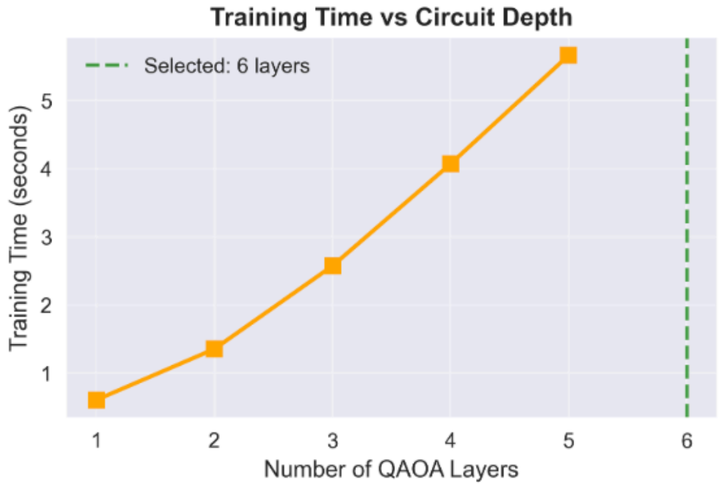


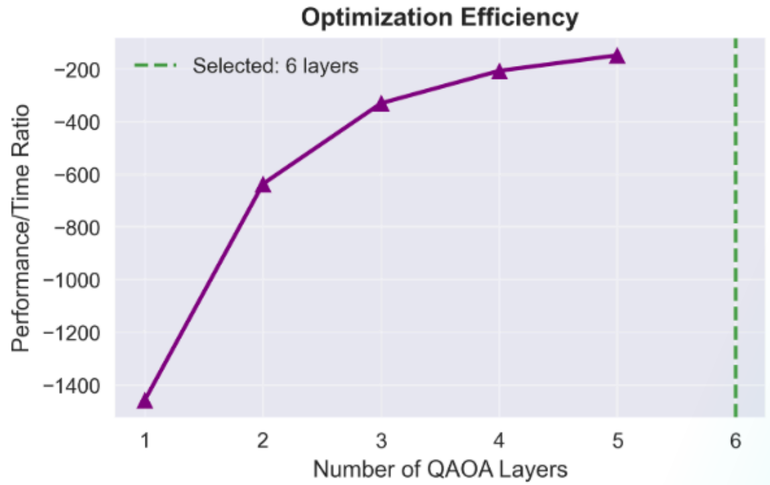


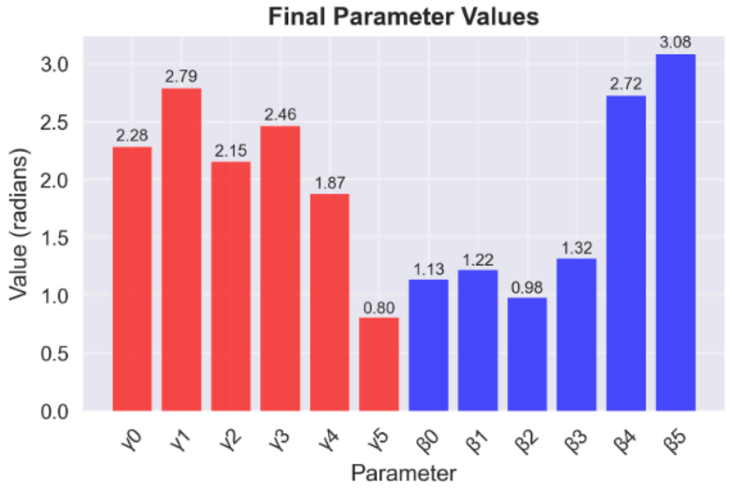
Step 3: Building the VQE with the optimized QAOA circuit

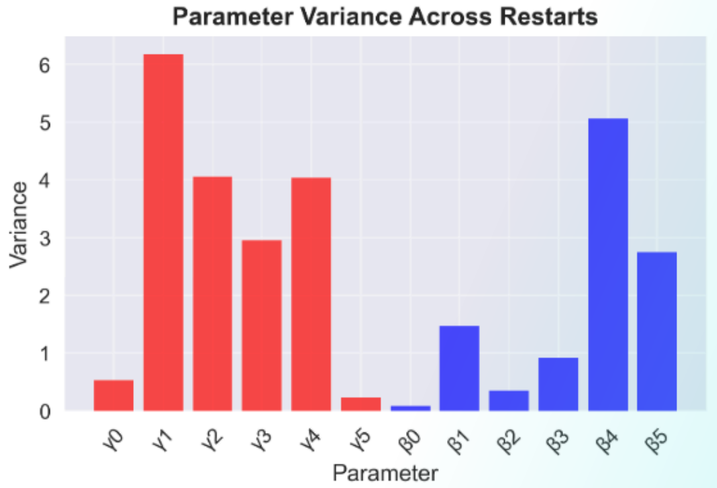
- **Parameterization: 2p parameters ($\gamma_1 \ldots \gamma p, \beta_1 \ldots \beta p$)**
- **Layers: Adaptive p = 2-6 based on problem size**
- **Gate Optimization: Significant gates filtering**

note visualizations are for a run with just 10 qubits to make them easy to follow









Step 4: Hardware-optimized backend selection

Memory Analysis:

- State Vector: 2^n × 16 bytes (complex128)
- 24 qubits: ~268 MB
- 31 qubits: ~32 GB (memory limit!)

Backend Priority:

1. Lightning GPU
(n≤28, <8GB GPU memory)
2. Lightning CPU
(state_memory < 70% RAM)
3. Default.qubit (fallback)

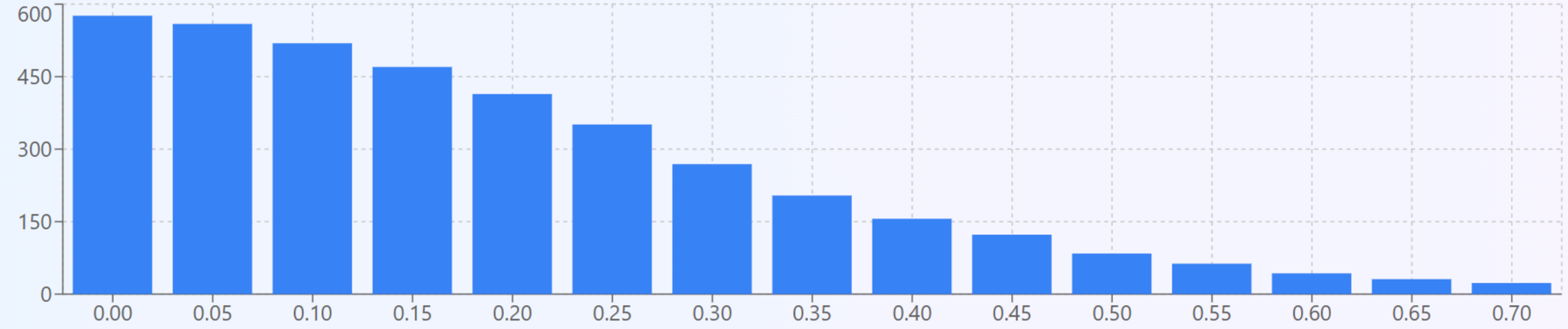
You can see that simulating 31 qubits classically is very computationally heavy - however, high fidelity, effective 31 qubits are capable of solving optimization problems extremely well. As you will see in the results, qubit capabilities scale exponentially - just like the problem difficulty.

Step 5: Optimizing Circuit

1. Threshold Filtering

Significance Threshold: 0.32

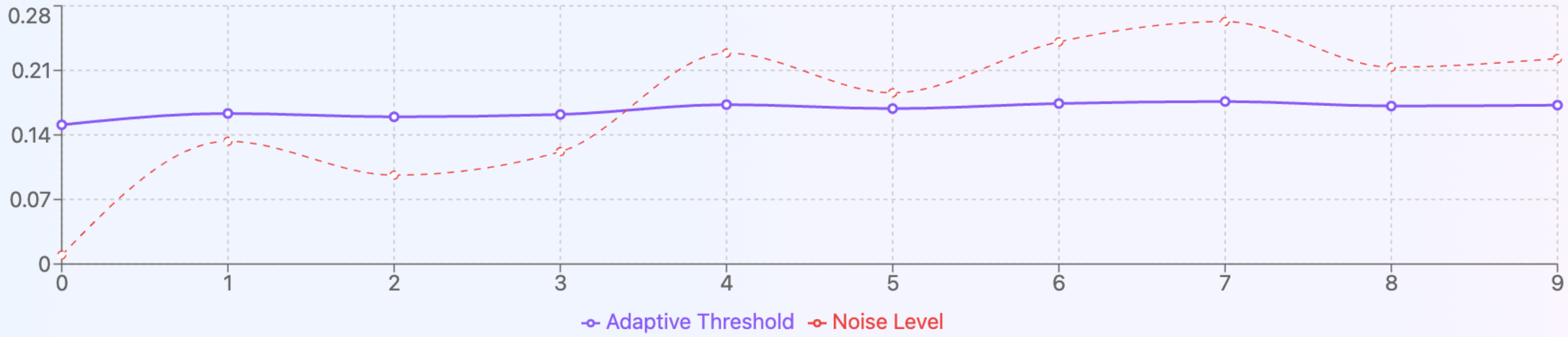
Before filtering: 576 gate operations (24 qubits)
After filtering: 238 operations
Reduction: 58.7%



Step 5: Optimizing Circuit

2. Adaptive Threshold Tuning

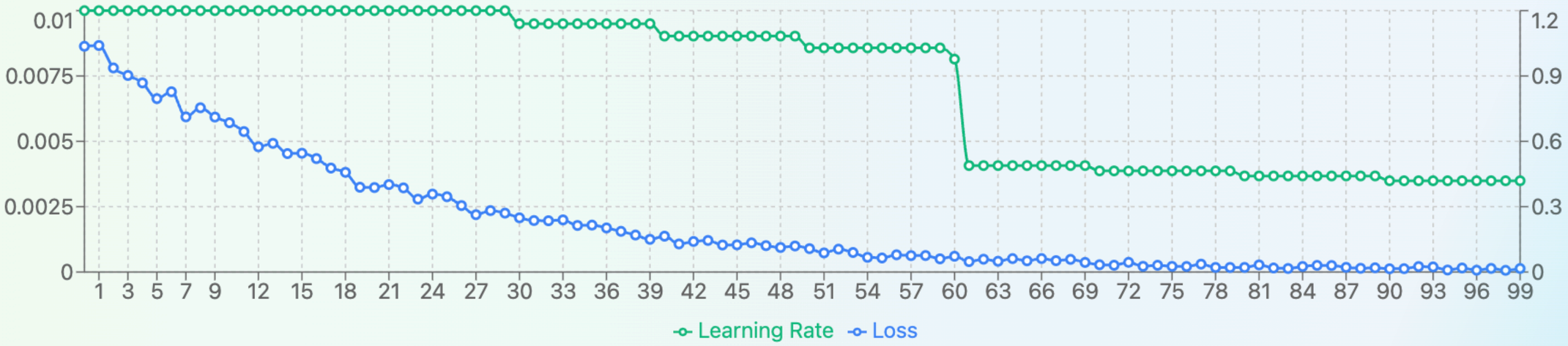
Current Threshold: 0.100
Purpose: Helps cancel out noise
Adaptation: Responds to circuit performance



Step 5: Optimizing Parameter

3. Adaptive Learning Rate

Schedule: Exponential decay
Decay rate: 0.95 every 10 epochs
Additional decay: 0.5x at epoch 60
Purpose: Fine-tune convergence



Quick Comparison of Classical vs Quantum Data Run #5

Constraint Analysis of 24 bonds, basket size 8 (2 QAOA layers deep)

- More graphs under v_quantum_analysis-4 folder in the GitHub

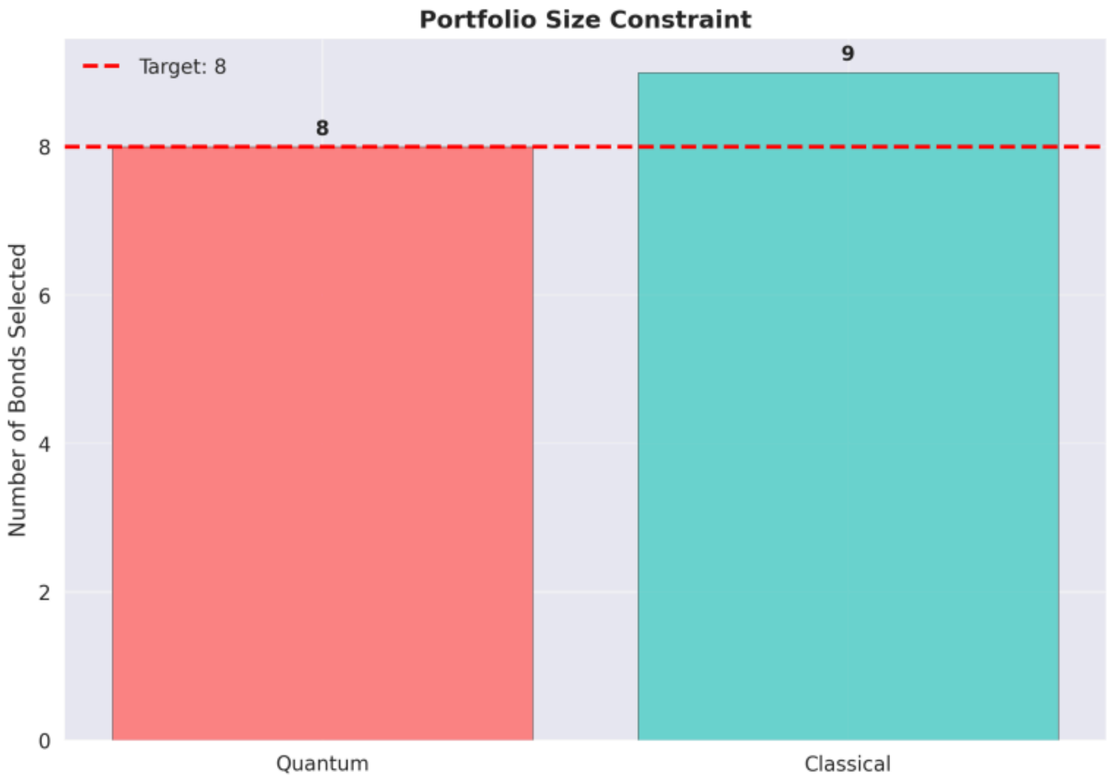
Basket Size: min(8, max(5, n // 3)), where n is the actual number of bonds loaded

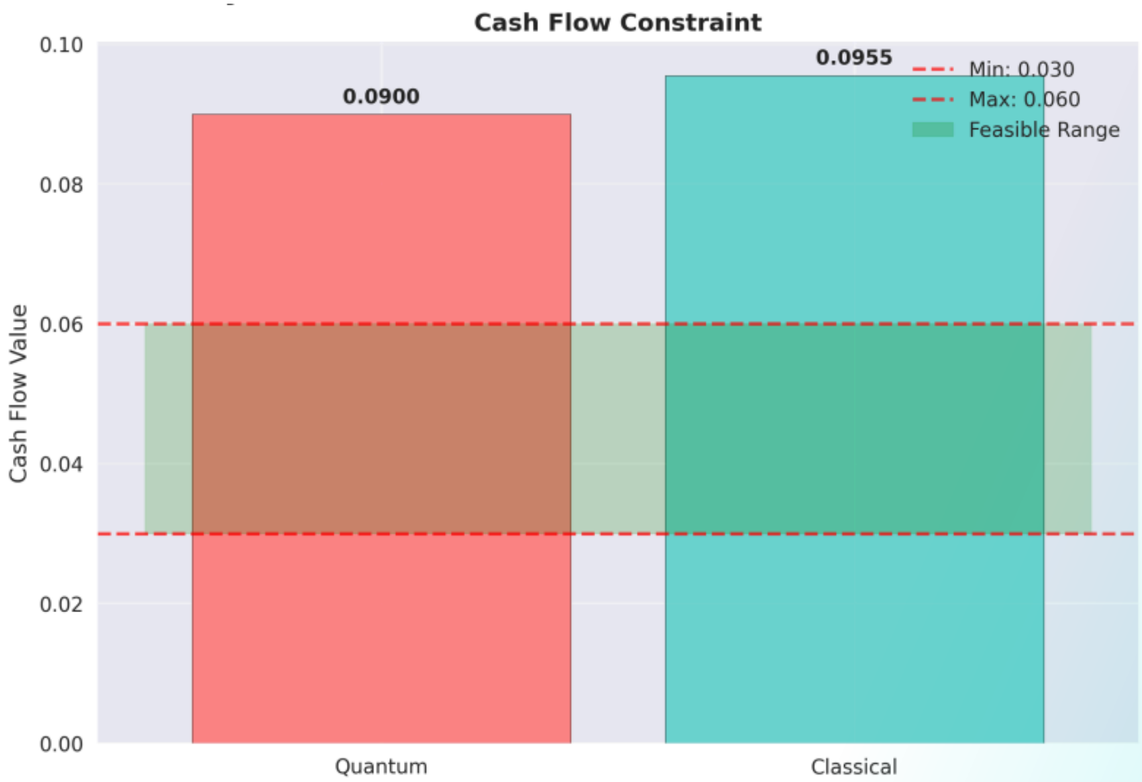
- So if we have 24 bonds --> min (8, max(5, 8)) --> min(8,8) --> 8 basket size

Solution	Basket Size	Cash Flow	Characteristic	Total Violation	Cost
Quantum	8	0.0900	0.3478	**0.0822**	-72117
Classical	9/8	0.0955	0.3793	**1.0562**	-72122

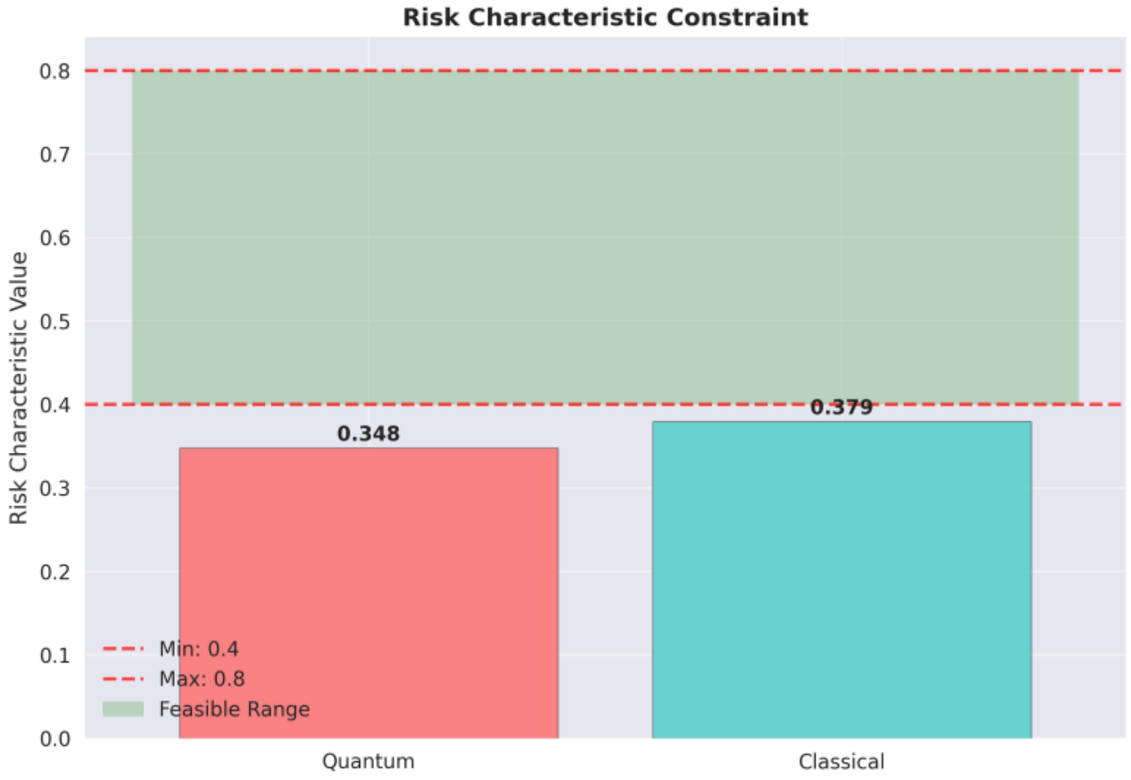
- 24 bonds (qubits) took 10+ hours on a Nvidia 4060 TI GPU! (using Lightning.gpu as well on Pennylane)
- 20 bonds took 30 minutes on the same GPU

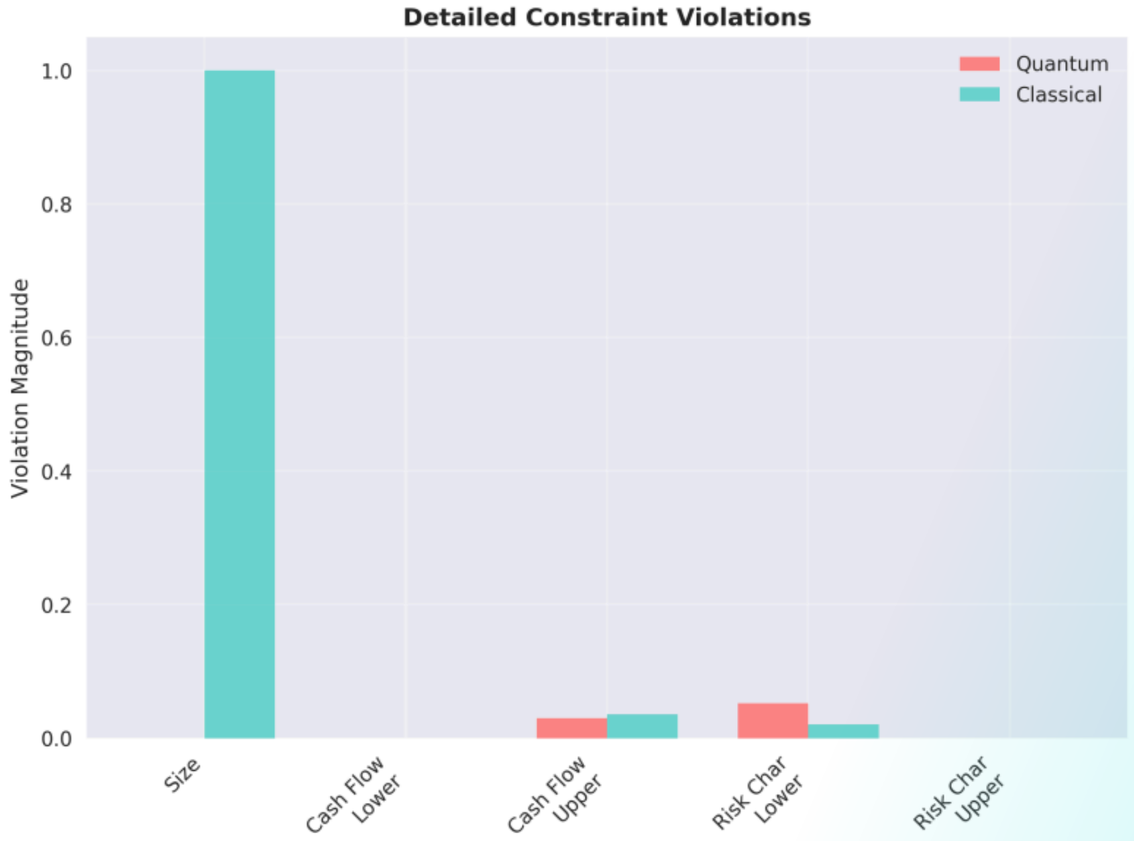
Quick comparison: visualizations of constraint violation (pt 1 of 2)



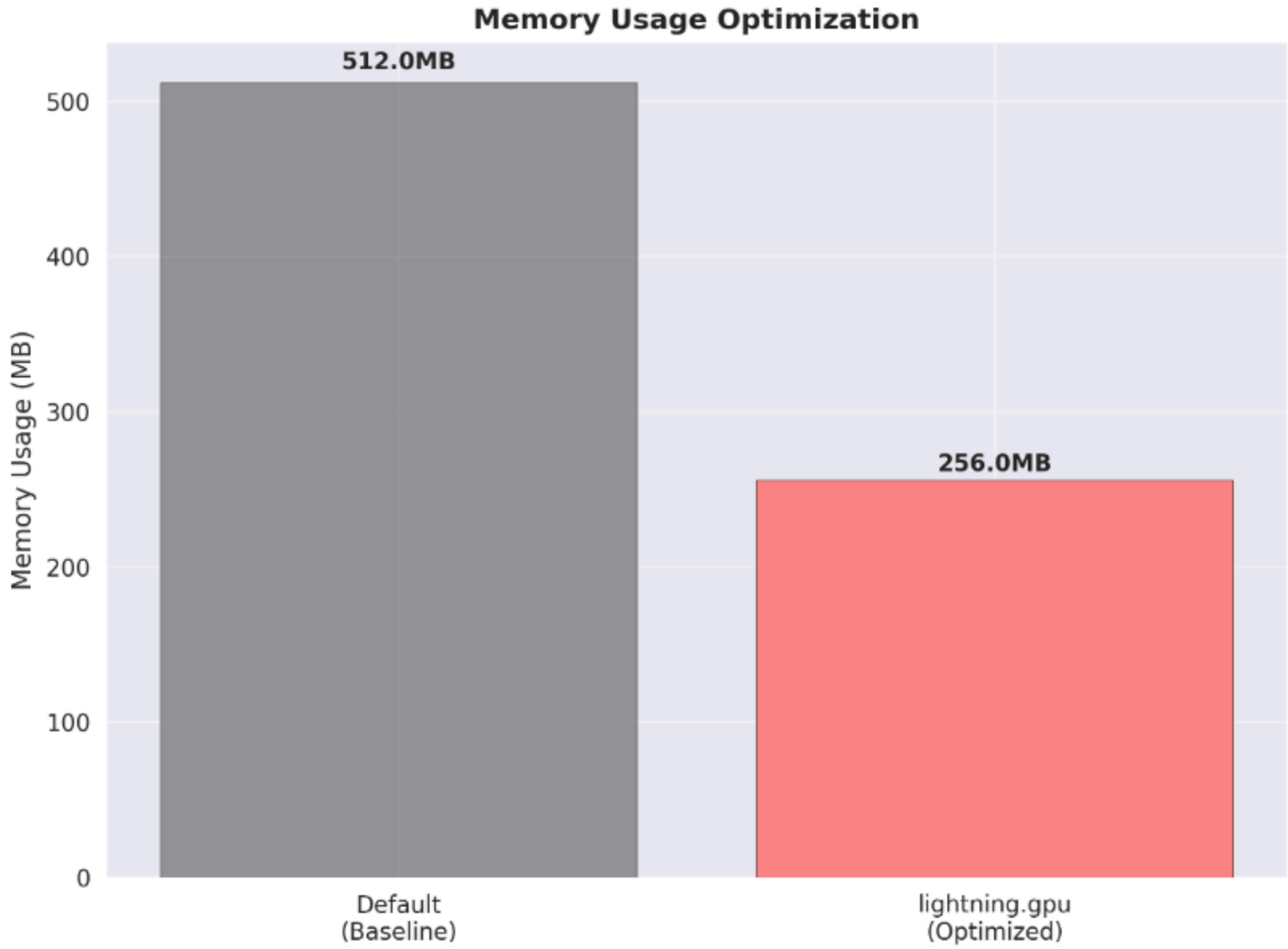


Quick comparison: visualizations of constraint violation (pt 2 of 2)





Using Lighning.gpu to speedup computation



Scalability Analysis

Current Implementation:

- 16 qubits (20-24 qubits for lightning.GPU): Substantial classical simulation requirements
- 65,536+ possible solutions: Classical enumeration becomes challenging
- Complexity: Exponential classical search space vs polynomial VQE optimization

Scaling Potential:

- Real-world portfolios: 50-500+ assets (using the data now!)
- Quantum advantage: Expected at 24+ qubits where classical optimization becomes intractable
- Hybrid approaches: Quantum sampling with classical post-processing proves effective (You can see more of this in the part 3 py file)

What can we do to further explore the code?

1) Run with greater memory so we can run more qubits (~32). As you can see, there's only so much potential we can capture with a limited number of qubits.

2) Look further into adaptive circuits

Thank you for your time!